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Mixing of doorway states via fine-structure states and possible application to the study of fission isomers

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The coupling of two doorway states via common fine-structure states is calculated numerically in a multichannel model using the K matrix. As an example of application the population of isomeric fission states via isobaric analog resonances is discussed.

[NUCLEAR REACTIONS Numerical multichannel model study, discussion of application.]

I. INTRODUCTION

Some nuclear states are populated in nuclear reactions with very low cross sections because the overlap of these states with the target ground state is very low. An example is the isomeric fission states[1] where the cause of the small overlap is the big difference in deformation of the target ground state and the isomeric fission states. This fact makes quantitative spectroscopic work concerning these states very difficult. To outline that this difficulty can possibly be overcome by the use of analog states is the purpose of the present note. To be definite we will take isomeric fission states in the actinide region as an example.

Consider (Fig. 1) a target nucleus with isospin T_0, atomic number A, and charge Z and having a double humped barrier. The states of low and strong deformation are denoted by T_1 and T_2, respectively. A (d,p) reaction will populate strongly a state P_1 in the first well of the final nucleus, whereas the transition to the state P_2 in the second well will be strongly inhibited. The analogs of the parent states P_1 and P_2 are denoted by A_1 and A_2, respectively. The partial proton widths will normally obey the same rules as the (d,p) reaction. This means, the state A_1 will be strongly populated by the entrance channel but the state A_1 will have a very small partial width (for simplicity we will put it equal to zero in the following discussion) for the deexcitation to the state T_2. The inverse holds for the state A_2. Therefore, the (p,p') reaction cross section leading to the state T_2 should be zero.

The states in the second well are at 2–3 MeV [1] above ground state, and the width of analog states is expected to be 300 keV.[2] Therefore, it will be a quite common situation that there are some states A_1 and A_2 having the same spin and parity and at a distance less than the total width. Then these two can possibly mix forming a compound nucleus state having some part of both components and the reaction cross section leading to the state T_2 could attain appreciable values. An experimental example of such a mixing has been found in ^{139}La(p,p').[3] If one uses a two level, two channel formula[4] to calculate the cross section the result is zero; that means that there is no mixing in this approximation. If there are more open channels (e.g., neutron channels) in the formula of Ref. 4 a mixing arises from a coupling of two states by *common* open channels. But this term involves a coherent sum over products of square roots of partial widths which will have fluctuating signs; therefore, the total is expected to be very small. Thus from these considerations one expects the mixing of the two states to be very small even if the two states are largely overlapping. But these considerations neglect an important feature of analog states.

II. MODEL

Analog states are intermediate structures[5]; they couple to the great number of $T_<$ states which they are imbedded in. The total width Γ_T of the analog states can be split into two parts[5]

$$\Gamma_T = \Gamma^{\uparrow} + \Gamma^{\downarrow}, \tag{1}$$

where $\Gamma^{\uparrow}$ is the sum over all proton partial widths and $\Gamma^{\downarrow}$ is the spreading width of the analog state over the $T_<$ states. We assume here weak absorption in all proton channels. If absorption is not weak, the proton partial widths in (1) must be corrected for absorption.[5] To see the influence of the fine structure states on the problem of two overlapping states we made a model calculation using the methods of Ref. 6.

Consider an unperturbed Hamiltonian H_0 with

$$H_0|A_i\rangle = E_i|A_i\rangle, \quad i=1,2,$$
$$H_0|3,n\rangle = E_n|3,n\rangle, \quad n=3,N, \tag{2}$$

