One- and two-step processes in single-nucleon pickup*

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The reaction ${}^{12}C(p,d){}^{11}C$ at high momentum transfers $(q \gtrsim 200 \text{ MeV}/c)$ has been examined with respect to single- and two-step processes. Our coupled-channel Born-approximation analysis of the reaction at 185 MeV shows that peculiarities observed in experimental angular distributions can be explained with the two-step process or the interference between the single-step and two-step processes. The analysis requires the presence of small 1d and $1f_{7/2}$ admixtures in the ${}^{12}C$ ground state along with the basic $1s^{4}1p^{8}$ configuration. The different energy dependence in the single-step and two-step processes is discussed and illustrated by data at 185 and 700 MeV.

[NUCLEAR REACTIONS ¹²C(p, d), E_p =185 MeV; calculated $\sigma(\theta)$; CCBA analysis.]

I. INTRODUCTION

The analysis and interpretation of pickup measurements such as (p, d) depend on a knowledge of the reaction mechanism. The direct single-step (SS) process is usually the prevailing one for pickup reactions at intermediate incident energies leading to the prominently excited states of the residual nucleus; these reactions are conventionally treated within the distorted-wave Born approximation. The SS process may be retarded if there is a lack of appropriate single-particle amplitudes in the ground-state wave function of the target nucleus, i.e., if the single-particle wave function has a small amplitude or is lacking in the momentum components that the reaction is sensitive to. One can still encounter sizable pickup cross sections for these cases due to two-step (TS) processes. The TS process¹ usually considered consists of a nucleon transfer followed or preceded by an inelastic excitation which can be treated in the coupled-channel Born-approximation (CCBA) calculation. One may anticipate that such a TS pickup reaction would manifest itself in the data. First, the TS process would give rise to differential cross sections whose angular distributions differ in shape from those of the SS process. Second, one expects a particular reaction dependence in the absolute cross section of the TS process, viz. a dependence upon projectile energy. In this paper we shall consider these two aspects of the TS process with respect to the (p,d) reaction on ¹²C. For this reaction, data exist at $E_{b} = 185$ (Ref. 2) and 700 MeV (Ref. 3) which lend themselves to demonstrate the role of the TS process in single nucleon reactions

carrying large momentum transfers.

In the (p, d) reaction on ¹²C one finds² that the low-lying $\frac{3}{2}$ and $\frac{1}{2}$ states in ¹¹C (see Fig. 1) are the predominantly excited ones. These excitations are naturally ascribed to 1*p* neutron removal from the basic 1s⁴1*p*⁸ configuration of the ¹²C ground state. Many of the other states in ¹¹C below E_x = 8.7 MeV are also populated although their J^{*} val-



FIG. 1. The level diagrams (Ref. 4) of ¹²C and ¹¹C; only the low-energy parts are shown leaving out the ¹¹C states which were not observed (Ref. 2) in ¹²C(p, d) ¹¹C at 185 MeV (the $J^{\pi} = \frac{7}{2}^{+}$, $\frac{5}{2}^{+}$ doublet was unresolved).

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ues prohibit access by neutron removal from the basic $1s^41p^8$ configuration. The excitation of these states requires either that the ¹²C ground state contains a small admixture of higher-lying shells, such as neutrons in the 1d, 1f, and 1g shell-model orbits, or that the (p, d) reaction is not of the SS type. In principle, most of the ¹¹C states under consideration could be reached by the TS process since the J^{π} values of these states can be furnished by coupling a $1p_{3/2}$ state to a monopole, quadrupole, or octupole excitation either in ¹²C or ¹¹C. An indicative estimate of the cross sections for the TS process ${}^{12}C \rightarrow {}^{12}C^* \rightarrow {}^{11}C^*$ may be obtained from the cross section for the inelastic scattering step $^{12}C(p,p')^{12}C^*$ and that of the succeeding pickup step ${}^{12}C^*(p',d){}^{11}C^*$; and similarly for the process ${}^{12}C \rightarrow {}^{11}C \rightarrow {}^{11}C^*$ (Fig. 2). It is plausible to assume that the (p, d) step involves the removal of a $1p_{3/2}$ neutron and that the intermediate state in the latter reaction is the ¹¹C ground state (the prominent $1p^{-1}$ state in ¹²C). A study of the cross sections for the partial reactions may then suggest which states are likely candidates for access by the TS process. This will be discussed later in the context of the reaction dependent effects. When both TS and SS processes feed a state they contribute coherently to the (p, d) cross section and the interference between the two will affect both the cross section magnitude and the shape of the angular distribution. One finds in the 185-MeV (p, d) data² several examples of angular distribution shapes which are anomalous to the direct SS pickup process; such examples are the excitation of the 4.33- $\left(\frac{5}{2}\right)$, 6.48- $(\frac{7}{2})$, and 6.91- $(\frac{5}{2})$ MeV states (Figs. 3-5). As will be seen, these anomalous shapes are caused by contributions from the TS process. The angular distributions obtained at 700 MeV will not be considered here since the TS signature is less obvious at higher energies and the CCBA analysis is harder to perform. It should be mentioned, however, that the 700 MeV data have been analyzed³ taking into account inelastic scattering in the entrance channel only; the TS process proved to be important in improving the relative intensity between different excitations (this is further discussed below).

