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Radiative transitions and isospin mixing in ^{12}C†

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γ-ray decays of the 12.7 and 16.1 MeV states of ^{12}C are investigated in a coincidence study of the ^{10}B(^{3}He,$p\gamma$) reaction. We obtain $\Gamma_{\gamma 0}/\Gamma = (1.93 \pm 0.12)\%$, $\Gamma_{\gamma 1}/\Gamma_{\gamma 0} = (15.0 \pm 1.8)\%$ and $\Gamma_\alpha/\Gamma = (97.8 \pm 0.1)\%$ for ^{12}C(12.7), and $\Gamma_{\gamma 1}/\Gamma = (2.42 \pm 0.29) \times 10^{-3}$ for ^{12}C(16.1). Relative γ-ray branching ratios of ^{12}C(16.1) were measured using the $E_p = 163$ keV ^{11}B(p,γ) resonance. We find $\Gamma_{\gamma 0}/\Gamma_{\gamma 1} = (4.6 \pm 0.7)\%$, $\Gamma_\gamma(16.1 \rightarrow 9.6)/\Gamma_{\gamma 1} = (2.4 \pm 0.4)\%$, and $\Gamma_\gamma(16.1 \pm 12.7)/\Gamma_{\gamma 1} = (1.46 \pm 0.25)\%$. This information, together with existing data on $M1$ transitions and single nucleon transfer reactions, is used to determine the isospin mixing between the 12.7 and 15.1 MeV levels of ^{12}C. A charge dependent matrix element of 110 ± 30 keV is deduced.

NUCLEAR REACTIONS ^{10}B(^{3}He,$p\gamma$), $E = 4.1$ MeV, p,γ coincidence. Deduced Γ_γ for ^{12}C levels at 12.7, 16.1 MeV. ^{11}B(p,γ), $E = 163$ keV resonance. Measured Γ_γ for transitions to 0.0, 4.4, 9.6, and 12.7 MeV in ^{12}C.
NUCLEAR STRUCTURE ^{12}C, deduced isospin mixing between 12.7 and 15.1 MeV levels.

I. INTRODUCTION

The doublets of $T=0$ and $T=1$ levels in ^{8}Be and ^{12}C have attracted much attention as an area for quantitative studies of isospin mixing. Good wave functions are available in both cases and the two levels in each doublet are known to have very similar space structures. Off-diagonal matrix elements of 150 keV (Ref. 1) and 250 ± 50 keV (Ref. 2) have been deduced in ^{8}Be and ^{12}C, respectively. Standard calculations of Coulomb effects give matrix elements of only ~60 keV (Refs. 1 and 3) in the two cases. This seems to support Negele's analysis of displacement energies which require a sizable $\Delta T = 1$ component of the short-range nuclear force.[4] However, Miller[5] had estimated an upper limit of 33 keV for the contributions of Negele's $\Delta T = 1$ force to the off-diagonal matrix element in ^{12}C so that the large matrix elements remain unexplained.

These considerations have led us to reexamine the isospin mixing between the 12.7 MeV J^π, $T = 1^+, 0$ and the 15.1 MeV 1^+, 1 state in ^{12}C. Our new analysis is necessary because the current experimental evidence on the isospin mixing is inconclusive. In the three years since Braithwaite, Bussoletti, Cecil, and Garvey (BBCG) reported an off-diagonal matrix element of 250 ± 50 keV (Ref. 2) a variety of experiments have been performed to check this value. Unfortunately the different approaches[6-9] yielded inconsistent results and none of the experiments by itself is completely convincing.

This paper contains two separate parts. In the first we present new experimental results on the decays of the 12.7 and 16.1 MeV levels of ^{12}C. In the second part we relate these quantities to the isospin mixing. Our strategy here is to winnow from the large number of experimental results which are affected by isospin mixing in ^{12}C those quantities which are in fact well suited for measuring the isospin mixing. We conclude that certain radiative and single particle transfer reactions do give a reasonable measure of the isospin mixing. We will show that there is clear evidence for isospin mixing in the 1^+ doublet, but that the magnitude is much smaller than reported in Ref. 2. A brief account of this work has already been published.[10]

II. EXPERIMENTAL RESULTS

A. ^{10}B(^{3}He,$p\gamma$) coincidence studies

Coincidence measurements of the γ-ray branching ratios of ^{12}C(12.7) and ^{12}C(16.1) were performed at the University of Washington as a subsidiary part of an experiment reported elsewhere.[11] The ^{12}C levels were populated in the ^{10}B(^{3}He,p) reaction by bombarding a ~150 μg/cm^2 self-supporting foil of enriched ^{10}B with 4.1 MeV ^{3}He ions. Protons were detected at 0° in a telescope consisting of a 300 mm^2 200 μm thick surface barrier detector followed by a 2.4 mm deep Si(Li) detector with an area of 350 mm^2. The surface barrier detector was used to obtain a timing signal which was derived from a fast charge sensitive preamplifier.[12] Slow signals from the two detectors were summed without particle identification to give the energy pulse. The telescope was placed behind a 9 mg/cm^2 Ni foil followed by a 7 mg/cm^2 Al foil. The foils stopped the incident ^{3}He beam while permitting the protons to pass through. Two different materials were used in the stopping foils in order to have the lowest Z consistent with a Coulomb barrier well above the local ^{3}He energy. Material with the lowest possible Z gives the least energy loss straggling and the most favorable ratio of dE/dx for ^{3}He compared with dE/dx for protons. The 45 msr solid angle of the proton detector was defined by a 1.59 cm diam collimator located 6.7 cm from the target. The above apparatus was contained in a 25 cm diam spherical scattering chamber with 0.8 mm thick aluminum walls. The beam was collimated by apertures of 4.8 and 6.4 mm diam located 1.17 and 0.81 m upstream from the target. Protons were detected at 0° in order to simplify the angular correlation of the decay γ rays. In this geometry the γ-ray correlation has the form $W_\gamma(\theta)=A_0+A_2P_2(\cos\theta)$ if the decaying level has $J\leq 1$ and/or the radiation is pure dipole. γ rays were detected at $\theta_\gamma=125°$ in a 25.4 × 25.4 cm NaI(Tl) spectrometer with anticoincidence shielding.[13] Since $P_2(\cos\theta)$ vanishes at $\theta\approx 125°$ a single measurement at this angle can give the branching ratio $\Gamma_\gamma/\Gamma=4\pi(N_c/N_s)/(\eta\Delta\Omega_\gamma)$ where N_c and N_s are the coincident and singles counts, η is the γ-ray detection efficiency, and $\Delta\Omega_\gamma$ is the solid angle of the γ-ray detector. Our NaI spectrometer has a resolution of 3.2% for a well collimated beam of 15.1 MeV γ rays. In this work the detector was used with an effective aperture about 15 cm in diameter at a distance of approximately 36 cm from the target resulting in a solid angle $\Delta\Omega_\gamma=130$ msr. Under these conditions the resolution at 15.1 MeV was 4.2%.

Particle-γ-ray coincidences were recorded event by event and written on magnetic tape by an on-line SDS 930 computer. The tapes were then played back for analysis. A block diagram of the electronics is shown in Fig. 1. Three parameters were recorded for each coincident event, a γ-ray energy signal, a charged particle energy signal, and the output of a time-to-amplitude converter which was started by the NaI detector and stopped by the particle detector. The γ-ray pulse heights were routed into two spectra, denoted "accept" and "reject." The accept spectrum contains those γ-ray events which did not produce a count in the anticoincidence shield, while the reject spectrum contains those events where some energy has escaped from the NaI and been detected by the anticoincidence shield. The time resolution of our

