$T = 1$ positive parity states in ^{28}Si and ^{28}P

C. Miehé, A. Huck, G. Klotz, and G. Walter
Centre de Recherches Nucléaires, 67037 Strasbourg Cedex, France
(Received 3 August 1976)

The ^{27}Al$(p,\gamma)^{28}$Si, ^{27}Al$(d,n\gamma)^{28}$Si, ^{26}Mg$(^3$He$,n\gamma)^{28}$Si, and ^{28}Si$(p,n\gamma)^{28}$P reactions have been used to investigate the $T = 1$ states of ^{28}Si and ^{28}P. The γ decay of the levels at 10.27 and 11.43 MeV (doublet) in ^{28}Si and of seven levels in ^{28}P has been studied. Limits on lifetimes for the ^{28}Si levels and for the states at 1.13 and 2.10 MeV in ^{28}P were determined using the Doppler shift attenuation method. From six isobaric triplets identified in $A = 28$ nuclei, Coulomb displacement energies, isovector, and isotensor Coulomb energies have been deduced. Excitation energies of the members of the multiplets are compared with theoretical values obtained from Coulomb displacement energy calculations, using shell model wave functions with particles in the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ subshells.

[NUCLEAR REACTIONS ^{27}Al$(d,n\gamma)$, E=5.8 MeV; ^{26}Mg$(^3$He$,n\gamma)$, E=6.0 MeV; measured E_γ, Doppler shift attenuation, n-γ coin. ^{27}Al(p,γ), E=1439 keV; measured E_γ. ^{28}Si deduced levels, τ_m, γ branching. ^{28}Si$(p,n\gamma)$, E=23 MeV; measured E_γ, Doppler shift attenuation, n-γ coin. ^{28}P deduced levels, τ_m, γ branching, analog states. Natural and enriched targets.]

I. INTRODUCTION

Study of isobaric states of the nuclei A =28 is of special interest. The T_z=0 member of the isospin triplet is an even-even nucleus and corresponds in the simple shell model picture, to the $d_{5/2}$ subshell closure where the deformation of the nuclei changes from prolate to oblate. From the measurements of the excitation energies of the T =1 members of an isobaric triplet (T_z=1, 0, −1) Coulomb energy differences can be deduced. Because the T =1 levels in A =28 have been described in terms of the shell model,[1-3] these quantities test the properties of the wave functions. Experimental investigations of T =1 states in ^{28}Si and ^{28}Al have been extensively performed by means of single-nucleon transfer reactions such as (d,n), (d,p), and (p,γ).[4-9] Only two experiments leading to excited states in ^{28}P have been reported.[10,11] The location of some levels has been performed by means of the ^{28}Si$(^3$He$,t)^{28}$P reaction.[10] Available information about electromagnetic decay in that nucleus is limited to the results obtained from the ^{28}Si$(p,n\gamma)^{28}$P reaction by Moss *et al.*[11] In order to get more evidence for the isobaric triplet identification in A =28 isobars and to deduce Coulomb displacement energies, new measurements were necessary.

The present paper describes experiments to study T =1 states in ^{28}Si(T_z=0) and ^{28}P(T_z=−1). The positive parity states of ^{28}Si have been reached through the ^{27}Al$(d,n\gamma)^{28}$Si and ^{26}Mg$(^3$He$,n\gamma)^{28}$Si reactions and for one particular level by the ^{27}Al$(p,\gamma)^{28}$Si reaction. Information on T =1 negative parity states obtained by proton radiative capture in another set of experiments will be presented separately.[12] The nucleus ^{28}P has been investigated by means of the ^{28}Si$(p,n\gamma)^{28}$P charge exchange reaction. From our results and previously published data[13] on ^{28}Al, isobaric triplets can be identified. Experimental Coulomb displacement energies over the triad are compared with theoretical values obtained with the method introduced by De Meijer[14] using the wave functions recently elaborated by Meurders[3] for A =27 and 28.

II. EXPERIMENTAL TECHNIQUE

Proton, deuteron, and ^{3}He beams from 3 and 7 MV Van de Graaff accelerators (Strasbourg) with E_p =1.439 MeV, E_d =5.8 MeV, and $E_{^3\mathrm{He}}$ =6 MeV and proton beams of E_p =23 MeV from the cyclotron of the Institut des Sciences Nucléaires (Grenoble) have been used in these experiments with intensities in the order of 10, 1, 0.1, and 0.005 μA, respectively. In these experiments a 270 μg/cm^2 ^{27}Al, a 400 μg/cm^2 ^{26}Mg, and a 1.4 mg/cm^2 Si foil were used as targets. For the (p,γ) work a layer of 30 μg/cm^2 ^{27}Al on gold backing was used. In order to analyze the γ rays emitted in coincidence with the neutrons, an experimental setup with two Ge(Li) counters (sensitive volume around 80 cm^3) and an annular neutron detector placed at 0° was used. Annular neutron detection geometry combines large solid angle with low kinematic spread and allows the beam to be carried on through the counter at some distance from the target. The neutron detector we have built is a cylindric container of 10 cm diam and 10 cm depth, filled with NE213 liquid scintillator. An axial tube with a diameter of 30 mm allows the beam to pass through the counter to a Faraday cup. The inner surface of the cell is coated with an optical diffusor material (titane dioxyde). A vertical internal partition separates the container into two regions. These are viewed by two phototubes placed at 90° with respect to the beam axis on opposite ends of a diameter of the cell; the photocathodes are in direct contact with the scintillator. The outputs of the phototubes are added in such a manner that the two parts of the annular counter work like a single one, as well for timing as for pulse shape analysis for particle identification. Fast coincidences are registered between the scintillator and the two Ge(Li) detectors, with a typical resolution of 10 ns. In our experiments, distances between target and detectors are such that the spread introduced by the differences in time of flight for neutrons is kept lower than 1 ns. A slow coincidence circuit provides the identification of the γ counter involved in the n-γ event and allows the storage of the data by the corresponding multichannel analyzer, for each Ge(Li) counter. This method enables the simultaneous measurement of γ rays emitted in two different directions for Doppler shift attenuation measurements. In the $(p, n\gamma)$ experiments we extended the experimental setup in order to be able to measure at the same time γ-γ coincidences.