II. ANGULAR DISTRIBUTIONS

The CCBA calculations were performed with the computer code CHUCK written by Kunz.⁵ The proton and deuteron optical potentials and the bound neutron potential of Ref. 2 were used which gave a "best" fit to the angular distribution of the reaction ${}^{12}C(p,d){}^{11}C(g.s.)$. The first derivative of the real part of the optical potential was used for inelastic scattering form factors. The deformation



FIG. 2. Reaction channels considered in the description of ${}^{12}C(p, d) {}^{11}C$. Nucleon transfer amplitudes are indicated by α and inelastic scattering amplitudes by β (the nuclear deformation parameter); $\alpha = (122.5)^{1/2} \cdot \sqrt{S}$ (apart from a phase factor) where S is the spectroscopic factor.

parameters β for the 2⁺ and 3⁻ states in ¹²C were chosen so that the (p, p') cross sections⁶ were reproduced; this gave $\beta_2 = 0.62$ and $\beta_3 = 0.44$ (ignoring signs here) which are close to the results from other analyses (see Ref. 7). For deuteron scattering from ¹¹C (or ¹¹B) no appropriate data are available to fix the β 's so reasonable estimates had to be relied on. For the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states the β 's were assumed to be proportional to the square root of the corresponding (p, p') cross section, ⁶ σ (¹¹B), so the adopted β_2 's for (d, d') in ¹¹C were

$$\beta_2(^{11}B) = \left[\frac{\sigma(^{11}B)}{\sigma(^{12}C)}\right]^{1/2} \beta_2(^{12}C).$$

To obtain β_3 for the $\frac{5}{2}^*$ state, the weak-coupling model was applied, giving

$$\beta_3 = \left(\frac{6}{28}\right)^{1/2} \beta_3(^{12}\text{C}).$$

The $\beta_2(^{12}C)$ and $\beta_3(^{12}C)$ values were taken as averages of the values given in Ref. 7 which gave β_2 =0.30, β_2 =0.22, and β_3 =0.16 for deuteron scattering from the $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{5}{2}$ states, respectively. The nucleon transfer amplitudes were fixed in a similar way; α'' was fixed to reproduce the measured cross section ${}^{12}C(p,d){}^{11}C(g.s.)$ and to determine the α' 's the full $1p_{3/2}$ amplitude was assumed $(D_0\sqrt{S} = 122.5 \cdot 2 = 245 \text{ where } D_0 \text{ is the zero-}$ range deuteron strength and S is the spectroscopic factor) and was distributed on the ¹¹C states according to the weak-coupling model. Values of α'' = 146 and α' = 108, 155, 91 were then obtained for the $\frac{3}{2}$, $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{5}{2}$ states, respectively. Once determined, all the amplitudes were kept fixed in the calculations except for the SS amplitude α and the phase between the $\alpha'\beta'$, $\alpha''\beta''$, and α channels.

Besides the uncertainties in the β 's for the (d, d')scattering, the calculations are affected by ambiguities in the deuteron optical potential. In order to provide a check on the latter uncertainty, distorted-wave Born-approximation calculations were performed for deuteron scattering from the 2⁺ and 3⁻ states in ¹²C using the above-mentioned potentials and the deuteron potential of Ref. 8 obtained from a fit to elastic scattering data (potential II in Table IV of Ref. 8 with volume absorption). Calculations using the latter potential gave cross sections which were a factor of 2 lower, while the angular distributions remained essentially unchanged in shape. It should also be mentioned that the (d, d') cross sections are a factor of 5 to 10 larger than those of (p, p'), rendering a greater importance to the ${}^{12}C + {}^{11}C^*$ portion of the TS process as compared with that of ${}^{12}C + {}^{12}C^* + {}^{11}C^*$ (cf. Fig. 3).