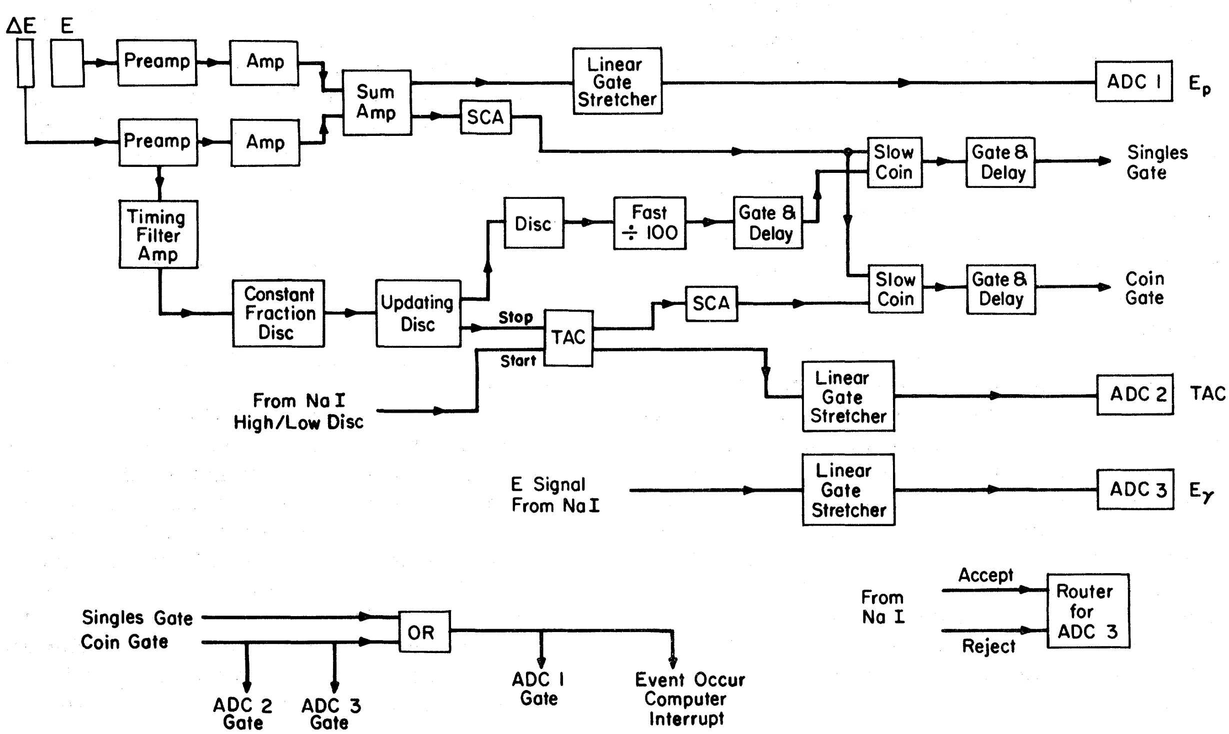

FIG. 1. Block diagram of the electronics used in the $^{10}B(^3He,p\gamma)$ coincidence experiment.

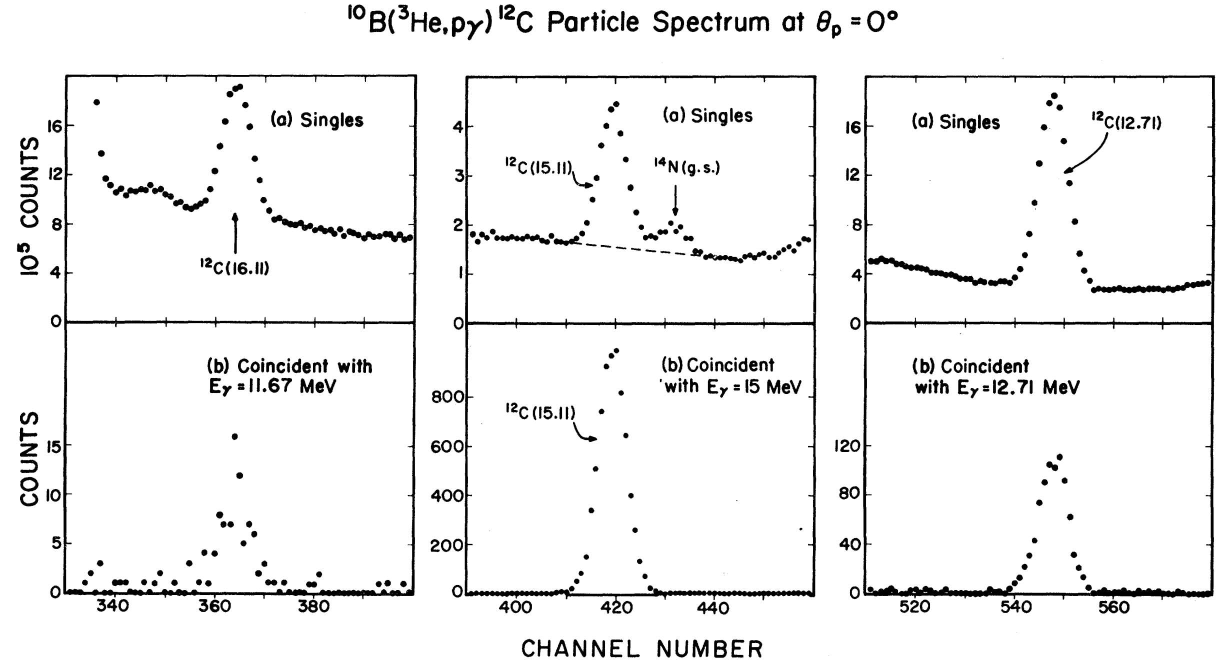

FIG. 2. Partial spectra of charged particles from the $^{10}B(^3He,p\gamma)$ measurement. Identical regions of the singles and coincidence spectra are shown. The group labeled ^{14}N(g.s.) arises from the ($^3He,p$) reaction on a ^{12}C contaminant in the target. The spectrum containing the $^{12}C(15.1)$ peak was taken with $\theta_\gamma = 125°$. Spectra containing the $^{12}C(12.7)$ and $^{12}C(16.1)$ peaks also include data at $\theta_\gamma = 30°$ and $\theta_\gamma = 90°$.

coincidence system is ~3 ns full width at half maximum. To facilitate the measurement of absolute branching ratios, singles and coincidence proton spectra were accumulated simultaneously in the same analog-to-digital converter. The singles and coincidence channels were both driven by the same fast logic signal from the ΔE detector and the slow coincidence requirement on the particle energy signal was common to both channels. This insured that dead time corrections to the branching ratios are negligible and singles and coincident proton spectra can be matched channel for channel. Portions of the singles and coincidence charged particle spectra are displayed in Fig. 2. Spectra of γ rays in coincidence with protons populating the 12.7 and 16.1 MeV levels of ^{12}C are shown in Figs. 3 and 4. Accidental coincidences have been subtracted. The spectrum of $^{12}C(12.7)$ decays displayed in Fig. 3 was obtained at $\theta_\gamma = 125°$. The smooth curve is a fitted line shape described below. Similar spectra were taken at $\theta_\gamma = 30°$ and 90°. Data were fitted to an angular distribution $W(\theta) = A_0 + A_2P_2(\cos\theta)$ in order to obtain the branching ratio. The spectrum of $^{12}C(16.1)$ decays [Fig. 4(a)] was obtained by summing separate spectra taken at $\theta_\gamma = 30°$, 90°, and 125°. Only the strong $M1$ transition ($16.1 \rightarrow 4.4$) is observed. The apparent weak yield of 15.1 MeV γ rays is actually in coincidence with a background under the $^{12}C(16.1)$ proton group as displayed in Fig. 4(b). A 4% background correction was made to the final $^{12}C(16.1)$ γ-ray yield on the basis of Fig. 4(b). Due

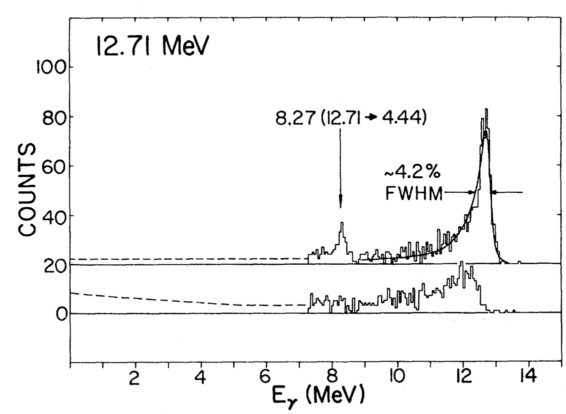

FIG. 3. Spectrum of γ rays in coincidence with protons from the $^{10}B(^3He,p)^{12}C(12.7)$ reaction. The spectrum is not shown below $E_\gamma \sim 7$ MeV due to strong background γ rays. The spectrum accepted by the anticoincidence shield is shown above the spectrum rejected by the shield.

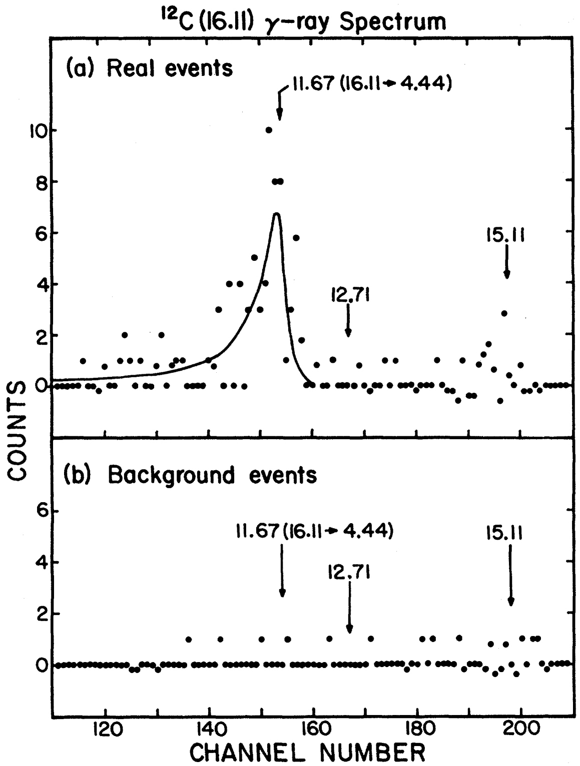

FIG. 4. (a) Spectrum of γ rays in coincidence with protons from the ^{10}B(^{3}He,p)^{12}C(16.1) reaction. The solid curve results from a least squares fit using the line shape measured at 15.1 MeV (see Fig. 5). (b) γ-ray spectrum in coincidence with a region of the particle spectrum adjacent to the ^{12}C(16.1) proton group. It is clear that the apparent yield of 15.1 MeV γ rays in Fig. 3(a) is not due to decays of ^{12}C(16.1). The nonintegral counts arise from the procedure used to subtract the random coincidences.

to the small number of coincidence counts, the yield of 11.7 MeV γ rays was obtained by summing counts in the region of the photopeak rather than from a line shape fit. The γ-ray branching ratio was determined by fitting the data obtained at $\theta_\gamma = 30°$, 90°, and 125° to the expression $W(\theta) = A_0 + A_2P_2(\cos\theta) + A_4P_4(\cos\theta)$. Since the $16.1 \to 4.4$ transition is isovector, it is expected to be primarily $M1$ with a small $E2$ admixture.

