Mass extrapolation of the amplitude for pion production in N-N collision

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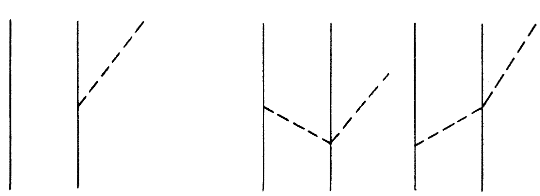
The s-wave pion production process (at $\vec{q}_\pi = 0$) is calculated as a function of the mass of the produced pion. The invariant matrix element is found to be surprisingly constant over a large range of pion mass.

[NUCLEAR REACTION Threshold $\pi^- d \to nn$, off-mass-shell extrapolation.]

I. INTRODUCTION

A considerable understanding of pion production in the reaction $pp \to \pi^+ d$ has been obtained in the last 25 years. Apart from the simple impulse approximation first proposed [Fig. 1(a)],[1] one also needs to include at least one rescattering of the pion [Fig. 1(b)].[2,3] [That is to say, the series is usually truncated at Fig. 1(b), but higher order terms may be important.[4]] These diagrams were evaluated for realistic N-N wave functions in the initial and final state for the case of the s-wave pion at threshold by Koltun and Reitan (KR).[5] More recently this model has been extended with more success to include p-wave production through the resonance region.[6]

Of course, the approaches which we have described involve a rather complex model dependent description at the microscopic level. Indeed, the large momentum transfer involved renders quantitative reliance on any one model rather dubious. For this reason, attempts to derive the pion production cross section in the soft pion limit are of considerable interest.[7-11] In analogy with Low's theorem for low energy photons, one can express the amplitude for producing a pion of zero mass in terms of purely on-shell quantities. Certainly the extrapolation to the real mass requires a model, but experience in other reactions at first suggested such corrections might be small.[12]



(a) (b)

FIG. 1. Diagrams included in the KR (Ref. 5) method for calculating pion production. (a) Lowest order diagram (SS). (b) Rescattering diagram (DS).

Recently there has been an interest in the soft pion approach for a different reason. Bernabéu *et al.*[13] have investigated μ capture in nuclei in the timelike region—corresponding to low neutrino momentum. In the zero momentum limit they observe that the cross section depends only on an incoherent sum of $|\langle\vec{A}\rangle|^2$, $|\langle A_0\rangle|^2$, $|\langle\vec{V}\rangle|^2$—the axial-vector and vector weak currents. Although the cross section vanishes as $\vec{q}_\nu^{\,2}$, one might hope to determine the coefficient of $\vec{q}_\nu^{\,2}$ in a model independent way from other experiments.

In particular, using partial conservation of axial vector current (PCAC) for a process with zero three-momentum transfer one finds

$$q_0\langle f|A_0|i\rangle \propto \langle f|\phi_\pi|i\rangle, \tag{1}$$

and hence the matrix element of A_0 is related to the matrix element for s-wave pion absorption. One might therefore hope to use the measured widths of pionic atom 1s states to get $|\langle A_0\rangle|^2$, if the four-momentum transfer (m_π^2 and m_μ^2, respectively) were equal, or if it were a slowly varying function of momentum transfer.

Unfortunately it has been known for several years that soft pion theory fails qualitatively for $pp \to \pi^+ d$. Numerical results based on the KR model indicate that for a reasonable deuteron D-state probability the parts of Fig. 1(a) leading to S and D states effectively cancel.[5,14] Therefore the rescattering diagram Fig. 1(b), which goes almost entirely to the S state, dominates. On the other hand, the nice feature of the soft pion limit is that external emission should dominate. Clearly, there are drastic changes during the extrapolation. It is certainly not obvious that one could reliably extrapolate the pion absorption amplitude even from m_π^2 to m_μ^2.

It is true that the most detailed calculations of pion production have been carried out for $pp \to \pi^+ d$, and the above considerations are based entirely on this. However, the phenomenological two-nucleon model of pion absorption (production) seems fairly successful in complex nuclei,[15,16] and the operator in this model is written in terms of the absorption on the quasideuteron. Clearly within such a model, poor extrapolation properties of the elementary amplitude will be reflected in the full nuclear amplitude.

