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Model calculation of relativistic corrections to the triton binding energy

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We compute relativistic corrections to the triton binding energy (B_T) using an effective s-wave potential which yields, in the nonrelativistic case, $B_T = 7.6 \pm 0.2$ MeV. In the framework of Vinogradov's formulation of the three-body problem in Lobachevsky space, we find $B_T = 8.3 \pm 0.15$ MeV. We use two-body t matrices that give the same two-body phase shifts and binding as the nonrelativistic t matrices. Our results for relativistic corrections are substantially larger than those previously reported.

[NUCLEAR STRUCTURE ^3H; binding energy calculated with nonrelativistic and relativistic three-body theories; average s-wave potential.]

Recent work[1] on the calculation of the triton binding energy (B_T) suggests that nonrelativistic three-body calculations with phenomenological potentials might be closer to experimental data than previously thought. This fact makes it very important to have an estimate of relativistic corrections to B_T, in order to determine how they modify the nonrelativistic results. Previous computations[2-5] have yielded relativistic corrections of approximately +0.25 MeV,[2] +0.5 MeV,[3] and −0.25 MeV.[4] It is, however, difficult to compare the results obtained by Jackson and Tjon[2] in the framework of relativistic Faddeev-like equations[6] on the one hand, and by Mitra and coworkers[3,4] on the other hand. The latter work[3] indeed does not readjust the parameters of the relativistic two-body problem so as to leave two-body phase shifts unchanged and expands the energy parameter in powers of v/c. It is therefore of interest to examine how other approaches to the relativistic three-body problem will affect the above results.

We wish to report here an exploratory calculation of the triton binding energy. It is based upon a formulation of the relativistic three-body problem in Lobachevsky space,[7,8] which extends to the three-body problem the work by Kadyshevsky and coworkers[9] on the relativistic two-body problem. This approach is unrelated to the Bethe-Salpeter equation.[9] Although the interaction is not introduced via field theory, the formulation of the two-body problem in Lobachevsky space allows for a natural generalization of the nonrelativistic (Euclidean) interaction.[9] Thus, this approach avoids the main defect of non-field-theoretic approaches, i.e., the arbitrariness of the interaction term. We discuss the assumptions we make, the equations we use, and the binding energies we find for our model triton.

A complete relativistic three-body problem is much more complicated than the corresponding nonrelativistic problem. The first difficulties are due to the use of relativistic variables[10] and we discuss these below. In addition, care is needed in obtaining the required two-body scattering amplitudes. By this we mean the scattering amplitudes may be required for energies outside of physical regions or in regions not reached by straightforward analytic continuation.[11] However, when spinless particles interact, all the required two-body t matrices are found from the integral equation associated with the two-body Kadyshevsky equation. Since our intent is to provide an illustrative estimate of relativistic effects with the Vinogradov-Kadyshevsky three-body formulation, we use spinless nucleons. The appropriate relativistic two-body equation is discussed in Ref. 12.

We further simplify the problem by using an average s-wave potential. Such potentials have been used in various three-body calculations.[4,13,14] Our potential in coordinate space is

$$V(r)=\gamma[-14.947\exp(-0.7r) -2358.0\exp(-2.8r) +9263.1\exp(-4.9r)]/r \qquad (1)$$

and the parameters are those of the $^1S_0$ nucleon-nucleon Reid potential.[15] Here $\gamma = 1.22535$ for the nonrelativistic case. We find the same phase shifts and binding energy when $\gamma = 1.18145$ and the parameters of Eq. (1) are used in the two-body Kadyshevsky equation. We expect that this assumption of an average s-wave interaction is adequate for our present purposes.