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The results of the CCBA calculations are shown in Figs. 3-5 together with experimental data. The three angular distributions considered are well described allowing for the couplings indicated in Fig. 2. The $\frac{5}{2}$ state thus appears to be predominantly excited by the TS process while both processes are obviously contributing to the excitation of the $\frac{7}{2}$ and $\frac{5}{2}$ * states (Figs. 4 and 5). Neither the







FIG. 3. Calculated differential cross sections for ${}^{12}C(p, d) {}^{11}C$ at 185 MeV populating the $\frac{5}{2}$ state at 4.33 MeV compared with the experimental data of Ref. 2. The effect of including different channels and changing the deuteron optical potential is shown.

FIG. 4. Calculated differential cross sections for ${}^{12}C(p, d) {}^{11}C$ at 185 MeV populating the $\frac{7}{2}$ state of 6.48 MeV compared with the experimental data of Ref. 2. The effect of changing the relative composition of SS and TS processes is shown.



FIG. 5. Calculated differential cross sections for ${}^{12}C(p, d) {}^{11}C$ at 185 MeV populating the $\frac{5}{2}^+$ state at 6.91 MeV compared with the experimental data of Ref. 2. The effect of changing the phase between $\alpha'\beta'$ and $\alpha''\beta''$ and the relative composition of SS and TS processes is shown.

angular distribution does not help determine the $\alpha'\beta'-\alpha''\beta''$ phase leading to the excitation of the $\frac{5}{2}^-$ state although this phase is crucial for the cross section magnitude. If, however, the $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states are described⁹ as a $1p_{3/2}$ hole coupled to the 2⁺ state in ¹²C, their $\alpha'\beta'-\alpha''\beta''$ phases should be opposite, i.e., it should be positive for the $\frac{5}{2}^-$ state. This indicates that the larger of the CCBA cross sections is applicable for the $\frac{5}{2}^-$, $\frac{7}{2}^-$, and $\frac{5}{2}^+$ states would therefore scale as 2:5:1 in cross-section magnitude.

From the CCBA analysis we obtain spectroscopic factors (S) for the $\frac{7}{2}$ and $\frac{5}{2}$ states of S = 0.7 and S = 0.4 if the value for the ¹¹C ground state is set to S = 100 while for the $\frac{5}{2}$ state we can state an upper limit of S = 0.1. The results for the 1*f* pickup strength in ¹²C(*p*, *d*)¹¹C can be compared with the calculations of Kurath¹⁴ which give S = 0.023 and S = 1.3 for the $\frac{5}{2}$ and $\frac{7}{2}$ states. Our S values, however, are only qualitative since the cross-section magnitudes are not reproduced by the CCBA calculations. The calculated TS cross section for the $\frac{5}{2}$ state, for instance, is low by a factor of 2.5. Such discrepancies may be symptomatic of faulty potentials, e.g., the one used to generate the deuteron waves (Fig. 3 shows an example of the effect of altering the deuteron potential). The underestimate of the TS cross sections, however, can also originate from effects due to momentum transfer sharing (discussed below) that are not fully accounted for in CCBA. When both SS and TS amplitudes contribute to a cross section, this problem could easily be magnified, which may be what we observe for the $\frac{7}{2}$ and $\frac{5}{2}$ states; the difference between measured and calculated cross sections is here as large as a factor of 13. Despite the quantitative ambiguities, the present analysis has given valid qualitative results. The TS process is present in the excitation of the $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{5}{2}$ states considered. Equally interesting is the result that the two latter excitations contain SS amplitudes which suggest $1f_{7/2}$ and $1d_{5/2}$ configurations in the ¹²C ground state; one would also claim some $1d_{3/2}$ admixture as judged from the similarity² of angular distributions for the $\frac{5}{2}$ state and the $\frac{3}{2}$ state at 7.51 MeV (not analyzed here).

III. REACTION DEPENDENCE

The reaction dependence of the (p, d) cross section can be expected to be specifically different for the SS and TS reaction mechanisms. The SS process shows a strong variation with momentum transfer q (Fig. 6); q is the momentum transfer in the lab system, $q = |p_p - p_d|$. From comparison of the (p,d) cross sections at different incident energies one may say that a general q dependence is indicated in the data. The momentum of the picked-up neutron and the (p, d) cross section would then largely reflect the single particle momentum density in the nuclear wave function (apart from a weaker q dependent factor reflecting the deuteron momentum density). A comparison between the (p, d) and (e, e'p) data¹¹ (Fig. 6) shows that this is approximately true for momentum transfers corresponding to deuterons in the extreme forward direction; for larger angles there are deviations probably due to distortion effects in (p,d).

