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Elastic scattering of protons by helium 4: New experiments and analysis*

D. C. Dodder, G. M. Hale, Nelson Jarmie, J. H. Jett, P. W. Keaton, Jr., R. A. Nisley,† and K. Witte

University of California, Los Alamos Scientific Laboratory, Los Alamos, New Mexico 87545
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Accurate measurements of p-α elastic scattering cross sections were made at energies of 11.157, 12.040, 13.600, and 14.230 MeV. The average relative error is about 0.6% and the scale error is 0.37%. These data and all available cross section and spin-dependent measurements of p-α scattering between 0 and 18 MeV were collected and prepared for input to a general purpose R-matrix analysis program. Strict statistical criteria were used for the elimination of data. The resulting search on 1131 data produced a unique fit with a χ^2 per degree of freedom of 1.001 which is within one standard deviation in χ^2 space. Arbitrary normalizations to the data were not allowed; a normalization was treated as another datum restrained by a scale error obtained from the experimental information. The parameter space was made up of background contributions in $S, P, D,$ and F states with an additional level each in the $P_{3/2}$ and $P_{1/2}$ states. There were 14 free parameters. For the first time, the reduced widths of the P-wave resonance states come out almost equal. Comparisons are made to the R-matrix analysis of Stammbach and Walter and to the phase shift analysis of Arndt, Roper, and Shotwell.

[NUCLEAR REACTIONS ^{4}He(p,p), E=11–14 MeV; measured $\sigma(\theta)$, θ(c.m.) =19–167°, $\Delta\theta$=0.03°, $\Delta\sigma$=0.06%; calculated R-matrix parameters for all ^{4}He(p,p)^{4}He data, E=0–17 MeV.]

I. INTRODUCTION

The elastic scattering of protons by ^{4}He has become one of the classic reactions for both experimental and theoretical study in low energy nuclear physics. There have been continual and interacting improvements in experimental accuracy and in the precision with which phenomenological analysis fits the experimental observations. The present work continues in this tradition, reporting new measurements of high accuracy and giving results of the first genuinely "global" R-matrix analysis of all significant p-α measurements in the 0- to 18-MeV range. The fit to these data is satisfying in that it fulfills expected statistical criteria and results from physically reasonable resonance parameters.

There have been a number of previous energy-dependent analyses of p-^{4}He elastic scattering. One of the first attempts in this direction was an optical model analysis of N-α scattering below 20 MeV by Satchler *et al.*[1] Schwandt, Clegg, and Haeberli[2] used an effective range parametrization for a limited set of p-α scattering data between 3 and 18 MeV. Also using effective range parametrizations were Arndt and Roper for 0–2-MeV (Ref. 3) and 0–21-MeV (Ref. 4) n-α scattering, Arndt, Roper, and Shotwell[5] for 0–23-MeV p-α scattering, and Arndt, Long, and Roper[6] for 0–3-MeV n-α and 0–5-MeV p-α scattering. Finally, Stammbach and Walter[7] have used an R-matrix formulation for N-α scattering from 0–20 MeV. The present analysis, using the R-matrix formalism, is the first such analysis in which the observables are directly calculated from R-matrix parameters, all of which are searched on simultaneously.

The N-^{4}He reactions have invited attention because of the large spin polarizations which are a consequence of the large spin-orbit splitting. This attention is warranted both because of intrinsic interest in the effect and because of the obvious utility of the elastic scattering as a polarization analyzer for nucleons. These considerations in turn call for careful and precise phenomological analysis both as a step toward further theoretical study and as an aid to efficient use of the polarization analyzers.

The information that is contained in the experimental results is customarily expressed in terms of a few energy-dependent phase shifts. The work described in this report improves the knowledge of these phase shifts and sheds more light on their values. The means employed are here briefly listed and will be described more fully in the main body of the paper:

1. New and more accurate data have been included in the analysis. Spin rotation parameters

$A(\theta)$ and $R(\theta)$ have been used for the first time. (In terms of recent notation[8] for polarization transfer coefficients, $A = K_z^{x'}$ and $R = K_x^{x'}$.) New accurate differential cross sections are reported here and used.

2. A more critical evaluation of all existing data has been made with special attention paid to error estimates. Rejection of data follows, with a few inconsequential exceptions, strict statistical rules.

3. A statistically reliable energy-dependent phase-shift analysis, using the R-matrix formalism, has been made. This has resulted in a set of 14 independent R-matrix parameters that predicts the final selected 1131 experimental values that comprise the consistent body of data with a χ^2 per degree of freedom of 1.001, which is within one standard deviation[9] in χ^2 space.

II. EXPERIMENTAL MEASUREMENTS

A. Introduction

The p-α elastic scattering cross section measurements were done as part of a continuing program of accurate cross section measurements. Measurements were made at laboratory proton beam energies of 11.157, 12.040, 13.600, and 14.320 MeV, each with an uncertainty of ±0.015 MeV. The energies were chosen to coincide with those of previously existing experiments. The average relative error is about 0.6% and the scale error is 0.37%. The largest section of data was at 12.040 MeV where preliminary analysis had indicated difficulties in fitting the existing data.

B. Method

The proton beam from the Los Alamos tandem Van de Graaff accelerator passed through a gas target with thin Havar foil windows, and the scattered protons were detected by a single E-ΔE detector arrangement using solid-state detectors. Amplified pulses gated by the E-ΔE coincidence were digitized and sent to an on-line computer for mass analysis and storage. The resulting spectra were later analyzed for the yield.

The experimental apparatus used is described in detail by Jarmie *et al.*[10] and Detch.[11] The methods and techniques used for this experiment are as described in those references with the following exception. The helium target-gas purity, as determined both by detection of particles scattered by contaminants and by mass-spectrographic analysis, ranged from 0.996 to 0.998 with an uncertainty of ±0.002. Dead time and background corrections for this experiment were usually very small.

TABLE I. Errors.

Scale Source	% error	Relative Source	% error
Pressure	0.1	Yield	$1/\sqrt{N}$
Temperature	0.07	Background	≈0.1
Purity	0.2	Dead time	0.1
G	0.2		
N_b	0.2		

C. Errors

The reader should refer to Refs. 10 and 11 for extensive details about corrections and errors. The central energy of the proton beams used was known to ±15 keV and had an energy spread of 20 keV full width at half maximum, including the effects of foil and target-gas straggling and machine energy resolution. The beam was purified by magnetic analysis, and its purity was confirmed by failure to find any experimental evidence of impurities. The laboratory scattering angle was known to ±0.03°. The contributions to the scale error, i.e., the normalization, include uncertainties in the pressure, temperature, purity, number of beam particles, and the geometrical factors. Contributions to the relative error are primarily from the yield. The relative and scale errors are combined to give the absolute error according to:

$$(\text{rel. error})^2 + (\text{scale error})^2 = (\text{abs. error})^2 .$$

Table I is a summary of the various sources of error which are not negligible. All error values here and in the results are standard deviations.

D. Experimental results

The data are presented in Table II–V. Significant differences between the present data and previous measurements would not be visible in a figure showing the data directly. In order to display the differences the figures show the deviations of the data, in percent, from the final R-matrix fit. These figures show the unrenormalized data; the actual fitting procedure adjusts the scale factor so that the fit to the R-matrix values is better. Figure 1 shows the percent differences at 11.157

TABLE II. Differential cross sections for $p+\alpha$ elastic scattering 11.157 MeV, scale error = 0.37%.

θ_{lab} (deg)	$\sigma(\theta)_{\text{lab}}$ (mb/sr)	$\theta_{\text{c.m.}}$ (deg)	$\sigma(\theta)_{\text{c.m.}}$ (mb/sr)	Relative error (%)
20.08	509.0	25.08	332.6	0.52
28.17	422.0	35.06	283.1	0.56
39.68	301.6	49.01	213.0	0.49
57.67	140.3	70.05	110.8	1.0

TABLE III. Differential cross sections for $p+\alpha$ elastic scattering 12.040 MeV, scale error = 0.37%.

