Proton-neutron correlations and shell model for collective motions

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A new truncation method for shell-model spaces is proposed, based on the proton-neutron correlations. The dominance of the quadrupole force in the p-n interactions is established by the investigation of empirical matrix elements in the pf shell. A truncation scheme based on the quadrupole component of the p-n interactions is applied to the $N = 30$ nuclei and compared with exact shell-model calculations. Satisfactory agreement between the two results suggests the wide validity of the present truncation method when applied to collective phenomena in nuclei. A relation between the shell model and the particle-vibration model is discussed.

[NUCLEAR STRUCTURE ^{54}Cr, ^{55}Mn, ^{56}Fe calculated levels J, π. Monopole and quadrupole calculation.]

I. INTRODUCTION

Many phenomenological facts suggest that collective motions appear more clearly in a nucleus which has active protons and neutrons in different shells outside a doubly closed core (to be referred to as a proton-neutron nucleus), than in a singly closed nucleus. For example, excitation energies of the first 2^+ state (2_1^+) in the Cd isotopes (proton-neutron nuclei) are about half of those in the Sn isotopes (singly closed nuclei). Another example is the relative ratio between the excitation energies of the 2_1^+ and the 4_1^+ states.[1] Almost all singly closed nuclei have a value of less than 2.0 for this ratio, while most proton-neutron nuclei have a value larger than 2.0. It is also well known that rotational spectra do not appear in singly closed nuclei but in proton-neutron nuclei.[2] The enhancement of nuclear collective motions in the proton-neutron nuclei points to strong correlation between the protons and the neutrons in nuclei.

Recently the proton-neutron correlations have been theoretically studied for nuclei in the $1f$-$2p$ shells.[3] Nuclei with $N=30$, such as ^{56}Fe, have two neutrons and Z-20 protons outside the ^{48}Ca core. The configuration $(1f_{7/2})^{Z-20}{}_p$, $(2p_{3/2}, 2p_{1/2}, 1f_{5/2})^2{}_n$ can be assumed for them and the use of empirical two body interactions for the shell-model Hamiltonian leads to satisfactory agreement between the calculated spectra and the experimental ones. The resulting wave functions show strong coupling between the proton and the neutron states. The breakdown of the pairing scheme is obvious in the 0^+ ground states of even-even nuclei. A large amount of the excited component $|J_p=2_1^+, J_n=2_1^+; J=0^+\rangle$ is contained in the ground state wave function besides the main component $|J_p=0_1^+, J_n=0_1^+; J=0^+\rangle$. The wave functions of the excited levels suggest that collective motions are much more enhanced in the proton-neutron nuclei than in the singly closed nuclei.

In this paper we reinvestigate the results of this shell-model work for the $N=30$ nuclei and try to understand simply how the proton-neutron interactions induce the nuclear collectivity. The proton-neutron interactions which are used in the shell-model calculations are described by a simple force in Sec. II and a new truncation scheme based on the proton-neutron interactions is proposed in Sec. III. In Sec. IV, we mention the relation between the shell-model Hamiltonian and the collective Hamiltonian. Our main conclusions are summarized in Sec. V.

II. SIMPLE FORCES FOR THE PROTON-NEUTRON INTERACTIONS

The proton-neutron interactions (the p-n interactions) between the $1f_{7/2}$ proton and the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ neutrons have been determined by a least-square fit to the observed spectra of the $N=29$ nuclei.[4] These empirical matrix elements hint at the character of the p-n interactions. The values of the diagonal matrix elements $\langle 1f_{7/2}2p_{3/2}|V_{pn}|1f_{7/2}2p_{3/2}\rangle_J$ are -0.787 MeV ($J=2$), -0.444 ($J=3$), -0.141 ($J=4$), and -1.026 ($J=5$). These values typically suggest the following two points:

(i) the negative value of the average energies $\overline{V}(j_p j_n)$, which is defined by

$$\overline{V}(j_p j_n) = \sum_J \frac{(2J+1)\langle j_p j_n|V_{pn}|j_p j_n\rangle_J}{(2j_p+1)(2j_n+1)}; \tag{1}$$

(ii) the large negative matrix elements of the states with the spin $J=|j_p-j_n|$ and $J=j_p+j_n$.