III. RESULTS

A. ^{28}Si nucleus

1. ^{27}Al$(d,n\gamma)^{28}$Si reaction

Proton transfer processes like (d,n), leading to the formation of even-even nuclei, populate preferentially the levels of isospin $T=1$ resulting from $l_p=0$ transfer.[4] For ^{28}Si the reaction ^{27}Al$(d,n\gamma)^{28}$Si gives information complementary to radiative capture work by exciting directly proton bound levels ($E_x \leq 11.58$ MeV). In this work a point of interest was to establish the existence of a doublet at 11.43 MeV populated by the (d,n) reaction. From earlier (d,n) work by Lawergren,[4] and by Bohne *et al.*[5] indications for a $T=1$, $(2,3)^+$ state at 11 418 keV have been given. From (e,e') (Ref. 15) and (γ,γ') (Ref. 16) reactions, a $T=1$, $J^\pi=1^+$ level has been located at 11 410± 30 and 11 420± 20 keV, respectively. More recently, excitation energies of levels in ^{28}Si have been determined using the ^{28}Si$(p,p')^{28}$Si reaction.[17] In this experiment the existence of one level at 11 434 ±3 keV has been established and there was no evidence for a level at 11 418 keV. The γ decay of levels at that energy has been reported by several authors, and has led to different conclusions. From the (p,γ) work of Meyer, Venter, and Reitmann[18] two levels are located at 11 434 keV each one being observed separately in distinct resonances. From one of them a transition to the 1.78 MeV excited state is observed while the other decays to the states at 6.27, 8.59, and 9.32 MeV. In the (d,n) and $(d,n\gamma)$ work done up to now, only one level around 11.43 MeV has been reported and in particular Lyttkens *et al.*[19] attribute the four γ branches reported above to a unique level at 11 434.1 ±0.7 keV.

From the analysis of γ rays in the present work, it can be established that two different levels at 11 432.1 ±2.0 and 11 432.3 ±1.5 keV are populated in the (d,n) reaction. The spectrum taken at 150° is shown in Fig. 1 where one notices strong transitions from the two first $T=1$ states in ^{28}Si located at 9.32 and 9.38 MeV and from the doublet at 11.43 MeV. Doppler shift attenuation has been measured by comparing the γ-ray spectra at 90° and 150° (Fig. 2). The transition 6.27→1.78 MeV was used as reference in energy. The long lifetime of the $E_x=6.27$ MeV level [$\tau_m=1.37\pm0.13$ ps (Ref. 20)] and the low recoil velocity do not give rise to a noticeable Doppler shift of the 6.27→1.78 MeV line. On the ground of the lifetime values reported in Table I, it appears that transitions from 11.43 to 1.78 MeV and 11.43 to 9.32 MeV have to be related to two different levels. Using γ-ray spectra recorded at 45°, 90°, and 135° in another set of experiments, branching ratios have been measured for the decay of the 11.43 MeV levels. Transition strengths deduced from these measurements in the case of $M1$ emissions, are reported in Table II.

Lifetimes measured in the $(d,n\gamma)$ experiments are in agreement with previous results except for the level at 11 432.3 keV for which Dalmas, Leccia, and Aleonard[21] reported the value 20 ±5 fs using the (p,γ) reaction. The other member of the doublet we located at 11 432.1 keV has a short lifetime ($\tau_m \leq 30$ fs) and probably corresponds to the level populated in inelastic electron scattering experiments where a width of 20.8 eV was measured and (J^π, T) values $(1^+,1)$ were assigned.[15]

2. ^{26}Mg$(^3$He$,n\gamma)^{28}$Si reaction

The reaction ^{26}Mg$(^3$He$, n\gamma)^{28}$Si, as a two-proton transfer process, may excite states which are not populated in one-proton transfer reactions.[5] In particular, it seems a suitable tool for the search of the first $J^\pi=0^+, T=1$ state in ^{28}Si. Boerma[6] has indicated that the first $J^\pi=0^+$ level in ^{28}Al has a strong $(d_{5/2})^{-2}(s_{1/2})^2$ component. Up to now the γ emission of the ^{26}Mg$(^3$He$,n\gamma)^{28}$Si reaction has not

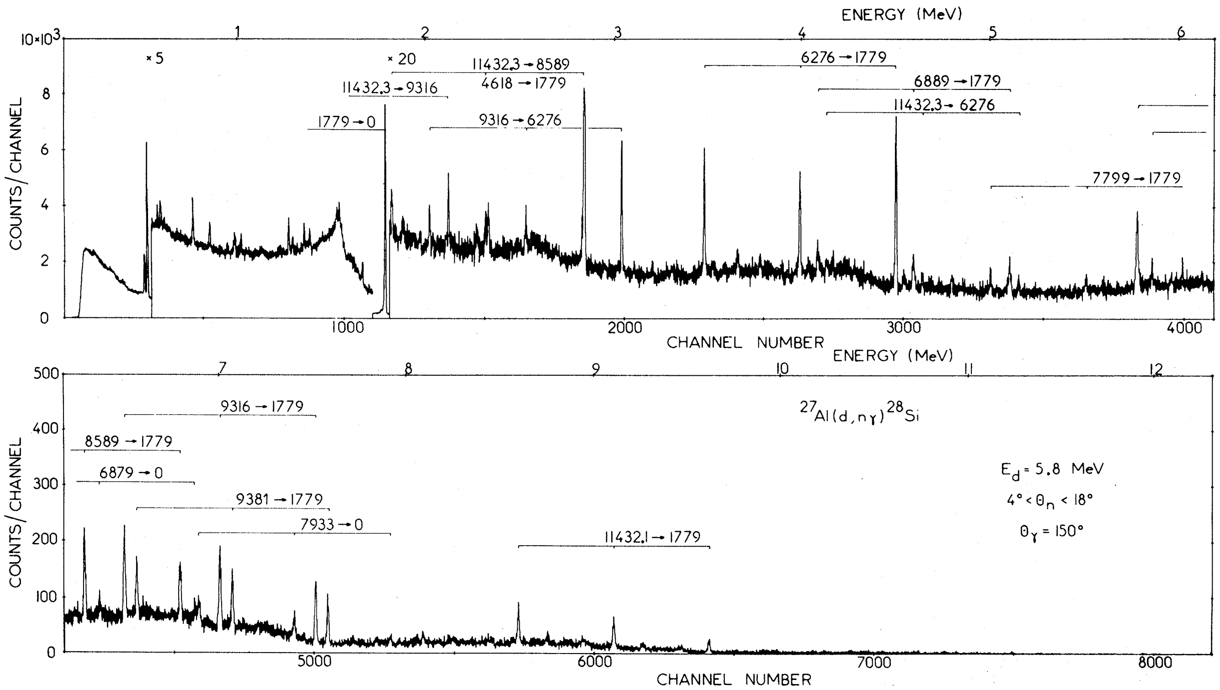

FIG. 1. γ-ray spectrum from the ^{27}Al$(d,n\gamma)$ reaction taken at 150° in coincidence with neutrons ($4° < \theta_n < 18°$) at E_d =5.8 MeV.

