Improved optical-potential calculation of the elastic and charge-exchange scattering of intermediate-energy pions from the helium isotopes*

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The elastic scattering of intermediate energy pions from ^{3}He and ^{4}He and the single charge exchange scattering from ^{3}He are calculated with a theoretical momentum space optical potential including spin, nucleon recoil, Fermi motion and binding, and a Lorentz invariant angle transformation. Realistic nuclear form factors and a πN T matrix with a finite range are employed. An examination is made of the π-^{4}He and π-^{3}He total and differential cross section data of several experimental groups and the possibility of extracting information on the magnetic moment form factor of ^{3}He is investigated. Comparisons are made with the procedures and results of several theoretical groups to determine the relevant physical differences and the areas in need of improvement.

[NUCLEAR REACTIONS ^{4}He($\pi^\pm,\pi^\pm$), ^{3}He($\pi^\pm,\pi^\pm$) elastic, ^{3}He(π^-,π^0)^{3}He; E =100–260 MeV; $\sigma(\theta)$ and σ_{TOT}; theoretical calculation, momentum space optical potential; binding, recoil, Fermi motion, angle transformation; compare data and other calculations.]

I. INTRODUCTION

The elastic scattering of intermediate energy pions from nuclei has received such extensive theoretical attention[1-9] that there seems little left to learn from another paper, even if its theory agrees well with the data. Indeed, the oft-quoted successes of so many diverse and apparently incompatible theories obscure rather than illuminate any picture developed of the π-nucleus interaction, make it difficult to focus on what are still the unanswered questions and on which avenues of investigation may be the most fruitful to follow.

In this paper we attempt to modify the above situation; hopefully we succeed by at least a slight amount. We examine several data sets on the He isotopes, we analyze the relevant physical differences between our calculation and those of other investigators, and in this way deduce the theoretical and experimental areas in need of improvement.

Specifically, in Section II we update our previous construction of the optical potential[10] in order to examine certain nucleon binding, Fermi motion and recoil effects, and in light of recent discussion, take another look at the "angle transformation." In Section III we make use of these improvements to examine the recently published π^--^{4}He data of the CERN group[11] and the $\pi^\pm$-^{4}He and $\pi^\pm$-^{3}He data of the Dubna-Torino group[12] with the specific aim of learning new physics. In addition, we extend our calculation and examine the single charge exchange reaction of ^{3}He. In Section IV we conclude with a summary of our calculation and a general comparison with the work of other groups. In a separate paper[13] we discuss several questions of particular importance for low energy (<100 MeV) pi-nucleus scattering.

II. THEORETICAL OPTICAL POTENTIAL

We calculate pi-nucleus ($\pi\eta$) scattering amplitudes by solving a Lippman-Schwinger integral equation,

$$T'_{L\pm}(k'|k) = U_{L\pm}(k'|k) + \frac{2}{\pi}\int_0^\infty \frac{dp\ p^2\ U_{L\pm}(k'|p)T'_{L\pm}(p|k)}{E(k_o) - E(p) + i\varepsilon}\ , \tag{2.1}$$

where the complex optical potential U is non-local, energy-dependent, and spin-dependent,[14] and $E(p)=(m_\pi^2+p^2)^{\frac{1}{2}}+(m_A^2+p^2)^{\frac{1}{2}}$. Since U is constructed from elementary pi-nucleon (πN) amplitudes, a solution of (2.1) contains all orders of multiple scattering (and multiple spin flip for a spin $\frac{1}{2}$ nucleus). Since a general description of the optical potential we use in (2.1) has been given before,[4,10] we concentrate on several procedures which are of current interest.

A. Optimal Factored Approximation

In "impulse approximation" the collision matrix which describes a pion scattering from a bound nucleon is an operator in only pion and active nucleon coordinates. The first order optical potential in the pion-

nucleus c.m. is then

$$U(\vec{k}'|\vec{k}) = (A\text{-}1) \sum_\alpha \int \psi^*(\vec{p}_1-\vec{q},\vec{p}_2,\ldots\vec{p}_A)\, O_\alpha \langle \vec{k}',\ \vec{p}_1-\vec{q}|t_\alpha(\omega)|\vec{k},\vec{p}_1\rangle \times \psi(\vec{p}_1,\vec{p}_2,..\vec{p}_A)\delta(\sum_{i=1}^{A}\vec{p}_i + \vec{k})\, d^3p_1 d^3p_2 \ldots d^3p_A \quad . \tag{2.2}$$

Here $\vec{k}$ and $\vec{k}'$ are initial and final pion momenta, $\vec{q} = \vec{k}'-\vec{k}$, t_α is a πN T matrix appropriate to the spin-isospin operator α, and ψ is a nuclear many body momentum space wave function. The delta function in (2.2), insures momentum conservation and, guarantees there are only A-1 independent coordinates for the nucleons, by accounting for it in our theory we include c.m. motion effects.