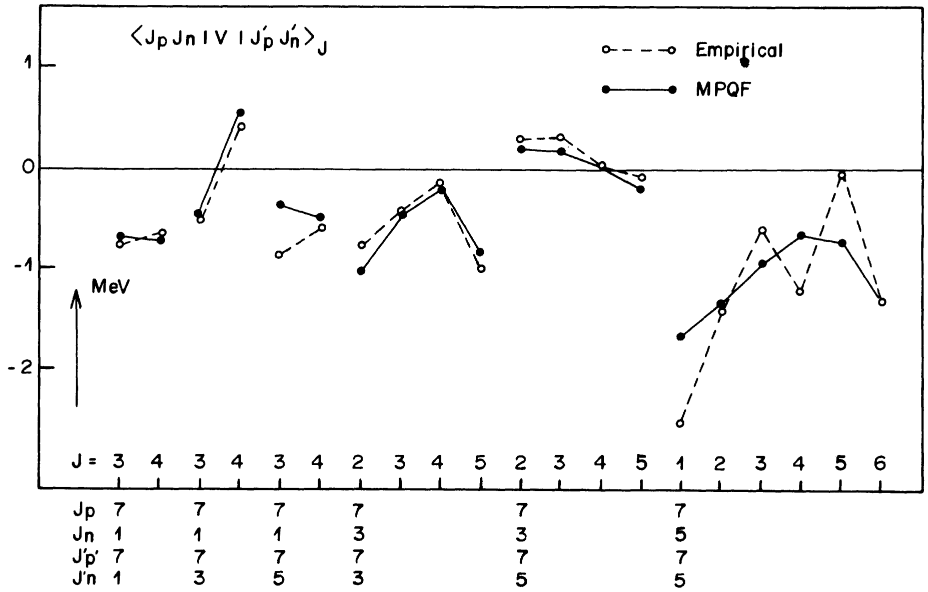

FIG. 1. Values of the p-n matrix elements calculated from the monopole plus quadrupole force (MPQF) are compared with matrix elements obtained empirically (see Ref. 4). The nlj values are specified by 7 for $1f_{7/2}$, 3 for $2p_{3/2}$, 1 for $2p_{1/2}$, and 5 for $1f_{5/2}$.

These two points are also clear for the other diagonal matrix elements $\langle 1f_{7/2}1f_{5/2}|V_{pn}|1f_{7/2}1f_{5/2}\rangle_J$. Furthermore, it should be noticed that such tendencies are common to the diagonal matrix elements of the p-n interactions in other shells. For example, the matrix elements $\langle 1d_{3/2}1f_{7/2}|V_{pn}|1d_{3/2}1f_{7/2}\rangle_J$ deduced from the ^{38}Cl nuclei and $\langle 1g_{9/2}2d_{5/2}|V_{pn}|1g_{9/2}2d_{5/2}\rangle_J$ from the ^{92}Nb nuclei have a behavior similar to these.[5]

Let us consider what kind of forces can simply reproduce these properties of the effective p-n interactions. The first property (i) suggests that the monopole force contained in the p-n interactions is strongly attractive. As far as the second property (ii) is concerned, we know that the quadrupole force gives the same tendency to the p-n interactions. Now we try to fit all empirical matrix elements of the p-n interaction between the $1f_{7/2}$ proton and the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ neutron by a monopole plus quadrupole force defined as

$$V_{pn} = k_0 + k_2(Y_p^{(2)} \cdot Y_n^{(2)}) r_p^2 r_n^2 ,$$

where k_0 and k_2 are strength of the monopole force and the quadrupole force, respectively.

First we determine the value of k_0. It is already known that average energy $\overline{V}(j_p j_n)$ depends on the neutron orbit.[4] Values of k_0 determined from the empirical average energies are -0.62, -0.73, and -1.01 MeV for $j_n = 2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$.

Once the parameter k_0 is fixed, the remaining part of the diagonal elements and the whole part of the nondiagonal elements of the p-n interactions should be explained by the quadrupole force. The strength of the quadrupole force k_2 is treated as a free parameter and given its best-fitted value $k_2 = -0.027$ MeV fm^{-4}. In Fig. 1, the matrix elements calculated by the monopole plus quadrupole force with the best-fitted parameters k_0 and k_2 are compared with the empirical ones. It is remarkable to see that not only does this simple force reproduce the diagonal matrix elements but also the nondiagonal ones. There remains a small discrepancy in the matrix elements $\langle 1f_{7/2}1f_{5/2}|V_{pn}|1f_{7/2}1f_{5/2}\rangle_J$. Dipole and higher multipole forces such as hexadecapole can probably improve the agreement between them. However, we omit such higher multipoles in this paper since we are interested in collective correlations of quadrupole character.