θ_{lab} (deg)	$\sigma(\theta)_{lab}$ (mb/sr)	$\theta_{c.m.}$ (deg)	$\sigma(\theta)_{c.m.}$ (mb/sr)	Relative error (%)
15.00	547.0	18.76	353.0	0.60
20.00	477.4	24.98	311.8	0.49
25.00	423.4	31.16	280.7	0.45
30.00	377.9	37.29	255.2	0.47
35.00	331.1	43.37	228.5	0.44
40.00	279.3	49.39	197.6	0.50
45.00	235.5	55.34	171.2	0.45
50.00	192.4	61.21	144.2	0.52
55.00	151.9	67.00	117.7	0.52
65.00	88.04	78.29	73.42	0.51
75.00	46.64	89.18	42.22	0.72
85.00	22.93	99.63	22.66	0.85
95.00	12.94	109.62	14.01	1.01
105.00	12.76	119.17	15.15	1.50
115.00	17.32	128.27	22.48	1.02
125.00	23.82	136.97	33.60	0.82
135.00	29.11	145.31	44.23	0.69
145.00	34.60	153.35	56.00	0.55
155.00	38.58	161.14	65.59	0.60
165.00	41.28	168.75	72.60	0.46

MeV, and the poor agreement with Barnard (BA64) is apparent. The key (e.g., BA64) for the experimental data references used in the analysis is given in Table VI. Figure 2 compares the present data at 12.04 MeV with Sanada (SA59). The discrepancy here is apparently mainly in the scale factor and is outside the indicated scale errors.

In the immediate neighborhood of 14.32 MeV there have been three previous experiments: (SA59), (BR57), and (GA69). Brockman's (BR57) data agree well, but their errors seem to be overestimated. The data of Sanada (SA59), Fig. 3, are in disagreement, and their failure to agree with our values and with the R-matrix calculated values especially at low angles led to this data set being discarded for the final fit. The Garreta (GA69) data are not in agreement at the largest angles (see Fig. 4).

TABLE IV. Differential cross sections for $p+\alpha$ elastic scattering, 13.600 MeV, scale error = 0.37%.

θ_{lab} (deg)	$\sigma(\theta)_{lab}$ (mb/sr)	$\theta_{c.m.}$ (deg)	$\sigma(\theta)_{c.m.}$ (mb/sr)	Relative error (%)
20.41	415.8	25.50	271.7	0.41
39.95	251.4	49.35	181.9	0.44
50.00	171.6	61.22	128.5	0.42
70.00	57.49	83.80	49.90	0.68

TABLE V. Differential cross sections for $p+\alpha$ elastic scattering. 14.320 MeV, scale error = 0.37%.

θ_{lab} (deg)	$\sigma(\theta)_{lab}$ (mb/sr)	$\theta_{c.m.}$ (deg)	$\sigma(\theta)_{c.m.}$ (mb/sr)	Relative error (%)
19.78	403.5	24.71	263.2	0.41
45.00	203.0	55.36	147.5	0.48
60.00	100.0	72.71	80.27	0.50
75.00	38.14	89.20	34.52	0.71
105.00	9.601	119.19	11.40	2.00
145.00	28.57	153.36	46.27	0.63
165.00	34.61	168.76	61.07	0.60

III. USE OF THE R MATRIX

The R-matrix formalism[12] is appropriate for an energy-dependent analysis of p-α scattering for a number of reasons. First, the presence of known levels in this reaction suggests the use of a parametrization in which resonance behavior is made explicit. Second, by taking advantage of the short range of nuclear forces, the R-matrix formalism parametrizes the strong-interaction energy dependence of asymptotic quantities (phase shifts) entirely in terms of the internal wave functions. The energy dependence that comes from the external (Coulomb) wave functions is accounted for explicitly through the penetrability and shift functions. This means that many of the features of scattering—the tendency of phase shifts to decrease in

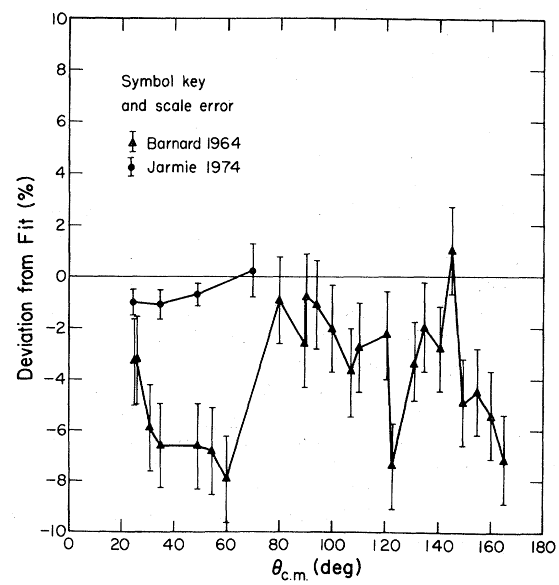

FIG. 1. Unrenormalized data at 11.157 MeV of Barnard *et al.* (BA64) and the present experiment. The data are presented as percent deviation from the final R-matrix fit. Note that the symbol key also shows the experimental scale (or normalization) error.

TABLE VI. 0–18-MeV p-α selected references.

$\sigma(\theta)$

1. BA64: A. C. L. Barnard, C. M. Jones, and J. L. Weil, Nucl. Phys. 50, 604 (1964).
2. BR51: C. H. Braden, Phys. Rev. 84, 762 (1951).
3. BR56: K. W. Brockman, Phys. Rev. 102, 391 (1956).
4. BR57: K. W. Brockman, Phys. Rev. 108, 1000 (1957).
5. FR49: G. Freier, E. Lampi, W. Sleator, and J. H. Williams, Phys. Rev. 75, 1345 (1949).
6. GI57: W. M. Gibson, D. J. Prowse, and J. Rotblat, Proc. R. Soc. 243, 237 (1957).
7. JA74: N. Jarmie and J. H. Jett, present data (1974).
8. KR54: W. E. Kreger, W. Jentschke, and P. G. Kruger, Phys. Rev. 93, 837 (1954).
9. MI58: P. D. Miller and G. C. Phillips, Phys. Rev. 112, 2043 (1958).
10. PU56: T. M. Putnam, J. E. Brolley, Jr., and L. Rosen, Phys. Rev. 104, 1303 (1950).
11. SA59: J. Sanada, J. Phys. Soc. Jpn. 14, 1463 (1959).
12. WI55: J. H. Williams and S. W. Rasmussen, Phys. Rev. 98,56 (1955).

$P(\theta)$

1. AD67: B. P. Ad'yasevich, V. G. Antonenko, Yu. P. Polunin, and D. E. Fomenko, Yad. Fiz. 5, 933 (1967) [Sov. J. Nucl. Phys. 5, 665 (1967)].
2. BR63: R. I. Brown, W. Haeberli, and J. X. Saladin, Nucl. Phys. 47, 212 (1963).
3. BR67: L. Brown and W. Trächslin, Nucl. Phys. A90, 334 (1967).
4. DR64: L. Drigo, C. Manduchi, G. C. Nardelli, M. T. Russo-Manduchi, and G. Zannoni, Nucl. Phys. 60, 441 (1964).
5. DR66: L. Drigo, G. Manduchi, G. C. Nardelli, M. T. Russo-Manduchi, G. Tornielli, and G. Zannoni, Nuovo Cimento B42, 363 (1966).
6. GA69: D. Garreta, J. Sura, and A. Tarrats, Nucl. Phys. A132, 204 (1969).
7. HA76: R. A. Hardekopf and G. G. Ohlsen, preceding paper, Phys. Rev. C 15, 514 (1976).
8. JA67: M. F. Jahns and E. M. Bernstein, Phys. Rev. 162, 871 (1967).
9. OH71C: G. G. Ohlsen, D. D. Armstrong, P. W. Keaton, Jr., G. P. Lawrence, and J. L. McKibben, Phys. Rev. Lett. 27, 599 (1971); private communication.
10. PL68: D. J. Plummer, T. A. Hodges, K. Ramavataram, D. G. Montague, and N. S. Chant, Nucl. Phys. A115, 253 (1968).
11. RO61: L. Rosen, J. E. Brolley, Jr., M. L. Gursky, and L. Stewart, Phys. Rev. 124, 199 (1961).
12. SC58: M. J. Scott, Phys. Rev. 110 (1958) 1398.
13. SC70: P. Schwandt, T. B. Clegg, and W. Haeberli, Nucl. Phys. A163, 432 (1971).
14. WE66: W. G. Weitkamp and W. Haeberli, Nucl. Phys. 83, 46 (1966).