Absolute γ-ray branching ratios were obtained from our data using a γ-ray efficiency calibration based on the well-known decays of ^{12}C(15.1) and ^{12}C(4.4). The calibrations were obtained simultaneously with the decay data on ^{12}C(12.7) and ^{12}C(16.1) discussed above. The decays of ^{12}C(15.1) form a convenient calibration because the γ-ray branching is large and well known, the correlation is simple $[W(\theta) = A_0 + A_2P_2(\cos\theta)]$ and the γ-ray energy is in the useful range. We assume that for ^{12}C(15.1) $\Gamma_{\gamma_0}/\Gamma = (88.2 \pm 2.1)\%$, based on a relative γ_0 γ-ray branch of $(92 \pm 2)\%$ (Ref. 14) and a recent measurement[7] of $\Gamma_\alpha/\Gamma = (4.1 \pm 0.9)\%$ for the α-decay branch of ^{12}C(15.1). The relevant portions of the singles and coincidence charged particle spectra for the 15.1 MeV calibration are displayed in Fig. 2. The resulting spectrum of γ rays from the decay of ^{12}C(15.1) is shown in Fig. 5. The energy dependence of the γ-ray detection efficiency was estimated from the absorption of γ rays in the material between the target and the NaI detector along with the measured accept/reject ratio. The estimate is in reasonable accord with our two data points at 15.1 and 4.4 MeV. Our resulting values for the γ-ray branching ratios of ^{12}C(12.7) are $\Gamma_{\gamma_0}/\Gamma = (1.93 \pm 0.12)\%$, $\Gamma_{\gamma_1}/\Gamma_\gamma = (15.0 \pm 1.8)\%$. For the ^{12}C(16.1) decays we obtain $\Gamma_{\gamma_1}/\Gamma = (2.42 \pm 0.29) \times 10^{-3}$.

B. ^{11}B(p,γ) measurement at the 163 keV resonance

The radiative width for decay of ^{12}C(16.1) to ^{12}C(12.7) was measured using the ^{11}B(p,γ) reaction at the $E_p = 163$ keV resonance. The 3 MV electrostatic generator at the Caltech Kellogg Laboratory was used to bombard a ≈ 35 μg/cm^2 target of enriched ^{11}B with a 10–15 μA (electrical) beam of H_3^+ ions. The ^{11}B was evaporated directly onto a Ta beam stop which was water cooled. The target chamber is similar to one described by Trautvetter and Rolfs[15] with the liquid nitrogen cooled shroud removed. γ rays were detected at 45° in a 15% Ge(Li) detector. A γ-ray spectrum accumulated at an energy below the resonance for 20 mC (electrical) of integrated charge established that the nonresonant γ-ray yield was negligible. Two spectra were accumulated on resonance, one for

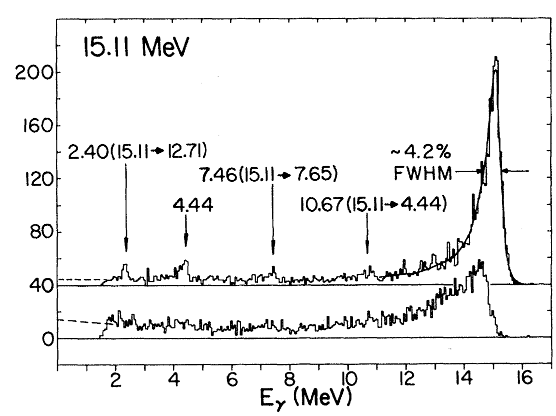

FIG. 5. Spectrum of γ rays in coincidence with protons from the ^{10}B(^{3}He,p)^{12}C(15.1) reaction. The spectrum of γ rays accepted by the anticoincidence shield is shown above the spectrum rejected by the shield.

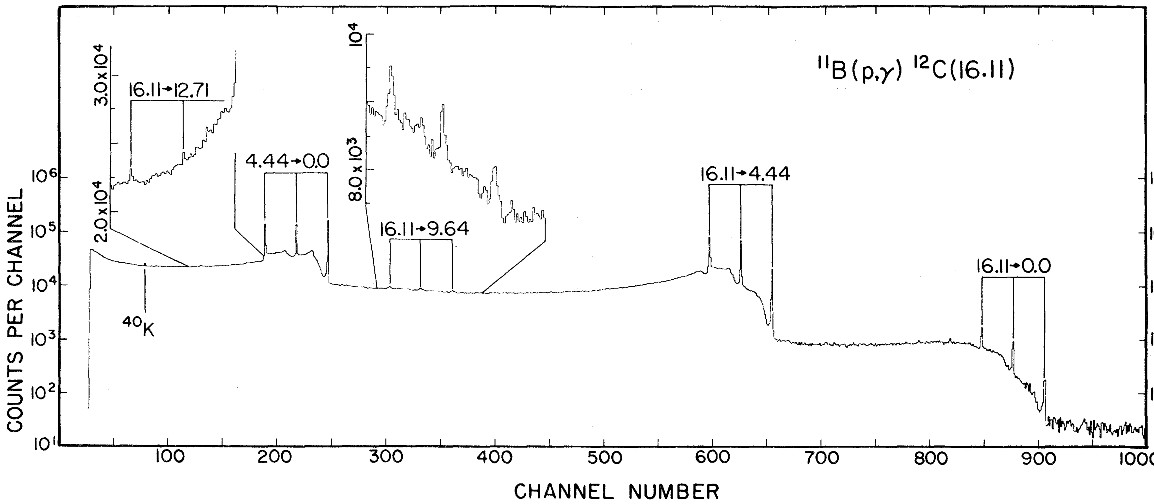

FIG. 6. Spectrum of γ rays from the 163 keV $^{11}B(p,\gamma)$ resonance observed in a 15% Ge(Li) detector.

20 mC and one for 100 mC of integrated charge. The 100 mC spectrum is shown in Fig. 6. γ rays corresponding to transitions from the 16.1 MeV level to ^{12}C states at 0.0, 4.4, and 9.6 MeV are clearly visible as well as a weak transition to the 12.7 MeV state. The 16.1 MeV γ-ray group contains an appreciable component of summed 4.4 and 11.7 MeV γ rays from the cascade via the 4.4 MeV level. This can be seen by the apparent presence of "three escape" and "four escape" peaks.

The relative efficiency of our Ge(Li) detector for γ rays with energies between 0.847 and 11.67 MeV was measured with well known decay schemes of a ^{56}Co source and $^{23}Na(p,\gamma\gamma)$ resonances at E_p = 1318 and 1416 keV. The $^{23}Na(p,\gamma\gamma)$ resonances were studied using the apparatus and geometry employed in the $^{11}B(p,\gamma)$ work. The measured detection efficiency, shown in Fig. 7, was obtained from the relative ^{56}Co γ-ray intensities used in Ref. 16 and from the relative γ-ray intensities at the $^{23}Na(p,\gamma\gamma)$ resonances reported in Ref. 17. In addition our $^{11}B(p,\gamma\gamma)$ data were used to give the relative detection efficiency at 4.44 and 11.67 MeV since the number of 4.44 MeV γ rays in our spectrum is essentially equal to the number of 11.67 MeV γ rays. From our $^{11}B(p,\gamma)$ data we find $\Gamma_\gamma(16.1\to 0.0)/\Gamma_\gamma(16.1\to 4.4) = (4.6\pm 0.7)\%$ $\Gamma_\gamma(16.1\to 9.6)/\Gamma_\gamma(16.1\to 4.4) = (2.4\pm 0.4)\%$, and $\Gamma_\gamma(16.1\to 12.7)/\Gamma_\gamma(16.1\to 4.4) = (1.46\pm 0.25)\%$. A 16% correction for summing has been applied to the ground state branching ratio. Our measurement is based on spectra taken at 45°. Since the γ ray decays are not isotropic, corrections to our results for angular correlation effects were necessary. These were made by assuming pure dipole decays for the 16.1 → 12.7, 16.1 → 9.6, and 16.1 → 4.4 transitions. Let us consider two cases in detail. For an isolated p-wave $^{11}B+p$ resonance the γ-ray angular distribution has the form $W(\theta) = A_0 + A_2 P_2(\cos\theta)$. If we assume pure dipole decays,

$$\frac{W(\theta)_{16.1\to 12.7}}{W(\theta)_{16.1\to 4.4}} = \frac{1-0.35[(1-x)/(1+x)]P_2(\cos\theta)}{1+0.35[(1-x)/(1+x)]P_2(\cos\theta)},$$

where x is the channel-spin-2 to channel-spin-1 intensity ratio. Thomson *et al.* found that $x = 0.42 \pm 0.02$,[18] which indicates that a 7% correction should be applied to our relative branching ratios. After correction for the finite-geometry attenua-