been studied. A typical spectrum taken at $E_{^3\mathrm{He}}=6$ MeV and $\theta_\gamma=128°$ in coincidence with neutrons emitted near 0°, is presented in Fig. 3; it corresponds to a total collected charge of 1.7×10^{-2} C. The strong lines due to ^{24}Mg and ^{27}Al can be explained by the exoenergetic reactions ^{26}Mg(^{3}He, $n\alpha\gamma$)^{24}Mg and ^{26}Mg(^{3}He, $np\gamma$)^{27}Al. Although much weaker, we observe the decay of the ^{28}Si levels at E_x =7.38, 7.42, 9.48, 10.27, and 10.59 MeV, as well as the main branch of the deexcitation of the E_x =8.33 MeV level which was not seen in the neutron time-of-flight spectrum registered by Bohne *et al.*[5] and which is populated by γ rays cascading onto it. In our spectra, a 1944 keV γ ray was attributed to a transition 10.27→8.33 MeV. Assignment of the 1944 keV γ ray to the decay of the 10.27 MeV level has been corroborated by the study of the ^{27}Al$(p,\gamma)^{28}$Si reaction at E_p =1439 keV (see next section). The (J^π, T) assignment $(0^+,1)$ of the 10.27 MeV state suggested by several authors[13] is consistent with the decay scheme deduced from our spectra: 10.27→1.78 MeV $(2^+,0)(70\pm20)$ and 10.27→8.33 MeV $(1^+,0)(30\pm20)$. In this case, the last transition would have an $M1, \Delta T=1$ character. The Doppler shift of the 1944 keV γ ray has been measured from spectra taken at 90° and 128°. Its value $[F(\tau)\geq0.7]$ corresponds to a short lifetime ($\tau\leq60$ fs). The γ-ray decay properties of the 10.27 MeV level will be discussed in the next section. It should be noted that the small cross section of the (^{3}He, $n\gamma$) reaction and the contribution to the coincidence spectra of the competing reactions (^{3}He, $n\alpha\gamma$) and (^{3}He, $np\gamma$) prevent accurate measurements of the γ decays of the ^{28}Si levels.

3. ^{27}Al$(p,\gamma)^{28}$Si reaction. Resonance at E_p = 1439 keV

In order to check the decay scheme of the level at E_x =10.27 MeV, we investigated the resonance at E_p =1439 keV, which is the only one feeding the 10.27 MeV level reported in the literature. According to Forsblom,[22] this state is populated from the resonant level (E_x =12 972 keV, $J^\pi=1^-$) with an intensity of 15%. The resonance strength has been measured as $\omega_\gamma=0.16\pm0.06$ eV (Ref. 23) and in our work γ-ray spectra have been taken at θ_γ =0°, 55°, and 90° using Ge(Li) counters and with a total charge of one Coulomb at each angle. An off-resonance spectrum was taken at E_p =1435 keV in order to identify background lines. From the resonant level transitions to the states at 0, 4.98, and 10.27 MeV were observed, with intensities as reported in Table III. The results are in disagreement with the work of Forsblom[22] where weak transitions to five $J=2$ and 3 states were also given; the corresponding γ rays are present in our

