Dramatic nuclear structure effects in $(\pi, \pi N)$ reactions

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Ratios of pion-induced nucleon knockout cross sections are discussed utilizing a modified nucleon charge exchange model which takes into account nuclear structure effects on the effective number of nucleons and on the various probabilities for charge exchange. The availability of particle-stable analog states in the charge exchange transitions can give factor of 2 differences in the neutron knockout ratio between isotopes for a given Z. On the other hand, proton knockout ratios are largely independent of nuclear structure. Predictions of the model are in reasonable semiquantitative agreement with all experimental data.

[NUCLEAR REACTIONS Ratios of $\sigma(\pi, \pi N)$, $T_\pi \approx 190$ MeV, when residual nucleus observed: ^{12}C; ^{11}B, ^{19}F, ^{31}P; ^{58,60,62,64}Ni, ^{64,66,68,70}Zn; ^{16,17,18}O. Dependence on availability of particle-stable IAS's.]

I. INTRODUCTION

The ratio of the $^{12}\mathrm{C}(\pi^\mp, \pi N)^{11}\mathrm{C}$ cross sections near the (3,3) resonance is smaller than the impulse approximation prediction of about 3 by a surprisingly large amount.[1-3] Recently we showed that this could be understood with a semiclassical model which assumes that a nucleon knocked out by the incident pion can charge exchange before leaving the nucleus.[4] Subsequently[5] this model predicted with good success the corresponding ratios for ^{14}N, ^{16}O, and ^{19}F. Monahan and Serduke[6] have also shown that similar reductions in the ^{58,60}Ni nucleon removal ratios[7] were in agreement with this model. This is remarkable, since the neutron removal ratio for ^{64}Zn is apparently rather larger.[8] Another peculiarity, recently noted by Karol,[9] is that the $^{11}\mathrm{B}(\pi^+, \pi^0 n)^{10}\mathrm{C}$ cross section[1] is some 5 times smaller than our simple model would predict.

In this paper we will show that all these apparently conflicting observations can be understood, at least qualitatively, by consideration of the detailed nuclear structure involved in each case. In particular, in our earlier work we assumed that the probabilities for several distinct charge exchange processes were equal. Taking into account the large differences between analog and nonanalog charge exchange cross sections invalidates this approximation. Furthermore, in addition to bringing the model's predictions into agreement with the experimental data noted in the first paragraph, this refinement makes a dramatic qualitative prediction. Specifically, in experiments which measure cross sections to particle-stable states of the residual nucleus, the neutron removal ratio for certain elements can jump by a factor of 2 in going from one isotope to the next.

In this paper we restrict our attention to those pion knockout experiments in which the residual nucleus is observed either through activation techniques[1-3] or by observation of deexcitation γ rays.[7] Many of the ideas we discuss here appear explicitly or implicitly in the lengthy work by Robson,[10] but without the emphasis on the role of nucleon charge exchange.

II. NUCLEON KNOCKOUT CROSS SECTIONS–GENERAL REMARKS

To illustrate our semiclassical picture of the manner in which the $(\pi, \pi N)$ reaction takes place, we consider explicitly the case of a π^- beam scattering quasielastically from nucleons in a target nucleus (Z,N), leaving behind a residual nucleus of one less neutron $(Z, N-1)$. As the pion enters the nucleus, it can hit both neutrons and protons, scattering elastically or charge exchanging with cross sections given by the free πN cross sections: $\sigma(\pi^- n \to \pi^- n)$, $\sigma(\pi^- p \to \pi^- p)$, and $\sigma(\pi^- p \to \pi^0 n)$.

The only struck neutrons which can count in this reaction are those which are not too deeply bound, so that the $(Z, N-1)$ residual nucleus is left in a particle-stable state. Hence the relevant number of neutrons is not N but some smaller number N_{eff} (which we define more precisely in Sec. III). Of these N_{eff} struck nucleons, only those which leave the nucleus without charge exchanging will contribute to the reaction producing $(Z, N-1)$. Those that do charge exchange, $n + (Z, N-1) \to p + (Z-1, N)$, correspond to depletion of the desired product nucleus. Let P_1^d be the probability for this charge exchange (whence $1-P_1^d$ is the probability for *no* charge exchange). We are interested here only in the probability that a particle-stable state of $(Z,N-1)$ disappears, but we do not care, in this case, whether the resulting $(Z-1,N)$ state is particle stable or not. We write this schematically as

$$P_1^d = P[(Z,N-1)_s \to (Z-1,N)_{s+u}] , \tag{1a}$$

where s and u stand for "particle-stable" and "unstable," respectively.

As for the Z protons, the π^- can scatter ($\pi^- p \to \pi^- p$) or charge exchange ($\pi^- p \to \pi^0 n$), with the residual nucleus after the interaction being $(Z-1,N)$. For the reaction considered here, the pion charge exchange process does not count, since the residual nucleus is not $(Z,N-1)$ regardless of whether the recoil neutron charge exchanges on the way out. For the elastic scattering, however, the recoil proton can produce the desired $(Z,N-1)$ nucleus by charge exchange as it leaves, but to give an *observable* $(Z,N-1)$ nucleus the nucleon charge exchange process must leave the residual nucleus in a particle-stable state. In the notation of Eq. (1a), the probability for this charge exchange process is

$$P_2^e = P[(Z-1,N)_{s+u} \to (Z,N-1)_s] , \tag{1b}$$

where the e denotes an enhancement of the desired product nucleus.