The cross section maximum occurs for minimum momentum transfer (only higher energies are considered), i.e., collinear p and d momenta, which is fixed for each energy. In the TS process, however, this constraint is relaxed since the total momentum transfer is shared between the partial pickup and scattering steps and the intermediate momentum p' need be neither of the asymptotic momenta p_p and p_d where the modulus of the in-



FIG. 6. Differential cross sections for ${}^{12}C(p, d) {}^{11}C$ leading to the ${}^{11}C$ ground state plotted vs the momentum transfer $q = |p_p - p_d|$ (lab system). The data (Refs. 10, 2, and 3) obtained at $E_p = 100$, 185, and 700 MeV are shown together with the 1*p* proton momentum density determined (Ref. 11) in ${}^{12}C(e, e'p) {}^{11}B$.

termediate momentum p' may differ from p (apart from distortion effects) in the TS process; specifically, for the reaction ${}^{12}C + {}^{12}C^* + {}^{11}C$ the momentum transfer is

$$q = \left| \underline{p}_{p} - \underline{p}_{d} \right| = \left| \underline{p}_{p} - \underline{p}_{p'} + \underline{p}_{p'} - \underline{p}_{d} \right|$$

with the possibility of having $|\underline{p}_{p'} - \underline{p}_{d}| < |\underline{p}_{p} - \underline{p}_{d}|$. The TS process thus allows lower nuclear singleparticle momenta to be probed in the reaction, enhancing the pickup cross section in the TS process as compared with the SS one. If the maximum cross sections (σ) of the SS ($1p_{3/2}$ pickup) and TS process are related, one would expect to encounter an energy dependence in the relative cross section $\sigma(TS)/\sigma(1p_{3/2})$ since the effect of momentum transfer sharing will increase with increasing q, i.e., E. The inelastic scattering in the TS process would also introduce an energy dependence. The maximum of the (p, p') cross section varies roughly as the phase space factor (p_{p}^{2}) at higher energies (one can, e.g., compare the data^{6,12} at 185 and 1040 MeV) while its location in momentum transfer is guite energy independent (it occurs in the range q = 200-300 MeV/c for the considered quadrupole and octupole excitations in ¹²C). Between 185 and 700 MeV the (p, p') cross section would increase a factor of 4 to 5 and if the (d, d')cross section scales accordingly (no data available) a similar difference would be transferred to the TS (p,d) cross section at the two energies. Increasing the incident energy would therefore favor the TS excitation of a state nl_i^{-1} (through a $1p_{3/2}$ neutron pickup) over the SS nl_i neutron pickup provided the $1p_{3/2}$ and nl_j momentum distributions are similar.

IV. COMPARISON OF (p,d) CROSS SECTIONS AT 185 AND 700 MeV

The excitations of a certain state may involve pure SS or TS amplitudes, or it may be a mixed SS + TS transition. In the latter case, the reaction cross section will be dominated by either of the amplitudes, or both amplitudes may affect the cross section where the weight of each amplitude in the cross section is reaction dependent. For (p,d) at 185 MeV, the excitations of the three lowlying $\frac{3}{2}$ and $\frac{1}{2}$ states are governed by the SS process. A possible TS contribution to these excitations is certainly smaller than that for the $\frac{5}{2}$ state and will not affect the maximum cross section which we shall consider. The TS process, on the other hand, dominates the excitation of the $\frac{5}{2}$ state at 185 MeV and a possible SS amplitude admixture can be estimated to be $\alpha < 5$. Such a small amplitude cannot account for the relatively large cross section observed³ for this state in (p, d) at 700 MeV. Therefore, the excitation of the $\frac{5}{2}$ state can be taken as representative of the TS process at both 185 and 700 MeV as the ground state excitation is a proper representative of the SS process.¹³ Below we refer to these as "typical" TS and SS processes when comparing the relative maximum cross sections.