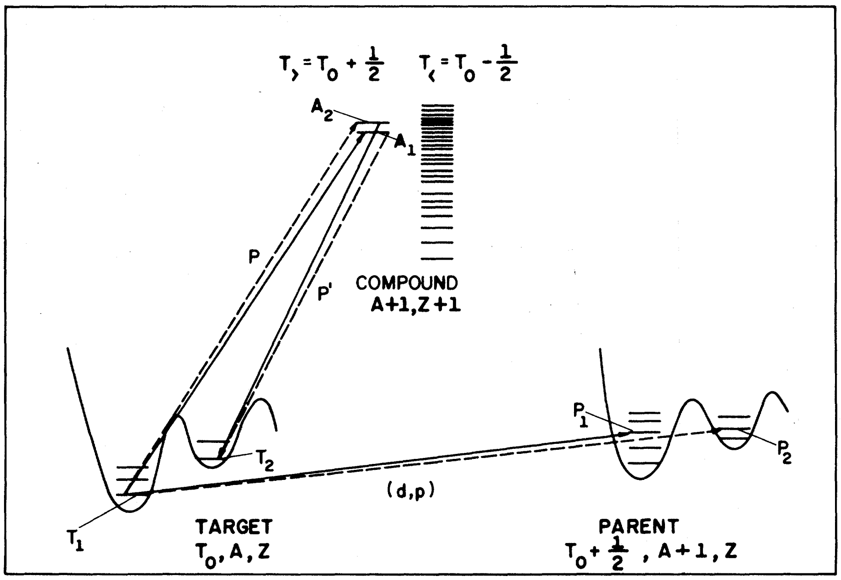

FIG. 1. Illustration of the relation of target, parent, and compound nucleus for nuclei having a double humped barrier. A_1 and A_2 are the analogs of the parent states P_1 and P_2, respectively. Normal transitions are indicated by continuous lines, strongly inhibited transitions by broken lines.

where $|A_1\rangle$ and $|A_2\rangle$ are the analog states and $|3,n\rangle$ are the $T_<$ states of the fine structure. Then we introduce a perturbation ΔV which couples the states $|A_1\rangle$ and $|A_2\rangle$ to the $T_<$ states with the only nonzero matrix elements

$$\langle A_i|\Delta V|3,n\rangle = M_{i3}(n), \quad n=3,\ldots,N; \quad i=1,2. \tag{3}$$

Then the N eigenvalues and eigenvectors of the Hamiltonian $H=H_0+\Delta V$ are denoted by $\mathcal{E}_\mu$ and $|\psi_\mu\rangle$ where

$$|\psi_\mu\rangle = \alpha_1^\mu|A_1\rangle + \alpha_2^\mu|A_2\rangle + \sum_{n=3}^{N}\alpha_n^\mu|3,n\rangle. \tag{4}$$

The K matrix is a real matrix defined by

$$K_{cc'} = \frac{1}{2\pi}\sum_{\mu=1}^{N}\gamma_c^\mu\gamma_{c'}^\mu/(E-\mathcal{E}_\mu). \tag{5}$$

We take the overlap of the channels T_1, T_2 with the $T_<$ states to be zero, as of T_1 with A_2 and of T_2 with A_1 and the widths are

$$\gamma_1^\mu = \langle T_1\otimes\chi_p(E_1)|V_c|\psi_\mu\rangle = \alpha_1^\mu\gamma_1'\langle T_1|A_1\rangle = \alpha_1^\mu\gamma_1, \tag{6a}$$

$$\gamma_2^\mu = \langle T_2\otimes\chi_p(E_2)|V_c|\psi_\mu\rangle = \alpha_2^\mu\gamma_2'\langle T_2|A_2\rangle = \alpha_2^\mu\gamma_2, \tag{6b}$$

where $\chi_p(E)$ is the scattering wave function of the proton, V_c the interaction which couples the analog state to the continuum, and $\gamma_{1,2}'$ and $\gamma_{1,2}$ the single particle width and the single particle width multiplied by the spectroscopic amplitude of the analog state $A_{1,2}$ with respect to the channel $T_{1,2}$. For simplicity we put $\gamma_1=\gamma_2$ in most of the calculations.

The $T_<$ states $|3,n\rangle$ will deexcite in general to a great number of neutron and fission channels. For simplicity of language we will not make a distinction between neutron and fission channels and the expression neutron channels will include possible fission and other reaction channels other than proton channels. Denoting these channels by $\langle c_i|$ and neglecting direct coupling to the analog states one can write the partial widths in these channels

$$\begin{aligned}\gamma_{c_i}^\mu &= \langle c_i\otimes\chi_{c_i}(E_{c_i})|V_c|\psi_\mu\rangle\\ &= \sum_{n=3}^{N}\alpha_n^\mu\langle c_i\otimes\chi_{c_i}(E_i)|V_c|3,n\rangle\\ &= \sum_{n=3}^{N}\alpha_n^\mu\gamma_{c_i,n}.\end{aligned} \tag{7}$$