Putting these ingredients together we have

$$\begin{aligned}\sigma_n^{(-)} &\equiv \sigma[{}^AZ(\pi^-,\pi^- n)^{A-1}Z] \\ &\propto N_{\text{eff}}\sigma(\pi^- n \to \pi^- n)(1-P_1^d) + Z\sigma(\pi^- p \to \pi^- p)P_2^e \\ &\approx \tfrac{1}{9}\sigma_{(3,3)}[9N_{\text{eff}}(1-P_1^d)+ZP_2^e] .\end{aligned} \tag{2a}$$

The approximate equality comes from the assumption of (3,3) dominance for the πN cross sections. Using similar arguments, the cross sections for the five other single-nucleon knockout reactions which are possible with charged pion beams are

$$\begin{aligned}\sigma_n^{(+)} &\equiv \sigma[{}^AZ(\pi^+,\pi^+ n+\pi^0 p)^{A-1}Z] \\ &\propto N_{\text{eff}}[\sigma(\pi^+ n \to \pi^+ n)(1-P_1^d) \\ &\qquad + \sigma(\pi^+ n \to \pi^0 p)(1-P_3^d)] \\ &\quad + Z\sigma(\pi^+ p \to \pi^+ p)P_2^e \\ &\approx \tfrac{1}{9}\sigma_{(3,3)}[N_{\text{eff}}(3-P_1^d-2P_3^d)+9ZP_2^e] ,\end{aligned} \tag{2b}$$

$$\begin{aligned}\sigma_p^{(+)} &\equiv \sigma[{}^AZ(\pi^+,\pi^+ p)^{A-1}(Z-1)] \\ &\propto Z_{\text{eff}}\sigma(\pi^+ p \to \pi^+ p)(1-P_2^d) + N\sigma(\pi^+ n \to \pi^+ n)P_1^e \\ &\approx \tfrac{1}{9}\sigma_{(3,3)}[9Z_{\text{eff}}(1-P_2^d)+NP_1^e] ,\end{aligned} \tag{2c}$$

$$\begin{aligned}\sigma_p^{(-)} &\equiv \sigma[{}^AZ(\pi^-,\pi^- p+\pi^0 n)^{A-1}(Z-1)] \\ &\propto Z_{\text{eff}}[\sigma(\pi^- p \to \pi^- p)(1-P_2^d) \\ &\qquad + \sigma(\pi^- p \to \pi^0 n)(1-P_4^d)] \\ &\quad + N\sigma(\pi^- n \to \pi^- n)P_1^e \\ &\approx \tfrac{1}{9}\sigma_{(3,3)}[Z_{\text{eff}}(3-P_2^d-2P_4^d)+9NP_1^e] ,\end{aligned} \tag{2d}$$

$$\begin{aligned}\sigma_{\text{up}} &\equiv [{}^AZ(\pi^+,\pi^0 n)^{A-1}(Z+1)] \\ &\propto N\sigma(\pi^+ n \to \pi^0 p)P_3^e \approx \tfrac{2}{9}\sigma_{(3,3)}NP_3^e ,\end{aligned} \tag{2e}$$

$$\begin{aligned}\sigma_{\text{down}} &\equiv \sigma[{}^AZ(\pi^-,\pi^0 p)^{A-1}(Z-2)] \\ &\propto Z\sigma(\pi^- p \to \pi^0 n)P_4^e \approx \tfrac{2}{9}\sigma_{(3,3)}ZP_4^e .\end{aligned} \tag{2f}$$

The last two reactions can be characterized as "knight's moves" on the Chart of the Nuclides chessboard, namely, $(Z,N)\to(Z+1,N-2)$ and $(Z,N)\to(Z-2,N+1)$, respectively. The additional charge exchange probabilities in Eqs. (2b)–(2f) are defined as

$$P_1^e = P[(Z,N-1)_{s+u} \to (Z-1,N)_s] , \tag{1c}$$

$$P_2^d = P[(Z-1,N)_s \to (Z,N-1)_{s+u}] , \tag{1d}$$

$$P_3^e = P[(Z,N-1)_{s+u} \to (Z+1,N-2)_s] , \tag{1e}$$

$$P_3^d = P[(Z,N-1)_s \to (Z+1,N-2)_{s+u}] , \tag{1f}$$

$$P_4^e = P[(Z-1,N)_{s+u} \to (Z-2,N+1)_s] , \tag{1g}$$

$$P_4^d = P[(Z-1,N)_s \to (Z-2,N+1)_{s+u}] . \tag{1h}$$

The various charge exchange processes for isobars of $A-1$ nucleons are illustrated for a typical level scheme in Fig. 1.

If we set $N_{\text{eff}}=N$, $Z_{\text{eff}}=Z$, and all $P_i=P$, we re-

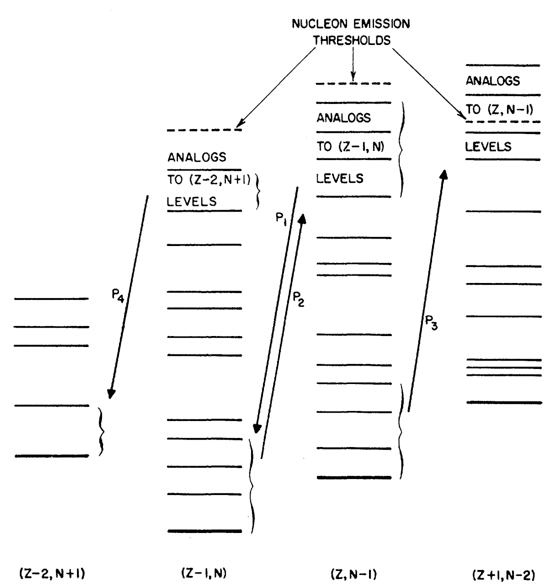

FIG. 1. Schematic level diagram for $(\pi,\pi N)$ reactions on a nucleus (Z,N) indicating the nuclei which the charge exchange probabilities P_i connect.

cover the results of the simpler model of Ref. 4. The remainder of the paper consists mainly of estimates of $N_{\rm eff}$, $Z_{\rm eff}$, and the $P_i^{d,e}$ for various interesting cases. But first we make some general remarks about these quantities that we have introduced.

The reason why Eq. (2) has proportionality signs instead of equalities is that there are attenuations in both pion and recoil nucleon fluxes. These can be estimated, for example, in the Glauber model[11] or, alternatively, in a more semiclassical approach.[12] However, our main interest here is in *ratios* of cross sections, for which the attenuation due to these distortions will basically cancel. On the other hand, effects due to nuclear structure differences between the $(Z,N-1)$ and $(Z-1,N)$ residual nuclei, such as the number of available particle-stable states, will not cancel.