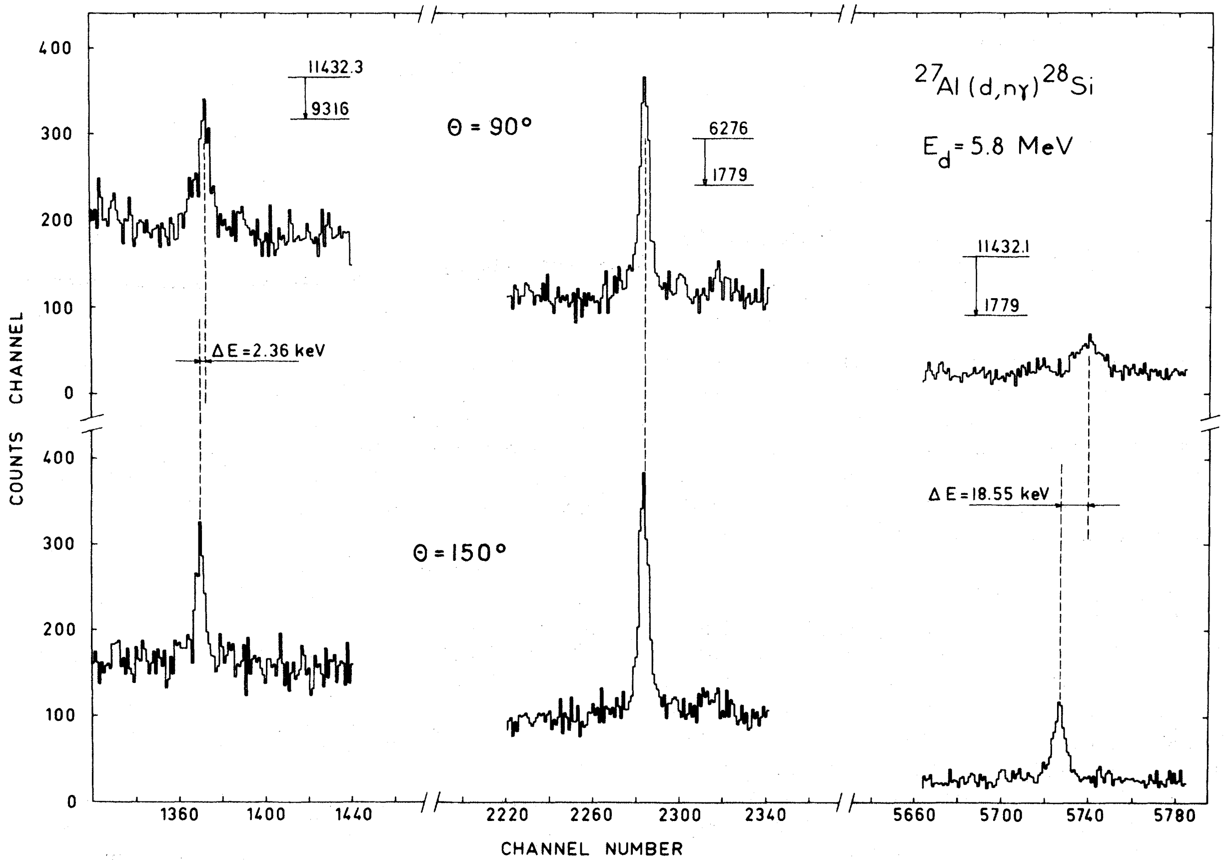

FIG. 2. Doppler shifts for the lines from the 6.27 MeV level and the 11.43 MeV doublet observed between 150° and 90° in the ^{22}Al($d, n\gamma$) reaction at E_d = 5.8 MeV, in coincidence with neutrons ($4° < \theta_n < 18°$).

TABLE I. Lifetimes deduced from the Doppler shift measurement of the γ rays observed in the ^{27}Al($d, n\gamma$)^{28}Si reaction.

Observed transitions				τ (fs)	
E_x^i (keV)	J^π, T	E_x^f (keV)	$F(\tau)$	Present work	Previous results
9316.1 ± 0.5 [a]	3^+, 1	1779	≥0.74	≤30	13 ± 10 [b]
		6276			
9381.2 ± 0.6 [a]	2^+, 1	1779	0.82 ± 0.16	20^{+30}_{-17}	5 ± 3 [c]
10917 ± 3	(2^+, 3^-, 4^+), 0	1779			
11432.1 ± 2.0	(1, 2), (1)	1779	0.90 ± 0.10	≤30	≤100 [d]
11432.3 ± 1.5	(3, 4), (0)	6276			
		8589 [e]			
		9316	0.52 ± 0.20	125^{+130}_{-60}	20 ± 5 [f]

[a] Energy value taken from Ref. 13 and used for calibration.
[b] Reference 27.
[c] Reference 28.
[d] Reference 19, in this work the two levels at 11.43 MeV were not distinguished.
[e] At θ_γ = 90° and 150°, the transition to the 8.59 MeV level cannot be distinguished from the decay of the 4.62 MeV level.
[f] Reference 21.

TABLE II. Transition strengths for transitions from the doublet at $E_x = 11432$ keV in ^{28}Si.

	Transition				Branching	Γ_γ/Γ_w ($M1$) [a]
E_x^i (keV)	J^π, T	E_x^f (keV)	J^π, T		ratio (%)	(W. u.)
11432.1	(1, 2), (1)	1779	2^+, 0		100	$\geq 1.1\times10^{-3}$
11432.3	(3, 4), (0)	6276	3^+, 0		25 ± 12	0.4×10^{-3}
		8589	3^+, 0		21 ± 12	2.3×10^{-3}
		9316	3^+, 1		54 ± 15	1.4×10^{-2}

[a] Pure $M1$ transitions are assumed.

off-resonance spectrum. The decay scheme of the 10.27 MeV level is in agreement with that resulting from the (^{3}He, $n\gamma$) spectra analysis; it should be noted that only the 10.27 → 1.78 MeV branch was quoted in previous works.[13,22] Due to experimental difficulties resulting from the overlap of lines at several angles it was not possible to measure the angular distributions of the primary transition, 12.97 → 10.27 MeV, and of the 10.27 → 8.33 and 10.27 → 1.78 MeV γ rays. The transition strengths which are reported in Table III are deduced from the ω_γ value mentioned above and from the lifetime determined in the (^{3}He, n) experiment. Their comparison with a recent review of experimental transition strengths[24] gives new arguments in favor of a $J^\pi = 0^+$, $T = 1$ assignment to the 10.27 MeV level. For the transition 12.97 → 10.27 MeV only a lower limit of the γ-ray strength can be derived from the resonance strength as the α partial width is not negligible. This value is consistent with $E1$ isovector deexcitation while for the 12.97 → 0 and 12.97 → 4.97 MeV transitions, strength limits are smaller and do not exceed the mean value for isoscalar $E1$ emission. The limit [2.9×10^{-2} W.u. (Weisskopf units)] obtained for the 10.27 → 8.33 (1^+, 0) MeV γ decay, compared with the recommended upper limit of 3.0×10^{-2} W.u. for $M1$ isoscalar transitions,[24] suggests an $M1$ isovector deexcitation.

B. ^{28}P nucleus. Reaction ^{28}Si($p,n\gamma$)^{28}P

The reaction ^{28}Si(p, $n\gamma$)^{28}P was studied at $E_p = 23$ MeV using a natural silicon target, 1.4 mg/cm^2