$R(\theta), A(\theta)$

1. KE72: P. W. Keaton, Jr., D. D. Armstrong, R. A. Hardekopf, P. M. Kurjan, and Y. K. Lee, Phys. Rev. Lett. 29, 880 (1972).

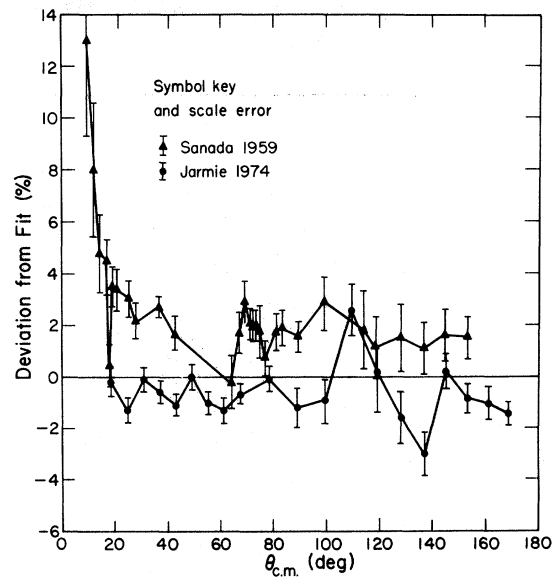

FIG. 2. Unrenormalized data at 12.04 MeV of Sanada *et al.* (SA59) and the present experiment. See the caption of Fig. 1 for details.

magnitude for increasing l, the shapes of broad resonances, the effects of coupling to channels below and through thresholds—come accurately and automatically from the R-matrix parametrization. The last feature will be of particular importance in eventually extending the present analysis near and through the $d+{}^3He$ threshold. Finally, the de-

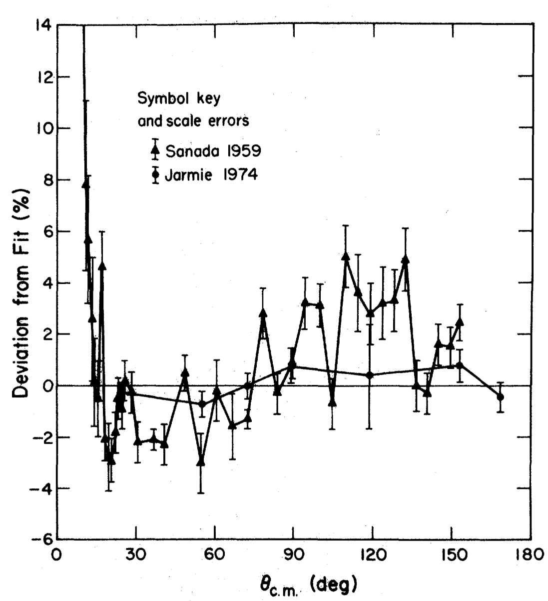

FIG. 3. Unrenormalized data at 14.32 MeV of Sanada *et al.* (SA59) and the present experiment. See the caption of Fig. 1 for details. Note the deviation of the Sanada data here and at 12.04 MeV at low angles.

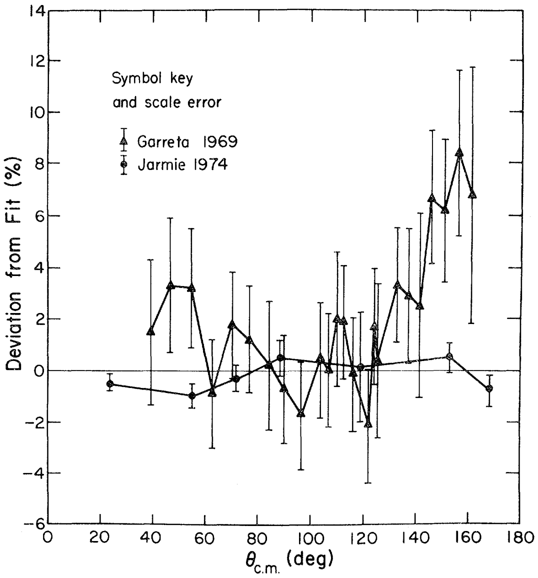

FIG. 4. Unrenormalized data at 14.23 MeV of Garreta *et al.* (GA69) and the present experiment. See the caption of Fig. 1 for details.

scription of interfering and overlapping levels, and the separation of "resonant" and distant levels, or "background" contributions is done simply and most directly in the R matrix.

It is not necessary to describe the R-matrix formalism; this has been done many times. The most complete description still probably is the work of Lane and Thomas.[13] It is, however, convenient to list the notation for a few quantities that occur. The R function for our case of a single channel is given by

$$R=\sum_{\lambda}\frac{\gamma_{\lambda}^{2}}{E_{\lambda}-E},$$

where the γ_λ are the reduced widths and E_λ are the eigenenergies. The boundary conditions are defined at channel radii a as the values B which the logarithmic derivatives take at the eigenenergies. (For the single channel case a and B are functions only of l, the orbital angular momentum.)

The phase shifts for a given l can be calculated from the R function using Eq. (1.15a), page 273, of Ref. 13 (Lane and Thomas):

$$\delta_l=\tan^{-1}[R_lP_l/(1-R_lS_l)]-\phi_l+\omega_l\,.$$

The penetration factor P_l, shift function S_l, and hard-sphere phase shifts ϕ_l are all energy-dependent functions that can be computed [Eqs. (4.4b), (4.4a), and (4.5b) on page 271 of Ref. 13] from standard Coulomb functions, tables of which are listed in Ref. 13. The ω_l are Coulomb phase shifts [Eq. (2.13c), page 269 of Ref. 13]. The energy dependence of the phase shifts is seen to result from two effects: the resonance behavior of the R functions and the often rapid energy variation of the Coulomb functions. Both are very important in the present case. It is also important to note that the nonresonant "background" contributions are not lumped together with the hard-sphere phase shifts ϕ_l, but are included with the resonant contributions directly in the R function. An example of the importance of this is given by Plattner, Bacher, and Conzett.[18] Previous attempts to find the R-matrix parameters for the low energy P-wave resonances have ignored the above consideration. This difference may explain why the near equality of the two P-wave reduced widths, which we shall show later, has not been found before.

IV. FITTING PROGRAM

The R-matrix parameters were determined using the computer program EDA [14] on a CDC 7600. EDA is a general multichannel, multilevel search program that directly and simultaneously fits the results of measurements over a range of energies of all possible quantum mechanical observables in spin space (differential cross sections, polarizations, and polarization transfers in the p-α case) by minimizing the quantity:

$$\chi^2=\sum_i w_i(N_iX_i-Y_i)^2+\sum_j w_j(N_j-1)^2+\sum_k w_k(E_k-\epsilon_k)^2\,, \tag{1}$$

where Y_i is the measured observable with relative error ΔY_i, w_i is the experimental weight for Y_i, $\equiv(\Delta Y_i)^{-2}$, X_i is the calculated observable, N_j is the normalization for jth *set* of data with experimental scale error ΔN_j, N_i is the appropriate N_j for measurement Y_i, w_j is the experimental weight for N_j, $\equiv(\Delta N_j)^{-2}$, E_k is the experimental nominal energy with error ΔE_k, ϵ_k is the adjusted energy, and w_k is the experimental weight for $E_k\equiv(\Delta E_k)^{-2}$.