Turning to the nucleon charge exchange probabilities P_i, we note that, experimentally, charge exchange transitions between isobaric analog states are typically an order of magnitude larger than nonanalog transitions.[13] This is understood to arise from the forward peaking of the $pn\to np$ cross section, giving a preference to those nuclear transitions that can occur with small momentum transfer. These are the analog transitions, since the nuclear wave functions involved differ only in their isospin projection quantum number. (Hence there is a collective enhancement of the transition as well.) In contrast, the nonanalog states are, because of the Pauli principle, necessarily also different in their space and spin properties. Thus they require larger momentum transfer to effect the more extensive nuclear rearrangements required.[14] The various probabilities P_i will be "large" when many analog transitions are available and "small" when otherwise.

III. SPECIFICATION OF EFFECTIVE NUCLEON NUMBER AND CHARGE EXCHANGE PROBABILITIES

The probability of reaching a particular excited state in the residual $(Z,N-1)$ or $(Z-1,N)$ nucleus by knocking a nucleon out of the target (Z,N) ground state is related to the spectroscopic factor.[15] This provides the clue as to how to define $N_{\rm eff}$ and $Z_{\rm eff}$. Note that the total number of neutrons and protons for a nucleus of isospin $T=\frac{1}{2}(N-Z)$ is given in terms of the spectroscopic factors[15] by

$$\begin{aligned} N=&\sum_{I_0\alpha_0 j} S(I,T,\alpha;\; I_0,T_0=T-\tfrac{1}{2},\alpha_0,j)\\ &+(2T+2)^{-1}\sum_{I_0\alpha_0 j} S(I,T,\alpha;I_0,T_0=T+\tfrac{1}{2},\alpha_0,j)\;,\\ Z=&(2T+1)(2T+2)^{-1}\sum_{I_0\alpha_0 j} S(I,T,\alpha;I_0,T_0=T+\tfrac{1}{2},\alpha_0,j)\;, \end{aligned} \tag{3}$$

where the sums go over all final states of the residual nucleus and all quantum numbers of the struck nucleon. Thus we define

$$\begin{aligned} N_{\rm eff}=&\sum_i{}' S_i(T_0=T-\tfrac{1}{2})\\ &+(2T+2)^{-1}\sum_i{}' S_i(T_0=T+\tfrac{1}{2})\\ \equiv& N_{\rm eff}^{(-)}+N_{\rm eff}^{(+)}\;, \end{aligned} \tag{4}$$

$$Z_{\rm eff}=(2T+1)(2T+2)^{-1}\sum_i{}' S_i(T_0=T+\tfrac{1}{2})\;,$$

where the prime on the sums for $N_{\rm eff}$ mean restriction to the particle-stable states of $(Z,N-1)$ and, likewise, for $Z_{\rm eff}$, to the particle-stable states of $(Z-1,N)$. Here $N_{\rm eff}^{(-)}$ refers to the sum of spectroscopic factors with $T_0=T-\frac{1}{2}$; $N_{\rm eff}^{(+)}$ refers to those with $T_0=T+\frac{1}{2}$. [Of course, if $T=0$, the undefined $T_0=T-\frac{1}{2}$ spectroscopic factors do not appear in the sums of Eqs. (3) and (4)].

The S_i are also useful in estimating the charge exchange probabilities P_i. We will assume that, if a particular state in the initial $(Z,N-1)$ or $(Z-1,N)$ residual nucleus has an isobaric analog in the final nucleus, the probability for charge exchange for that state is P, the semiclassical probability calculated in Ref. 4. If there is *no* analog, we will assume for simplicity a probability for charge exchange of zero for that particular state.[16] Thus each P_i is a weighted average, with the weights for each level in the average being the spectroscopic factors. Let us thus write

$$P_i^{d,e}=f_i^{d,e}P\;, \tag{5}$$

where each fraction f_i will be the ratio of two sums of spectroscopic factors. We consider the possible cases separately:

(1) For the fraction f_1^d, representing depletion of the particle-stable states of $(Z,N-1)$ by charge exchange to any state of $(Z-1,N)$, the numerator is the sum of S_i for particle-stable states in $(Z,N-1)$ which have analogs in $(Z-1,N)$, stable or otherwise. The denominator is the sum over all particle-stable states in $(Z,N-1)$. Thus,

$$f_1^d=N_{\rm eff}^{(+)}/N_{\rm eff}\;. \tag{6a}$$

The special case of $T=0$ has $N_{\rm eff}=N_{\rm eff}^{(+)}$, whence

$$f_1^d(T=0)=1\;. \tag{6b}$$

An exactly parallel argument applies to the fraction f_2^d, representing depletions of stable states of $(Z-1,N)$, but here

$$f_2^d=Z_{\rm eff}/Z_{\rm eff}=1\;, \tag{7}$$

regardless of T. (We are assuming $N-Z\geqslant 0$ for all target nuclei.)

(2) For the enhancement fraction f_1^e, the numerator involves those states of $(Z, N-1)$ which have particle-stable analogs in $(Z-1, N)$, while the denominator sums over *all* the states of $(Z, N-1)$. Thus,

$$f_1^e = (2T+1)^{-1} Z_{\text{eff}} / N \ . \tag{8}$$

The parallel argument for f_2^e gives

$$f_2^e = (2T+1) N_{\text{eff}}^{(+)} / Z \ . \tag{9a}$$

The special case of $T=0$ reduces to

$$f_2^e(T=0) = N_{\text{eff}} / Z \ . \tag{9b}$$

(3) For the depletion fraction f_3^d, representing charge exchange from $(Z, N-1)$ to $(Z+1, N-2)$, we must consider three separate cases.
(a) For $T=0$, the attainable $(Z, N-1)$ states have isospin $T_0 = \frac{1}{2}$, while the states of $(Z+1, N-2)$ have $T_0 \geq \frac{3}{2}$. No analogs exist, so

$$f_3^d(T=0) = 0 \ . \tag{10a}$$

(b) For $T = \frac{1}{2}$, the states of $(Z+1, N-2)$ have $T_0 \geq 1$. Thus

$$f_3^d(T=\tfrac{1}{2}) = N_{\text{eff}}^{(+)} / N_{\text{eff}} \ , \tag{10b}$$

by the same argument as for f_1^d. Indeed, $f_3^d = f_1^d$ in this case.
(c) For $T \geq 1$, the states of $(Z+1, N-2)$ have $T_0 \geq T - \frac{3}{2}$ while those of $(Z, N-1)$ have $T_0 = T \pm \frac{1}{2}$. Thus all the particle-stable states of the $(Z, N-1)$ nucleus have (high lying) analogs in the $(Z+1, N-Z)$ nucleus, and therefore

$$f_3^d(T \geq 1) = (N_{\text{eff}}^{(-)} + N_{\text{eff}}^{(+)}) / N_{\text{eff}} = 1 \ . \tag{10c}$$