III. TRUNCATION METHOD BASED ON THE p-n INTERACTIONS

The fact that the characteristic feature of the p-n interactions can be described by the monopole plus quadrupole force allows the following approximation. The shell-model Hamiltonian of the proton-neutron nucleus is generally written as

$$H = H_p + H_n + V_{pn} , \qquad (2)$$

where $H_p(H_n)$ is Hamiltonian of the proton (neutron) system and V_{pn} is the p-n interactions. According to the previous discussion, the p-n interactions can be approximated by the monopole plus quadrupole force, i.e., $V_{pn} = k_0 + k_2(Q_p \cdot Q_n)$. Since the monopole part in the p-n interactions modifies single particle energies in H_p and H_n, we can write the Hamiltonian in the following form:

$$H = H'_p + H'_n + k_2(Q_p \cdot Q_n) , \quad (3)$$

where $H'_p(H'_n)$ is Hamiltonian of the proton (neutron) system with single particle energies modified by the monopole force. The matrix elements of the third term in Eq. (3) is written as

$$\langle \alpha J_p \cdot \beta J_n | k_2(Q_p \cdot Q_n) | \alpha' J'_p \cdot \beta' J'_n \rangle_J = k_2(-1)^{J_p + J'_n - J} W(J_p J_n J'_p J'_n ; J_2) \times \langle \alpha J_p \| Q_p \| \alpha' J'_p \rangle \langle \beta J_n \| Q_n \| \beta' J'_n \rangle , \quad (4)$$

where αJ_p and $\alpha' J'_p$ (βJ_n and $\beta' J'_n$) are proton (neutron) wave functions and J is the spin of the total system. This equation tells us how the p-n interaction correlates the motions of protons and neutrons. The qualitative feature of this matrix element depends mainly on the reduced matrix elements of the quadrupole operator, i.e., $\langle \alpha J_p \| Q_p \| \alpha' J'_p \rangle$ and $\langle \beta J_n \| Q_n \| \beta' J'_n \rangle$. This suggests that two states between which the reduced matrix element of Q_p or Q_n is large are strongly coupled by the p-n interactions. For example, it is well understood through this equation why the excited component $|J_p = 2^+, J_n = 2_1^+; J = 0^+\rangle$ has a large percentage (about 20%) of the 0^+ ground state wave function of an even-even nucleus such as ^{56}Fe. Since usually the $J_p = 2_1^+$ and $J_n = 2_1^+$ states exhaust most of the total sum of the $E2$ excitation from the ground states, $J_p = 0_1^+$ and $J_n = 0_1^+$, respectively,[6] in other words, since the reduced matrix elements $\langle J_p = 2_1^+ \| Q_p \| J_p = 0_1^+ \rangle$ and $\langle J_n = 2_1^+ \| Q_n \| J_n = 0_1^+ \rangle$ are large, the matrix element $\langle J_p = 2_1^+, J_n = 2_1^+ | (Q_p \cdot Q_n) | J_p = 0_1^+, J_n = 0_1^+ \rangle_{0^+}$ becomes the biggest nondiagonal element of the Hamiltonian matrix for $J = 0$. Almost equally weighted mixing of the $|J_p = 2_1^+, J_n = 0_1^+; J = 2^+\rangle$ and the $|J_p = 0_1^+, J_n = 2_1^+; J = 2^+\rangle$ components in the 2_1^+ state of the even-even proton-neutron nuclei can be explained in a similar way.