Since only a single nucleon's coordinate appears in the T matrix in (2.2), the nucleus can be described by giving the probability amplitude of finding a nucleon with momentum $\vec{p}_1$ and the rest of the nucleus (the core) with $\vec{P}$, $\psi(\vec{p}_1,\vec{P})$. Accordingly, the integration in (2.2) removes the delta function and U is written in terms of the wave function ϕ for relative nucleon-core motion[13]:

$$U(\vec{k}'|\vec{k}) = (A\text{-}1) \sum_\alpha \int \phi^*(\vec{p} - \tfrac{A-1}{A}\vec{q})\, \phi(p) \times \langle \vec{k},\vec{p}-\vec{q}-\vec{k}/A|t_\alpha(\omega)|\vec{k},\vec{p}-\vec{k}/A\rangle d^3p \quad . \tag{2.3}$$

The same result is obtained by assuming an independent particle model for the nucleus.

Although (2.3) represents a viable means of calculating the first order optical potential, it does not immediately incorporate the rapidly varying nuclear form factors into the theory and in application would require the wave function ϕ to first fit the experimental form factors - we thus investigate another form for U. If the nuclear wave functions were Gaussian, $\phi(p) = \exp(-p^2/2\alpha)$, the form factor with the c.m. constraint,

$$F(q) = \int \psi^*(\vec{p}_1-\vec{q},\ \vec{p}_2\ldots\vec{p}_A)\psi(p_1,\ p_2\ldots p_A) \times \delta(\sum_{i=1}^{A} p_i+\vec{k})d^3p_1 d^3p_2\ldots d^3p_A \ , \tag{2.4}$$

would be $\exp\{-[(A-1)/A]^2q^2/4\alpha\}$. In this case the F(q) could be factored out of the wave functions in (2.3) leaving [4,13]:

$$U(\vec{k}'|\vec{k}) = (A\text{-}1) \sum_\alpha F_\alpha(\vec{k}'-\vec{k}) \int d^3p|\phi(p)|^2 O_\alpha \langle k',\ \vec{p}+\vec{p}_o-\vec{q}|t_\alpha(\omega)|\vec{k},\vec{p}+\vec{p}_o\rangle \ , \tag{2.5}$$

where

$$\vec{p}_o = -\vec{k}/A + (\tfrac{A-1}{2A})\vec{q} \ . \tag{2.6}$$

This form is illuminating since the important q-dependence describing the nuclear size is separated off into the nuclear form factor $F_\alpha(q)$, which also incorporates the c.m. constraint; in using explicit nuclear wave functions to calculate (2.2), the delta function constraint is difficult to include.

If the nucleon momentum ($\vec{p}$) variation of the T-matrix is much slower than that of $|\phi(p)|^2$ (T and ϕ scale as the πN and $\pi\eta$ sizes), the T-matrix could be removed from the integral and evaluated at some optimal "average" value, of $\vec{p}$, (2.6)[13]:

$$U(\vec{k}'|\vec{k}) \simeq (A\text{-}1)\sum_\alpha F_\alpha(q)\langle \vec{k}',\ \vec{p}_o-\vec{q}|t_\alpha(\omega)|\vec{k},\ \vec{p}_o\rangle\, O_\alpha \ . \tag{2.7}$$

The same result was deduced previously[10] in a more intuitive way by a direct examination of (2.2).

In our calculations we assume (2.5-.6) is a good approximation even for non-Gaussian wave functions; we use experimental nuclear matter and spin form factors (which include c.m. motion-effects) - but harmonic oscillator relative wave functions for ϕ. This should be an excellent approximation for light nuclei like He and C. We also compare the form (2.7) with the "folded" form (2.5).

