Relativistic effects in a one-nucleon model for proton induced pion production

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Pion production near threshold is investigated in a simple relativistic one-nucleon model, using the pseudoscalar πN interaction operator and four-component wave functions both for the projectile and for the description of bound nucleons. Compared to a purely nonrelativistic treatment, based on the Galilean invariant πN interaction, the differential cross section in this model exhibits significant differences, even in the most simple approximation.

[NUCLEAR REACTIONS Relativistic effects in (p,π) near threshold.]

Recently, the problem of the nonrelativistic reduction of the pseudoscalar πN interaction operator has been investigated by several authors.[1-5] The starting point for these investigations came mainly from the study of the recoil-free proton induced pion production on light nuclei near threshold (for example, compare Ref. 6): under specific kinematical situations, particularly for π emission in the forward direction, there is some hope of testing various approximations for the nonrelativistic πN interaction, such as the importance of higher order nonstatic contributions to the πN vertex. The main criticism of a nonrelativistic model is twofold: firstly, the nonrelativistic treatment of the typically relativistic pseudoscalar πN interaction operator is certainly not adequate for all kinematical situations; secondly, it is probably also questionable to treat wave functions of bound nucleons in a purely nonrelativistic fashion, especially in processes involving high momentum transfer of typically 500 MeV/c or more; under such kinematical conditions, the small component of a relativistic wave function is no longer significantly suppressed, since the relation $k/E \ll 1$ is no longer valid.

Of course, it is well known from various investigations of proton induced pion production that the pure one-nucleon model (ONM) cannot describe adequately typical features of the angular distributions of (p,π) processes. On the contrary, microscopic calculations point out that the production mechanism near threshold involves dominantly two nucleons: the momentum, transferred to the residual nucleus, is shared between the projectile and one bound nucleon by π and ρ meson rescattering (a brief survey of different versions of the one- and two-nucleon model can be found in Refs. 7 and 8). The only chance to separate the contribution of the ONM from the generally dominating background seems possible only for a special kinematical situation: for pion emission in forward direction the momentum transferred on the residual nucleus in (p,π) processes near threshold (for $m_\pi \lesssim T_p \lesssim 200$ MeV) is an extremely slowly varying function of the projectile energy; the two-nucleon model, therefore, predicts a very smooth variation of, for example, $d\sigma/d\Omega$ $(\theta_\pi = 0°)$ as a function of T_p. In contrast to that, the contribution from the ONM is expected to be very sensitive to a slight variation of the kinematics and should be a strongly varying function of the proton energy. Thus, a significant variation of the forward production cross section near threshold possibly allows one to extract information about the importance of the one-nucleon model.

From these arguments it seems worthwhile to investigate relativistic effects in proton induced pion production near threshold even on the basis of a very crude relativistic model for the generation of bound state nuclear wave functions. The transition amplitude in the differential cross section for pion production on a closed shell target nucleus

$$d\sigma = \frac{1}{(2\pi)^2}\frac{1}{V}\frac{1}{2}\sum_{fi}|T_{fi}|^2\,\delta(E_f - E_i)\delta(\vec{k}_f - \vec{k}_i)d\vec{k}_\pi d\vec{k}_{rec} \tag{1}$$

is given by the expression

$$T_{fi} = \langle 0^+; nljm_j | H_{\pi N} | \vec{k}_p\mu; 0^+\rangle, \tag{2}$$

where $|0^+\rangle$ denotes the target nucleus; $|\vec{k}_p\mu\rangle$ and $|nljm_j\rangle$ respectively, characterize the projectile in the initial state and the bound single particle state which is populated by the projectile in the production process:

$$|k_p\mu\rangle = \left(\frac{E_p + m_p}{2m_p}\right)^{1/2}\begin{bmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{k}_p}{E_p + m_p}\end{bmatrix} e^{i\vec{k}_p\cdot\vec{r}}\,|\tfrac{1}{2}\mu\rangle \tag{3}$$

and

$$|nljm_j\rangle = \begin{bmatrix} \dfrac{ig_{nlj}(r)}{r} \\ & |ljm_j\rangle \\ \dfrac{f_{nlj}(r)}{r}(\vec{\sigma}\cdot\hat{\vec{r}}) \end{bmatrix}. \tag{4}$$