According to the discussion above a truncation method of the shell-model space is now proposed and its results compared with the exact ones of Ref. 3 mentioned above. A study of the states on the $(2p_{3/2}, 2p_{1/2}, 1f_{5/2})^2{}_n$ configuration, e.g., the nucleus ^{58}Ni, reveals to us that the $J_n = 2_1^+$ state nearly exhausts the sum rule $\sum_i B(E2; 0_1^+ \to 2_i^+)$ for the ground state $E2$ transition.[6] It is also true in the $N = 30$ system that the $J_n = 4_1^+$ state mostly exhausts the sum rule $\sum_i B(E2; 2_1^+ \to 4_i^+)$. From this fact we can know that the reduced matrix elements of $\langle J_n = 2_1^+ \| Q_n \| J_n = 0_1^+ \rangle$ and $\langle J_n = 4_1^+ \| Q_n \| J_n = 2_1^+ \rangle$ are much larger than others such as $\langle J_n = 2_2^+ \| Q_n \| J = 0_1^+ \rangle$ or $\langle J_n = 4_2^+ \| Q_n \| J_n = 2_1^+ \rangle$. Furthermore, the $J_n = 2^+$ wave functions yield diagonal element $\langle J_n = 2_1^+ \| Q_n \| J_n = 2_1^+ \rangle$ much larger than the nondiagonal ones such as $\langle J_n = 2_2^+ \| Q_n \| J_n = 2_1^+ \rangle$. Now we can select the most important neutron states for the calculation of the energy spectra in the proton-neutron nuclei with $N = 30$. Those are the states $J_n = 0_1^+$, 2_1^+, and 4_1^+, because coupling between them will be very strong owing to the quadrupole force in the p-n interactions. Instead of taking into account the 14 neutron states produced by the $(2p_{3/2}, 2p_{1/2}, 1f_{5/2})^2{}_n$ configuration and adopted in the exact shell-model calculations, we keep only 3 neutron states $J_n = 0_1^+$, 2_1^+, and 4_1^+, which are simply obtained by diagonalization of the Hamiltonian H'_n and couple them with all proton-states of the $(f_{7/2})_p^{Z-20}$ configuration.

Results for ^{56}Fe are shown in Fig. 2 together with the experimental[7] and the exact shell-model spectra. The truncated shell-model calculation successfully reproduce the spectrum obtained by the exact shell model. In other words, this truncation method explains well the observed spectrum. Difference in the ground state energy between the exact and the truncated calculations is 0.16 MeV. Overlaps between wave functions are 0.967, 0.962, 0.970, 0.983, and 0.997 for $J = 0_1^+$, 2_1^+, 4_1^+, 6_1^+, and 8_1^+ levels. These numbers show that this truncation is quite accurate especially for the calculation of the first yrast states. The one exception in Fig. 2 is the 0_2^+ level. The difference between two calculated excitation energies is 0.8 MeV. But this discrepancy is understandable because the 0_2^+ wave function obtained by the exact calculation shows that its main component is the $|J_p = 0_1^+, J_n = 0_2^+; J = 0^+\rangle$ one and the $J_n = 0_2^+$ state is omitted in the truncated calculation.

In the case of the nucleus ^{54}Cr ($Z = 24, N = 30$), further truncation of the proton states is possible. The proton configuration $(f_{7/2})^4{}_p$ gives eight states, i.e., $J = 0^+$ with seniority $v = 0$, $J = 2^+$, 4^+, and 6^+ with $v = 2$, and $J = 2^+$, 4^+, 5^+, and 8^+ with $v = 4$. But in this configuration, reduced matrix elements of the even-rank tensor operator vanish between the two states with same seniority, because the $1f_{7/2}$ shell is half filled: The example matrix elements $\langle 2^+ v = 2 \| Q_p \| 4^+ v = 2 \rangle$ and $\langle 4^+ v = 2 \| Q_p \| 6^+ v = 2 \rangle$ are exactly zero. So we select five proton states among the eight states. Those are the states of $J_p = 0^+ (v = 0)$, $2^+ (v = 2)$, $4^+ (v = 4)$, $6^+ (v = 2^+)$, and $8^+ (v = 4)$. Quadruple matrix elements between these are much larger than the others. These five proton states are coupled to the three neutron states ($J_n = 0_1^+$, 2_1^+, and 4_1^+ obtained from the Hamiltonian H'_n). The result of this truncated calculation for the nucleus ^{54}Cr is compared in Fig. 3 with the exact shell-model calculation, the calculation with truncated neutron state, and the experi-