As can be seen from Fig. 7, the low-lying $\frac{3}{2}$ and $\frac{1}{2}$ states appear with the same relative cross sections at 185 and 700 MeV which is what is expected for predominant SS processes. The TS process manifests itself in a conspicuous difference in relative cross section, i.e., it is 25 times larger at 700 MeV than at 185 MeV for the $\frac{5}{2}$ state. Similar differences are observed for the states at 6.48 ($\frac{7}{2}$) 8.11 ($\frac{3}{2}$), and around 8.5 MeV indicating significant TS amplitudes in the respective excitations (i.e., it is true for at least one of the unresolved $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{5}{2}$ * states around 8.5 MeV). This is consistent

FIG. 7. Comparison of relative maximum cross sections for the excitation of various states in ¹¹C in the (p, d) reactions at 185 and 700 MeV; the data from Refs. 2 and 3 have been used. The 700-MeV cross sections for the higher-lying states $(E_x > 6.7 \text{ MeV})$ are only rough estimates from the spectra given in Ref. 3 and represent lower limits. The cross section at $E_x \approx 8.5 \text{ MeV}$ comprises the excitation of the unresolved states at 8.43 $(\frac{5}{2}^{-})$, 8.66 $(\frac{7}{2}^{+})$, and 8.70 $(\frac{5}{2})$ MeV.

with the observation made in Ref. 2 that these states are at least partly reached by TS processes through quadrupole (the $\frac{7}{2}$ state), monopole (the $\frac{3}{2}$ state), and octupole (the $\frac{7}{2}$ and/or $\frac{5}{2}$ states) excitations in (p, d) at 185 MeV (the monopole excitation is, of course, a higher order one). However, some of these are mixed SS+TS excitations for which the ratio of the cross sections at 185 and 700 MeV may vary, such as for the $\frac{7}{2}$ and $\frac{5}{2}$ states. From the CCBA analysis of the 185-MeV data the TS cross sections are in the ratio 2:5:1for the $\frac{5}{2}$, $\frac{7}{2}$, and $\frac{5}{2}$ states. These ratios are also reflected in the corresponding cross sections at 700 MeV which indicate that the TS process dominates these excitations at 700 MeV. It therefore seems that the SS amplitude in the $\frac{5}{2}$ excitation affects the 185-MeV cross sections sufficiently to wash out the specific TS signature of the energy dependence of the relative cross section, while that is not the case for the $\frac{7}{2}$ state. These observations may be extended to a general comment about the energy variation of the relative cross section. For the SS process, one would expect little variation with bombarding energy

while the relative maximum cross section would increase with increasing energy for the TS process. The same behavior would be observed for a mixed SS + TS process as long as one process is dictating the cross section. In the energy interval where the SS and TS contributions are comparable, interference effects are important. The constructive interference will depress the cross section and may give an energy variation showing a minimum where the cancellation is maximum.

Finally, we observe that the energy dependence of the TS cross section is very strong; the relative cross section increases a factor of about 25 between 185 and 700 MeV. This seems to be in excess of what can be attributed to the energy variation in the inelastic scattering cross section alone since the free (p, p') cross section varies only a factor of 5. The sharing of the momentum transfer may very well make up the missing factor of 5 in the cross section ratio.

V. CONCLUSIONS

In this paper it has been shown that the TS process is important in the (p, d) reaction at intermediate energies. The TS process can allow excitations which are prohibited in the SS process and it can enhance the cross section of mixed SS + TS processes. The relative importance of the TS process is reaction dependent. The comparison of the (p,d) reactions at 185 and 700 MeV suggests that the TS process becomes increasingly important with increasing incident energy and momentum transfer. Some plausible reasons for this have been pointed out. In related single-nucleon removal reactions such as (γ, n) and (π^*, p) or for proton removal, (γ, p) and (π, n) , the TS process should in general be less favored-than in (p, d) for comparable momentum transfer; the reason being generally smaller inelastic cross sections. A primary need in the analysis of (p, d) and other related reactions is a knowledge of the reaction mechanism and here the different dependences of the SS and TS processes can be exploited as demonstrated in this paper. The TS process is a complex one, carrying a corresponding richness in information relating the involved partial reactions and the involved nuclear states. It has been shown that some peculiar features in ${}^{12}C(p,d){}^{11}C$ data at 185 MeV can be qualitatively accounted for within CCBA. The analysis requires the ¹²C ground-state configuration to be a mixed one, $(1 - a^2)^{1/2} 1s^4 1p^8$ $+als^{4}1p^{6}(1d, 1f_{7/2})^{2}$ where *a* is in the range 0.1-0.2; SS pickup from the small 1d and $1f_{7/2}$ admixtures competes with the TS 1p pickup through octupole and quadrupole excitations.



*Work done under the auspices of the U.S. Energy Research and Development Administration.

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