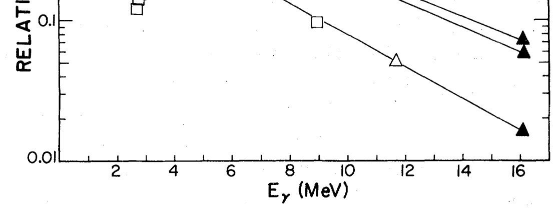

FIG. 7. Measured efficiency of the Ge(Li) detector. The procedure used to obtain these points is described in the text. The $^{11}B(p,\gamma)$ points at $E_\gamma = 16.1$ MeV were only used to obtain the ratios of the single escape and double escape peaks to the full energy peaks.

TABLE I. Decay properties of the 12.7 and 16.1 MeV states of ^{12}C.

^{12}C(12.7)	^{12}C(16.1)
$\Gamma_{\gamma_0}/\Gamma=(1.93\pm0.12)\times10^{-2}$ [a]	$\Gamma=6.7\pm0.5$ keV [e]
$\Gamma_{\gamma_1}/\Gamma_{\gamma_0}=0.150\pm0.018$ [a]	$\Gamma_{\gamma_1}/\Gamma=(2.42\pm0.29)\times10^{-3}$ [a]
$\Gamma_\alpha=(17.7\pm2.8)$ eV [b]	$\frac{\Gamma_\gamma(16.1\to0.0)}{\Gamma_\gamma(16.1\to4.4)}=(4.6\pm0.7)\%$ [a]
$\Gamma_{\gamma_0}=0.35\pm0.05$ eV [c] $=0.008$ W.u.	$\frac{\Gamma_\gamma(16.1\to9.6)}{\Gamma_\gamma(16.1\to4.4)}=(2.4\pm0.4)\%$ [a]
$\Gamma_{\gamma_1}=0.053\pm0.010$ eV [d] $=0.005$ W.u.	$\frac{\Gamma_\gamma(16.1\to12.7)}{\Gamma_\gamma(16.1\to4.4)}=(1.46\pm0.25)\%$ [a]
	$\Gamma_\gamma(16.1\to0.0)=(0.75\pm0.16)$ eV $=0.53$ W.u.
	$\Gamma_\gamma(16.1\to4.4)=(16.2\pm2.3)$ eV $=0.49$ W.u.
	$\Gamma_\gamma(16.1\to9.6)=(0.39\pm0.09)$ eV $=4.0\times10^{-3}$ W.u.
	$\Gamma_\gamma(16.1\to12.7)=(0.24\pm0.05)$ eV $=0.29$ W.u.
	$\Gamma_p=(27.2\pm4.2)$ eV [f]

[a] This work.
[b] Assuming $\Gamma_\alpha+\Gamma_{\gamma_0}+\Gamma_{\gamma_1}=\Gamma$.
[c] See Ref. 6.
[d] Combining Ref. 6 and present work.
[e] See Ref. 21.
[f] See Refs. 21, 22 and present work.

tion of the correlation we find a 5% correction. The assumption of an isolated (p,γ) resonance is not valid for the 16.1 → 0.0 transition since this transition displays interference with a broad 1^- level. In this case we obtained the total cross section using the γ_0 angular distribution measured in Ref. 19.

C. Results

Our results for the decay properties of the 12.7 and 16.1 MeV states of ^{12}C, along with other quantities deduced by combining our values with previously available data, are summarized in Table I. Our value for the γ ray branching ratio of the 12.7 MeV level, $\Gamma_{\gamma_0}/\Gamma=(1.93\pm0.12)\%$, is in mild disagreement with an earlier less precise measurement by Reisman, Connors, and Marion[3] of $\Gamma_{\gamma_0}/\Gamma=(2.4\pm0.3)\%$. Our measurement of the relative strength of the γ_1 and γ_0 transitions from ^{12}C(12.7) is $\Gamma_{\gamma_1}/\Gamma_{\gamma_0}=0.150\pm0.018$. Due to background effects it would not have been possible for us to have seen transitions to levels in ^{12}C at and above 7.65 MeV. If we assume that these unobserved transitions have negligible strength we obtain relative γ-ray branches of $(13.0\pm1.4)\%$ and $(87.0\pm1.4)\%$ for transitions to ^{12}C(4.4) and ^{12}C(0.0), respectively. These results are in good agreement with values of $(15\pm4)\%$ and $(85\pm4)\%$ found by Alburger and Wilkinson,[14] and in acceptable agreement with the results of Ref. 3, which gave $(17\pm3)\%$ and $(83\pm3)\%$, respectively. The α branching of ^{12}C(12.7) inferred from our results, $\Gamma_\alpha/\Gamma=(97.8\pm0.1)\%$, may be combined with our γ-ray branching ratio and a recent measurement[6] of $\Gamma_{\gamma_0}=(0.35\pm0.05)$ eV to give $\Gamma=(18.1\pm2.8)$ eV, $\Gamma_{\gamma_1}=(0.053\pm0.010)$ eV, and $\Gamma_\alpha=(17.7\pm2.8)$ eV.

Our result for $\Gamma_\gamma(16.1\to0.0)/\Gamma_\gamma(16.1\to4.4)$ is consistent with previous measurements of $(3.7\pm0.3)\%$,[20] $(3.3\pm1)\%$,[19] and ~4%.[21,22] However, a previous measurement of $\Gamma_\gamma(16.1\to9.6)/\Gamma_\gamma(16.1\to4.4)=(1.0\pm0.3)\%$[18] is inconsistent with our results. Anderson *et al.*[23] recently attempted to measure branching ratios using coincidence techniques but were unsuccessful. However, from a study of the ^{11}B(p,γ) and ^{11}B(p,α) resonances they concluded that $\Gamma_{\gamma_0+\gamma_1}=(21.6\pm3.3)$ eV and $\Gamma_p=(21.7\pm1.8)$ eV, approximately 3 times larger and smaller, respectively, than previously accepted values. We obtain partial widths of $\Gamma_p=(27.2\pm4.2)$ eV and $\Gamma_{\gamma_0+\gamma_1}=(17.0\pm2.4)$ eV, based on our branching ratio and previous values for Γ[21] and $\Gamma_p\Gamma_\gamma/\Gamma$.[22,23] Our measurement of $\Gamma_{\gamma_0}(16.1)=(0.75\pm0.13)$ eV is in good agreement with an electron scattering measurement of $\Gamma_{\gamma_0}=0.83\pm0.06$ eV.[24]

The results of this work and of Ref. 23 support the suggestion by Monahan *et al.*[25] that, based on a comparison of nucleon widths of analog states of

^{12}B and ^{12}C, the previously accepted proton width for ^{12}C(16.1) must be too large by as much as a factor of 5. Using the analysis of Monahan *et al.*[25] and our values for Γ_p we obtain $\gamma_n^2/2\gamma_p^2=0.46$, which is to be compared with the expected value of unity and values of 0.68–0.99 observed[25] for other $T=1$ pairs in mass 12. This residual discrepancy almost certainly does not reflect a significant isospin impurity in the 16.1 MeV level. Instead, it probably is an artifact of the analysis used for extracting reduced widths, which was estimated in Ref. 25 to have an accuracy of ~30%.