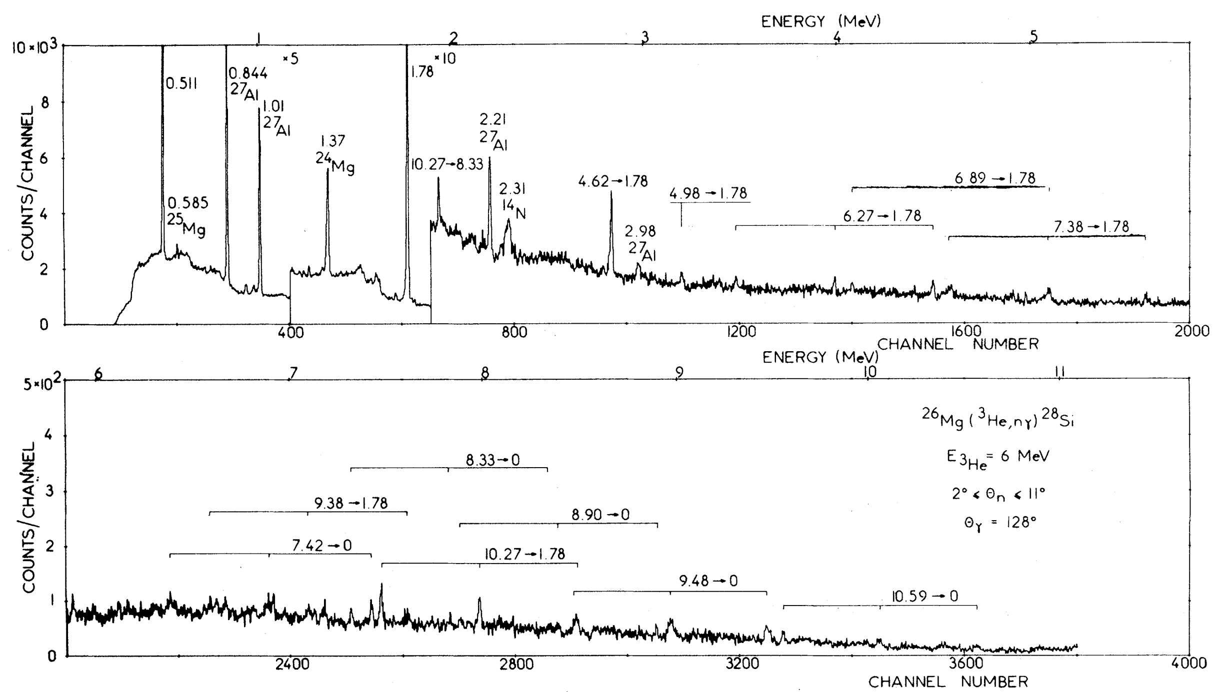

FIG. 3. γ-ray spectrum from the ^{26}Mg(^{3}He, $n\gamma$) reaction taken at 128° in coincidence with neutrons ($2° < \theta_n < 11°$) at bombarding energy $E_{^3\mathrm{He}} = 6$ MeV.

TABLE III. Relative intensities and transition strengths for transitions from the $E_x = 12.97$ and 10.27 MeV states in ^{28}Si.

E_x^i (keV)	Transition J^π, T	E_x^f (keV)	J^π, T	Branching ratio (%)	Transition strength (W. u.)
12972±2	1^-, 0	0	0^+, 0	56±3	Γ_γ/Γ_w $(E1) \geq 0.2\times10^{-4}$
		4979	0^+, 0	21±3	Γ_γ/Γ_w $(E1) \geq 0.35\times10^{-4}$
		10272	$(0)^+$, (1)	23±3	Γ_γ/Γ_w $(E1) \geq 1\times10^{-3}$
10272.0±1.5	$(0)^+$, (1)	1779	2^+, 0	58±15	Γ_γ/Γ_w $(E2) \geq 3.6\times10^{-2}$
		8328	1^+, 0	42±15	Γ_γ/Γ_w $(M1) \geq 2.9\times10^{-2}$

thick. With such a target thickness, beam straggling is kept low enough to use the annular neutron detector placed at 0°, the beam being stopped 5 m behind the target. Due to the high neutron background at this energy a Lucite target chamber was used to minimize the γ-ray background due to neutron induced reactions in surrounding materials. The measurements were done with the experimental setup described in the preceding section, the Ge(Li) counters being placed at 90° and 135° to the beam direction. n-γ and γ-γ coincidences were measured simultaneously. A spectrum of the γ rays in coincidence with the decay of the first excited state of ^{28}P ($E_x = 106$ keV), was taken at 90°, the $E_\gamma = 106$ keV line being selected by the 135° detector. Spectra registered between beam pulses allowed identification of γ rays from induced radioactivity. The spectrum of the ^{28}Si$(p, n\gamma\gamma)^{28}$P reaction is presented in Fig. 4. Contamination γ rays are labeled by the associated isotope. Lines attributed to ^{28}P are indicated by the corresponding transitions. The small intensity of the γ rays from ^{28}P levels makes determinations of the decay scheme and of Doppler shifts difficult. Excitation energies, branching ratios, and lifetimes are given in Table IV and compared with the results obtained by Moss *et al.*[11] One has to note that in the present work and in Ref. 11 the branching ratios result from n-γ correlation measurements where the γ detector is placed at 90° with the neutron counter at forward angles. Our values are in fair agreement with those given in Ref. 11 except for the decay of the 1313 keV state where we report two branches, respectively, to the 106 and 877 keV states, while only the last branch is indicated by Moss *et al.* From the observed γ rays and taking into account previous data, we propose a decay scheme for two levels at 1516 and 1567 keV. In the analysis of the ^{28}Si$(\tau, t)^{28}$P reaction, Mangelson *et al.*[10] found a level at $E_x = 1540 \pm 40$ keV. From neutron time-of-flight measurements, Moss *et al.* have located levels at 1602 ±19 and 1520 ±13 keV; our interpretation of the 1516 keV line as the direct transition from the level at 1516 keV to the ground state is in agreement with these measurements. A γ ray at 1461 keV present in the n-γ and γ-γ coincidence spectra suggests the existence of a level at 1567 keV decaying to the first excited state. Due to the presence of γ ray at 1595 keV, also observed in Ref. 11, the existence of a level

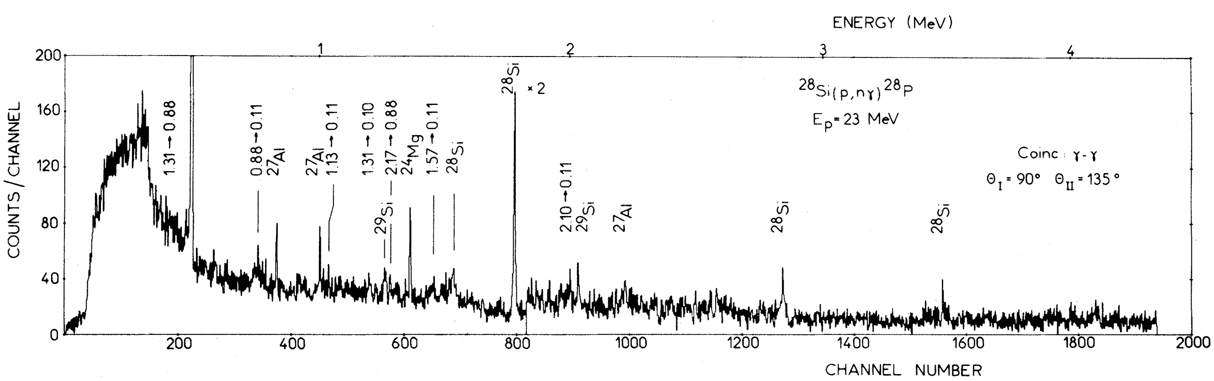

FIG. 4. γ-ray spectrum measured at 90° in coincidence with the decay of the first excited state of ^{28}P, the 106 keV line being detected at 135°.