It is this uncertainty in the extrapolation of the matrix element of $A_0(q^2)$ which has led us to the present work. We investigate the dependence of the pion production matrix element on the external pion mass. Although we work only within an extended version of the KR method, the result is so beautiful that we feel it may be much more general.

II. METHOD OF CALCULATION

Our evaluation of Fig. 1 follows the KR[5] formulation rather closely. That is, the system is described by a fully interacting N-N Hamiltonian, plus a πN interaction. [There is no double counting because the πN interaction is only included in first order.] The πN interaction is a phenomenological form based on a nonrelativistic reduction of the pseudoscalar interaction

$$H_{\pi N}=K_0+K_1+K_2 . \tag{2}$$

The first term is the usual Galilean invariant term, linear in the pion field, which leads in lowest order to single pion production or absorption. The last two terms describe the isoscalar (ϕ^2) and isovector ($\vec{\tau}\cdot\vec{\phi}\times\vec{\pi}$) s-wave πN interactions, with strengths (λ_1 and λ_2, respectively) adjusted to fit the πN scattering lengths.[5]

In evaluating the matrix element for the production of an off-mass-shell pion, we need to know the scattering amplitude for the case of one pion on shell and one off shell. There is very little information on the behavior of the scattering amplitude in this case. (Usually both the incident and outgoing pions are taken to zero mass.) We have chosen to use a smooth extrapolation, with variation arising solely from the normalization in the field operators,

$$\langle O\,|\,\vec{\tau}\cdot\vec{\phi}(\vec{x})|\,\pi^+,\,(\vec{q},q_0=\mu)\rangle=-\tau_-\frac{e^{i\vec{q}\cdot\vec{x}}}{(2\mu)^{1/2}} \tag{3}$$

$$\langle O\,|\,\vec{\tau}\cdot\vec{\pi}(\vec{x})|\,\pi^+,\,(\vec{q},q_0=\mu)\rangle=+i\tau_-(\tfrac{1}{2}\mu)^{1/2}e^{i\vec{q}\cdot\vec{x}} . \tag{4}$$

Using Eqs. (3) and (4) at the vertex which gives rise to the external pion, the entire KR analysis can be repeated. The coupling strength in K_0 is assumed constant at (f/m_π). Then the total dependence on the external pion mass of the resulting invariant matrix element is

$$|M_{\rm inv}|^2=A\left|\sum_{i=1}^{6}\tilde{I}_i\right|^2 \tag{5}$$

where A is a constant depending on the pion, nucleon, and deuteron masses and the πN coupling constant $f^2=0.079$. (Although we call it an invariant matrix element, the nucleons are treated nonrelativistically.) Once again the integrals $\tilde{I}_1$ and $\tilde{I}_2$ correspond to direct production [Fig. 1(a)] leading to the S and D state of the deuteron, respectively. The other integrals ($\tilde{I}_3\cdots\tilde{I}_6$) involve rescattering of the produced pion [Fig. 1(b)]. They are given in Eq. (13) of KR except that: (a) μ is now the external pion "mass" or the fourth component of the pion four-momentum in which we are analytically continuing the production amplitude off mass shell; (b) the N-N ($^3P_1$) scattering wave function is calculated at a c.m. kinetic energy of $\mu-\epsilon_d+(m_p-m_n)$ where ϵ_d is the deuteron binding energy and (m_p-m_n) is the proton-neutron mass difference. (c) the rescattering strength $(\lambda_1+\frac{3}{2}\lambda_2)$ is replaced by

$$\lambda_1\left(\frac{\mu}{m_\pi}\right)+\tfrac{3}{2}\lambda_2\left(\frac{\mu}{m_\pi}\right)^2 \tag{6}$$

with $\lambda_1=0.002\,33$ and $\lambda_2=0.043\,67$ corresponding to the lastest values of s-wave πN scattering length[17]; (d) the effective pion propagator in the intermediate state is

$$\tilde{f}(r)=\exp(-\tilde{\mu}r)/r \tag{7}$$

with

$$\tilde{\mu}=(m_\pi^{\,2}-\tfrac{1}{4}\mu^2)^{1/2} .$$

Equation (5) has been evaluated for external pion masses μ in the range 0–140 MeV, using both the Reid soft core (RSC) and Bryan-Scott (BS) N-N