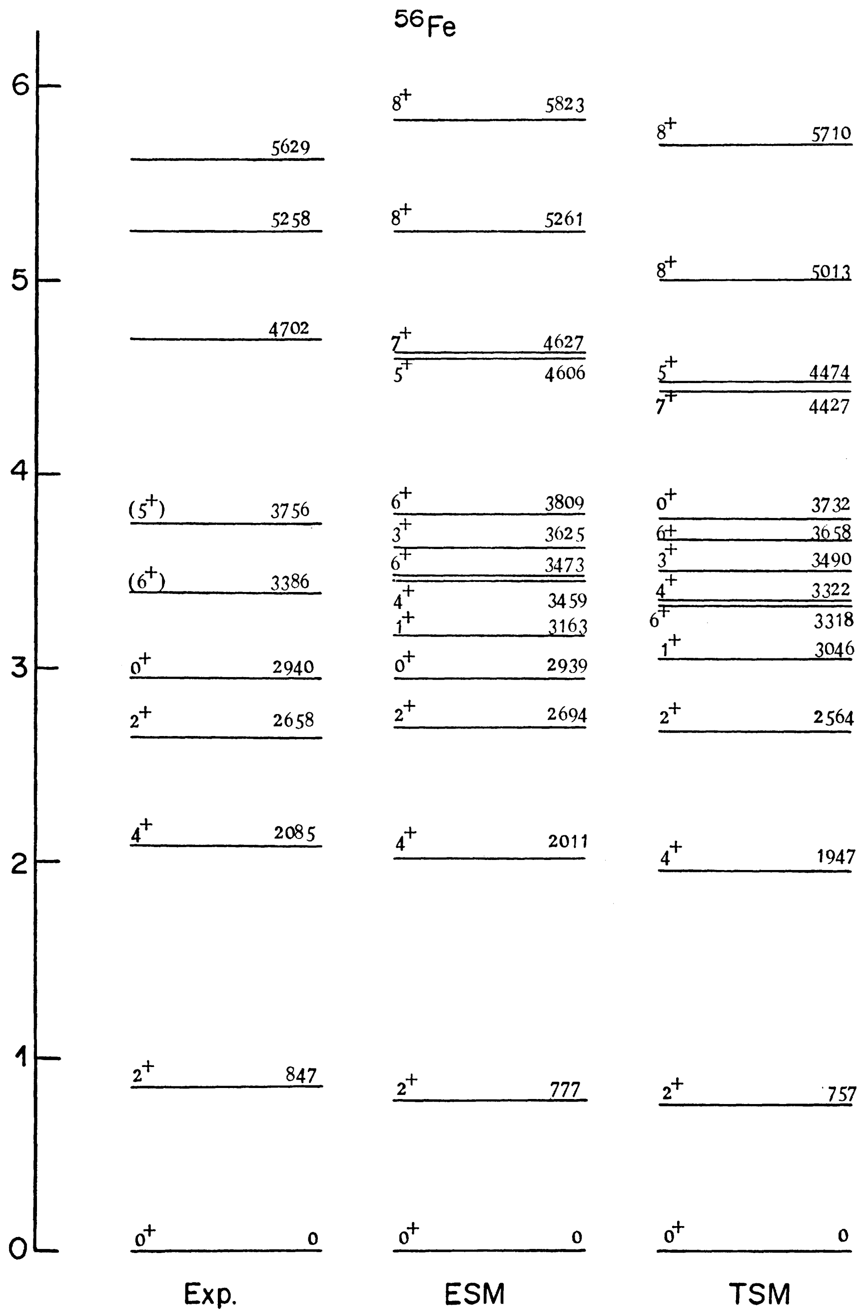

FIG. 2. Energy spectra of ^{56}Fe (in MeV). Result of the truncated shell-model calculation (TSM) is compared with the experiment and the exact shell-model calculation (ESM) (see Ref. 3).

mental spectrum. It is remarkable that agreement of this highly truncated calculation with the exact one and also with the experiment, is satisfactory. In the highly truncated calculations the dimensions of the Hamiltonian matrix are very much reduced from those of the exact calculations. For example, the dimension of the $J=4^+$ Hamiltonian matrix is reduced from 70 to 9.