The partial widths $\gamma_{c_i,n}$ will in general fluctuate in magnitude and sign. In the context here it is sufficient to retain the fluctuating sign and thus $\gamma_{c_i,n}=\operatorname{sgn}(c_i,n)\gamma_3$ where $\operatorname{sgn}(c_i,n)$ is a computer generated random number. Because of the fluctuating sign one can write

$$\begin{aligned}\gamma_{c_i}^\mu &= \sum_{n=3}^{N}\alpha_n^\mu\operatorname{sgn}(c_i,n)\gamma_3\\ &= (1-\alpha_1^{\mu 2}-\alpha_2^{\mu 2})^{1/2}\operatorname{sgn}(c_i,\mu)\gamma_3.\end{aligned} \tag{8}$$

Up to 10 neutron channels were included in the calculation. The S matrix is obtained by the relation

$$S=(1-i\pi K)(1+i\pi K)^{-1}. \tag{9}$$

Omitting the direct part of the cross section and well known geometrical factors,[4] the cross section is given by

$$\sigma_{cc'}=|1-S|^2. \tag{10}$$

Because of the complicated analytical structure of the S matrix (9) no analytical evaluation of (10) seemed possible whereas a numerical calculation is quite straightforward. A FORTRAN program has been written for the IBM 360/44 of the Instituto de Física, São Paulo. It was useful to calculate the resonance integral defined by

$$R_{cc'}=\int_{-\infty}^{+\infty}\sigma_{cc'}(E)\,dE \tag{11}$$

because this integral is less than $\sigma_{cc'}(E)$ subject to fluctuations introduced by the use of random numbers (see Fig. 2). Therefore, the resonance integral more easily permits study of the model.

In numerical calculation one can integrate only over a limited domain and one has to correct for this effect. This has been estimated in the following way. We suppose that the behavior of $\langle\sigma_{cc'}\rangle$ can be approximated by a Breit-Wigner form with a width Γ_T. Then one can write

$$\begin{aligned} C&=\int_{-\infty}^{+\infty}\sigma_{cc'}\,dE\Big/\int_{E'}^{E''}\sigma_{cc'}\,dE\\ &\simeq\pi/\{\arctan[2(E''-E_R)/\Gamma_T]\\ &\quad-\arctan[2(E'-E_R)/\Gamma_T]\}. \end{aligned} \tag{12}$$

Because the description by a Breit-Wigner form of $\langle\sigma_{cc'}\rangle$ is only approximative, C should not be very different from one, to keep the correction low. In the results presented the correction used was less than 10%.

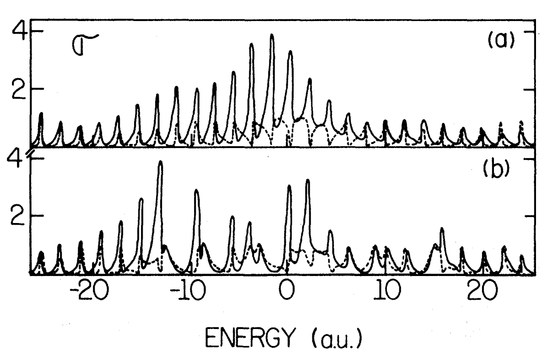

FIG. 2. Calculation of the cross section as a function of energy (arbitrary units) for $E_1=-2$, $E_2=2$, and $N=52$, spacing of the $T_<$ states $d=2$, $M=2.5$, $\gamma_1=\gamma_2=3$, and $\gamma_3=0$. The continuous line corresponds to $\sigma_{T_1T_1}$, the broken line to $\sigma_{T_1T_2}$. For clarity the cross section $\sigma_{T_2T_2}$ which is very similar to $\sigma_{T_1T_1}$, is not shown. (a) and (b) correspond to the statistical assumptions of Eqs. (24a) and (24b), respectively.