TABLE I. The dependence of the invariant matrix element for pion production M on the external pion mass μ for two different N-N interactions (Ref. 17). $M^{\rm SS}$ is the direct contribution of Fig. 1(a). Finally

$$\Delta=\{|M_{\rm BS}|^2-|M_{\rm RSC}|^2\}/\{|M_{\rm BS}|^2+|M_{\rm RSC}|^2\}$$

measures the difference between these two models.

μ (MeV)	$\lvert M(\mu)\rvert^2/\lvert M(m_\pi)\rvert^2$ BS	RSC	Δ (%)	$M^{\rm SS}(\mu)/M(\mu)$ BS (%)	RSC
m_π	1.00	1.00	8.8	16	−3
130	1.01	1.03	7.6	21	4
120	0.98	1.03	6.5	24	10
m_μ	0.97	1.04	5.0	30	20
90	0.97	1.07	3.9	39	32
70	0.91	1.02	2.6	48	45
50	0.82	0.94	1.7	60	58
30	0.62	0.73	0.4	75	72
10	0.26	0.31	−1.2	88	88

interactions.[18] (In view of the comments in Refs. 14 and 19, the same model of the N-N interaction is always used in both the $^3P_1$ and deutron channels.) The results of these calculations are summarized in Table I and discussed in the following section.

III. DISCUSSION OF RESULTS

It is immediatley apparent from the table that $|M(\mu)|^2$ is very slowly varying over a wide range of masses μ. Of course, for very small μ the Adler condition [that $M(\mu)\to 0$ as $\mu\to 0$] takes over. (A brief examination of the forms of the integrals $\tilde{I}_k$ shows that each goes to zero at least as quickly as $\mu^{3/2}$.) Nevertheless the variation of less than 10% between 140 and 60 MeV is remarkable.

A second important observation is that the single scattering (SS) term dominates as μ decreases. Qualitatively, this is rather easily understood, for as we decrease the mass μ, we lower the momentum transfer, and hence the rescattering mechanism becomes less important. Furthermore, since the S-wave deuteron component has a higher percentage of the wave function at low momentum as compared with the D state, the decrease in the momentum transfer leads to a larger contribution from the S wave ($\tilde{I}_1$), while the D-wave contribution ($\tilde{I}_2$) decreases. Thus the cancellation responsible for the well known sensitivity to the deuteron D-state probability (P_D) goes away. (This also increases the SS contribution.)

Certainly an important factor contributing to the decrease of the double scattering (DS) term, is that the effective isovector coupling is $\lambda_2(\mu/m_\pi)^2$. [From πN data λ_1 is very small for real pions (~5% of λ_2), and would be identically zero for a soft pion.] Therefore, using a different off-shell extrapolation could alter our results—at least quantitatively. Nevertheless, if we accept the present results (for lack of a better extrapolation), it is a remarkable fact that the decrease in both the D-state and rescattering contributions is largely responsible for the constancy of the production matrix element over a range of pion masses μ. The close connection of the one-pion exchange tensor force to the deuteron D state, suggests to us that this may not be coincidental, but we have been unable to establish the deeper connection.

As a final positive comment, we observe that our results encourage the use of pionic atom widths to put limits on the A_0 contribution to the timelike μ capture (as discussed earlier). In particular, if the quasideuteron model of pion absorption has some validity, the extrapolation from the pion to the muon mass should be essentially constant. This smoothness of extrapolation is essential to the recent work of Bernabéu *et al.* [13] in which the ρ^2 term in the pion potential is used, with the same coefficient (B_0), to describe a pion of mass m_μ as was needed in pionic atom studies.

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