(4) The argument for the enhancement fraction f_3^e goes similarly in three separate cases:

$$f_3^e(T=0) = 0 \ , \tag{11a}$$

$$f_3^e(T=\tfrac{1}{2}) = (2T+2)^{-1} \sum_i{}'' S_i(T_0 = T + \tfrac{1}{2}) / N$$

$$\equiv N_s^{(+)} / N \ , \tag{11b}$$

$$f_3^e(T \geq 1) = \sum_i{}'' S_i(T_0 = T - \tfrac{1}{2}) / N$$

$$\equiv (N_s^{(-)} + N_s^{(+)}) / N \ , \tag{11c}$$

where the double prime on the sum means to include all $(Z, N-1)$ states of that T_0 which have particle-stable analogs in $(Z+1, N-2)$. The appearance of N in the denominators comes about in the same way it did in Eq. (8).

(5) For the fractions f_4^d and f_4^e, representing transitions from $(Z-1, N)$ to $(Z-2, N+1)$, note that the latter nucleus has $T_0 = T + \frac{3}{2}$. As long as the pion contributes no isospin to the reaction (i.e., three-step processes are negligible), there will be no excitation of $T + \frac{3}{2}$ states when the proton is knocked out to form $(Z-1, N)$. Thus

$$f_4^d = f_4^e = 0 \ . \tag{12}$$

To summarize, we have seen how the necessary ingredients for calculating the cross sections—the effective nucleon numbers, N_{eff} and Z_{eff}, and the charge exchange probabilities P_i—can be obtained from spectroscopic factors for the target nucleus. These may be taken either from experimental data on pickup reactions or from theoretical nuclear wave functions.

IV. NUMERICAL RESULTS FOR SPECIAL CASES

A. $T=0$ targets

From Eqs. (2) and the formulas for the f_i $(T=0)$ given in the last section, we find that the knockout ratios for pion energies near the (3,3) resonance take the forms

$$R_n \equiv \sigma_n^{(-)} / \sigma_n^{(+)} = (9 - 8P)/(3 + 8P) \ , \tag{13a}$$

$$R_p \equiv \sigma_p^{(+)} / \sigma_p^{(-)} = (9 - 8P)/(3 + 8P) \ , \tag{13b}$$

$$R_{\text{up}} \equiv \sigma_{\text{up}} / \sigma_n^{(+)} = 0 \ , \tag{13c}$$

$$R_{\text{down}} \equiv \sigma_{\text{down}} / \sigma_p^{(-)} = 0 \ . \tag{13d}$$

The formula for R_n is exactly that given by Hewson[17] in his earlier optical model treatment of the $^{12}C(\pi, \pi N)^{11}C$ problem. It differs from the result of Ref. 4 in that the denominator is $3+8P$ instead of $3+6P$ (in the present case $P_3^d = 0$ rather than P).

The case of ^{12}C is the one for which the most experimental information is available.[2,3] Using the value $P \approx 0.25$ as calculated in Ref. 4 for energies near the resonance, we find a 10% smaller value for R_n than obtained originally. A minor adjustment of our one free parameter[18] within its expected range is sufficient to restore agreement with the experimental ratio at all energies.

The R_n's for other $T=0$ targets, in particular ^{14}N and ^{16}O, remain likewise in agreement with experiment.[5]

Formulas for the R_i can also be easily written down for cases when the πN cross sections are *not* dominated by the (3,3) resonance. Even though we do not dwell on this point in this paper, we remind the reader that the energy dependence of the ratios $R_i(T_\pi)$ is a distinctive feature of the nucleon charge exchange model, since the probability $P(T_\pi)$ falls off rapidly with increasing energy.[4]

B. $T=\frac{1}{2}$ targets

Again collecting the results for the $f_i(T=\frac{1}{2})$ and substituting into Eqs. (2), we find the knockout ratios near the (3,3) resonance have the following remarkably simple forms:

$$R_n = (9 - 7\nu P)/(3 + 15\nu P) \ , \quad (14a)$$

$$R_p = (9 - 8.5P)/(3 + 3.5P) \ , \quad (14b)$$

$$R_{up} = 2\nu' P/(3 + 15\nu P) \ , \quad (14c)$$

$$R_{down} = 0 \ , \quad (14d)$$

where

$$\nu = N_{eff}^{(+)}/N_{eff} \ , \quad (15a)$$

$$\nu' = N_s^{(+)}/N_{eff} \quad (15b)$$

can be obtained in terms of spectroscopic factors, known either theoretically or experimentally. Note that the proton knockout ratio R_p is *independent of nuclear structure*, the factors of Z_{eff} having canceled.

For comparison with Eqs. (14), the simpler model of Ref. 4 gives

$$\begin{aligned} R_n^{old} &= [9 - (9 - r)P]/[3 + (9r - 3)P] \ , \\ R_p^{old} &= [9 - (9 - 1/r)P]/[3 + (9/r - 3)P] \ , \\ R_{up}^{old} &= 2P/[3 + (9r - 3)P] \ , \\ R_{down}^{old} &= 2P/[3 + (9/r - 3)P] \ , \end{aligned} \quad (16)$$

where $r = Z/N$.