B. πN Subenergy

One of the more important ingredients in constructing the optical potential is the value chosen for the πN subenergy ω in (2.5) or (2.7). The most obvious choice (called "optimal" in Ref. 10) sets ω equal to the Lorentz invariant c.m.

energy of the π and the N:

$$\omega_o^2 = s_{in} = (P_\pi^\mu + P_N^\mu)^2 \qquad (2.8)$$

$$= (E_\pi(k) + E_N(\vec{p}+\vec{p}_o))^2 - (\vec{k}+\vec{p}+\vec{p}_o)^2 \ ,$$

where $\vec{k}$ is taken to be $\vec{k}_o$, the on-shell value of pion momentum, and $\vec{p}_o$ is the "optimal" nucleon momentum (2.7). We now call this energy ω_o the "two-body" energy, since two particles are used to define it. For non-relativistic nucleons, ω_o has the form

$$\omega_o \simeq E_\pi(k) + m_N + (\vec{P}+\vec{k})^2/2m_N$$

$$-P^2/2(E_\pi(k) + m_N) \ . \qquad (2.9)$$

Another choice of energy, which Thomas and Landau[13] showed is important for low energy scattering, incorporates the average potential and kinetic energy of a bound nucleon into the theory. This is based on a three body picture of (2.5) in which a pion ($\vec{k}$) interacts with an active nucleon ($\vec{p}$ + $\vec{p}_o$) outside of a passive core ($\vec{P} = -\vec{k}-\vec{p}-\vec{p}_o$), and thus the nucleon 4-momentum is taken as the 4-momentum of the nucleus minus the 4-momentum of the core:

$$\omega_{3B}^2 = (P_\pi^\mu + P_\eta^\mu - P_c^\mu)^2 \ . \qquad (2.10)$$

For on-mass and energy shell scattering ω_{3B} and ω_o would of course be identical, yet for historical reasons we now take the total energy of the nucleus and core as a mass plus binding energy (eigenvalue of kinetic plus potential energy). In the non-relativistic nucleon limit, ω_{3B} then has the familiar form,[15]

$$\omega_{3B} \simeq E_\pi(k) + m_N + k^2/2Am_N - P^2/2(A-1)m_N$$

$$- P^2/2(E_\pi(k) + m_N) - |E_B| \ , \qquad (2.11)$$

i.e. the energy of the pion, plus the rest energy of a free nucleon, plus the kinetic energy of the nuclear c.m., minus the kinetic energy of the core, minus the kinetic energy of the c.m. motion of the πN pair, and minus the effective binding energy of the active nucleon (nucleus-core binding energy).

The essential difference between ω_o and ω_{3B}, after substituting for $\vec{P}$, lies in a $p^2/2\mu$ term, a $q^2/8\mu$ term [$\mu^{-1} = (E_\pi+m_N)^{-1} + ((A-1)m_N)^{-1}$] and the binding energy term. All three terms shift ω_{3B} <u>down</u> to a lower energy the first by $\sim$ 16 MeV, the second by an amount which increases with momentum transfer, and the binding energy, by an amount one might expect[16] to be $\sim$ 22 MeV. However, it has been shown in model calculations by several authors[17] that the interaction between the nucleon and the core, which is ignored in our impulse approximation calculation, can reduce considerably the downward shift suggested by a naive interpretation of (2.11). For this reason $|E_B|$ was treated as a parameter for low energy scattering and found to have a value of $\sim$ 5 MeV.[13] In the intermediate energy study presented here we still treat $|E_B|$ as a parameter in an effort to determine the need for binding corrections at these higher energies.

Regardless of which prescription for the subenergy is used, ω_o or ω_{3B}, the πN T matrix must still be evaluated at an energy which <u>increases</u> with pi-nucleus scattering angle ($\vec{p}_o$ is $\vec{q}$ dependent). In addition, (2.50), (2.7), (2.9) and (2.11) all require an independent variation of the momentum variables $\vec{k}$ and $\vec{k}'$ from the energy ω (which sometimes goes below threshold). Since we calculate the optical potential in momentum space, and use a separable model for the off-shell behavior of the πN center-of-mass T matrix:

$$\langle \kappa'|t^\alpha[\omega(\kappa_o)]|\kappa\rangle = \langle\kappa_o|t^\alpha[\omega(\kappa_o)]|\kappa_o\rangle$$

$$\times g_\alpha(\kappa')g_\alpha(\kappa)/g_\alpha(\kappa_o)^2 \ , \qquad (2.12)$$

these requirements are straightforward to include. However, for these reasons, and others, it would be extremely difficult for a coordinate space optical potential to accurately describe the important physics contained in (2.5), (2.6), (2.11), (2.12) and (2.13). As we indicate in Section III, the r-space potentials do not provide as good agreement with data.