The γ-ray widths of the 12.7 and 16.1 MeV levels are compared with the Weisskopf estimates in Table I. The $16.1\rightarrow 9.6$ $\Delta T=1$ $E1$ transition has $\Gamma_\gamma=(0.39\pm0.09)$ eV. The $16.1\rightarrow 0.0$ $\Delta T=1$ $E2$ transition is one of the few known examples of isovector $E2$'s, with a strength of 0.53 ± 0.11 Weisskopf units (W.u.). As expected it is not enhanced (for comparison the isoscalar $E2$ $4.4\rightarrow 0.0$ transition has a strength of ~5 W.u.). The isovector $M1$ transitions have quite typical strengths.[26]

III. ANALYSIS OF THE ISOSPIN MIXING

A. Introduction

The 15.1 MeV 1^+, $T=1$ and 12.7 MeV 1^+, $T=0$ levels of ^{12}C are an interesting system for quantitative studies of isospin violation.[3] These analog and antianalog levels have a simple well-known structure and it is often proposed[2,3,6,8] that the isospin impurities in these levels can be approximated by simple two state mixing with

$$|15.1\rangle=\alpha|1\rangle+\beta|0\rangle\ ,$$

$$|12.7\rangle=-\beta|1\rangle+\alpha|0\rangle\ ,$$

where

$$\beta/\alpha=\langle 1|H_{CD}|0\rangle/2.40\text{ MeV and }\alpha^2=1-\beta^2.$$

However, the difficulties with this system result from the 2.40 MeV energy splitting which causes the mixing amplitude β to be quite small. It is correspondingly difficult to measure the mixing reliably, and the validity of the two state mixing approximation and other assumptions used in the analysis must be carefully examined. In this section we examine critically those experiments which have been used to estimate β. Our idea is to determine what kinds of experiments provide the most reliable measure of the isospin mixing coefficients β. We then establish experimentally the general correctness of the charge independent wave functions[27] of Cohen and Kurath (CK). We use these wave functions to estimate the validity of the two-state mixing assumption as applied to various probes of the isospin mixing. Finally, we show that the data which are expected to give the most reliable measure of β do, in fact, demonstrate appreciable isospin mixing between the 15.1 and 12.7 MeV levels but that the magnitude is much smaller than reported previously.[2]

B. Measurements based on particle reactions

Most published estimates of the mixing coefficient β were deduced from particle reactions. Two classes of reactions have been employed: isospin forbidden probes which are proportional to the intensity of the isospin admixtures, and consequently display very small effects (~1%), and isospin allowed probes which detect the mixing via interference of $T=0$ and $T=1$ amplitudes and thus display correspondingly larger effects. Estimates of the mixing obtained from isospin forbidden reactions such as ^{12}C$(d,d')^{12}$C(15.1),[2,28] ^{14}N$(d,\alpha)^{12}$C(15.1),[8] and ^{10}B$(\alpha,d)^{12}$C(15.1)[9] are made under the assumption that the small cross sections observed in these experiments are due entirely to a direct population of the small $T=0$ component of the 15.1 MeV state. Isospin violation in the reaction mechanism and in the compound nucleus is ignored. This is probably not justified when one is dealing with 1% effects. A possible mechanism for isospin violation in a two-step reaction process has been discussed by Iachello and Singh.[29] A two-step calculation of the isospin violating reaction ^{14}N$(d,d')^{14}$N(2.31) is in good agreement with the experimental results.[30]

Because of the ambiguities involved in the interpretation of isospin forbidden particle reactions we do not consider these probes to be well suited for determining β^2. Nevertheless, we summarize such results here. BBCG have deduced $|\beta|=0.106\pm0.013$ from a study of the ^{12}C(d,d') reaction at bombarding energies of 27.2 and 28.0 MeV.[2] A more extensive study of ^{12}C(d,d') at $E_d=26.2$ to 28.8 MeV using a quadrupole-dipole-dipole-dipole magnetic spectrometer yields $|\beta|\leq 0.14\pm0.02$,[28] where the upper limit results from evidence that precludes attributing all of the observed isospin violation to final state mixing. A ^{14}N(d,α) experiment performed at $E_d=40$ MeV with relatively poor energy resolution yielded $\sigma_{15.1}/\sigma_{12.7}<0.003$ which corresponds to $|\beta|<0.05$.[8] On the other hand, a study[9] of the ^{10}B(α,d) reactions via the yield of 12.7 and 15.1 MeV γ rays gives $|\beta|=0.109$, which the authors of Ref. 9 consider an upper limit because of possible isospin violation in the compound nucleus or reaction mechanism. It should be noted that even if a small cross section for populating ^{12}C(15.1) is found in such a reaction, this need not mean that the 12.7-15.1 mixing is small. If other $T=0$ admixtures are present in the 15.1

state and contribute to the direct reaction amplitude, the coherence of these different contributions may result in a small apparent β.

Estimates of β have also been made by comparing the isospin forbidden α decay of ^{12}C(15.1) with the isospin allowed α decay of ^{12}C(12.7).[3,7] If the α width of ^{12}C(15.1) is dominated by an admixture of ^{12}C(12.7), then β is given by

$$\frac{\beta^2}{1-\beta^2}=\frac{\gamma_\alpha{}^2(15.1)}{\gamma_\alpha{}^2(12.7)}=\frac{P(12.7)\Gamma_\alpha(15.1)}{P(15.1)\Gamma_\alpha(12.7)} ,$$

where the P's are Coulomb penetration factors. Balamuth, Zurmuhle, and Tabor[7] made a careful remeasurement of Γ_α/Γ for ^{12}C(15.1) and obtain $|\beta|=0.143\pm0.024$ using the two-state mixing approximation. We revise this estimate using experimental values obtained above. We compute

$$\Gamma_\alpha(15.1)=\frac{\Gamma_\alpha}{\Gamma}\,\Gamma_{\gamma_0}\left\{\left(\frac{\Gamma_{\gamma_0}}{\Gamma_{\gamma\mathrm{TOT}}}\right)\left[1-\left(\frac{\Gamma_\alpha}{\Gamma}\right)\right]\right\}^{-1}$$

$$=1.72\pm0.38 \text{ eV} ,$$

based on $\Gamma_{\gamma_0}=(37.0\pm1.1)$ eV (Ref. 31) γ-ray and α-particle branching rates from Refs. 14 and 7, respectively. Similarly we find

$$\Gamma_\alpha(12.7)=\Gamma_{\gamma_0}\left[\left(\frac{\Gamma_{\gamma_0}}{\Gamma}\right)^{-1}-\left(\frac{\Gamma_{\gamma_1}}{\Gamma_{\gamma_0}}\right)-1\right]$$

$$=(17.7\pm2.8) \text{ eV}$$

based on our γ-ray branching ratios and the value of Γ_{γ_0} from Ref. 6. Balamuth *et al.*[7] calculate that $P(12.7)/P(15.1)=1/6$ with an uncertainty of 25%. Combining these results we find $|\beta|=0.126\pm0.023$ or $H_{CD}=\beta(1-\beta^2)^{-1/2}\Delta E=(300\pm55)$ keV.

As pointed out in Ref. 7, the weak point in this approach is the assumption of two-state mixing. Although the α decay of ^{12}C(12.7) is isospin allowed it is dynamically forbidden to the extent that the ^{8}Be 2^+ residual state has the [4] configuration in LS coupling. Consequently, $\Gamma_\alpha(12.7)$ is only 18 eV and it is not clear that $\Gamma_\alpha(15.1)$ is dominated by an admixture of ^{12}C(12.7). On the other hand, these arguments inhibit the α decay to ^{8}Be(2^+) of any $1p$ shell $J^\pi=1^+$ level of ^{12}C. In fact, the $1p$ shell calculations of CK predict that the 12.7 MeV level of ^{12}C has a larger value of $\gamma_\alpha{}^2$ than any other 1^+ level.[27] However, the inclusion of $2s1d$ shell configurations is expected to greatly increase the $\gamma_\alpha{}^2$ values of higher lying levels of ^{12}C, so we do not expect the α decay of ^{12}C(15.1) to give a good "handle" on the mixing parameter β. We conclude that none of the isospin forbidden reaction experiments can be expected to give a reliable quantitative measure of β.

C. Measurements based on electromagnetic transitions

Cecil *et al.*[6] have argued that the $M1$ decays of ^{12}C(12.7) and ^{12}C(15.1) provide a crisp, albeit model dependent, measure of the mixing parameter β. This is possible because the isovector $M1$ transitions are intrinsically 100 times faster than isoscalar transitions. Thus a small $T=1$ admixture into ^{12}C(12.7) will have a big effect on the $M1$ decay rate of that level. Cecil *et al.*[6] obtained $\Gamma_{\gamma_0}(12.7)=0.35\pm0.05$ eV from an inelastic electron scattering measurement. They inferred $|\beta|=0.19\pm0.01$ or 0.05 ± 0.01 using the relation

$$\Gamma_{\gamma_0}(12.7)=\frac{2.76}{3}\left(\frac{E_\gamma}{10}\right)^3|\alpha M_0+\beta M_1|^2 \text{ eV} ,$$

where $\alpha^2=1-\beta^2$. The isoscalar matrix element M_0 was taken from the CK shell model calculation[27] while the isovector matrix element M_1 was obtained from the known radiative width of ^{12}C(15.1). Although the analysis in Ref. 6 is unnecessarily crude, the basic idea is excellent since the isospin mixing leads to large effects in the $M1$ decay of ^{12}C. Furthermore, one is dealing with electromagnetic rather than strong transitions so there is very little ambiguity in extracting matrix elements from the experimental data.

In this section we refine the analysis of Ref. 6. We consider the γ_0 and γ_1 decays of ^{12}C(12.7) and investigate the following questions:

(1) How accurate are the $M1$ observables given by the charge independent CK calculation?