IV. SHELL MODEL AND PARTICLE-VIBRATION MODEL COUPLING

Several eigenstates of the Hamiltonian H_p' or H_n' are adopted as basic states in the truncated shell model mentioned in the previous section. It is, however, not our intention here to insist on how to select basic states among the eigenstates of H_p' and H_n'. Rather, we wish to stress another way to

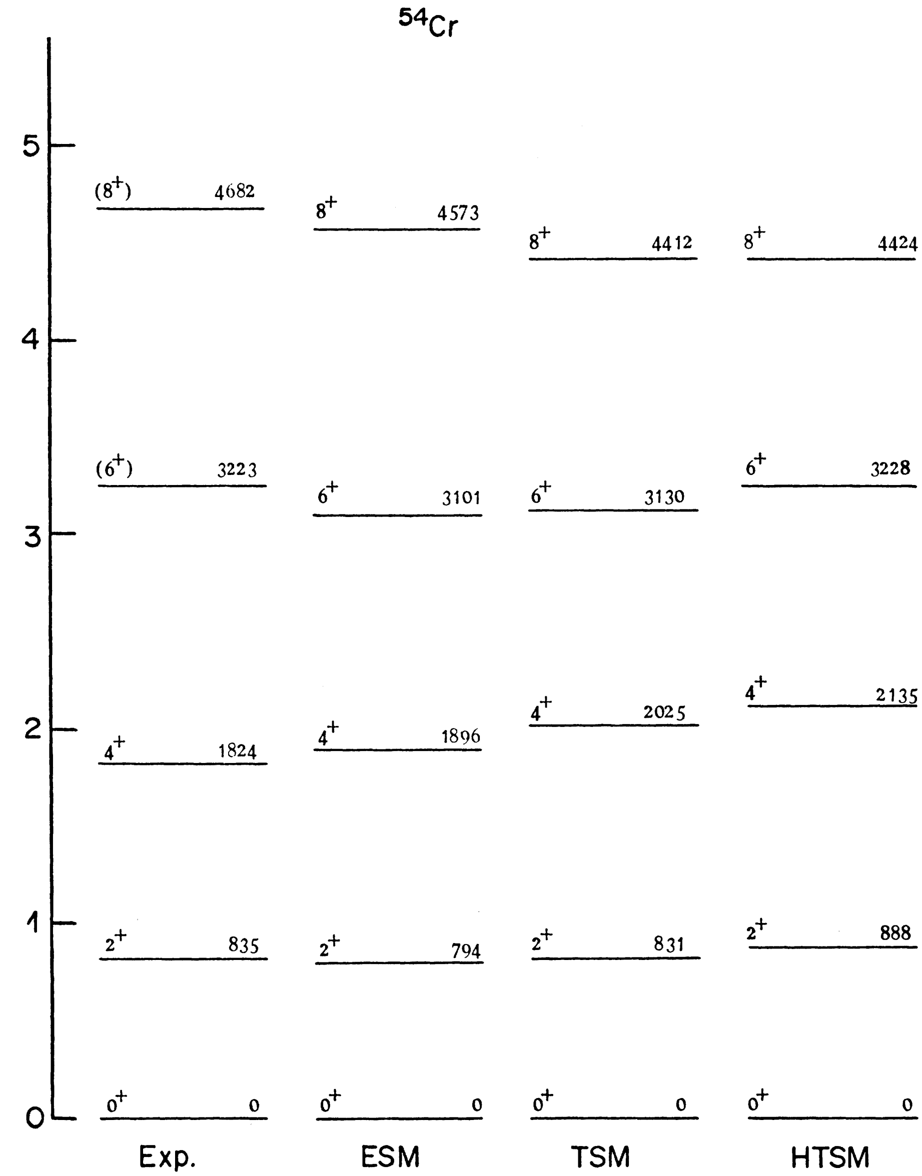

FIG. 3. Energy spectra of ^{54}Cr (in MeV). Results of the truncated shell model (TSM) and highly truncated shell model (HTSM) where proton states as well as neutron states are truncated, are compared with the experiment (see Ref. 7) and the exact shell model (ESM) (see Ref. 3). Only the lowest J = even spin levels are shown in the figure.