Here the angular functions $|ljm_j\rangle$ are defined by

$$|ljm_j\rangle \equiv Y_{ljm_j} = \sum_{m_l m_s} \langle lm_l \tfrac{1}{2}m_s | jm_j\rangle |lm_l\rangle |\tfrac{1}{2}m_s\rangle, \tag{5}$$

while $g_{nlj}(r)$ and $f_{nlj}(r)$ denote the radial behavior of the large and small component in the nuclear wave function, respectively. Equation (4) clearly indicates that the large and the small component have opposite parity. For the production of positive pions the pseudoscalar πN interaction operator is given by

$$H_{\pi N} = ig_\pi \gamma_0 \gamma_5 \frac{1}{\sqrt{E_\pi}} e^{i\vec{k}_\pi\cdot\vec{r}}, \tag{6}$$

with $g_\pi^2/4\pi = 14.6$. With those relations the nuclear transition amplitude is given by

$$T_{fi} = -\frac{g_\pi}{\sqrt{E_\pi}} \frac{E_p+m_p}{2m_p} \sum_\mu \int e^{i\vec{q}\vec{r}} d\vec{r}\, \{ig_{nlj}(r) Y^*_{ljm_j}(\hat{\vec{r}})(\sigma k_p/E_p+m_p)X_{1/2\,\mu} - f_{nlj}(r)(\vec{\sigma}\hat{\vec{r}})Y_{ljm_j}(\hat{\vec{r}})X_{1/2\,\mu}\}, \tag{7}$$

where $\vec{q} = \vec{k}_p - \vec{k}_\pi$ denotes the momentum transfer on the target nucleus. Choosing $\vec{k}_p$ parallel to the z axis, the total transition amplitude is easily evaluated; the final result for the differential cross section then turns out to be

$$\frac{d\sigma}{d\Omega_\pi} = \frac{1}{2\pi} \frac{E_p k_\pi}{k_p} (2j+1) g_\pi^2 \frac{E_p+m_p}{2m_p} \times \left[\left(\frac{k_p}{E_p+m_p}\right)^2 I_g^2 + I_f^2 + (-1)^l 4\sqrt{6}\,[j(j+1)]^{1/2} \frac{k_p}{E_p+m_p} \begin{pmatrix} j+\frac{1}{2} & j-\frac{1}{2} & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ j-\frac{1}{2} & j+\frac{1}{2} & j \end{Bmatrix} I_g I_f (\hat{\vec{q}}\cdot\hat{\vec{k}}_p) \right], \tag{8}$$

with the radial integrals defined by

$$I_g(q) = \int g_{nlj}(r) j_l(qr) r\, dr, \tag{9a}$$

$$I_f(q) = \int f_{nlj}(r) j_\lambda(qr) r\, dr, \tag{9b}$$

where λ is defined by $\lambda = l \pm 1$ for $j = l \pm \frac{1}{2}$.

The main problem remaining is the determination of the large and small component of the bound state wave function $|nljm_j\rangle$ of Eq. (4). Since a completely relativistic description of the nucleus is not available, we use the following simple model: we assume that the ratio of the large to the small components in nuclear wave functions in momentum space is strictly determined by the free spinor at the energy of the bound nucleon for all momentum components, i.e.,

$$|nljm_j\rangle = \frac{1}{(2\pi)^{3/2}} \int \phi_{nlj}(\vec{k}) |ljm_j\rangle \begin{pmatrix} 1 \\ \dfrac{\vec{\sigma}\cdot\vec{k}}{E_b+m_p} \end{pmatrix} e^{i\vec{k}\vec{r}} d\vec{k}, \tag{10a}$$

with the normalization condition

$$\int \phi_{nlj}^2(k) \left[1 + \left(\frac{k}{E_b+m_p}\right)^2\right] k^2 dk = 1, \tag{10b}$$

where the behavior of $\phi_{nlj}(k)$, especially at high momentum components, is determined by fits to electron scattering. The total energy of the bound nucleon E_b can be chosen in different ways. One possibility is to assume that E_b is given by

$$E_b = m_N - |E_B|, \tag{11a}$$

where E_B denotes the binding energy of the bound nucleon in the nuclear state $|nlj\rangle$. On the other side, the momentum q of the bound nucleon is far above the Fermi momentum, i.e., $q \gg k_F$; thus binding corrections should play a minor role and E_b would be given in that approximation by

$$E_b = (q^2 + m_N^2)^{1/2} \tag{11b}$$

which is roughly equal to the total energy of the projectile at threshold.