(2) Is the assumption of two-level mixing with ^{12}C(15.1) valid for the γ_0 and γ_1 decays?

(3) Are both of the solutions for β obtained in Ref. 6 valid?

(4) What is the mixing parameter β?

It is clear that our accuracy in determining β is limited by our confidence in the isoscalar matrix elements involved in the $M1$ decay of ^{12}C(12.71). While these cannot be determined directly from experiment we can test the general validity of the CK wave functions. We do this in Table II where we tabulate the experimental values of 11 electromagnetic observables in the $A=12$ nuclei and the corresponding theoretical values from the 8-16 POT calculation of CK,[27] labeled theory A. Included are two magnetic moments, eight $M1$ transitions, and one $\Delta T=1$ $E2$ transition. The magnetic moments can be brought into good agreement with experiment by a very small change in the wave function of the lowest 1^+ $T=1$ level (we label this as theory B): $|15.1\rangle=|11_1\rangle+0.04|11_2\rangle-0.07|11_3\rangle$, where the CK states are labeled by $|JT_i\rangle$. It can be seen that this change does not materially affect any other transitions (see theory B in Table II) and hence we will base the rest of our discussion on the original 8-16 POT wave functions (theory A). The CK calculation is in very good agreement with experiment for seven out of nine γ-ray transitions with a maximum discrepancy between theory and experiment of 17%. On the other hand the "isoscalar" $M1$ transitions from the 12.7 MeV level are faster than expected by a factor of ~3. Since there is no reason why the CK isoscalar transition speeds should be less accurate than the isovector speeds (we discuss this point in some detail below) we follow Ref. 6 and assume that the anomalously fast isoscalar transitions contain isovector components. For $\Delta T=1$ $M1$ transitions isovector matrix elements are intrinsically faster than isoscalar ones by a factor

$$\approx \frac{\mu_p-\mu_n+\frac{1}{2}}{\mu_p+\mu_n-\frac{1}{2}}=13.7.$$

Therefore a small isovector component has a big effect in the $M1$ rate. Isovector components can arise from $T=1$ impurities in either the initial or the final state. In the CK framework the isospin mixed $M1$ matrix elements are

$$\begin{aligned}\langle 0.0|M1|12.7\rangle &= \langle 00_1|M1|10_1\rangle \\ &+\sum_i \frac{\langle 11_i|H_{CD}|10_1\rangle}{E_1^{10}-E_2^{11}}\langle 00_1|M1|11_i\rangle \\ &+\sum_j \frac{\langle 00_1|H_{CD}|01_j\rangle}{E_1^{00}-E_j^{01}}\langle 01_j|M1|10_1\rangle ,\end{aligned}$$

$$\begin{aligned}\langle 4.4|M1|12.7\rangle &= \langle 20_1|M1|10_1\rangle \\ &+\sum_i \frac{\langle 11_i|H_{CD}|10_1\rangle}{E_1^{10}-E_i^{11}}\langle 20_1|M1|11_i\rangle \\ &+\sum_j \frac{\langle 20_1|H_{CD}|21_j\rangle}{E_1^{20}-E_j^{21}}\langle 21_j|M1|10_1\rangle .\end{aligned}$$

The $12.7\to 0.0$ and $12.7\to 4.4$ transitions are useful probes of the isospin mixing between the 12.7 and 15.1 MeV levels to the extent that the isovector impurities in these transitions are dominated by admixture of the 15.1 MeV level into the 12.7 MeV state. We evaluate the validity of the two-state mixing approximation by the following procedure. We take the energy denominator (when unknown experimentally) and the $M1$ matrix elements from CK. Since we do not know *a priori* the type of charge dependent interaction responsible for the mixing but instead want to *measure* the mixing without bias, we make the conservative assumption that all the matrix elements of H_{CD} are comparable to $\langle 11_1|H_{CD}|10_1\rangle$. We then can decide quantitatively if the two-level mixing approximation is valid.

As expected we find that the isovector impurity in the $12.7\to 0.0$ transition is dominated by the admixture of the 15.1 MeV level into the 12.7 MeV state. This occurs in part because of the small energy denominator, but more importantly because the 15.1 MeV transition nearly exhausts the isovector $M1$ strength from the ^{12}C ground state. Therefore this transition provides an excellent way to measure the 12.7-15.1 mixing. On the other hand, the $12.7\to 4.4$ transition does not provide a good measure of β because our assumptions about the possible size of $\langle H_{CD}\rangle$ imply that the isospin impurity in this transition has four significant components: admixtures into ^{12}C(12.7) of the two lowest 1^+ $T=1$ levels and admixtures into ^{12}C(4.4) of the two lowest 2^+ $T=1$ levels. The admixtures with large energy denominators are important because of their large $M1$ matrix elements. The unsuitability of the $12.7\to 4.4$ transition is due to the slowness of the $15.1\to 4.4$ rate.

We obtain $\beta=0.046\pm 0.012$ by fitting the observed ratio $\Gamma_\gamma(12.7\to 0.0)/\Gamma_\gamma(15.1\to 0.0)$ to that calculated from the CK matrix elements (see Fig. 8). Our error includes a 20% uncertainty in the theoretical isoscalar rate. This is reasonable in light of the agreement between theory and experiment displayed in Table II. The second solution to the quadratic equation, with negative β, is not consistent with transfer reaction data or with expectations based on general considerations (see below). We believe that our procedure of using the CK matrix elements for both the $1^+, T=0\to 0^+$, $T=0$ and 1^+, $T=1\to 0^+$, $T=0$ transitions is superior to that of

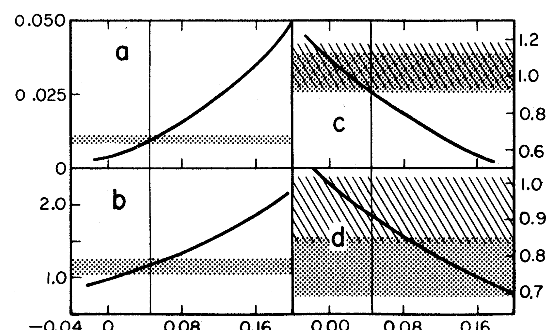

FIG. 8. Experimental (shaded area) versus calculated (solid line) quantities as a function of β. (a) $\Gamma_{\gamma_0}(12.7)/\Gamma_{\gamma_0}(15.1)$. (b) $C^2S(12.7)/C^2S(15.1)$ for proton stripping on ^{11}B. (c) $C^2S(12.7)/C^2S(15.1)$ for neutron pickup on ^{13}C. Cross-hatched area is an average of results from ^{13}C(p,d) experiments of Refs. 34 and 35. Dotted area is from the ^{13}C(d,t) experiment of Ref. 2. (d) $\{C^2S[^{12}\text{B}(0.0)]/C^2S[^{12}\text{B}(0.95)]\}/[C^2S(15.1)/C^2S(16.1)]$. Cross-hatched area based on neutron pickup on ^{13}C from Refs. 34 and 35 and proton pickup on ^{13}C from Ref. 41. Dotted area based on neutron and proton pickup data from Ref. 2. We arbitrarily assume that relative spectroscopic factors have ±10% errors.

TABLE II. Comparison of $A=12$ electromagnetic observables with theory.

Transition	Expt. Γ_γ (eV)	Theory A[a] Γ_γ (eV)	Theory B[a] Γ_γ (eV)
^{12}B(0.95 → 0.00)	$(2.19\pm0.24)\times10^{-3}$ [b]	1.71 ± 10^{-3}	2.32 ± 10^{-3}
^{12}C(16.11 → 0.00)	0.75 ± 0.16 [c]	0.607	0.607
^{12}C(16.11 → 4.44)	16.2 ± 2.3 [c]	11.8	11.8
^{12}C(16.11 → 12.71)	0.24 ± 0.05 [c]	0.215	0.215
^{12}C(15.1 → 0.0)	37.0 ± 1.1 [d]	30.8	30.3
^{12}C(15.1 → 4.4)	0.92 ± 0.36 [d,e]	1.32	1.01
^{12}C(15.1 → 12.7)	0.56 ± 0.16 [d,e]	0.47	0.48
^{12}C(12.71 → 0.0)	0.35 ± 0.05 [f]	0.11	0.11
^{12}C(12.71 → 4.4)	0.053 ± 0.010 [f,c]	0.015	0.015
State	**Expt.[g] μ (μ_N)**	**Theory A[a] μ (μ_N)**	**Theory B[a] μ (μ_N)**
^{12}B(0.0)	$+1.003\pm0.001$	+0.762	+0.948
^{12}N(0.0)	$+0.4571\pm0.0005$	+0.611	+0.418

[a]See Ref. 27.
[b]J. W. Olness and E. K. Warburton, Phys. Rev. 166, 1004 (1968).
[c]This work.
[d]See Ref. 31.
[e]See Ref. 14.
[f]See Ref. 6.
[g]K. Sugimoto *et al.*, J. Phys. Soc. Jpn. 25, 1258 (1968).