prepare basic wave functions for the truncation. It is not always true that real eigenstates exhaust mostly the $E2$ sum rule. For example a shell-model study of ^{60}Ni reveals us that values of $\langle 4_1^+ \| Q_n \| 2_1^+ \rangle$ and $\langle 4_2^+ \| Q_n \| 2_1^+ \rangle$ are comparable.[9] In this case it is better to make a new $J_n = 4^+$ wave function which is a linear combination of the $J_n = 4_1^+$ and 4_2^+ states so as to exhaust the sum rule for the quadrupole transition from the 2_1^+ wave function. This new $J_n = 4^+$ state is no longer an eigenstate of the neutron Hamiltonian, but is more meaningful in the proton-neutron nuclei. Generally speaking, motions of the like-nucleon system are rearranged in the proton-neutron nuclei by the p-n interactions whose strength favors preparing basic wave functions that exhaust the sum rule of the quadrupole operator. Such wave functions of the like-nucleon system already include the rearrangement effect due to the p-n interactions. For example, we can make a neutron state of the spin $J_n = 2^+$ within a given configuration space by operating with Q_n on a suitable $J_n = 0^+$ state. Then one can also construct wave functions of the spin $J_n = 4^+, 6^+, \ldots,$ up to the highest spin state which the

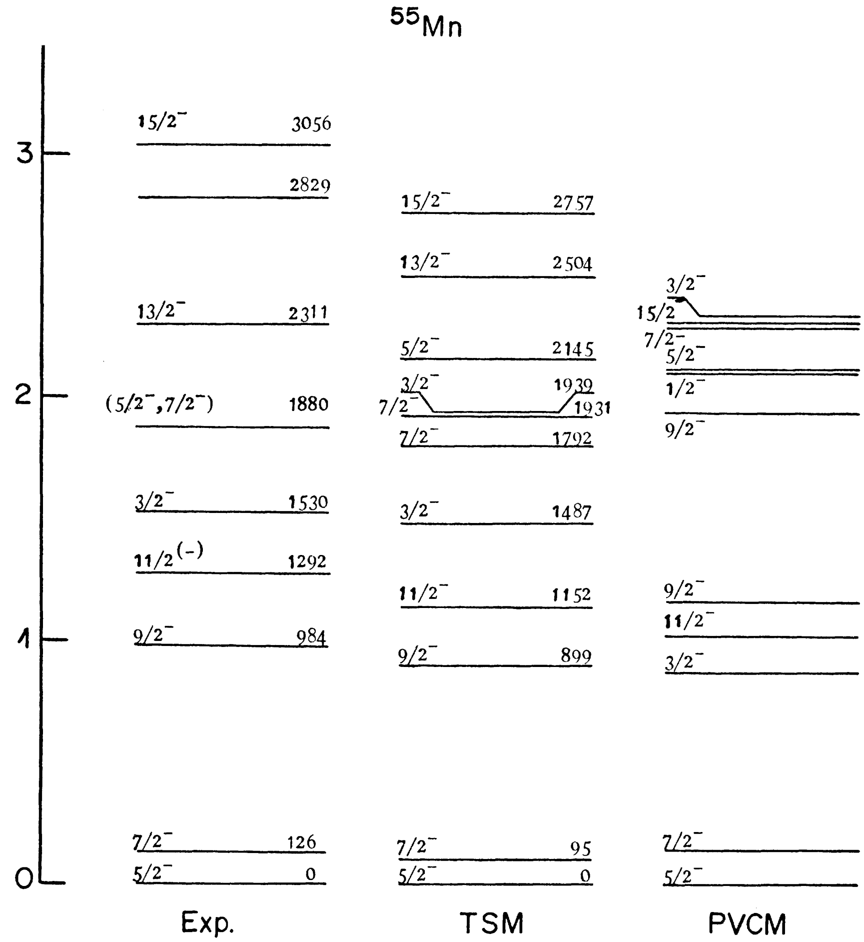

FIG. 4. Energy spectra of ^{55}Mn (in MeV). Two different predictions by the truncated shell model (TSM) and the particle-vibration coupling model (PVCM) (see Ref. 11) are compared with the experiment (see Refs. 12 and 13).

configuration gives, by operating with Q_n successively. These neutron states are coupled to the proton wave functions which are determined in a similar way. If it is necessary, we can prepare another series of wave functions which are constructed from a different initial 0^+ state. This truncation method will have wide validity in the shell-model calculation of the nuclei with complicated configurations. In the case of the $N=30$ nuclei, $J_n=0_1^+$, 2_1^+, and 4_1^+ states which are eigenstates of H_n' can be considered as being approximately the same as those obtained by the method just mentioned.