The first sum runs over all data points, the second over all *sets* of data that have common normalizations, and the third over those energies to be adjusted in the fitting procedure. The second sum in χ^2 accounts for adjustments in the normalization, or scale, of data sets included in the first sum ($N_i=N_j$ for all points i in the jth data set), implying that relative and scale errors can be provided separately. The third sum accounts for adjustments in the experimental energies that were found necessary in a few cases.

The actual "goodness of fit" indicator used was the weighted variance (χ^2 per degree of freedom), which has an expected value of 1.00. The number of degrees of freedom is given by $N_d - N_p$ in which N_d = number of data points and N_p is the number of independent resonance parameters. The normalizations and resonance parameters were adjusted by the automated search, while the three energy adjustments were made by hand.

The automated minimization routine is one that uses analytic first derivatives of χ^2 with respect to the parameters in a succession of steps that build up an approximation to the covariance matrix H_{ij}. Successive approximations differ by a rank one matrix in an algorithm based on Eqs. (31) and (32) of Ref. 15. The method differs from those suggested in Ref. 15 in not requiring a positive definite H_{ij} at intermediate stages of the search, but instead in modifying the direction of parameter changes for the subsequent step whenever H_{ij} has negative eigenvalues.

V. SELECTION OF DATA

The goal was to include all significant data published by January 1, 1976 in the energy region of interest (0–18 MeV). The selection of criteria for input data and the process of applying these criteria to the available data is critical because of the danger of biasing the result. The methods and criteria are different from other investigations, resulting in a rather different data selection. The criteria were as follows:

1. The data must be in "usable form." There must be a numerical table of the experimenters' best estimate of the observable in question, together with as complete as possible statement of the errors involved. Data in only a graphical form or without a meaningful error statement were not used.

2. Data superseded by later work of the *same* group of experimenters were not used.

These first two criteria were used in principle to determine an "*initial basic data*" set for the analysis. Several sets of data were eliminated. Using the reference coding in Arndt, Roper, and Shotwell's[5] paper to save space, the sets discarded were: D1, D4, D8, D11, P1, P2, P3, P9, and also the data of Ref. 16. It should be noted that data were *not* discarded for criteria sometimes used elsewhere: (a) Newer data were not given preference over older data *per se*, and (b) "redundant" data were not culled to save computer time.

3. Individual points were discarded if their χ^2 contribution to the analysis was greater than 9, *and* if they were isolated; that is, if their poor fit was not associated with a poor fits of points at nearby angles. Nine points out of about 1500 initial points were discarded in this fashion. This test is in agreement with Chauvenet's criterion[17] for this number of data points.

4. The criterion for the deletion of an entire data *set* is that the χ^2 per point for the set ($\chi^2{}_s$) was three or more standard deviations[9] in χ^2 space away from the expected $\chi^2{}_s$ of one. In actual fact, in this analysis there were no data sets between three and six standard deviations out, so that the effective criterion was six standard deviations. The only exception to this was the data of DR64 and DR66 which is discussed in Table VII. The fits of all deleted points and sets were reinspected using the final parameters obtained. There were, in fact, several sets of data so wholly out of range, or with errors so large (much greater by a factor of 3 than nearby data) that they were discarded almost immediately on beginning the analysis. These were, also in the reference code of Arndt, Roper, and Shotwell,[5] D7, P4, P6, P7, and P14. The references of the remaining sets of data are given in Table VI. This data set (initial basic

TABLE VII. Changes and deletions for the "*final selected data*" set from that of Table VI. All angles are c.m. angles. $\chi^2{}_s$ is the χ^2 per point for a given data set. R_s is the ratio of $\chi^2{}_s$ to the $\chi^2{}_s$ expected for one standard deviation in χ^2 space for that number of data points.

1. BA64: energy change; 1.997 → 2.007 MeV.
2. BR51: 5.10 MeV, 157.3° deleted; 18 standard deviations away from final solution.
3. BR56: 17.45 MeV, entire set deleted; $\chi^2{}_s$ = 3.32 for 32 pts, R_s = 7.7.
4. FR49: energy changes; 0.95 → 0.92 MeV, 1.49 → 1.48 MeV.
5. GI57: 9.550 MeV, entire set deleted; $\chi^2{}_s$ = 3.3 for 38 pts, R_s = 9.2.
6. KR54: 5.780 MeV, 66.68° deleted; χ^2 = 25.
7. MI58: 5.000 MeV, set deleted; $\chi^2{}_s$ = 4.5 for 15 pts, R_s = 8.1.
8. PU56: 7.50 MeV, delete 8 pts derived from α-particle detection (high χ^2 and marked disagreement with proton data); delete 15.6°, 121.5°, χ^2 = 11 for each.
9. PU56: 9.480 MeV, delete 12.30°, $\chi^2{}_s$ = 23; 12.8°, χ^2 = 10.4; 78.5°, χ^2 = 51.
10. SA59: 14.32 MeV, set deleted; $\chi^2{}_s$ = 6.0 for 39 pts, R_s = 19.8.
11. JA74: 12.040, delete 109.62°, χ^2 = 11.5.
12. BR63: 5.43 MeV, 1 pt. deleted; χ^2 = 9.5, 56.2°.
13. BR67: 3.006 MeV, 104.48° deleted; χ^2 = 13.5.
14. DR64, DR66: Many of these 13 data sets had a very large $\chi^2{}_s$ (several more than 100 standard deviations out in χ^2 space). In addition the data of the two references were inconsistent with each other at nearby energies. It was judged that the probability of a gross systematic error in the experimental method was very high and the data of both references were deleted.
15. KE72: 12.000, 128.3° $A(\theta)$ deleted; χ^2 = 10.5.

data) was used to begin the detailed analysis.

It is important to note that each set of errors of each data set was separated into scale and relative errors (see Sec. IIC). This was important for proper analysis; the scale error became the error used for the normalization in χ^2 [see Eq. (1)], and the relative errors the proper errors used in weighting the individual data points. This separation of the scale error was not always easy, and was the result of study of the published papers and correspondence with several of the original authors. A detailed listing of the data sets and their errors is available on request.

A. Energy shift for data sets

It was realized during the course of the analysis that the early results of Freier (FR49) *et al.* showed certain systematic deviations from the calculated values. Therefore, attempts were made to fit the data assuming that their quoted energies were not exact. The fits to these data could indeed be significantly improved by small energy changes. Upon further investigation it was found that single energy phase shifts were also improved by shifting the energy, and the minimum χ^2 as a function of energy was the same for each energy in the single energy fits as it was for the energy-dependent fit. The energies found are assumed to be better ones and have been used in the data comparison tables. The shifts were $0.95 \rightarrow 0.92$ MeV (data set FR49, scale error 0.02 MeV), $1.49 \rightarrow 1.48$ MeV (data set FR49, scale error 0.02 MeV), and $1.997 \rightarrow 2.007$ MeV (data set BA64, scale error 0.01 MeV). The contribution to χ^2 (3.5 total from these three changes) from these shifts was included in the final value [see Eq. (1), third sum].

VI. DETAILS OF THE ANALYSIS

For the analysis of p-α scattering from 0 to 18 MeV it is sufficient to include partial waves through $l = 3$. The value of 3 for l_{max} chosen for the search minimized the χ^2 per degree of freedom. A search was made using $l_{max} = 4$ and while the total χ^2 was reduced, the χ^2 per degree of freedom *increased*. In addition, at every energy the values of the $l = 4$ phase shifts differed from 0 by less than their estimated errors. Even at the highest energy the G-wave phases were less than 0.1°. In any event, the changes in the remaining parameters caused by the addition of G waves were negligible, lying within the ranges of their error estimates. The small values of the G-wave phases are in agreement with the 20–40-MeV phase-shift analysis of Plattner, Bacher, and Conzett.[18] Because of the simple level structure in this energy region, a simple structure was assumed for the R matrix. For the $^2p_{3/2}$ state, the R-matrix expansion was truncated to include two levels—a low-lying level to produce the observed resonance and a high-lying level to act as a distant background. The same structure was assumed for the $^2p_{1/2}$ partial wave. All the other states were described in the R-matrix formalism as single level backgrounds.