We consider first the case of ^{11}B, for which the nuclear structure complications are not too severe.[19] For the purposes of estimating N_{eff} and Z_{eff}, we will assume that if the pion knocks out one of the $1s$-core nucleons, the residual $A = 10$ nucleus is particle unstable. For proton knockout, there are six particle-stable states in ^{10}Be, of which four have spin-parities that correspond to removal of a $p_{3/2}$ or $p_{1/2}$ proton from the $\frac{3}{2}^-$ ground state of ^{11}B. (The other states have negative parity and are attributed to configurations involving the $2s$-$1d$ shell.) All the ^{10}Be states have $T_0 = 1$, of course. For neutron knockout, there are 13 particle-stable states in ^{10}B, of which 10 have the "right" spin-parities. Of these, 2 have $T_0 = 1$ and the rest have $T_0 = 0$.

The predicted knockout ratios for ^{11}B are summarized in the first column of Table I. We have used the spectroscopic factors calculated for seven active nucleons in the $1p$ shell by Cohen and Kurath.[20] The semiclassical value of P for ^{11}B has been rescaled from that for ^{12}C (0.25 at resonance) because the nucleon removal energy is smaller. (This enters as a linear factor in the $np \to pn$ charge exchange cross section; see Appendix for details.) The neutron knockout ratio is some 20% larger than the prediction of Ref. 4 ($R_n^{old} = 1.98$).

There are no experimental ratios to compare to, since ^{10}B is stable and ^{10}Be is very long-lived, precluding an activation experiment. An experiment measuring deexcitation γ rays in the $A = 10$ daughters is feasible, however. In this case the summations in N_{eff}, and $N_s^{(+)}$ should also exclude the spectroscopic factors for the unobserved ground states.

If and when experimental ratios are available, the parameter β will have to be fitted.[18] We remark here that because we are now taking into account nuclear structure effects explicitly, the value of β obtained might be different from that expected on the basis of Ref. 4. We would still expect β to lie between the extreme values corresponding to no Pauli principle correction and to the estimate made with a zero-temperature Fermi gas model.

In connection with the ^{11}B case, Karol[8] has considered the reaction ^{11}B$(\pi^+, \pi^0 n)^{10}$C. The first column of Table I also shows the prediction for the ratio $R_{up} = \sigma_{up}/\sigma_n^{(+)}$. The present prediction is some 4 times smaller than the earlier result ($R_{up}^{old} = 0.094$). Using the measured ^{10}C cross section of 0.85 mb,[1] this corresponds to a $\sigma_n^{(+)}$ of about 35 mb, which is a typical cross section in this part of the Periodic Table.[2,3,5,21]

The nuclear structure involved in the case of ^{19}F is somewhat more complex, involving first of all a richer spectrum in the $A = 18$ system.[22] Let us first assume, as above, that if the pion knocks out a $1s$ or $1p$ nucleon from the ^{16}O core, the residual ^{18}F or ^{18}O will be too excited to be particle stable. Then the shell-model configuration is $(dsd)^3$ and we can take the necessary spectroscopic factors from the extensive Oak Ridge calculations.[23] Then, assuming $P = 0.25$, we have results for knockout ratios as shown in the second column

TABLE I. Calculated and observed ratios of pion-induced nucleon knockout cross sections, $R_n = \sigma_n^{(-)}/\sigma_n^{(+)}$, $R_p = \sigma_p^{(+)}/\sigma_n^{(-)}$, and $R_{up} = \sigma_{up}/\sigma_n^{(+)}$, for various nuclei with $T = \frac{1}{2}$. N_{eff}, Z_{eff}, ν, and ν' for ^{11}B and ^{19}F are calculated from theoretical spectroscopic factors as explained in text. For ^{31}P all nucleons in the $(1d, 2s)$ shell are assumed active.

	^{11}B	^{19}F(dsd)	^{19}F(pds)		^{31}P(dsd)
N_{eff}	3.210	1.923	3.30		8 (Assumed)
Z_{eff}	2.166	0.967	2.67		7 (Assumed)
ν	0.245	0.251	0.303		0.240
ν'	0.245	0.246	0.100		0.221
P	0.18	0.24	0.24	0.35	0.28
R_n	2.37	2.17	2.05	1.80	2.13
R_p	2.06	1.77	1.77	1.43	1.66
R_{up}	0.024	0.031	0.012	0.015	0.031
R_n^{exp}	...	1.68 ± 0.11 [a] 1.52 ± 0.05 [b] 1.78 ± 0.15 [c]			2.6 ± 0.5 [c]

[a] See Ref. 5. The error has been increased to include those of the ^{12}C cross sections used for normalization. $T_\pi = 178$ MeV.

[b] See Ref. 26. $T_\pi = 190$ MeV.

[c] See Ref. 8. $T_\pi = 184$ MeV.

in Table I. The ratio R_n is now 33% larger than with the old model (which fit experiment very well[5] at this value of P).

There is evidence, however, that the low-lying negative parity states in ^{18}O have large spectroscopic factors.[24] These states cannot be constructed from $(dsd)^2$ configurations, i.e., the shell-model space must be enlarged to accommodate them. McGrory and Wildenthal have calculated shell-model wave functions assuming, for ^{19}F, seven nucleons outside an inert ^{12}C core.[25] Using their spectroscopic factors, we get the results shown in the third column of Table I. The ratio R_n is smaller than that for the $(dsd)^3$ calculation. More striking is the difference between the "knight's move up" ratios R_{up}. Such reactions are evidently quite sensitive to details of the nuclear structure of the target nucleus.

The $(pds)^7$ model for ^{19}F gives a neutron knockout ratio which is still large with respect to the experimental values.[5,8,26] Stretching the β parameter[18] to its maximum value (no Pauli principle inhibition) gives the predictions listed in the fourth column of Table I; the ratio $R_n = 1.80$ is still a bit large. Perhaps this is due to the simplifying assumptions made in this paper and in the estimation of the semiclassical probability P. But it could also be an indication that some other mechanism may be involved in producing the deviation of R_n from its impulse approximation value.[27]

To our knowledge, the only other $T=\frac{1}{2}$ nucleus for which any experimental knockout information exists is ^{31}P. The $A=30$ level schemes[28] show a paucity of negative parity states, so we presume the ^{16}O core is inert. If we assume that the particle-stable states in ^{30}Si and ^{30}P saturate sum rules corresponding to Eqs. (3) for the (dsd) shell, $N_{\rm eff}=8$ and $Z_{\rm eff}=7$. Knowing the single-particle emission thresholds[28] and the relative spectroscopic factors to $T_0=1$ states measured in a $(d,{}^3\text{He})$ experiment,[29] we can estimate ν and ν'. Scaling the probability P, as in the Appendix, the longer path length in ^{31}P (as compared, say, with that in ^{19}F) gives $P=0.28$ near resonance. The results for ^{31}P are summarized in the last column of Table I.