TABLE IV. Excitation energies, branching ratios, and lifetimes for states in ^{28}P.

E_x^i (keV) Present work	E_x^i (keV) Ref. 11	E_x^f (keV)	Branching ratios determined at 90° Present work	Branching ratios determined at 90° Ref. 11	Lifetimes (fs) this work
106 ± 1	105.64 ± 0.10	0	100	100	
877 ± 2	879 ± 3	106	100	> 49	
1134.0 ± 0.5	1134 ± 3	0	69 ± 16	53 ± 14	< 1500
		106	31 ± 16	47 ± 14	
1313 ± 2	1313 ± 4	106	46 ± 20		
		877	54 ± 20	100	
1516 ± 2	1516 ± 3	0	100	100	
1567 ± 3		106	100		
2104 ± 1	2104 ± 5	0	39 ± 16	41 ± 14	< 125
		106	61 ± 16	59 ± 14	

at that energy cannot be excluded. In neutron time-of-flight measurements, Moss *et al.* have located a level at 2180 ± 40 keV, for which they did not report any electromagnetic deexcitation. In the mirror nucleus ^{28}Al, a level is known at 2201 keV ($J^\pi = 1^+$) which decays to the states at 972 and 31 keV. In our n-γ and γ-γ spectra a γ ray at 1289 keV could correspond to a transition from a level at $E_x = 2166$ keV (2166 → 877 keV), for which no other branch has been found. Nine unbound levels above 2 MeV are reported in Ref. 10. In our spectra the only γ ray which could account for the decay of high energy levels is the 2587 keV line (2587 → 0 keV). These decay modes can be compared with the decay scheme of the ^{28}Al levels. We have reported in Fig. 5 the level diagram of the mirror nuclei ^{28}Al and ^{28}P. From excitation energies and electromagnetic decay properties, despite the fact that no angular distribution could be measured and that only two lifetimes could be determined for ^{28}P, correspondences between some $T = 1$ levels can be established.

C. Comparison of experimental and theoretical Coulomb displacement energies in $A = 28$ triplets

The Coulomb displacement energy (or Coulomb energy differences) ΔE_C between any two members of the same isobaric multiplet is defined by

$$\Delta E_C(A, T, \tilde{T}_z | T_z) = E_C(A, T, \tilde{T}_z) - E_C(A, T, T_z).$$

The Coulomb energy E_C can be expressed in terms of scalar, vector, and tensor contributions as[25]:

$$E_C(A, T, T_z) = E_C^{(0)}(A, T) - T_z E_C^{(1)}(A, T) + [(3T_z^2 - T(T+1)] E_C^{(2)}(A, T).$$

It can be seen from these expressions that the measurement of ΔE_C over an isobaric triplet yields the isovector and isotensor terms of the Coulomb energy:

$$E_C^{(1)}(A, 1) = \tfrac{1}{2}[\Delta E_C(A, 1, -1|0) + \Delta E_C(A, 1, 0|+1)],$$

$$E_C^{(2)}(A, 1) = \tfrac{1}{6}[\Delta E_C(A, 1, -1|0) - \Delta E_C(A, 1, 0|+1)].$$

From experimental results obtained up to now, a comparison of the excitation energies of the members of the six first isobaric triplets in the mass $A = 28$ has been made (Table V). The values reported are those obtained in the present work as far as ^{28}P and the 10 272 keV level of ^{28}Si are concerned. For ^{28}Al the energies are taken from the compilation of Endt and van der Leun.[13] For the $J^\pi = 1^+, T = 1$ levels of ^{28}Si, the excitation energies are those measured by Jelley *et al.*[26] in the decay of the first $T = 2$ state. Isovector and isotensor Coulomb energies deduced from the experimental excitation energies for each multiplet are reported in the last columns; the quoted errors take into account the uncertainties on excitation energies and mass excesses. The mean values of $E_C^{(1)}$ and $E_C^{(2)}$ are consistent with those reported in Ref. 25 where the variations of these quantities as a function of the mass number are discussed by Jänecke. It appears that in mass 28 the isovector Coulomb energy does not change by more than 1% from its mean value, over the six considered multiplets. The isotensor Coulomb energy presents larger relative variations but it still remains lower than the corresponding quantity defined for the triplets of mass 26 and 30; this does not affect the oscillatory pattern of the $E_C^{(2)}$ values in the region $6 \leq A \leq 40$, for isobaric triplets.

Using the shell model description, Coulomb displacement energies in mass 28 have been calculated by De Meijer, Van Royen, and Brussaard[14] as a sum of single-particle Coulomb displacement energies. Displacement energies result from changing a neutron outside a core (^{27}Al or ^{27}Si) into a proton with the same quantum numbers. The wave functions used in Ref. 14 to describe the relevant levels were those obtained by Wildenthal *et al.*[2] with a