In the early stages of the analysis, the channel radii were assigned a common value of 3.0 fm based on the relation $a = 1.2\ \text{fm} \times (A_p^{1/3} + A_\alpha^{1/3})$. An attempt was made to determine a better value by treating the common radius as a free parameter. It was found that the program was unable to determine a unique value of the radius, i.e., the fit to the data was quite independent of the choice of this radius. However, in a related analysis[19] in which p-α and n-α data were simultaneously fitted, a value of 2.9 fm was preferred. Consequently, throughout the results given here, a channel radius of 2.9 fm is used, and was not considered a free parameter.

For final results the boundary condition $B = 0$ was used in all states. This choice is nonrestrictive in the sense that the predicted observables are independent of the boundary condition. That is, once a set of R-matrix parameters have been found with a specified boundary condition, the predicted observables can be reproduced exactly for a different boundary condition by an appropriate transformation of the R-matrix parameters.[20]

This boundary condition was chosen because of its usefulness for a combined analysis of p-α and n-α systems. The common choice $B = -l$ is really appropriate only for uncharged particles, while the choice $B = S(E_r)$, where S is the shift function of Wigner and Eisenbud[12] as defined by Lane and Thomas,[13] makes the values of the R-matrix parameters dependent on the presence or absence of the external Coulomb field.

The background E_λ's for the P waves were fixed at 1000 MeV. The search routine was unable to determine these parameters when they were treated as free parameters and no significant improvements were achieved by placing these levels at lower energies. The effect of placing the background E_λ's very high is to make the background contribution essentially constant over the energy range of interest in the analysis.

Similarly, the F wave E_λ's were fixed at 100 MeV. Again no improvement was noted by raising or lowering these levels.

VII. RESULTS OF THE ANALYSIS

An R-matrix fit to the initial basic data set (see Table VI) was obtained with a weighted variance

TABLE VIII. R-matrix parameters. The quantities are as defined in Ref. 13. E_λ and γ_λ^2 are in MeV. Values marked (fixed) were not varied in the searches. The data sets are described in the text.

		Final selected data	Initial basic data
$S_{1/2}$	E_1	27.56 ± 1.09	28.13 ± 0.83
	γ_1	2.083 ± 0.052	2.120 ± 0.042
$P_{3/2}$	E_1	-8.73 ± 0.29	-9.31 ± 0.23
	γ_1	4.70 ± 0.11	4.929 ± 0.086
	E_2	1000.0 (Fixed)	1000.0 (Fixed)
	γ_2	25.55 ± 0.89	27.13 ± 0.59
$P_{1/2}$	E_1	3.296 ± 0.053	3.387 ± 0.033
	γ_1	4.655 ± 0.055	4.498 ± 0.041
	E_2	1000.0 (Fixed)	1000.0 (Fixed)
	γ_2	28.99 ± 0.66	28.07 ± 0.60
$D_{5/2}$	E_1	29.6 ± 1.2	31.2 ± 1.4
	γ_1	3.15 ± 0.12	3.36 ± 0.13
$D_{3/2}$	E_1	30.2 ± 1.0	32.2 ± 1.3
	γ_1	3.01 ± 0.10	3.24 ± 0.11
$F_{7/2}$	E_1	100.0 (Fixed)	100.0 (Fixed)
	γ_1	8.66 ± 0.63	7.74 ± 0.34
$F_{5/2}$	E_1	100.0 (Fixed)	100.0 (Fixed)
	γ_1	7.47 ± 0.41	6.97 ± 0.25

of 3.72. In addition to the 14 free R-matrix parameters, 75 normalizations were adjusted for best fit. Starting from this solution further data were eliminated following the guidelines given in Sec. V on data selection. All changes and deletions are listed in Table VII. A fit to the final selected data set of 1131 experimental values, 75 normalizations, and 3 energy shifts was obtained with a χ^2 per degree of freedom of 1.001 which is well within one standard deviation[9] in χ^2 space. All changes and deletions were checked with respect to the final fit and still satisfied the criteria for exclusion.

Table VIII gives the R-matrix parameters and associated standard errors for the final selected data set as well as those obtained for the initial basic data set. In addition to the values of the parameters, errors on the parameters are also given. The errors are correlated and are defined by the χ^2+1 rule; i.e., when a parameter is changed by its error and all the remaining parameters are searched to optimum values, the χ^2 increases by one. Table IX gives the complete covariance matrix for the 14 free parameters of the solution to the final selected data. The error of a parameter is the square root of the diagonal ele-

TABLE IX. Covariance matrix for the R-matrix parameters fitting the final selected data. The matrix given here is actually twice the inverse of the matrix of second derivatives of χ^2 with respect to the parameters, evaluated at the minimum of χ^2. Refer to the text for further explanation. The parameters are indexed as follows: $1=S_{1/2},E_1$; $2=S_{1/2},\gamma_1$; $3=P_{3/2},E_1$; $4=P_{3/2},\gamma_1$; $5=P_{3/2},\gamma_2$; $6=P_{1/2},E_1$; $7=P_{1/2},\gamma_1$; $8=P_{1/2},\gamma_2$; $9=D_{5/2},E_1$; $10=D_{5/2},\gamma_1$; $11=D_{3/2},E_1$; $12=D_{3/2},\gamma_1$; $13=F_{7/2},\gamma_1$; $14=F_{5/2},\gamma_1$.

	1	2	3	4	5	6	7
1	1.185 715 02						
2	−0.055 202 50	0.002 654 13					
3	0.275 101 38	−0.012 559 44	0.084 398 06				
4	−0.111 818 59	0.005 039 19	−0.033 350 38	0.013 312 18			
5	−0.871 932 97	0.038 577 96	−0.252 585 23	0.102 067 76	0.794 607 96		
6	0.008 659 77	−0.000 360 08	0.000 012 05	−0.000 157 10	−0.002 867 13	0.002 813 51	
7	0.025 020 37	−0.000 921 83	0.006 278 92	−0.002 858 67	−0.025 239 18	−0.001 040 40	0.003 073 95
8	−0.337 177 11	0.016 020 04	−0.112 870 89	0.042 807 26	0.307 991 91	−0.012 429 11	0.015 415 56
9	−0.062 889 75	0.005 555 49	0.095 454 96	−0.034 048 59	−0.228 383 50	−0.019 335 37	−0.008 949 93
10	−0.056 847 50	0.002 588 10	−0.005 913 92	0.003 047 98	0.029 525 34	−0.002 248 30	−0.003 188 69
11	−0.090 834 15	0.006 268 08	0.060 079 71	−0.021 237 10	−0.140 771 86	−0.016 676 31	−0.003 283 58
12	−0.048 789 97	0.002 193 29	−0.006 602 51	0.003 133 67	0.028 553 75	−0.001 817 77	−0.002 159 46
13	−0.590 227 26	0.025 131 38	−0.150 170 78	0.062 672 15	0.507 009 28	−0.004 849 16	−0.020 443 77
14	−0.385 038 16	0.016 389 79	−0.098 647 12	0.041 147 97	0.332 717 95	−0.003 297 07	−0.012 972 17

	8	9	10	11	12	13	14
8	0.434 245 06						
9	−0.369 195 79	1.461 929 86					
10	−0.021 036 41	0.123 912 66	0.014 365 11				
11	−0.212 439 24	1.086 688 82	0.091 868 72	1.088 283 42			
12	−0.077 236 37	0.082 886 26	0.010 111 55	0.086 075 64	0.009 358 67		
13	0.156 283 18	−0.100 794 04	0.027 353 18	−0.071 524 03	0.023 749 38	0.397 046 99	
14	0.111 467 94	−0.064 578 01	0.017 954 88	−0.049 322 57	0.015 471 92	0.256 319 49	0.172 252 76

ment of the covariance matrix. Because of the large correlations among the R-matrix parameters, the many significant figures here are often necessary in calculating uncertainties of quantities derived from the parameters.