It is amusing that this time, in contrast with the ^{19}F case, the model predicts a value of R_n which is perhaps smaller than experiment. It seems clear that more experimental information will be needed before one can make any definite statements about the validity of the nucleon charge exchange model for $T=\frac{1}{2}$ targets.

C. $T>\frac{1}{2}$ targets

Finally, collecting the results for $f_i(T>\frac{1}{2})$ and substituting them into Eqs. (2), we find the ratios have a simple form

$$R_n=\frac{9-(8-2T)\nu P}{3+[(18T+8)\nu-2]P}\,, \tag{17a}$$

$$R_p=\frac{9-(18T+8)(2T+1)^{-1}P}{3+(8-2T)(2T+1)^{-1}P}\,, \tag{17b}$$

$$R_{up}=\frac{2\nu' P}{3+[(18T+8)\nu-2]P}\,, \tag{17c}$$

where

$$\nu'=(N_s^{(-)}+N_s^{(+)})/N_{\rm eff} \tag{18}$$

and ν is defined as in (15a). We note that ν', which is associated with P_3^e, has a different definition than in (15b) because for $T=\frac{1}{2}$, the $T_0=0$ states in the $(Z,N-1)$ nucleus do not have analogs in the $(Z+1,N-2)$ nucleus, whereas for $T>\frac{1}{2}$, the $T_0=T-\frac{1}{2}$ states do. For the same reason the expression for R_n given in (17a) does not reduce to (14a) for $T=\frac{1}{2}$.

The ratio R_p is again independent of nuclear structure effects and depends only on P and on isospin. Thus for a given Z, R_p will increase slightly as the neutron excess increases.

In contrast R_n and R_{up} can change from nucleus to nucleus. In general more and more of the $T_0=T+\frac{1}{2}$ analog states in the $(Z,N-1)$ nucleus will become particle unstable, as the neutron excess increases. Thus ν will decrease and R_n will increase, in some cases dramatically.

Likewise, more and more of the $T_0=T-\frac{1}{2}$ analog states in the $(Z+1,N-2)$ nucleus will become particle unstable and ν' and R_{up} will decrease. Special cases in this regard are $T=1$ nuclei because the lowest states of the $(Z+1,N-2)$ nucleus will have the same isospin, $T_0=\frac{1}{2}$, as the lowest levels of the $(Z,N-1)$ nucleus. Thus we expect R_{up} to be largest for such nuclei.

We have made calculations for several nickel and zinc isotopes. Since at least the entire $(1f,2p)$ shell, and very likely the $(1d,2s)$ and $1g_{9/2}$ shells as well, are active for such nuclei, the available shell-model calculations are inadequate. Hence we have used spectroscopic factors derived from (p,d), (d,t), and $({}^3\text{He},\alpha)$ reactions on the target nucleus.[30-32] Generally the normalization of experimental spectroscopic factors is questionable but since we need only the ratios this shortcoming does not affect our results.

The predictions are compared with known experimental data for nickel in Table II and for zinc in Table III. The value $P=0.34$ was used for all of these nuclei. The tables show only the values of ν and ν' calculated from $({}^3\text{He},\alpha)$ reaction data; similar values are obtained from spectroscopic factors for the other pickup reactions.

The calculated R_p are in reasonable agreement

TABLE II. Calculated and observed ratios of pion-induced nucleon knockout cross sections for nickel isotopes of different isospin T. ν and ν' are estimated from (^{3}He, α) reaction data, except where noted.

	$^{58}_{28}Ni_{30}$	$^{60}_{28}Ni_{32}$	$^{62}_{28}Ni_{34}$	$^{64}_{28}N_{36}$
T	1	2	3	4
R_p	1.6	1.8	1.9	2.0
R_p^{exp}	1.0 ± 0.3 [a]	1.7 ± 1.1 [a]	···	···
ν	0.22	0.17	0	0
R_n	2.0	1.8	3.9	3.9
R_n^{exp}	1.6 ± 0.4 [a]	1.1 ± 0.4 [a]	···	···
ν'	$\leqslant$0.20	$\leqslant$0.46	$\leqslant$0.49 [b]	$\leqslant$0.75
R_{up}	$\leqslant$0.03	$\leqslant$0.07	$\leqslant$0.14 [b]	$\leqslant$0.22

[a] See Ref. 7. $T_\pi=220$ MeV. The authors of this work warn us that these ratios may be contaminated by secondary $(n,2n)$ background.
[b] From (p,d) reaction data.

with the data, but the experimental errors are too large to establish the predicted increase in R_p as the neutron excess increases.

In the nickel isotopes the ratio R_n is predicted to change dramatically going from ^{60}Ni to ^{62}Ni. The reason for this is that the analogs of the low-lying states of $^{59}_{27}$Co in $^{59}_{28}$Ni are particle stable. However, the analog of the ground state of $^{61}_{27}$Co in $^{61}_{27}$Ni is about 1.7 MeV above the neutron threshold. We will assume that isospin selection rules and an angular momentum barrier do not significantly inhibit particle decay of this state. Then $\nu=0$ and R_n will be large. Likewise for ^{64}Ni; here the ground state analog is certainly not particle stable, since it is about 5 MeV above the neutron threshold. Unfortunately there are as yet no data available on these two targets to test for this change.

For the zinc isotopes R_n changes less dramatically as the neutron excess increases. This gradual change results from the fact that there are a number of strongly excited $T_0=T+\frac{1}{2}$ states, namely, the $J^\pi=\frac{3}{2}^-$, $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ states. For ^{64}Zn the first few $T_0=T+\frac{1}{2}$ states are stable and the rest are within three MeV above the proton threshold. These states may be inhibited to decay by the Coulomb barrier and hence a range of R_n is given in Table III. For ^{66}Zn only the $J^\pi=\frac{3}{2}^-$ state is below the neutron threshold, and since the other states have large spectroscopic factors, ν is small and therefore R_n is relatively large. For both ^{68,70}Zn all $T_0=T+\frac{1}{2}$ are particle unstable and R_n reaches its maximum value.