Cecil *et al.* who obtained their value of β from the CK value for 1^+, $T=0\to0^+$, $T=0$ transition and the experimental value for the $15.1\to0.0$ transition. One expects calculations to give the relative $B(M1)$'s for two similar states more accurately than the absolute $B(M1)$'s.

We must consider to what extent our value of β is dependent on the precise details of the nuclear shell model calculation. If the predicted isoscalar speeds are as accurate as the predicted isovector speeds the "slop" in the model has already been accounted for in our assigned 20% error in the theoretical isoscalar speed. Is there any reason why the calculated isoscalar speeds should be less valid than the isovector speeds? We think not. Unfortunately it is not possible to demonstrate this point by a direct comparison between the CK rates of "pure" isoscalar transitions and experimental values because no other $T=0$ $M1$ transitions are known in ^{12}C. The CK prediction for the isoscalar magnetic moment of the lowest 1^+, $T=1$ state $\mu_+=\mu(^{12}\mathrm{B})+\mu(^{12}\mathrm{N})=1.37\mu_N$ is in good agreement with the experimental value $\mu_+=1.460\mu_N$. In fact, except for other tests involving magnetic moments, there are no $\Delta T=0$ $M1$ observables in the $1p$ shell which are known with sufficient precision to adequately test the CK isoscalar predictions. A few $\Delta T=0$ $M1$ transitions are known in ^{10}B but these are very sensitive[32] to small changes in the CK wave functions and hence are not suitable for comparison since such extreme sensitivity does not occur in ^{12}C.

Therefore we must argue theoretically. As mentioned earlier the 12.7 and 15.1 MeV states have very similar space structures. Both the 15.1 → 0.0 and 12.7 → 0.0 transitions are strong. The big difference in the $B(M1)$'s for the two transitions is due almost entirely to the difference in the nucleon isoscalar and isovector magnetic moments. In fact the 0.0 → 12.7 transition exhausts 94% of the sum rule for $\Delta T=0$ $M1$ excitations of the ^{12}C ground state. This sum rule

$$\sum_i B_{0\to i}(M1)=\frac{3}{4\pi}(\mu_p+\mu_n-\tfrac{1}{2})^2\langle0^+0|S^2|0^+0\rangle$$

given by Warburton and Weneser[33] can easily be evaluated using CK wave functions expressed in an LS basis (kindly supplied by Kurath[27]).

Therefore we need not be concerned that small changes in the wave function of ^{12}C(12.7) will significantly affect the isoscalar 12.7 → 0.0 rate. We must, however, consider the effect of small changes in the ground state wave function on our estimate of β. Kurath has pointed out that small admixtures of the 00_2 state into the 00_1 ground state significantly affect the rate of $M1$ transitions to that state because they alter the amount of the [431] space symmetry in the ground state (the isoscalar $M1$ transition operator $M_0'=(\frac{1}{2}-\mu_n-\mu_p)L_z$ and hence cannot change the space partition). However, such changes hardly change the relative speed of the $12.7 \to 0.0$ and $15.1 \to 0.0$ transitions. For example, if we arbitrarily mix enough (amplitude = 0.054) of the 00_2 state into the 00_1 state to bring the predicted $15.1 \to 0.0$ rate into agreement with experiment we find that the predicted ratio $\Gamma(12.7 \to 0.0)/\Gamma(15.1 \to 0.0)$ is changed by only 2%. Finally, we need not be concerned with possible collective effects neglected by CK since for $M1$ transitions the strength is all contained within the $1p$ shell itself.

We have shown that the predicted relative speeds of the $12.7 \to 0.0$ and $15.1 \to 0.0$ transitions are very stable against small changes in the CK wave functions. We conclude that the matrix element $H_{CD} = (110 \pm 30)$ keV extracted from the $M1$ decays is reliable and that the assigned error reasonably reflects the theoretical as well as the experimental uncertainties.

It is interesting to ask whether it is possible to reconcile the $12.7 \to 4.4$ rate with CK matrix elements and reasonable isospin mixing. The answer is yes, provided the other three important admixtures (see above) affect the decay rate constructively, and with an amplitude corresponding to $\langle H_{CD}\rangle \sim 70$ keV. A detailed theoretical calculation is required to prove that this is indeed the case.

D. Measurements based on single nucleon transfer reactions

In this section we examine single nucleon transfer data involving the 12.7 and 15.1 MeV levels of ^{12}C and ask if it is consistent with the value of β deduced from the $M1$ transition. We do not expect the β inferred from stripping and pickup to be as reliable as that obtained from electromagnetic transitions simply because our theories of particle reactions are not exact. Nevertheless, we assert that single nucleon transfer reactions are the least ambiguous of the particle probes of the isospin mixing because the deviations from the charge independent predictions are proportional to β rather than β^2, and hence the experimental effects are much larger for transfer reactions than for isospin forbidden reactions such as $^{12}\mathrm{C}(d,d')^{12}\mathrm{C}(15.1)$. Furthermore, we can use the transfer data to establish the sign of β which selects the correct solution of the quadratic relation involving the $M1$ strengths. We do not consider two nucleon transfer reactions because of the greater uncertainty associated with the reaction theory.

We shall interpret the transfer data using the single nucleon transfer CFP's from the CK calculation.[27] The two-state mixing approximation is valid for transfer reactions involving the 12.7 and 15.1 MeV levels because the CK stripping and pickup amplitudes are much larger for the 12.7 and 15.1 MeV states than for any other $J=1^+$ levels. We therefore find for single nucleon stripping and pickup that the relative strengths for populating the 12.7 and 15.1 MeV levels are

$$R=\frac{C^2S(12.7)}{C^2S(15.1)} = \frac{(\alpha A_0{}^{3/2}-\beta A_1{}^{3/2})^2+(\alpha A_0{}^{1/2}-\beta A_1{}^{1/2})^2}{(\alpha A_1{}^{3/2}+\beta A_1{}^{3/2})^2+(\alpha A_1{}^{1/2}+\beta A_0{}^{1/2})^2}\,,$$

where A_0 and A_1 are the CK amplitudes for $p_{3/2}$ and $p_{1/2}$ transfer to the lowest $T=0$ and $T=1$ levels, respectively. (We employ the conventional notation for transfer strength, C^2S, even though it is not completely appropriate for transitions to isospin mixed levels.) Since the 15.1 and 12.7 MeV states are an analog-antianalog pair with virtually the same structure, A_0 and A_1 have very nearly the same magnitude. The isospin mixing arising from Coulomb forces will cause the lower lying member of the doublet to have an enhanced parentage to $^{11}\mathrm{B}+p$ compared with $^{11}\mathrm{C}+n$, while the upper member of the doublet is preferentially $^{11}\mathrm{C}+n$. This phenomenon is well known from the celebrated case of the 16 Mev 2^+ states in ^{8}Be.[1] It is easily understood as follows. Consider the $T=0, T=1$ doublet to be a particle-hole excitation of the ground state. A proton excitation has a lower energy than the corresponding neutron excitation because of the Coulomb pairing energy. When an interaction mixes two levels they repel each other. Since the lower level drops in energy it must acquire an excess of proton excitation and hence be stronger in proton stripping. Conversely, the higher level must become stronger in neutron pickup. However, the summed strength to the two isospin mixed levels will be unaffected by the mixing.

In Table III we list relative experimental spectroscopic factors for (p,d) and (d,t) neutron pickup reactions[34,35,2] on ^{13}C as well as the (^{3}He, d) proton stripping reaction[36] on ^{11}B. Also shown are the charge independent predictions on CK.[27] Relative experimental spectroscopic factors have been normalized so that $S(12.7)+S(15.1)+S(16.1)$ is equal to the value given by CK. Note that this sum is unaffected by mixing between ^{12}C(12.7) and ^{12}C(15.1).

We have chosen experimental spectroscopic factors from the highest energy studies previously reported with good resolution and multipoint angular distributions. We also required that the distorted-wave Born-approximation (DWBA) fits to the angular distributions be in acceptable agreement with the data. The only suitable proton stripping data were those of the $^{11}\mathrm{B}(^3\mathrm{He},d)$ study

TABLE III. Comparison of $A = 12$ spectroscopic factors with theory.

State	$S^{\text{strip.}}_{\text{theo.}}$ a	$S^{\text{strip.}}_{\text{expt.}(^3\text{He},d)}$ b,c	$S^{\text{pickup}}_{\text{theo.}}$ a	$S^{\text{pickup}}_{\text{expt.}(p,d)}$ d,c	$S^{\text{pickup}}_{\text{expt.}(d,t)}$ e,c
^{12}C(0.0)	5.70	6.09	0.61	0.70	
^{12}C(4.44)	1.10	1.41	1.12	0.99	
^{12}C(12.71)	0.79	0.86	0.66	0.61	0.67
^{12}C(15.11)	0.83	0.76	1.81	1.71	1.98
^{12}C(16.11)	0.56	0.56	3.03	3.18	2.85

[a]See Ref. 27.
[b]See Ref. 36.
[c]Normalization: $[S(12.7)+S(15.1)+S(16.1)]_{\text{expt.}} = [S(12.7)+S(15.1)+S(16.1)]_{\text{theo.}}$.
[d]Average of Refs. 34 and 35.
[e]See Ref. 2.

by Miller *et al.*[36] taken at $E_{^3\text{He}} = 18$ MeV. Spectroscopic factors for ^{12}C(0.0) and ^{12}C(4.4) were not available at 18 MeV and were taken from 12 MeV data of Ref. 36. We adopt spectroscopic factors from the finite range DWBA analysis of Ref. 36.