The similarity of the two sets of R-matrix parameters given in Table VIII indicates that improvements in the χ^2 resulted from deletions of inconsistent data and not from any gross changes in the predicted observables. For the most part, the parameters obtained by fitting the initial basic data set and those obtained by fitting the final selected data set agree within their errors.

Because of the large number of parameters involved, a complete search of parameter space that would guarantee that a unique minimum had been reached was not made. Searches were made starting far from the solution given and the exact same solution was always reached. We feel that it is unlikely that another solution exists in the neighborhood of the one found.

The parameters in the R matrix in Table VIII show relatively smooth dependence on l and J. This might be expected from the fact that the data are also quite well described by a potential well model. The most striking feature is the similarity of the reduced width γ_λ for different J values of a given l. This is, of course, quite in line with results from a potential model but has not been apparent in previous R-matrix fits to phase shifts. The values of E_λ for the background P wave and for the D and F waves are somewhat arbitrary and represent a compromise between effects of the next E_λ in the infinite expansion and the sum of the effects of the rest of the more distant E_λ's. The data are not of sufficient precision to separate out these effects.

At first, in accordance with the choice of several previous workers the S-wave phase shift was parametrized to have purely hard-sphere behavior for a radius which turned out to be somewhat smaller than the common channel radius of the other states. It was found early in the analysis that a somewhat better fit would be obtained by maintaining the S-wave radius at the common value and adding a background level, and this parametrization was therefore adopted for the subsequent work.

VIII. STATISTICAL TESTS OF THE FIT

With the success of the fitting procedure and the apparent internal consistency of the data, it seemed appropriate to apply a goodness-of-fit test to investigate whether the data points exhibit a proper random distribution in relation to the R-matrix predictions. The hypotheses that are involved in this test are that the R-matrix predictions are accurate, that the data are behaving in a proper statistical fashion (no large systematic errors), and that the experimental errors are calculated properly. "Pearson's χ^2 test" was applied. The χ^2 for this test, which is denoted by $\chi^2{}_t$ to distinguish it from the χ^2 in Eq. (1) used in the R-matrix search, is a measure of the goodness-of-fit of the distribution of the individual data-point R-matrix solution χ^2 values to an expected distribution. The expected values of χ^2 are taken from a standard χ^2 distribution [for example Eq. (4-28), page 187, Ref. 17] with 0.924 degrees of freedom, the degrees of freedom per point for the present case. The expected distribution was divided into 35 (roughly the square root of the total number of data points) equal probability bins, resulting in approximately 32 χ^2 values in each bin. The χ^2 test criterion $P(\chi^2{}_t)$ for this case results in a value of 0.52. The standard interpretation of this number is that for repeating the entire set of experiments, the probability of obtaining the same $\chi^2{}_t$ or greater is 0.52, a satisfactory result. The test was applied with different numbers of equal probability bins from 20 to 50. The resulting χ^2 probabilities $P(\chi^2{}_t)$ also all fell well within the acceptable 0.05 to 0.95 confidence interval. Our final conclusion is that the χ^2 test indicates a statistically random distribution of the χ^2 contributions from the individual measured observables with respect to the R-matrix solution.

Table X lists the 91 data *subsets* in the final selected data set. $P(\chi^2{}_s)$ is the probability of $\chi^2{}_s$ for a given data subset. $P(\chi^2{}_s)$ is again the probability that the given $\chi^2{}_s$ or greater would occur were the measurements repeated. $P(\chi^2{}_s)$ is calculated in each case assuming $0.924N$ degrees of freedom. A study of the distribution of the subset $\chi^2{}_s$, similar to the above study of individual data-point χ^2, also gives a satisfactory result, indicating a statistically random distribution of the subsets.

$P(\chi^2)$ is not given for the norms. All the values are acceptable as can be seen by noting that, for one degree of freedom, $P(\chi^2)$ ranges from 0.99 to 0.01 for a χ^2 of 1.6×10^{-4} to 6.63, respectively. Where the χ^2 is written as 0.00 in the table it, in fact, falls well above 1.6×10^{-4}.

The case with the highest $\chi^2{}_s$ or the lowest $P(\chi^2{}_s)$ is that of BA64 11.157 MeV and corresponds to being three "standard deviations"[9] out in χ^2 space.

Note that in Table X there are several data sets whose $\chi^2{}_s$ is very small. For example, OH71C (11.93) has a $P(\chi^2{}_s)$ of 0.9999. The fit is much too good. Since the number of R-matrix parameters was chosen to find the lowest χ^2 per degree of freedom rather than the absolute lowest χ^2, it

TABLE X. The final selected data subsets. See Table VI for the Reference code. The energies and number of points per set, N, are given as corrected in Table VII. Norm is the normalization N_{ij} in Eq. (1) (Sec. IV) which multiplies the *predicted* observable. The norm is not given for a data set of a single point or when the experimenter did not give a scale error. It is marked "G" when it is ganged to the normalization at another energy for the same reference. $P(\chi^2{}_s)$ is the classic probability of χ^2 ("upper tailed"). For details see the text, Sec. VIII.