The R_{up} ratios are very sensitive to whether or not the $T_0=T-\frac{1}{2}$ states in the $(Z+1,N-2)$ nucleus are particle stable. For all cases these states are above the proton threshold, but again Coulomb inhibition may keep them particle stable. Hence upper limits are given in Tables II and III. For ^{62,64}Ni and ^{68}Zn, these ratios might become particularly large because $\nu=0$ and thus the $\sigma_n^{(+)}$ cross section is small.

The dramatic jump in R_n that is seen in the Ni isotopes (and, not so cleanly, in the Zn isotopes) is in fact a frequent phenomenon for medium weight nuclei. (It sometimes even occurs in light nuclei; see the next section.) This is a *qualitative* feature of the nucleon charge exchange model, and is not likely to be much affected by future refinements in, say, the estimation of the f_i or of P. In heavier nuclei ($A\gtrsim70$) the analog states are particle unstable generally, leading to the general rules that $R_n\approx9/(3-2P)\to4.5$ and $R_p\approx9(1-P)/(3-P)\to1.8$ in the limit of "infinitely large nuclei," $T\gtrsim10$ and $P\to\frac{1}{2}$.

TABLE III. Calculated and observed ratios of pion-induced nucleon knockout cross sections for zinc isotopes of different isospin T. ν and ν' are estimated from (^{3}He, α) reaction data.

	$^{64}_{30}Zn_{34}$	$^{66}_{30}Zn_{36}$	$^{68}_{30}Zn_{38}$	$^{70}_{30}Zn_{40}$
T	2	3	4	5
R_p	1.8	1.9	2.0	2.0
ν	$0.16\sim0.04$	0.03	0	0
R_n	$1.9\sim3.0$	3.1	3.9	3.9
R_n^{exp}	3.5 ± 1.5 [a]	···	···	···
ν'	$\leqslant$0.12	$\leqslant$0.64	$\leqslant$0.76	$\sim$0
R_{up}	$\leqslant$0.02	$\leqslant$0.14	$\leqslant$0.22	$\sim$0

[a] See Ref. 8. $T_\pi=184$ MeV.

D. Oxygen isotopes

To close this section on numerical results, we consider the three isotopes of oxygen. They illustrate each of the cases discussed in the subsections above, $T=0$ (^{16}O), $T=\frac{1}{2}$ (^{17}O), and $T>\frac{1}{2}$ (^{18}O).

We choose P so that R_n agrees with the experimental value[5] for ^{16}O and scale P according to the Appendix assuming, somewhat arbitrarily, that only the separation energy Q varies from one isotope to the next. (It is not clear that β might not also change.) The level schemes[33] establish ν for ^{17}O and ^{18}O, and ν' for ^{17}O. For ^{18}O ν' is found from experimental (p,d) spectroscopic factors.[34] The results are summarized in Table IV.

As in the case of the nickel isotopes, R_n jumps by a factor of 2 in going from ^{16}O to ^{18}O. We also point out, as already indicated in the last section, the $T=1$ ^{18}O target has quite a large R_{up}–some 2 to 7 times bigger than those, say, in Table I. In this case, moreover, the fact that $\nu=0$ even helps, since the denominator is now smaller than 3. The $^{18}O\to{}^{17}F$ cross section is thus predicted to be large, of the order of 3 mb. This situation is

TABLE IV. Calculated ratios of pion-induced nucleon knockout cross sections for oxygen isotopes. The charge exchange probability P is fixed for ^{16}O to fit the experimental ratio (Ref. 5) and calculated for the other isotopes taking into account separation energy differences.

	^{16}O	^{17}O	^{18}O
ν	1	0	0
ν'	0	0	0.55
P	0.20	0.14	0.17
R_n	1.61	3.00	3.38
R_p	1.61	2.24	2.25
R_{up}	0	0	0.070

typical for many light $T=1$ nuclei, up to ^{54}Fe, the last case with a stable $(Z+1, N-2)$ product.

V. SUMMARY AND CONCLUSIONS

We have seen that, with the inclusion of nuclear structure effects on the effective nucleon numbers and the various charge exchange probabilities that enter the model, we can resolve the apparent failures of the simpler nucleon charge exchange model of Ref. 4. Further, the qualitative features of the modified model are striking and can be readily tested:

(1) There is a strong dissimilarity between $T=0$ and $T\neq 0$ targets.[10]
(2) The neutron knockout ratio $R_n=\sigma_n^{(-)}/\sigma_n^{(+)}$ can jump by a factor of 2 between isotopes of the same element.
(3) The proton knockout ratio $R_p=\sigma_p^{(+)}/\sigma_p^{(-)}$ is, on the other hand, largely independent of nuclear structure, increasing slowly and smoothly with increasing target isospin.
(4) For $T=1$ targets, the "knight's move up" process $(Z,N)\to(Z+1,N-2)$ can have a relatively large cross section, $\sigma_{up}\sim 3$ mb. Targets with $T=0$ or $T=\frac{1}{2}$ have much smaller σ_{up}'s. The situation for $T>1$ is variable, depending on the number of stable $T_0=T-\frac{1}{2}$ analog states in $(Z+1,N-2)$.
(5) The corresponding cross sections σ_{down} for $(Z,N)\to(Z-2,N+1)$ are always small (of order 0.1 mb).
(6) Though not generally stressed in this paper, the energy dependence of the charge exchange probability gives a characteristic energy dependence for the ratios which differs considerably from that of the impulse approximation.

Confirmation of these predictions would demonstrate clearly the key role of nucleon charge exchange in pion-induced nucleon knockout reactions.