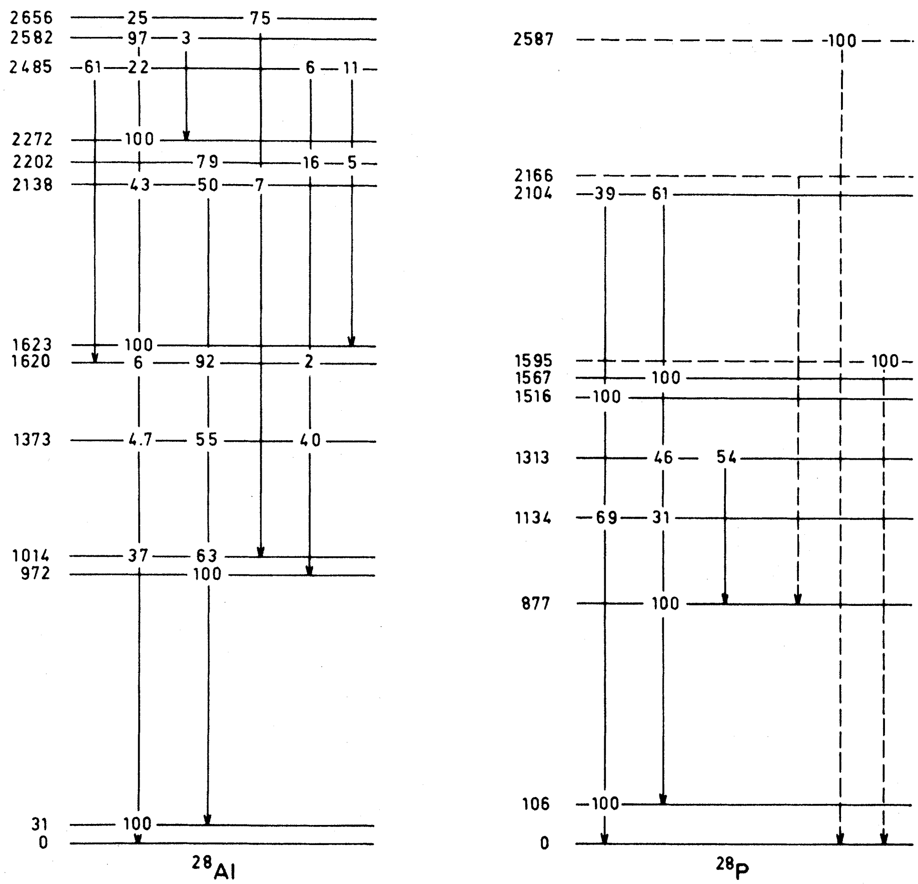

FIG. 5. Energy levels and γ decay in the mirror nuclei ^{28}Al and ^{28}P according to Ref. 13 and the present work.

configuration space limited to $1d_{5/2}$ and $2s_{1/2}$ shells. We have performed a new set of calculations using the wave functions recently calculated by Meurders[3] in the configuration space $(d_{5/2})^{n_1}$, $(s_{1/2})^{n_2}$, $(d_{3/2})^{n_3}$ with $6 \leq n_1 \leq 12$, $n_2 \leq 4$, $n_3 \leq 3$. All components of the wave functions with intensities exceeding 5% were considered. For the calculation, the nuclear potential is described as a Woods-Saxon potential where the well-depth parameter V_0 is determined from the requirement that the neutron separation energy is equal to the experimental value. The radius r_0, related to the nuclear radius by $R = r_0 A^{1/3}$, and the diffuseness parameter α, are defined by a least-squares fit from Coulomb

TABLE V. Excitation energies of the members of the isobaric multiplets in $A=28$ and the deduced isovector and isotensor Coulomb energies.

J^π	Excitation energies (keV) ^{28}Al ($T_z=1$) a	^{28}Si ($T_z=0$)	^{28}P($T_z=-1$) b	$E_c^{(1)}$ (keV)	$E_c^{(2)}$ (keV)
3^+	0	9316.1 ± 0.5 [a]	0	5630 ± 3	58 ± 1
2^+	30.641 ± 0.020	9381.2 ± 0.6 [a]	106 ± 1	5667 ± 3	58 ± 1
0^+	972.2 ± 0.2	102 72.0 ± 1.5 [b]	877 ± 2	5582 ± 6	47 ± 2
3^+	1014.0 ± 0.4	103 77.1 ± 1.0 [a]	1134.0 ± 0.5	5690 ± 4	62 ± 1
1^+	1372.8 ± 0.2	105 98 ± 3 [c]	1313 ± 2	5602 ± 8	78 ± 2
1^+	1620.1 ± 0.3	109 01 ± 3 [c]	1567 ± 3	5602 ± 8	60 ± 2

[a] Reference 13.
[b] Present work.
[c] Reference 26.

TABLE VI. Comparison between experimental and calculated Coulomb displacement energies for the six lowest $T=1$ levels in ^{28}Al, ^{28}Si, and ^{28}P.

J^{π}	$\Delta E_{C\,\mathrm{exp}}$ (keV)	$\Delta E_{C\,\mathrm{calc}}$ (keV) Ref. 14	$\Delta E_{C\,\mathrm{calc}}$ (keV) Present work	$\Delta E_{C\mathrm{exp}}-\Delta E_{C\mathrm{calc}}$ (keV) Ref. 14	$\Delta E_{C\mathrm{exp}}-\Delta E_{C\mathrm{calc}}$ (keV) Present work
			^{28}Al-^{28}Si		
3^+	5457±3	5328	5478	129	−21
2^+	5491±3	5317	5480	174	11
0^+ [a]	5441±4	5273	5494	168	−53
			5473		−32
3^+	5504±4	5375	5455	129	49
1^+	5366±6	5331	5485	35	−119
1^+	5422±6	5394	5489	28	−67
			^{28}Si-^{28}P		
3^+	5803±5	5742	5902	61	−99
2^+	5844±6	5731	5902	113	−58
0^+ [a]	5724±8	5697	5923	27	−199
			5898		−174
3^+	5876±6	5798	5873	78	3
1^+	5834±10	5784	5913	50	−79
1^+	5785±11	5812	5916	−27	−131

[a] The calculations of Meurders (Ref. 3) give two 0^+ levels at almost equal energies.

energies of "single-particle" states in $A=12-20$ nuclei as $r_0=1.28$ fm and $\alpha=0.63$ fm. The charge distribution is described by a homogeneously charged sphere. Table VI shows the results obtained previously by De Meijer and the present results. Both are compared with experimental Coulomb displacement energy values, $\Delta E_{C\mathrm{exp}}$, deduced from the excitation energies reported in Table V. It can be seen that the new description of the isobaric multiplets, involving the $1d_{3/2}$ shell, raises