For the discussion of the results of the model we need some formulas related to the resonance integral that we will derive here. If the cross section has a Breit-Wigner form one gets

$$R_{cc'}=\int\frac{\Gamma_c\Gamma_{c'}}{(E-E_R)^2+(\frac{1}{2}\Gamma)^2}\,dE=2\pi\Gamma_c\Gamma_{c'}/\Gamma_T. \tag{13}$$

For the total cross section in a channel c one has[4,5] using (10) and the unitarity of the S matrix

$$\sigma_{c,\text{total}}=\sum_{c'}\sigma_{c,c'}=2[1-\text{Re}(S_{cc})], \tag{14a}$$

$$\langle\sigma_{c,\text{total}}\rangle=2[1-\langle\text{Re}(S_{cc})\rangle]. \tag{14b}$$

Therefore, if $\langle S_{cc}\rangle$ has a Breit-Wigner form as has been obtained[5,6] for channels coupled directly to a single doorway state one has

$$\langle\sigma_{c,\text{total}}\rangle=\frac{\Gamma_c\Gamma_T}{(E-E_R)^2+(\frac{1}{2}\Gamma_T)^2} \tag{15}$$

independent of the form of $\langle S_{cc'}\rangle$ with $c'\neq c$ and the fluctuating part of S_{cc}. The resonance integral of the total cross section is then

$$R_{c,\text{total}}=2\int[1-\langle\text{Re}(S_{cc})\rangle]\,dE=2\pi\Gamma_c. \tag{16}$$

We can separate S in a smooth part and a fluctuating part

$$S_{cc'}=\langle S_{cc'}\rangle+S^{\text{fl}}_{cc'} \tag{17}$$

and therefore

$$\sigma_{cc'}=|\delta_{cc'}-\langle S_{cc'}\rangle|^2+|S_{\text{fl}}|^2=\sigma^{\text{se}}_{cc'}+\sigma^{\text{fl}}_{cc'} \tag{18}$$

and

$$\sigma_{cc'}\geqslant\sigma^{\text{se}}_{cc'}. \tag{19}$$

Here $\sigma^{\text{se}}_{cc'}$ and $\sigma^{\text{fl}}_{cc'}$ are the shape elastic and fluctuating part of the cross section, respectively. For channels that are directly coupled to the doorway state (proton channels) $\langle S_{cc'}\rangle$ has a Breit-Wigner form[5] and

$$R^{\text{se}}_{cc'}=\int(\sigma_{cc'}-\sigma^{\text{fl}}_{cc'})\,dE=2\pi\Gamma_c\Gamma_{c'}/\Gamma_T. \tag{20}$$

Using $\Gamma^\dagger=\sum\Gamma_{c_p}$ where the sum is over all channels directly coupled to the doorway denoted by c_p one gets

$$\sum_{c_p}R^{\text{se}}_{cc_p}=2\pi\Gamma_c\Big(\sum_{c_p}\Gamma_c\Big)\Big/\Gamma_T=2\pi\Gamma_c\Gamma^\dagger/\Gamma_T. \tag{21}$$

The mean branching ratio B_n to channels that are not directly coupled to the doorway state denoted by c_n can be obtained by

$$B_n = \sum_{c_n} R_{cc_n} \Big/ \sum_{c'} R_{cc'} = \Big(R_{c,\text{total}} - \sum_{c_p} R_{cc_p}\Big) \Big/ R_{c,\text{total}} \,. \qquad (22)$$

Using (16), (19), (21), and $\Gamma_T = \Gamma^{\uparrow} + \Gamma^{\downarrow}$ the following inequality is obtained

$$B_n \leq \frac{\Gamma_c - \Gamma_c \Gamma^{\uparrow}/\Gamma_T}{\Gamma_c} = \frac{\Gamma^{\downarrow}}{\Gamma_T} \,. \qquad (23)$$

The equality will arrive when $\sigma^{\text{fl}}_{c,c_p} = 0$; the fluctuating part of the cross section is zero. A more detailed discussion of these formulas will be published elsewhere.

The numerical evaluation of (2)–(12) is quite straightforward. However, one must pay attention to construct the model in such a way that it contains all important physical features. We supposed the $T_<$ states to be equally spaced. This should not be a critical point.

The spreading of the states A_1 and A_2 over the $T_<$ states is governed by the matrix elements $M_{13}(n)$ and $M_{23}(n)$. Thus the mixing of A_1 and A_2 is determined by these matrix elements and these have therefore to be considered with care.