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APPENDIX: HOW THE SEMICLASSICAL PROBABILITY FOR CHARGE EXCHANGE SCALES

The probability for nucleon charge exchange, as discussed in Ref. 4, is given by

$$P=\tfrac{1}{2}(1-e^{-x}) \ , \tag{A1}$$

$$x=\langle d\rangle\rho_0\sigma_{ex}(\langle T_N\rangle) \ , \tag{A2}$$

where $\langle d\rangle$ is the average path length traversed by the nucleon, and $\rho_0\sigma_{ex}$ is the inverse mean free path for charge exchange. The probability P depends on the size of the nucleus $R=r_0A^{1/3}$, the separation energy Q, and the Pauli inhibition parameter β. We show here how P changes, in first order, for changes in these quantities.

For an equivalent uniform sphere density,

$$\rho_0=3/4\pi r_0^{\,3} \ , \tag{A3}$$

where r_0 is chosen so that the sphere of radius $R=r_0A^{1/3}$ has the same rms radius as that measured in electron scattering. For example, $r_0\approx 1.4$ fm for ^{12}C but, for most heavier nuclei r_0 is more like 1.2 fm.

The average path length is given by

$$\langle d\rangle=\tfrac{4}{3}R[1-\tfrac{3}{2}y+O(y^3)] \ , \tag{A4}$$

where

$$y=\frac{\lambda_{\pi N}}{2R}=\frac{1}{2R\rho_0\sigma_{\pi N}}\leq 0.08 \tag{A5}$$

for energies near the (3,3) resonance. To first order $\langle d\rangle$ scales like R.

Finally, the variation of the charge exchange cross section is calculated as follows. The $np\to pn$ reaction is forward peaked[35] and can be represented well at each incident energy T_N by

$$\frac{d\sigma}{d\Omega}=a-b(1-\cos\theta) \ , \tag{A6}$$

where θ is the center of mass angle of the recoil proton relative to the incident neutron. The a and b are fitted to experiment[35] (and are functions of T_N). In order that the residual nucleus remain bound, we require that the recoil neutron energy

$$T_N'=\tfrac{1}{2}T_N(1-\cos\theta) \tag{A7}$$

be less than the average separation energy

$$Q=(NS_n+ZS_p)/A \ . \tag{A8}$$

That is, the scattering angle θ must be less than θ_{max}, where

$$\cos\theta_{max}=1-\frac{2Q}{T_N} \ . \tag{A9}$$

Then

$$\sigma_{ex} = 2\pi \int_0^{\theta_{max}} \sin\theta d\theta \frac{d\sigma}{d\Omega}$$

$$= \frac{4\pi Q}{T_N}\left(a - \frac{bQ}{T_N}\right), \quad \text{(A10)}$$

the factor of Q having come from the angular cutoff. The contribution of the b term is small.

The above derivation neglected the reduction due to the Pauli principle, however. We can estimate this effect in a zero-temperature Fermi gas model by including in the integrand in Eq. (A10) the factor[36]

$$R(\theta) = \begin{cases} \frac{3}{4} q_L (1 - q_L^2/12k_F^2), & q_L \leq 2k_F \\ 1, & q_L > 2k_F, \end{cases} \quad \text{(A11)}$$

where k_F is the Fermi momentum and q_L is the laboratory momentum transfer. For nonrelativistic scattering of equal mass particles,

$$q_L \approx p_L \sin\tfrac{1}{2}\theta, \quad p_L = (2mT_N)^{1/2}. \quad \text{(A12)}$$

Using $\sin\frac{1}{2}\theta$ as the integration variable one finds (for the case that q_L is always less than $2k_F$),

$$\sigma_{ex} = \beta \frac{4\pi Q}{T_N}\left(a - \frac{6bQ}{5T_N} - \frac{a\beta^2}{5} + \frac{2b\beta^2 Q}{7T_N}\right), \quad \text{(A13)}$$

where

$$\beta = (mQ/2k_F^2)^{1/2} \approx 0.3. \quad \text{(A14)}$$

Again, the terms in the bracket other than a are small, so we see the major effect of the Pauli principle is to reduce the cross section σ_{ex} given by Eq. (A10) by a factor of β. Of course, for incident energies such that q_L can exceed $2k_F$, the reduction will be less; as $k_F \to 0$ we must recover Eq. (A10). A more realistic estimate of σ_{ex} would involve a diffuse nuclear system in which there is a variation of nuclear density and Fermi energy. Because of all these complications we adopt for simplicity

$$\sigma_{ex} = \beta \frac{4\pi Q}{T_N}\left(a - \frac{bQ}{T_N}\right) \quad \text{(A15)}$$

and treat β as a parameter. In Ref. 4 it was chosen so that R_n fit experiment at $T_\pi = 180$ MeV; its value lay between the extremes of $\beta = 1$ (no Pauli principle) and $\beta \approx 0.3$ (zero-temperature Fermi gas).

The cross section σ_{ex} is to be evaluated at an average nucleon recoil energy given by

$$\langle T_N \rangle = (q_{\pi N}^2/m)[1 + t + O(t^2)], \quad \text{(A16)}$$

where $q_{\pi N}$ is the center of mass pion momentum in the πN system and

$$t = mQ/q_{\pi N}^2. \quad \text{(A17)}$$

For $Q \leq 10$ MeV, $t \leq 0.18$, so, to first order, $\langle T_N \rangle \approx \frac{1}{3} T_\pi$ is independent of Q. Further, the term $bQ/\langle T_N \rangle$ is small compared with a. Thus $\sigma_{ex}(\langle T_N \rangle)$ varies, for T_π fixed and near resonance, like βQ.

Combining all these factors, then,

$$x = \beta Q A^{1/3}/r_0^2 \times \text{constant}. \quad \text{(A18)}$$

Note that x does not depend much on whether the nucleon charge exchange occurs deep within the nucleus or in its periphery (where the density may be much smaller than ρ_0). In the thin-density region, r_0 is effectively larger, while β increases to its free-space value of 1. Thus β/r_0^2 is unchanged if r_0 is increased by a factor of $(0.3)^{-1/2} \approx 1.8$, which in turn corresponds to a reduction in density by a factor of about 6. Since β and r_0 are so clearly linked in x, we prefer to consider x as the parameter to be determined for each nucleus. Our intent here in presenting the variation of P (through x) on the various quantities involved—, Q, A, and r_0—is to be able to estimate what a reasonable change in P would be as the atomic mass changes.

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