The nonrelativistic three-body integral equation we solve is given by Eqs. (6) and (7) of Ref. 13. We solve these equations by the methods discussed there. For the potential of Eq. (1) we find

$$B_T(\text{nonrel}) = 7.6 \pm 0.2 \text{ MeV}. \qquad (2)$$

In order to get the relativistic counterpart of Eq. (6) of Ref. 13, we follow the work of Ref. 7 where all the variables we use are carefully defined. We introduce relativistic Jacobi variables $\vec{K}_{li}$ and $\vec{K}_j$, where $\vec{K}_{li}$ is the relativistic relative momentum of particles l and i, while $\vec{K}_j$ is the relative momentum of particle j relative to the pair (l,i). In the nonrelativistic limit, $\vec{K}_{li}$ and $\vec{K}_j$ reduce to the familiar Jacobi variables.[7] In terms of these variables, the total energy $\sqrt{S}$ of the three equal mass m particles is given by:

$$\begin{aligned}\sqrt{S} &= (S_{li} + \vec{K}_j^2)^{1/2} + (m^2 + \vec{K}_j^2)^{1/2} \\ &= \{[(m^2 + \vec{K}_{li}^2)^{1/2} + (m^2 + \vec{K}_{li}^2)^{1/2}]^2 + \vec{K}_j^2\}^{1/2} \\ &\quad + (m^2 + \vec{K}_j^2)^{1/2}. \end{aligned} \qquad (3)$$

In terms of the momenta of the individual particles, we have

$$S_{li} = (k_l + k_i)^2. \qquad (4)$$

Now, the invariant integration measure is given by

$$d\Omega_k = \frac{d^3k}{(1 + \vec{k}^2/m^2)^{1/2}}, \qquad (5)$$

whereas the analog of $\delta(\vec{p} - \vec{k})$ is

$$\left(1 + \frac{\vec{k}^2}{m^2}\right)^{1/2} \delta(\vec{p} - \vec{k}). \qquad (6)$$

The integral equation for the relativistic three-body problem is written in terms of $\vec{K}_{li}$ and $\vec{K}_j$, which play the role that $\vec{p}_j$ and $\vec{q}_j$ of Eq. (4) of Ref. 13 do in the nonrelativistic theory. When we carry out an angular-momentum decomposition we need to change basis and so we must determine the coefficients for

$$\vec{K}_{jl} = F\vec{K}_{li} + g\vec{K}_j, \qquad \vec{K}_i = \tilde{F}\vec{K}_{li} + \tilde{g}\vec{K}_j. \qquad (7)$$

We suppress the subscripts of the coefficients F, g, $\tilde{F}$, and $\tilde{g}$. The law of vector composition in Lobachevsky space is somewhat involved[7] and the coefficients in Eq. (7) involve two angle variables rather than one as the nonrelativistic theory does. In order to keep the three-body equations tractable, we find the coefficients and then evaluate them for the case that all the three-momenta are zero (i.e., we take their nonrelativistic limit). We proceed with the reduction to two variables of integration for the integral equation and find the analogs of the quantities of Eq. (7) of Ref. 13. In particular, the energy argument of the two-body scattering amplitude is

$$\sqrt{S} - (m^2 + K_j^2)^{1/2}.$$

We remark that with the variables of Vinogradov,[7] the two-body Kadyshevsky integral equation may be cast so it depends on variables which are invariants. This removes the need for a Lorentz transform of the two-body t matrices.

We have solved the analog of Eq. (6) of Ref. 13 to determine the three-body binding energy. Our two-body t matrices have the same binding energy and the same phase shifts from zero to over 300 MeV lab energy. Our result is

$$B_T(\text{rel}) = 8.3 \pm 0.15 \text{ MeV}, \qquad (8)$$

so that our relativistic correction is 0.7 ± 0.1 MeV. This is considerably larger than the result of Jackson and Tjon who, however, use a more realistic two-nucleon interaction that acts in the $^1S_0$ and $^3S_1$-$^3D_1$ channels. We also note substantial differences with the results of Mitra and coworkers.[3,4]

Our use of an effective s-wave interaction prevents us from drawing any definite conclusions on the cause of the discrepancy between the experimental B_T (8.48 MeV) and theory. We believe our result does suggest that the importance of relativistic corrections strongly depends on the underlying relativistic formalism used to describe two- and three-body problems. Recently varicus relativistic models of the two-nucleon interaction that include the important tensor force have appeared.[16] We hope that calculations of the triton binding energy are made with these models and a compatible three-body relativistic theory. This should clarify the role of relativistic corrections.

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