If the configuration space gives other $J_n=2^+$ and $J_n=0^+$ wave functions which have large quadrupole matrix elements with the $J_n=2^+$ state obtained from $Q_n|0_n^+\rangle$, we should include those 2^+ and 0^+ states in the truncated shell model. In such cases the situation resembles the particle-quadrupole-vibration-coupling Hamiltonian. So we can say the particle-vibration coupling model (or Alaga model[10]) corresponds to a special case of the truncated shell model.

Recently the particle-vibration coupling model has been applied to the nucleus ^{55}Mn ($Z=25$ and $N=30$)[11] described by the coupling of the three-proton–hole motion to a low frequency quadrupole vibration. In other words, the motion of the neutron system is assumed to be a quadrupole vibration and the $J=0_2^+$, 2_2^+, and 4_1^+ states in ^{58}Ni are regarded as two-phonon states. Three proton holes $(f_{7/2}, s_{1/2}, d_{3/2})^{-3}{}_p$ are coupled to this vibrational motion by the quadrupole interactions. In Fig. 4 the results of the particle-vibration coupling model are compared with the truncated shell model, we have kept the lowest $J_n=0_1^+$, 2_1^+, and 4_1^+ eigenstates of H_n' and coupled them to all proton states in the $(f_{7/2}{}^{-3})_p$ configuration. The essential difference between the particle-vibration coupling model and the truncated shell model lies in the assumptions about the neutron states. In the particle-vibration model the $J_n = 0_2^+$ and 2_2^+ states are included, but diagonal matrix elements $\langle 2_1^+ \| Q_n \| 2_1^+ \rangle$ or $\langle 4_1^+ \| Q_n \| 4_1^+ \rangle$ are neglected. Figure 4 simply shows us how these assumptions yield the different spectra. Both calculations reproduce the ground state with anomalous spin $J = \frac{5}{2}^-$ and the first excited state $J = \frac{7}{2}^-$. Around 1 MeV, three levels of $J = \frac{9}{2}^-$, $\frac{11}{2}^-$, and $\frac{3}{2}^-$ are also predicted by both models, but the detailed spacing of these three levels is predicted differently. The $J = \frac{3}{2}^-$ level is calculated at too low energy and the sequence of the levels of $J = \frac{9}{2}^-$ and $\frac{11}{2}^-$ is inversed in the particle-vibration model. From comparison with the experiment, the truncated shell model gives better prediction of the three levels than the particle-vibration coupling model. Inclusion of the $J_n = 0_2^+$ and 2_2^+ states is not essential in describing the ^{55}Mn nucleus and the introduction of the large quadrupole moment of the $J_n = 2_1^+$ state will improve the prediction by the particle-vibration coupling model.

Here we conclude that the $N = 30$ neutron system is not purely vibrational, in spite of the phononlike spectrum of ^{58}Ni. Thus the particle quadrupole-vibration coupling model appears to have a limit in applicability.

V. SUMMARY AND CONCLUSION

It has been shown that the monopole plus quadrupole forces can represent the empirical matrix elements of the p-n interactions in the pf shell. Strong attractive quadrupole force contained in the p-n interaction is indicated as one of the origins of the nuclear collectivity. Based on this force, a truncation method of the shell-model space is proposed and applied to the nuclei with $N = 30$ for which the exact shell-model calculations have already been carried out. Agreement between the exact and truncated shell-model calculations is satisfactory especially for so called yrast states. Since the proton-neutron correlation is strong in a proton-neutron nucleus, this new truncation method will have a validity in nuclei with configurations more complicated than the $N = 30$ nuclei. This method of truncation can be classified among those not based on unperturbed energies or diagonal matrix, but on the nondiagonal matrix elements[14] of the Hamiltonian. Hecht, McGrory, and Draayer[15] considered the truncation method based on the surface-δ interactions in some schematic cases. Our idea is similar to theirs, but much simpler. Since the surface-δ interaction contains higher multipole forces besides the quadrupole force, they therefore take into account the sum rule of the higher multipole operators. However, it is shown in the present study of the pf shell that the most important component of the p-n interaction for the collective motions is the quadrupole one. The inclusion of the higher multipole forces will improve results, but the simpler model presented here shall be applied to various nuclei in order to understand the nuclear collective motions like quadrupole vibration or quadrupole deformation by the use of the shell-model Hamiltonian.

Through this truncation scheme, we can easily discuss the applicability of the particle-vibration coupling model by comparing its Hamiltonian with a simplified shell model one.

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