Ref.	E (MeV)	N	Norm	χ^2 norm	$\chi^2{}_s$ set	$P(\chi^2{}_s)$	Ref.	E (MeV)	N	Norm	χ^2 norm	$\chi^2{}_s$ set	$P(\chi^2{}_s)$
AD66	0.222	1	1.054	0.58	0.29	0.56	JA67	7.890	2	···	···	0.11	0.93
AD66	0.300	1	G	···	0.38	0.50	BR63	7.890	4	0.988	0.49	1.76	0.74
AD66	0.390	1	G	···	2.56	0.10	OH71C	7.890	3	0.989	0.13	1.00	0.76
AD66	0.500	1	G	···	0.03	0.83	SC70	7.890	12	0.996	0.04	8.8	0.64
AD66	0.515	1	G	···	0.15	0.66	BA64	7.967	24	1.028	2.01	19.5	0.63
BR67	0.940	12	1.000	0.00	10.7	0.47	RO61	8.500	13	1.000	0.00	10.6	0.57
FR49	0.920	10	0.995	0.07	8.4	0.52	BA64	8.960	26	1.021	1.17	28.8	0.23
BR67	1.140	12	1.006	0.30	9.8	0.56	PU56	9.480	39	1.048	4.9	46.1	0.12
BR67	1.350	12	1.009	0.81	9.3	0.60	WI54	9.760	31	0.999	0.00	43.5	0.04
SC58	1.375	1	···	···	1.31	0.23	BR63	9.850	4	0.999	0.00	0.86	0.91
FR49	1.480	11	1.013	0.46	14.2	0.17	JA67	9.890	6	···	···	1.56	0.94
BR67	1.560	12	1.007	0.49	12.4	0.34	OH71C	9.890	6	0.999	0.01	1.91	0.90
FR49	1.700	11	0.981	0.85	21.3	0.02	SC70	9.890	12	1.003	0.03	9.0	0.62
BR67	1.765	12	0.997	0.07	28.2	0.003	BA64	9.954	24	1.024	1.50	19.7	0.61
BR67	1.970	11	1.004	0.17	14.0	0.18	PL68	10.00	9	0.987	0.43	10.5	0.25
BA64	2.007	25	0.986	0.48	25.4	0.33	JA73	11.157	4	1.006	2.26	3.1	0.49
FR49	2.020	12	0.986	0.48	14.5	0.21	BA64	11.157	24	1.037	3.48	48.7	0.001
SC58	2.020	1	···	···	4.01	0.04	JA67	11.160	2	···	···	0.22	0.87
BR67	2.180	12	1.005	0.29	7.3	0.78	BR57	11.420	4	1.006	0.10	0.86	0.91
FR49	2.220	12	0.984	0.64	13.4	0.27	BR57	11.650	7	0.993	0.11	2.51	0.90
FR49	2.530	15	1.001	0.00	25.3	0.03	BR63	11.900	4	1.008	0.12	1.17	0.85
BR67	2.590	3	0.993	0.45	2.28	0.47	OH71C	11.930	8	1.000	0.04	0.61	0.999
BR67	3.000	11	1.007	0.49	16.2	0.10	SC70	11.940	16	1.015	0.56	8.25	0.91
BA64	2.006	27	0.996	0.03	10.7	0.994	KE72	12.000	7A	1.001	0.00	4.6	0.65
MI58	3.030	14	0.975	1.06	13.6	0.39	KE72	12.000	6R	G	···	5.3	0.45
FR49	3.040	16	0.998	0.01	15.7	0.38	HA76	12.000	18	0.993	0.47	23.3	0.13
BR67	3.200	12	1.003	0.11	10.9	0.46	OH71C	12.030	1	···	···	0.35	0.52
MI58	3.510	15	0.978	0.82	15.0	0.37	JA73	12.040	19	1.007	3.2	25.0	0.11
FR49	3.580	17	1.010	0.25	15.4	0.47	SA59	12.040	29	0.979	4.3	50.7	0.003
SC58	3.580	1	···	···	3.2	0.06	GA69	12.040	26	1.000	0.00	30.9	0.16
BR63	3.650	1	···	···	0.86	0.32	BR57	12.490	6	0.984	0.61	0.49	0.996
BA64	4.006	25	1.000	0.00	13.5	0.94	BR57	13.300	9	1.001	0.00	3.51	0.91
MI58	4.020	15	0.965	2.14	22.3	0.07	JA73	13.600	4	1.003	0.66	12.0	0.013
BR63	4.220	1	···	···	1.03	0.28	GA69	14.230	24σ	0.987	0.39	24.4	0.33
MI58	4.500	15	1.014	0.36	19.6	0.14	GA69	14.230	26P	0.999	0.00	12.1	0.978
SC70	4.580	15	0.988	0.38	9.2	0.81	JA73	14.320	7	1.001	0.26	5.6	0.53
BR63	4.770	4	0.982	0.41	0.64	0.94	BR57	14.380	9	1.000	0.00	1.38	0.996
BA64	5.011	28	1.024	1.44	10.1	0.998	BR57	15.050	10	1.022	1.23	6.1	0.75
BR51	5.100	4	0.993	0.04	5.5	0.21	BR57	16.240	10	0.994	0.08	4.7	0.87
KR54	5.780	26	1.000	0.00	22.0	0.58	BR57	16.760	10	0.989	0.27	2.0	0.992
BR63	5.930	4	0.996	0.03	4.2	0.33	HA76	17.000	25	G	···	21.0	0.58
SC70	5.950	13	0.979	1.10	7.1	0.85	GA69	17.450	23σ	1.002	0.01	18.2	0.65
JA67	6.000	1	···	···	4.4	0.03	GA69	17.450	23P	1.018	0.85	11.6	0.95
BA64	6.016	28	1.025	1.64	9.6	0.998	WE66	17.510	6	0.991	0.82	3.1	0.75
BA64	6.977	28	1.033	2.80	12.9	0.984	BR57	17.840	10	0.987	0.41	6.5	0.71
PU56	7.500	28	1.023	3.18	39.5	0.04							

seems most likely that the cause of the low $\chi^2{}_s$ is overestimation of experimental errors. Experimenters are urged not arbitrarily to enlarge their errors when they suspect systematic errors. Doing so may result in their data being of little use and not affecting a phenomenological search. A separate discussion of systematic errors should be made. We note that these examples occur mostly in analyzing-power measurements.

Of statistical importance is the comparison of the parameter errors in Table VIII. It is significant that the errors for the final selected data

TABLE XI. Phase shifts in degrees. The rows marked "a" are the results of the present work. The errors are derived from the covariance matrix of χ^2 found during the search. The rows marked "b" are from Table V of Arndt *et al.*, Ref. 5.

E (MeV)		$S_{1/2}$	$P_{3/2}$	$P_{1/2}$	$D_{5/2}$	$D_{3/2}$	$F_{7/2}$	$F_{5/2}$
0.94	a	−11.034 ± 0.031	6.050 ± 0.018	1.539 ± 0.009	0.000 ± 0.000	−0.001 ± 0.000	0.000 ± 0.000	0.000 ± 0.000
	b	−10.988	6.175	1.458	0.000	0.000	0.000	0.000
2.02	a	−22.342 ± 0.057	49.982 ± 0.132	7.343 ± 0.039	0.004 ± 0.003	−0.004 ± 0.003	0.001 ± 0.000	0.001 ± 0.000
	b	−22.308	50.774	7.120	0.004	0.001	0.000	0.000
2.51	a	−26.480 ± 0.066	79.135 ± 0.155	11.331 ± 0.058	0.008 ± 0.006	−0.007 ± 0.005	0.003 ± 0.000	0.002 ± 0.000
	b	−26.470	79.767	10.921	0.007	0.002	0.001	0.000
3.006	a	−30.270 ± 0.073	97.046 ± 0.142	15.596 ± 0.077	0.015 ± 0.010	−0.009 ± 0.008	0.005 ± 0.001	0.004 ± 0.001
	b	−30.270	97.313	15.378	0.012	0.004	0.001	0.001
4.006	a	−36.952 ± 0.084	110.387 ± 0.120	25.425 ± 0.112	0.040 ± 0.021	−0.012 ± 0.017	0.015 ± 0.002	0.011 ± 0.002
	b	−36.993	110.376	25.349	0.028	0.009	0.003	0.002
5.011	a	−42.733 ± 0.094	113.491 ± 0.115	34.944 ± 0.142	0.085 ± 0.037	−0.007 ± 0.030	0.033 ± 0.004	0.025 ± 0.003
	b	−42.812	113.398	35.005	0.053	0.017	0.008	0.006
6.016	a	−47.819 ± 0.104	113.557 ± 0.123	42.782 ± 0.177	0.158 ± 0.057	0.012 ± 0.047	0.064 ± 0.008	0.048 ± 0.006
	b	−47.937	113.418	42.849	0.090	0.030	0.016	0.011
7.5	a	−54.358 ± 0.124	111.830 ± 0.157	50.680 ± 0.243	0.329 ± 0.095	0.080 ± 0.078	0.137 ± 0.016	0.103 ± 0.013
	b	−54.544	111.611	50.530	0.172	0.059	0.035	0.026
8.5	a	−58.235 ± 0.145	110.245 ± 0.194	53.962 ± 0.294	0.498 ± 0.125	0.162 ± 0.103	0.211 ± 0.025	0.158 ± 0.021
	b	−58.484	109.944	53.583	0.250	0.087	0.056	0.040
10.0	a	−63.389 ± 0.193	107.738 ± 0.268	56.811 ± 0.377	0.851 ± 0.177	0.359 ± 0.145	0.365 ± 0.044	0.273 ± 0.036
	b	−63.780	107.251	56.049	0.409	0.148	0.102	0.073
12.04	a	−69.302 ± 0.295	104.478 ± 0.403	58.323 ± 0.504	1.555 ± 0.260	0.807 ± 0.213	0.675 ± 0.081	0.503 ± 0.067
	b	−70.032	103.588	57.008	0.731	0.280	0.201	0.145
13.65	a	−73.170 ± 0.408	102.244 ± 0.531	58.541 ± 0.619	2.298 ± 0.331	1.321 ± 0.271	1.002 ± 0.122	0.746 ± 0.099
	b	−74.348	100.828	56.731	1.098	0.447	0.318	0.229
15.05	a	−76.032 ± 0.544	100.408 ± 0.669	58.424 ± 0.784	3.167 ± 0.403	1.957 ± 0.331	1.383 ± 0.169	1.027 ± 0.138
	b	−77.739	98.551	56.102	1.525	0.660	0.453	0.326
17.84	a	−80.328 ± 0.904	97.551 ± 0.987	58.082 ± 1.076	5.360 ± 0.568	3.667 ± 0.469	2.342 ± 0.291	1.733 ± 0.235
	b	−83.673	94.364	54.335	2.794	1.442	0.841	0.605

TABLE XII. Phase shifts in degrees. The rows marked "a" are from the present work. The rows marked "b" are from Table 4 of Stammbach and Walter, Ref. 7.