Isomeric fission states in the actinide region are at 2–3 MeV above ground state.[1] This situates the analog states at around 20–25 MeV excitation energy in the compound nucleus. At this excitation energy the density of $T_<$ states belonging to the first and second well will be about the same and the two configurations should be completely mixed, as shown by the presence of intermediate structures in subthreshold (n,f) reactions.[7] Thus one can expect that $\overline{|M_{13}|} = \overline{|M_{23}|}$ where the bar indicates the mean value. Because of the complicated nature of the $T_<$ states one expects that there is no correlation between M_{13} and M_{23} and thus $\overline{|M_{13} M_{23}|} = \overline{|M_{13}|}\,\overline{|M_{23}|}$. We made use of different statistical assumptions

$$M_{i3}(n) = M \,, \quad n = 3, \ldots, N \,; \quad i = 1, 2 \,, \qquad (24a)$$

$$M_{i3}(n) = \text{sgn}(n,i) M \,, \quad \text{sgn}(n,i) = \pm 1 \text{ random} \,, \qquad (24b)$$

$$M_{i3}(n) = X(n,i) M \,, \quad -2 \leq X \leq 2 \text{ random} \,. \qquad (24c)$$

The spreading width of Eq. (1) is related[5] to the matrix element M by the equation

$$\Gamma^{\downarrow} = 2\pi |M|^2 / d \,, \qquad (25)$$

where d is the distance between the fine structure states.

We did not take into account the absorptive part of the off-resonance scattering. In the model here this is represented by neglecting direct coupling of the $T_<$ states to the proton channels. We think that at least moderate absorption will change only quantitatively the results obtained here. The most important effect will be that one has to correct the partial widths and the sum rule (1) for absorption in a way similar to Ref. 5.

Numerical calculations were made for the statistical assumptions (24) and for various values for the spacing d of the $T_<$ states, the matrix element M, and the widths $\gamma_{1,2}$. The ratio γ_3/d was varied between 0 and 50, the last value corresponding to largely overlapping fine structure states. Up to 100 fine structure states were taken into account.

III. RESULTS

First we want to discuss the case when no neutron channels are open. A typical result for the cross sections is shown in Fig. 2. With no neutron channels open, it was found that the different statistical assumptions (24a)–(24c) gave the same result for the resonance integrals within the precision of the calculation (precision limited due to the fluctuation introduced by the use of random numbers). The result for different cases could be well parametrized by the formula for the resonance integral of the cross section leading from T_1 to T_2

$$R_{T_1,T_2} = 4\pi \frac{\Gamma_1 \Gamma_2}{\Gamma_1 + \Gamma_2} \, \frac{m}{1+m} \qquad (26a)$$

with

$$m = \frac{\Gamma^{\downarrow}}{\frac{1}{2}(\Gamma_1 + \Gamma_2) + \Gamma^{\downarrow}} \, \frac{\Gamma^{\downarrow 2}}{(E_1 - E_2)^2 + \Gamma^{\downarrow 2}} \,, \qquad (26b)$$

where $\Gamma^{\downarrow}$ is given by (25). The value of m expresses the mixing between the two analog states. The energy averaged cross section was found to have to a good approximation a Breit-Wigner form, and we can therefore write

$$\sigma_{T_1 T_2} = \frac{\Gamma_1 \Gamma_2}{(E - E_R)^2 + (\frac{1}{2}\Gamma_T)^2} \, \frac{\Gamma_T}{\frac{1}{2}(\Gamma_1 + \Gamma_2)} \, \frac{m}{1+m} \,, \qquad (27)$$

where $\Gamma_T = \Gamma^{\downarrow} + \frac{1}{2}(\Gamma_1 + \Gamma_2)$ and $E_R = \frac{1}{2}(E_1 + E_2)$. For analog states one has typically $\Gamma^{\downarrow}/\Gamma_T \simeq \frac{1}{2}$ (see discussion). Therefore, in the case when no neutron channels are open and $|E_1 - E_2| < \Gamma^{\downarrow}$, the mixing probability is very strong, and the cross section leading from T_1 to T_2 will be very strong too.