To get an idea for the effects resulting from a relativistic treatment of the nucleus and of the πN interaction, the differential cross section [Eq. (8)] is compared with the cross section derived from the so called Galilean invariant πN interaction, conventionally used in calculations:

$$H_{\pi N} = i\frac{g_\pi}{2m_p} \vec{\sigma}_N \cdot \left(\vec{\nabla}_\pi - \alpha \frac{m_\pi}{m_N} \vec{\nabla}_N\right) (\vec{\tau}_N \vec{\phi}_\pi) \tag{12}$$

in the static and nonstatic limits (i.e., $\alpha = 0$ and $\alpha = 1$), respectively. For both cases, the structure of the transition amplitude is very simple:

$$T_{fi} \alpha \frac{g_\pi}{\sqrt{E_\pi}} \langle nljm_j | \vec{\sigma} \cdot \vec{Q} | \vec{k}_p \mu \rangle. \tag{13}$$

The differential cross section is obtained in a straightforward way:

$$\frac{d\sigma}{d\Omega_\pi} = \frac{1}{2\pi} \frac{E_p k_\pi}{k_p} \left(\frac{g_\pi}{2m_p} \right)^2 (2j+1) Q^2 I_{nlj}{}^2(q), \tag{14a}$$

where $I_{nlj}(q)$ denotes the radial integral over the nonrelativistic bound state wave function $R_{nlj}(r)$,

$$I_{nlj}(q) = \int R_{nlj}(r) j_l(qr) r^2 dr, \tag{14b}$$

while Q^2 is given by

$$Q^2 = \left(\vec{k}_\pi - \alpha \frac{m_\pi}{m_N} \vec{k}_p \right)^2. \tag{15}$$

With the relations derived above, the differential cross section for $^{16}O(p,\pi^+)^{17}O$ has been calculated for different proton energies near π production threshold and for the excitation of different single particle levels in ^{17}O. For a comparison with experimental data the choice of the potential parameters for bound single particle wave functions is crucial. In our calculation the parameters were fixed by starting from a Woods-Saxon potential with $R = 1.25A^{1/3}$, a surface parameter $a = 0.60$ fm, and a spin orbit term $V_{so} = 4.46$ MeV.[9] The central depth was fixed from a fit to the experimentally known binding energies; furthermore, the form factor generated by those wave functions was compared with the results from elastic electron scattering. One result of such a calculation is shown in Fig. 1 for the excitation of the $\frac{5}{2}^+$ level in ^{17}O for