Several other (d,n) and $(^3\text{He},d)$ stripping studies exist in the literature, but they generally violate one or more of the above criteria for acceptability. For example, studies[23,37] of (d,n) for $E_d = 6$ MeV fall in a region where previous work[38] has demonstrated resonances in the yield of the neutron groups $n_0 - n_3$. (The lower energy neutron channels should be even more susceptible to resonance effects.) Other[39] (d,n) work at $E_d = 11.6$ MeV probably suffers similar problems; in any case, the DWBA fit to the angular distribution for the group populating the 15.1 MeV state fails to reproduce the shape of the forward angle maximum. The paucity of data in the region of the most important forward angle maxima in the $(^3\text{He},d)$ angular distributions reported in Ref. 40 causes us to reject this work. Thus, although an indiscriminate literature search[28] can turn up anomalous results for $S(12.7)/S(15.1)$, reasonable and objective criteria for acceptability eliminate these problem cases.

Three separate pickup studies are suitable for our purpose: ^{13}C(p,d) investigations by Taketani *et al.*[34] at $E_p = 55$ MeV and Scott *et al.*[35] at $E_p = 50$ MeV as well as the ^{13}C(d,t) work of BBCG[2] which was done at $E_d = 28$ MeV. The relative spectroscopic factors for the 12.7, 15.1, and 16.1 MeV levels obtained in Refs. 34 and 35 agree to within 5%. The (p,d) spectroscopic factors displayed in Table III are the average of those in Refs. 34 and 35. The BBCG spectroscopic factors disagree somewhat with the (p,d) results. We must place more confidence in the (p,d) spectroscopic factors because two independent experiments give similar results and because they were obtained at higher bombarding energies where DWBA analysis should be more reliable.

We plot the CK values for the relative strengths R for populating the 12.7 and 15.1 MeV levels on stripping and pickup in Figs. 8(b) and 8(c). Calculations are presented as a function of β. We note from Figs. 8(b) and 8(c) that all experimental data[2,34-36] cited are consistent with the β deduced from the $M1$ decays and that all are inconsistent with the value $\beta \approx 0.11$ reported by BBCG. So we conclude that our analysis of the $M1$ decays is supported by the transfer studies and that previously reported[2] large values of β are not correct. One should ask whether we have somehow inadvertently biased ourselves in our data selection process to results which yield small β. An examination of rejected data shows this is not the case. For example, the $(^3\text{He},d)$ data of Ref. 40, perhaps the best of the rejected lot, would yield $\beta \approx 0.00$.

How can we account for the large β deduced by BBCG[2] from the ^{3}C$(d,t)^{12}$C(15.1) and ^{13}C$(d,^3\text{He})$-^{12}B(0.0) reactions? If one assumes that the pickup amplitude leading to ^{12}C(16.1) has a negligible $T = 0$ component one can extract β by comparing the relative spectroscopic factors for pickup to ^{12}C(15.1) and ^{12}C(16.1) with the corresponding spectroscopic factors for the analog transitions leading to ^{12}B. In Fig. 8(d) we display theoretical values, calculated from the CK matrix elements, for the ratio

$$R' = \frac{C^2S[^{12}\text{B}(0.0)]}{C^2S[^{12}\text{B}(0.95)]} \bigg/ \frac{C^2S[^{12}\text{C}(15.1)]}{C^2S(16.1)}$$

as a function of β. The BBCG data (shown as the dotted area) requires $\beta = 0.14 \pm 0.06$ and is inconsistent with all the other data discussed in this paper. Although we cannot account for the BBCG results we can compute the same rates R' using data obtained at bombarding energies ≥ 50 MeV, much higher than were available to BBCG. If we adopt ^{12}B spectroscopic factors from ^{13}C$(d,^3\text{He})$ data at $E_d = 52$ MeV (Ref. 41) and ^{12}C spectroscopic factors from ^{13}C(p,d) studies at 50 and 55 MeV (Refs. 34 and 35) we obtain $R' = 0.93$. This requires $\beta \approx 0.04$ in excellent agreement with our interpretation of the γ-ray results. Since all the transfer data cited are consistent with $H_{CD} = 110 \pm 30$ keV with the single exception of the BBCG value for the $(d,t)/(d,^3\text{He})$ ratio, we claim it is justified to discard the large value of β obtained by BBCG.

Recently a more detailed study of the ^{13}C$(d,t)^{12}$C and ^{13}C$(d,^3\text{He})^{12}$B reactions has been made by Lind, Garvey, and Tribble (LGT).[28] Using the same technique as BBCG, LGT deduce $\beta = 0.074 \pm 0.031$

and the corresponding $\langle H_{CD}\rangle = 179 \pm 75$ keV from measurements between $E_d = 24.1$ and 27.5 MeV. LGT sought an explanation for the difference between BBCG's results obtained at $E_d = 28.0$ MeV (which yield[28] $\langle H_{CD}\rangle = 265 \pm 50$ keV) and their own, without success. Several problems are apparent in these new results, as discussed by LGT.[28] Some of the measured angular distributions show anomalously high forward angle points when compared with DWBA. Ratios of spectroscopic factors extracted from these data show several surprising energy dependences. Over the above energy range the 2^+ $T=1$ ratio $S[^{12}\text{B}(0.95)]/S[^{12}\text{C}(16.11)]$ changes by 10%. LGT place most confidence in results obtained by renormalizing relative spectroscopic factors for (d, t) compared with $(d, ^3\text{He})$ at each energy so as to remove this energy dependence. Since the cause of this energy dependence is unknown, it is not at all clear that such a renormalization is appropriate. Indeed, the "raw" results (without this renormalization) show a systematic decrease of 30% in the extracted $\langle H_{CD}\rangle$ in going from $E_d = 24.1$ to 27.5 MeV. This is what one might expect if the reaction becomes more direct at the higher energies, and suggests that measurements at even higher energies might yield smaller β. In addition, the ratio $S[^{12}\text{B}(0.95)]/S[^{12}\text{B}(0.0)]$ shows a variation of 20% over this energy range. That the ratio of spectroscopic factors extracted for the same reaction populating two strong states lying within 1 MeV of each other shows such a strong energy dependence must be taken as a serious failure of the reaction theory. If a similar anomalous variation were present in the $S[^{12}\text{B}(0.0)]/S[^{12}\text{C}(15.1)]$ ratio, the effect on β would be ~100%.

One must recognize the possibility that all the direct reaction studies cited in this paper may suffer problems similar to LGT (the problems may be more apparent in the case of LGT simply because their study has been more detailed). Thus it is important to look for consistency among different direct reaction studies, as we do above, in the expectation that different studies will not be affected in the same way by such problems.

It might be objected that LGT's analysis of the $(d, ^3\text{He})/(d, t)$ ratio, which did not require any theory for the nuclear structure, should be superior to our analysis which relies on the CK matrix elements. We must reply that LGT's and BBCG's analysis depends more sensitively on the reaction theory than does our analysis. In our analysis of the relative cross sections feeding ^{12}C(12.7) and ^{12}C(15.1) the DWBA need only account for a Q value difference. In LGT's and BBCG's analysis it must also properly compensate for two different reactions. In addition our ratios R are much more sensitive to β than is the BBCG ratio R'. Roughly speaking, the $R(\text{stripping})/R(\text{pickup}) \approx (1+4\beta)/(1-4\beta) \approx 1+8\beta$ while BBCG's ratio $R' \approx (1-2\beta)$. So a given error in the spectroscopic factors translates into a much larger error in the extracted value of β in LGT's and BBCG's case than in ours.

In light of the discussion above we must conclude that probes of isospin mixing based on particle reactions are subject to large errors in cases where the mixing is small. At present our understanding of nuclear reactions has not yet progressed to the point that 10% effects are understood. Trustworthy results can only be obtained if one has data from a variety of different reactions.

IV. CONCLUSIONS

We have established that the charge dependent matrix element connecting the 12.7 and 15.1 MeV levels of ^{12}C is 110 ± 30 keV. Although this value is less than half that reported previously[2] it is still much larger than expected from simple Coulomb effects. It is interesting that the charge dependent matrix element in ^{12}C is nearly equal to the 150 keV matrix element connecting the 16 MeV 2^+ states of ^{8}Be. Understanding the origin of the large matrix elements seen in ^{12}C and ^{8}Be must rank as one of the most important problems in the study of charge dependent effects in nuclei.

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