Before discussing the influence of the neutron channels on the mixing of two analog states, we want to consider a single analog state A_1. In Fig. 3, the resonance integrals $R_{T_1 T_2}$, $\sum_{c_n} R_{T_1,c_n}$, and $R_{T_1,\text{total}}$ are shown as a function of the number of open neutron channels ($\Gamma_3/d \gg 1$). The result for $R_{T_2,\text{total}}$ is in agreement with (16) and independent of the number N_n of open neutron channels. For $N_n = 0$ one has $R_{T_1,T_1} = 2\pi\Gamma_1$ and for a big number of open channels one has the asymptotic value $R_{T_1 T_1}$

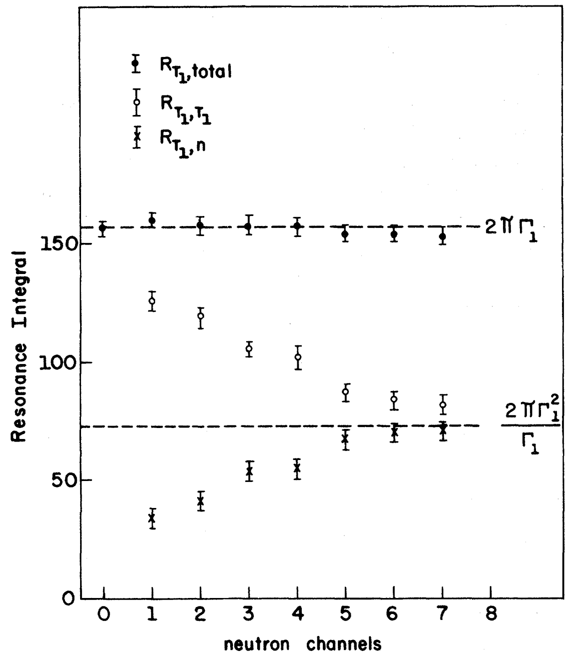

FIG. 3. Resonance integrals for a single doorway state as a function of the number of open neutron channels. The following parameters were used: $E_1=-2$, $N=51$, $d=4$, $M=4.3$, $\gamma_1=\gamma_3=5$. Shown are the results for the resonance integral of the total and partial cross section in the channel that is directly coupled to the doorway state, $R_{T_1,\mathrm{total}}$ and R_{T_1,T_1}, respectively, and of the cross section leading from the channel T_1 to the channels that are not directly coupled to the doorway state (neutron channels) $R_{T_1,n}$. The errors shown are uncertainties due to fluctuations in the numerical results introduced by the use of random numbers. Statistical assumption (24b) was used.

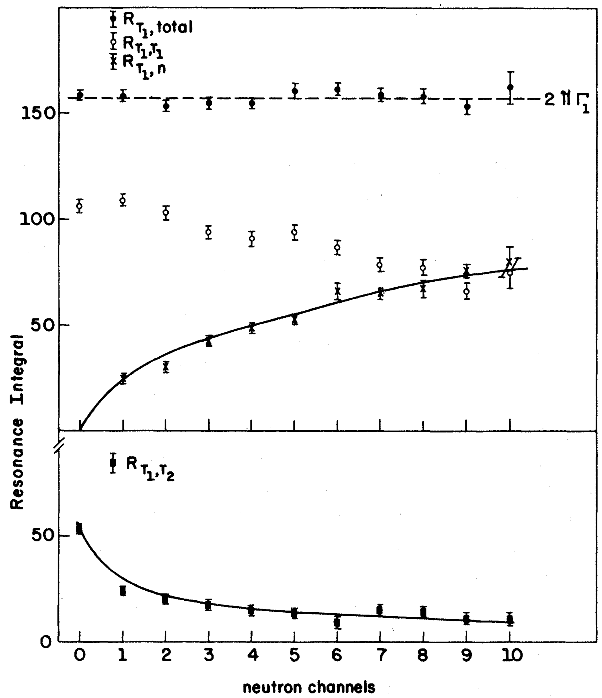

FIG. 4. Resonance integrals of $\sigma_{T_1,\mathrm{total}}$, σ_{T_1,T_1}, $\sigma_{T_1,n}$, and σ_{T_1,T_2} using $E_1=-2$, $E_2=2$, $N=52$, $d=4$, $M=4.3$, and $\gamma_1=\gamma_2=\gamma_3=5$. The smooth curve drawn through $R_{T_1,n}$ was used for the calculation of the branching ratio B_n in order to obtain from Eq. (28) the curve that is shown together with R_{T_1,T_2}. Statistical assumption (24b) was used.