E (MeV)		$S_{1/2}$	$P_{3/2}$	$P_{1/2}$	$D_{5/2}$	$D_{3/2}$	$F_{7/2}$	$F_{5/2}$
0.9	a	−10.53 ± 0.03	5.41 ± 0.02	1.40 ± 0.01	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
	b	−10.200	5.41	1.37	0.00	0.00	0.00	0.00
2.1	a	−23.05 ± 0.06	55.17 ± 0.14	7.92 ± 0.04	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00	0.00 ± 0.00
	b	−22.42	56.70	7.80	0.01	0.01	0.00	0.00
3.0	a	−30.23 ± 0.07	96.90 ± 0.14	15.54 ± 0.08	0.02 ± 0.01	−0.01 ± 0.01	0.00 ± 0.00	0.00 ± 0.00
	b	−29.46	98.88	15.42	0.04	0.02	0.00	0.00
4.0	a	−36.91 ± 0.08	110.35 ± 0.12	25.37 ± 0.11	0.04 ± 0.02	−0.01 ± 0.02	0.01 ± 0.00	0.01 ± 0.00
	b	−36.08	111.87	25.38	0.08	0.04	0.01	0.01
6.0	a	−47.74 ± 0.10	113.57 ± 0.12	42.67 ± 0.18	0.16 ± 0.06	0.01 ± 0.05	0.06 ± 0.01	0.05 ± 0.01
	b	−46.93	114.62	43.19	0.25	0.14	0.04	0.03
8.0	a	−56.34 ± 0.13	111.06 ± 0.17	52.49 ± 0.27	0.41 ± 0.11	0.11 ± 0.09	0.17 ± 0.02	0.13 ± 0.02
	b	−55.78	111.75	53.22	0.56	0.33	0.12	0.08
10.0	a	−63.39 ± 0.19	107.74 ± 0.27	56.81 ± 0.38	0.85 ± 0.18	0.36 ± 0.15	0.37 ± 0.04	0.28 ± 0.04
	b	−63.31	107.99	57.34	1.04	0.64	0.25	0.18
13.0	a	−71.69 ± 0.36	103.07 ± 0.48	58.52 ± 0.57	1.99 ± 0.30	1.10 ± 0.25	0.87 ± 0.10	0.65 ± 0.09
	b	−72.89	102.43	58.12	2.15	1.37	0.60	0.43
17.0	a	−79.23 ± 0.78	98.32 ± 0.88	58.16 ± 0.96	4.63 ± 0.51	3.08 ± 0.42	2.02 ± 0.25	1.50 ± 0.20
	b	−83.4	95.91	55.29	4.47	2.97	1.45	1.01

set are *larger* than those for the initial set. We interpret this to mean that the use of the covariance matrix to determine errors is reliable only if the data form a statistically consistent set without major systematic errors. One should always, then, view with suspicion errors derived from an analysis of unevaluated data.

IX. COMPARISON OF RESULTS WITH EARLIER WORK

It is difficult to compare satisfactorily the present fit to the data with any of the previous work because of the differences in data sets. The present fit is a faithful representation of the present data set, which does include more accurate experiments than previous ones. A pertinent question is one concerning the relative merits of different parametrizations. The present parametrization is completely satisfactory over the energy region under consideration, and has the merit of being easily and naturally extended above the ^{4}He$(p,d)^3$He threshold. In order to offer some comparison with the earlier work, Tables XI and XII show phase shifts and calculated errors at various energies along with some from Stammbach and Walter[7] and Arndt *et al.*[5]

Estimated errors for the earlier phase shifts are not given, so it is impossible to tell if the precision of the earlier work was such as to allow agreement within the errors. In general the present phase shifts agree reasonably well with both sets up to about 6 MeV, the agreement being better with Arndt *et al.* Above 6 MeV, where D waves begin to become more important, the agreements are not as good. Our D waves in general lie between those of the other two sets, while our F waves are larger than either agreeing better with Stammbach and Walter. Our S-wave phase shifts above 10 MeV lie below those of both the other sets, which agree with each other. Our $P_{3/2}$-wave phase shift above 10 MeV lies above the other two sets, but lies closer to Stammbach and Walter. Our $P_{1/2}$-wave phase shift above 10 MeV lies above both of the others but again is closer to that of Stammbach and Walter. The lowest energy phases of the 20–40-MeV analysis of Plattner *et al.*[18] are compatible with our highest energy phases.

Branden, Plattner, and Haeberli[21] very recently measured p-α observables from 2.2 to 8.9 MeV in a phase-shift study of the solutions of Refs. 1, 2, 5–7. Their results confirm our use of Refs. 5 and 7 as the best phase-shift sets for comparison. Their data were too recent for inclusion in our analysis.

X. ANALYZING POWER MAXIMA AND MINIMA, $A_y = \pm 1$

Plattner and Bacher[22] have shown that, because of the properties of the scattering amplitude in the complex plane, that A_y is expected to reach a maxima or minima of exactly ±1 at certain sets of energies and angles. Corresponding large asymmetries have often been observed experimentally. These extreme values of A_y are also of interest in the study of other spin observables, as the observables are often connected by quadratic relationships.[23] In Table XIII we show Plattner and Bacher's predictions for the elastic p-^{4}He case compared with the results of our final solution. The errors appearing in our values reflect only the accuracy to which we made the calculations. A calculation of a meaningful error statement appeared very difficult and was not attempted. The difference between our values and Plattner and Bacher's is not serious; their values were not intended to be definitive and also the maxima are very broad either in energy or angle.

XI. PREDICTED VALUES OF OBSERVABLES AND PHASE SHIFTS

The authors will be glad to provide tables of predicted values of cross sections, polarizations, and polarization transfers at various energies and angles to any potential user. It should be emphasized that reliable calculations of uncertainties in the predicted values must be made using the full covariance matrix for the parameters and cannot be reliably estimated from the uncertainties given for the phase shifts. Should any user require the complete covariance matrices for the phase shifts at particular energies, these can also be supplied. A set of predictions is available as an internal report (LA-6389-MS). The tables are given for energies from 250 keV to 5 MeV in 250-keV steps and from 5 to 17 MeV in 0.5-MeV steps.

Also given in (LA-6389-MS) are phase-shift predictions ($l = 0$ to 3) from 50 keV to 5 MeV in 50-keV steps, and from 5 to 17 MeV in 250-keV steps.

A contour plot of A_y is given on page 447 of Ref. 6. We do not present a contour plot of our results because the visual difference between it and that of Ref. 6 is not significant. Plots of the phase shifts are given in Ref. 3.

TABLE XIII. $A_y = +1$ points. Laboratory energy (MeV) and c.m. angles (degrees).

Plattner and Bacher[a]		Present work	
Energy	Angle	Energy	Angle
1.90 ± 0.02	88.00 ± 0.25	1.874 ± 0.001	86.45 ± 0.01
6.35 ± 0.04	128.80 ± 0.1	6.392 ± 0.001	128.91 ± 0.01
12.30 ± 0.04	125.50 ± 0.1	12.100 ± 0.001	125.53 ± 0.01

[a] Reference 22.

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†Present address: Tetra-Tech Inc., Arlington, Virginia 22209.

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