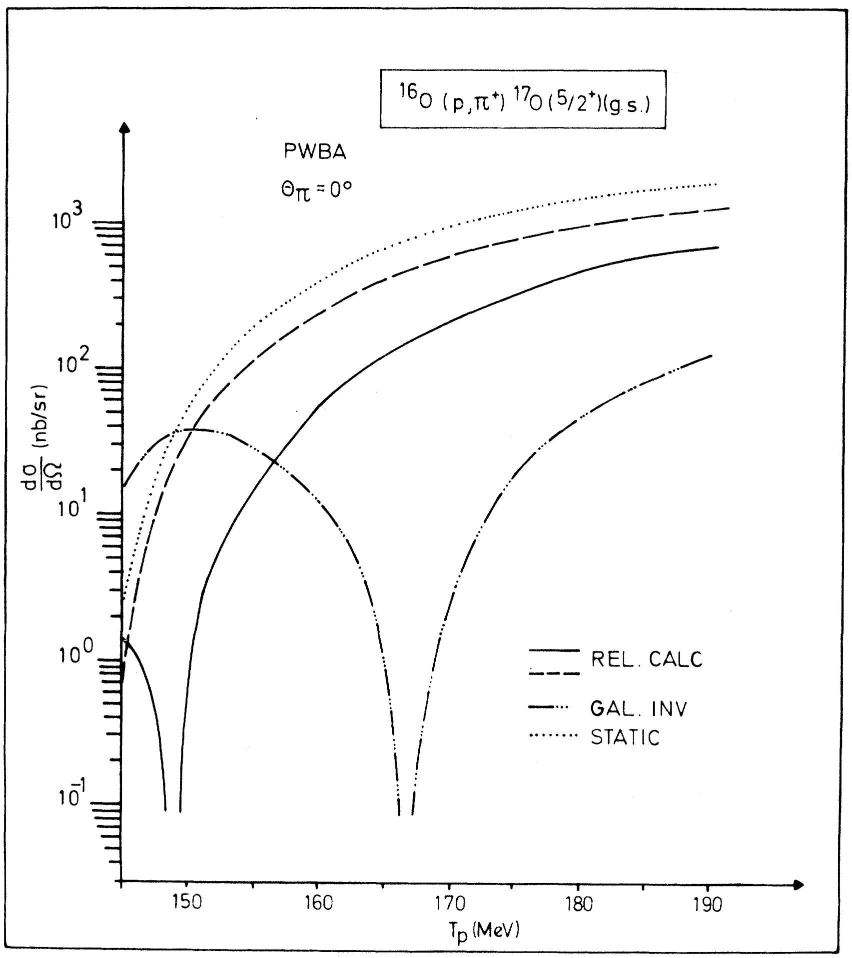

FIG. 1. Differential cross section for the reaction $^{16}O(p,\pi^+)^{17}O(\frac{5}{2}^+)_{g.s.}$ for forward pion emission ($\Theta_\pi = 0°$) as a function of the projectile energy. The full and the dashed lines (—— and ----) correspond to the relativistically derived cross sections [see Eq. (8)] for two different ratios of large to small components in the bound state nuclear wave function [see Eqs. (11a) and (11b), respectively]. The dashed-dotted (--- •••) and dotted lines (•••) represent the cross section calculated with the Galilean invariant interaction πN [Eq. (15)] in the static and nonstatic limit (i.e., for $\alpha = 0$ and $\alpha = 1$), respectively.

pion emission in the forward direction ($\theta_\pi = 0°$) at different projectile energies. From Fig. 1 it follows that the Galilean invariant πN interaction gives a strong reduction of the cross section in forward direction in contrast to the relativistic derivation from Eqs. (8) and (11a), where the cancellation in the forward direction is much less drastic; the increase is around one order of magnitude. This effect turns out to be even stronger for the normalization of the large to small components according to Eq. (11b), which nearly corresponds to the static limit in the nonrelativistic case. Furthermore, a comparison with the experimental cross section of typically 10 to 100 nb/sr [6] indicates that without the strong reduction due to the Galilean invariance of the πN interaction the single nucleon contribution may well be of the order of the experiment at least around a momentum transfer of $q \leq 500$ MeV/c, so that π production near threshold in the forward direction may be a test both for the importance of the one-nucleon model (ONM) as well as for the influence of relativistic effects. Of course, for a more definite result a systematic investigation of the ONM is necessary (for example, of the uncertainties resulting from the parametrization of the nuclear potential, center-of-mass corrections,[10] etc.). Such a test necessarily has to be performed in the frame of a relativistic theory. All investigations[1-5] performed so far are based on the description of a bound nucleon by the Dirac equation; they point out that the nonrelativistic reduction depends critically on the type of single particle potential, i.e., whether it is assumed to be either a scalar or the fourth component of a vector field; an interaction, however, derived from meson theory, is necessarily built up from pieces with different transformation properties, i.e.:

$$(\not{p} + m_N + g_s V_s + g_{ps}\gamma_5 V_{ps} + g_V(\gamma_\mu V_v^\mu + \gamma_\mu \gamma_5 V_v'^\mu) + g_T j_\mu j_v V_t^{\mu v})\psi = E\psi, \quad (16)$$

where s, ps, v, and t denote the scalar, pseudoscalar, vector, and tensor part of the single particle potential. Hopefully, the generation of a bound state wave function from such an equation as a generalization of the nonrelativistic Hartree-Fock procedure[11,12] might give an idea of relativistic effects in high momentum transfer reactions.

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