$\to 2\pi\Gamma_1^2/\Gamma_T$ which is identical to (20) where the fluctuating part of the S matrix has been neglected. This means that the fluctuating part of the cross section $\sigma_{T_1T_1}$ goes to zero as the number of neutron channels increases. The branching ratio to neutron channels reaches asymptotically the value $\Gamma^{\downarrow}/\Gamma_T$ in agreement with (23). This can be stated in the following way: The fluctuating part of $\sigma_{T_1,\mathrm{total}}$ disappears in the neutron channels when the number of open neutron channels is big ($\Gamma_3/d \gg 1$).

In Fig. 4 the results for the resonance integrals for two analog states are shown, as a function of the number of open neutron channels. Statistical assumption (24b) was used for this calculation. Assumptions (24b) and (24c) were found to give the same results, whereas assumption (24a), corresponding to constant matrix elements, gave different results. For (24a) the ratio $R_{T_1T_2}/R_{T_1T_1}$ was found to be independent of the number of open neutron channels. This is quite evident, because then the product $\alpha_1^\mu\alpha_2^\mu$ has no longer a fluctuating sign and there is no longer a principal difference between σ_{11} and σ_{12}. But it seems quite implausible that the matrix elements of (24) will have constant sign because of the complicated nature of the $T_<$ states. The results for the resonance integral $R_{T_1T_2}$ could be parametrized by the formula

$$R_{T_1,T_2}=2\pi\frac{\Gamma_1\Gamma_2}{\Gamma_T+\Gamma^{\downarrow}(a-1)}\left(\frac{m(1+b)^{-2}}{1+m(1+b)^2}\right) \tag{28}$$

[with $a=B_n\Gamma_T/\Gamma^{\downarrow}$ and $b=0$ if (24a) is valid; with $a=b=B_n\Gamma_T/\Gamma^{\downarrow}$ if (24b) or (24c) is valid] where $\Gamma_T=\Gamma^{\downarrow}+\frac{1}{2}(\Gamma_1+\Gamma_2)$ and $\Gamma^{\downarrow}=2\pi M^2/d$ and m is given by (26b).

As can be seen in Fig. 4, the branching ratio B_n reaches asymptotically the same value as for a single analog state (Fig. 3) as N_n increases even if this occurs more slowly than in Fig. 3. Therefore (23) seems to be valid here too, even if it has been derived for a single analog state. Then inequality (23) ensures that $a\leq 1$. A significant increase of the number of open channels was not possible because it implies the inversion of a matrix of a too big dimension at every energy step [see Eq. (9)] which becomes too time consuming. However, the branching ratio to neutron channels has already nearly reached the limit of Eq. (23) for 10 open neutron channels. In the example shown in Fig. 4, B_n has a value corresponding to 91% of the limit $\Gamma^{\downarrow}/\Gamma_T$; for other test cases it reached up to 96%. Therefore, even further increase of the number of open neutron channels should not modify Eq. (28).

In the ^{139}La(p,p') experiment,[3] the isobaric analog resonances are at 8 MeV above neutron threshold; thus a large number of neutron channels are open. The experimental results of Ref. 3 are in good agreement with Eq. (28), using $a=b=1$ and transforming the resonance integral (28) to a cross section in the same way as passing from (26) to (27). Unfortunately the experimental situation is not as clear as one would like to verify Eq. (28) because of the presence of other nearby resonances.

IV. DISCUSSION

The mixing of doorway states via common fine structure states was calculated numerically in a model, including up to 10 channels that are not directly coupled to the doorways (neutron and fission channels). An inequality connecting the branching ratio B_n of analog states to neutron channels and the spreading width was obtained and used to interpret the results of the numerical calculations. Formulas for the mixing and for the cross sections were obtained. These formulas must be considered to be approximate because they were not deduced analytically which would be preferable but very difficult because it was seen that a correct evaluation of the fluctuating part of the S matrix would be necessary. But even if these formulas are only approximate, they are surely good enough to make an estimation of cross sections for the population of isomeric fission states via analog resonances.