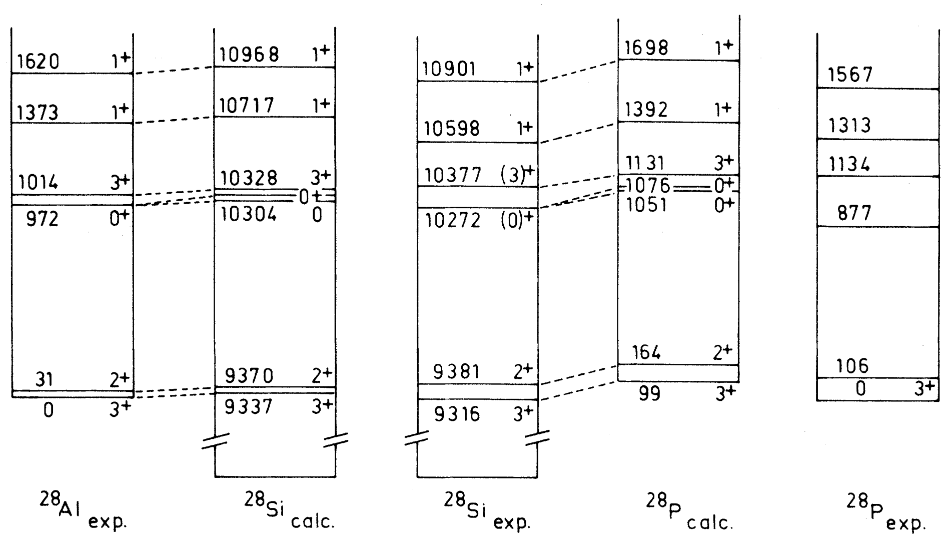

FIG. 6. Calculated and experimental excitation energies of $T=1$ levels in $A=28$ nuclei according to the present work and experimental values of Ref. 13.

the Coulomb energy differences which were too low in the previous calculations. Deviations between experimental and calculated values are smaller in the ^{28}Al-^{28}Si comparison than for ^{28}Si-^{18}P, but the overall mean deviation is not significantly improved by the present calculations. It should be noted that the Coulomb displacement energies are strongly dependent on the radius parameter r_0. For example, we obtain, for the low-lying $J^{\pi}=3^+$ levels, a $\Delta E_C(^{28}\text{Al}-^{28}\text{Si})$ value equal to 5478 keV for $r_0=1.28$ fm and 5544 keV for $r_0=1.26$ fm. An illustration of our results is given in Fig. 6 where the experimental excitation energies of the members of the triplets and the energies calculated from Coulomb displacement energies using wave functions from Ref. 3 and $r_0=1.28$ fm are compared. The levels presented in $^{28}\text{Si}_{\text{calc}}$ are obtained from the $\Delta E_C(^{28}\text{Al}-^{28}\text{Si})$ values of Table VI and the energies of the corresponding $^{28}\text{Al}_{\text{exp}}$ levels. In the same way the levels $^{28}\text{P}_{\text{calc}}$ result from the calculated Coulomb energy differences and the positions of the corresponding $^{28}\text{Si}_{\text{exp}}$ levels.

We wish to thank Dr. R. J. De Meijer for enlightening comments concerning Coulomb energy calculations and for making his CDE computer code available to us. We also thank Dr. F. Meurders for permission to use his wave functions prior to publication.

[1]M. J. A. De Voigt and B. H. Wildenthal, Nucl. Phys. A206, 305 (1973).
[2]B. H. Wildenthal, J. B. McGrory, E. C. Halbert, and P. W. M. Glaudemans, Phys. Lett. 26B, 692 (1968).
[3]F. Meurders, Fysisch Laboratorium der Rijksuniversiteit, Utrecht, The Netherlands (private communication).
[4]B. Lawergren, G. C. Morrison, and A. T. G. Ferguson, Nucl, Phys. A106, 455 (1968).
[5]W. Bohne, H. Fuchs, K. Grabisch, M. Hagen, H. Homeyer, J. Janetski, H. Lettav, K. H. Maier, H. Morgenstern, P. Pietrzik, G. Roschert, and J. A. Scheer, Nucl. Phys. A131, 273 (1969).
[6]D. O. Boerma and P. B. Smith, Phys. Rev. C 4, 1200 (1971).
[7]R. M. Freeman and A. Gallmann, Nucl. Phys. A156, 305 (1970).
[8]N. Ahmed, M. A. Rahman, S. Kahtun, M. A. Awal, M. Rahman, and H. N. Sen Gupta, Z. Phys. 257, 380 (1972).
[9]M. A. Meyer, N. S. Wolmarans, and D. Reitmann, Nucl. Phys. A144, 261 (1970).
[10]N. Mangelson, M. Reed, C. C. Lu, and F. Ajzenberg-Selove, Phys. Lett. 21, 661 (1966).
[11]C. E. Moss, N. S. P. King, A. B. Comiter, and R. A. Ristinen, Nucl. Phys. A179, 89 (1972).
[12]C. Miehé, A. Huck, and G. Walter (unpublished).
[13]P. M. Endt and C. van der Leun, Nucl. Phys. A214, 1 (1973).
[14]R. J. De Meijer, H. F. J. Van Royen, and P. J. Brussaard, Nucl. Phys. A164, 11 (1971).
[15]L. W. Fagg, W. L. Bendel, E. C. Jones, Jr., and S. Numrich, Phys. Rev. 187, 1378 (1969).
[16]H. W. Kuehne, P. Axel, and D. C. Sutton, Phys. Rev. 163, 1278 (1967).
[17]C. E. Moss and J. D. Sherman, Nucl. Phys. A259, 413 (1976).
[18]M. A. Meyer, I. Venter, and D. Reitmann, Nucl. Phys. A250, 235 (1975).
[19]J. Lyttkens, I. Bergqvist, G. Lodin, and L. Nilsson, Phys. Scripta 13, 96 (1976).
[20]F. A. Beck, T. Byrski, P. Engelstein, and C. Miehé, in Proceedings of the Conference on Nuclear Physics, New Delhi, India, 1974 (unpublished).
[21]J. Dalmas, F. Leccia, and M. M. Aleonard, Phys. Rev. C 9, 2200 (1974).
[22]I. Forsblom, Comment. Phys. Math. 40, 135 (1970).
[23]P. B. Lyons, J. W. Toevs, and D. S. Sargood, Nucl. Phys. A130, 1 (1969).
[24]P. M. Endt and C. van der Leun, At. Data Nucl. Data Tables 13, 67 (1974); Nucl. Phys. A235, 27 (1974).
[25]J. Janecke, in *Isospin in Nuclear Physics*, edited by D. H. Wilkinson (North-Holland, Amsterdam, 1969), p. 297.
[26]N. Jelley, N. Anyas-Weiss, M. R. Wormald, B. Y. Underwood, and K. W. Allen, Phys. Lett. 40B, 200 (1972).
[27]F. Huang, E. F. Gibson, and D. K. McDaniels, Phys. Rev. C 3, 1222 (1971).
[28]M. A. Meyer and N. S. Wolmarans, Nucl. Phys. A130, 663 (1969).