This cross section depends critically on the ratio $\Gamma^{\downarrow}/\Gamma_T$. To our knowledge no study of analog resonances has been made apart from a Coulomb-energy measurement[8] for ^{238}U. However, total widths as well as the ratio $\Gamma^{\uparrow}/\Gamma_T$ of analog resonances show a quite smooth behavior for heavy nuclei; thus one can confidently extrapolate from the lead region to get an estimate for these values. This results in $\Gamma_T=300$ keV and $\Gamma^{\uparrow}/\Gamma_T\simeq 0.5$, and using Eq. (1) $\Gamma^{\downarrow}/\Gamma_T\simeq 0.5$. For the case that the statistical assumption (26a) is valid, one expects for $|E_1-E_2|\ll\Gamma^{\downarrow}$ cross sections leading to isomeric fission states of the same order of magnitude as for normal (p,p') scattering via isobaric analog resonances, that is typically 100–1000 μb/sr for the on resonance cross section. In this case the cross-section estimation depends only within a factor of about 2 on the branching ratio to neutron channels. Because of the complicated nature of the $T_<$ states, probably the statistical assumptions (24b) and (24c) are more realistic. In this case the cross section $\sigma_{T_1T_2}$ depends more critically on the branching ratio B_n to neutron channels. Actually neutron decay of analog resonances has not been observed for nuclei more heavy than samarium. This seems to indicate that B_n is small. Another experimental methods is the determination of the branching ratio to proton channels B_p and then obtain B_n by the reaction $B_n=1-B_p$. This can be measured by (^{3}He,$d\bar{p}$) or $(p,n\bar{p})$ experiments. By the analysis of (p,p') data one gets $B_{pp'}=\sum\Gamma_{c_p}/\Gamma_T=\Gamma^{\uparrow}/\Gamma_T$. $B_{pp'}$ is only necessarily equal to B_p for an isolated single resonance. An example is ^{92}Mo(p,p') which gives for the analog of the ground state[9] $B_{pp'}=1.5/30$. For this resonance the neutron channels are not open and therefore B_p should be equal to one and actually a measurement of the (^{3}He,$d\bar{p}$) reaction gave[10] $B_p=1.01\pm0.03$. For more heavy nuclei the only measurement by the study of the reaction $(p,n,\bar{p})$ for ^{208}Pb is not very precise[11] and gives $B_p=1\pm0.3$ which results in $B_n=0\pm0.3$. For this nucleus[12] $B_{pp'}=0.6$ which gives $\Gamma^{\downarrow}/\Gamma_T=0.4$. Measurements of B_n by $(p,n\bar{p})$ reactions without observation of the intermediate n are not precise enough due to ambiguities in background substraction.[13]

With $\Gamma^{\downarrow}/\Gamma_T=0.5$, $B_n=0$, and $|E_1-E_2|<\Gamma^{\downarrow}\simeq 150$ keV one expects cross sections for the population of isomeric fission states via isobaric analog resonances (IAR) of the same order of magnitude as for normal (p,p') reactions via IAR, that is of the order of 100–1000 μb/sr corresponding to $\Gamma_1\simeq\Gamma_2\simeq 10$–$50$ keV in Eq. (28). If the branching ratio to neutron and fission channels has a value near the limit $\Gamma^{\downarrow}/\Gamma_T$, the cross section would be reduced by about one order of magnitude, resulting in 10–100 μb/sr, which is still 3 orders of magnitude bigger than the cross sections obtained in nonresonant reactions. This would permit experimental studies of these states using standard high resolution techniques. In an experiment looking for these resonances, the energy step should not be chosen too big. Following the numerical calculations, the width at half height of the resonance is about the same as for a simple resonance but the cross section goes much more rapidly to zero, if the branching to neutron channels is strong, than one would expect for a Breit-Wigner form. An interesting feature is contained in the Coulomb displacement energy ΔE_C. It depends on the deformation of the analog state.[14] For the big deformations predicted by theory[15] $\beta = 0.6$, a change of ΔE_C of ~700 keV is expected. This provides a means of direct measurement of the deformation of these states.

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