The on-shell amplitudes in (2.12) are calculated from the Salomon fit to the πN phases (T_π < 160 MeV) and the Almahed-Lovelace tabulation.[19] In the present calculation we use the Landau-Tabakin potentials in (2.12), but would obtain very nearly identical results with the Londergan-Moniz-McVoy potentials.[8] Although the details of the separable potentials are irrelevant for the intermediate energy elastic scattering study presented here, at lower energies, when the πN amplitude must frequently be determined for energies ω_{3B} below threshold, there is significant sensitivity to the low momentum behavior of the $g_\alpha(p)$, or to the phase shifts used to calculate the g's.[13]

C. Relations Between Reference Frames

To proceed further in evaluating the optical potential $U(\vec{k}'|\vec{k})$, it is necessary and important[4] to relate the off-shell πN T matrix which appears in (2.5) or (2.7) to the off-shell $\langle \kappa'|t|\kappa\rangle$ evaluated in the πN c.m., (2.12). Our procedure is

given by Eqs. (2.14)-(2.19) of Ref. 4. The Lorentz invariant (but not conserved) total momentum "s" is evaluated before and after the collision in order to determine κ and κ':

$$s_{in} = (k^\mu + p_o^\mu)^2 = (E_\pi(\kappa)+E_N(\kappa))^2 \tag{2.13}$$

$$s_{out} = (k'^\mu + p_o^\mu - q^\mu)^2 = (E_\pi(\kappa')+E_N(\kappa'))^2 \ .$$

This use of a different "s" for the initial and final states is an invariant, consistent procedure, even for energy-nonconserving collisions ((2.13) nowhere assumes energy is conserved). The two s's determine the same magnitude for the πN c.m.; this is equivalent to the prescription given in the relativistic potential theories described by Aaron et al.[20] and Heller et al.[21], and does not depend on any approximations in the theory.

The scattering angles in πN and pi-nucleus c.m. frames, $\theta_{\pi N}$ and $\theta_{\pi\eta}$ respectively, are assumed related by the Lorentz invariant $t = (k'_\mu - k_\mu)^2 = (\kappa_\mu - \kappa_\mu)^2$, for all values of k and k'. This gives

$$\cos\theta_{\pi N} = \frac{E_\pi(\kappa)E_\pi(\kappa') - E_\pi(k)E_\pi(k')}{\kappa\kappa'} + \frac{kk'}{\kappa\kappa'}\cos\theta_{\pi\eta} \ . \tag{2.14}$$

The amplitudes in the two frames are assumed related by the multiplicative factor which insures Lorentz invariance of probability for on-shell scattering:

$$\langle \vec{k}', \vec{p}_o - \vec{q} | t | \vec{k}, \vec{p}_o \rangle = \gamma \langle \kappa' | t | \kappa \rangle \tag{2.15}$$

$$\gamma = \left[\frac{E_\pi(\kappa)E_\pi(\kappa')E_N(\kappa)E_N(\kappa')}{E_\pi(k)E_\pi(k')E_N(p_o)E_N(\vec{p}_o - \vec{q})} \right]^{\frac{1}{2}} .$$

These last two relations, (2.14) and (2.15), are correct only if energy is conserved in the pi-nucleus and πN scattering processes, as then the same single Lorentz transformation relates the initial states in different reference frames as relates the final states. (The special choice of nucleon momentum $\vec{p}_o$, (2.6), guarantees that energy-conserving pi-nucleus collisions (k' = k) produces energy-conserving pi-nucleon collisions since $|\vec{p}_o - \vec{q}| = p_o$ for k' = k.)

The use of (2.14) and (2.15) for off-shell scattering, ($s_{in} \neq s_{out}$), is an assumption which we had hoped would be a good approximation. This hope was based on an examination of the optical potential, (2.5), in which it is argued that in general the rapid fall off of the nuclear, elastic form factor for k $\neq$ k' means that pi-nucleus elastic scattering is dominated by k = k' (though not necessarily = k_o) terms, i.e. diagonal collisions in which $s_{in} = s_{out}$ in (2.13).

We have verified this assumption by also calculating pi-nucleus scattering using the invariant, angle transformation procedure of Aaron, Amado and Young,[20] which is valid for off-shell transitions. The pi-helium phase shifts so calculated differ from our previous results by less than 1% at 50 or 180 MeV, whereas the total nuclear cross sections differ by less than 0.1%! Although these relativistic off-shell generalizations may appear to be of slight importance for our calculations, it should be emphasized that the use of a reasonable "angle transformation" is still important, the $\cos\theta_{\pi N} = \cos\theta_{\pi\eta}$ approximation of Ref. 1, e.g., produces quite different results.

III. DATA COMPARISON AND DISCUSSION

We now compare the cross sections computed with our theoretical optical potential to the new $\pi^\pm$ - ^{3}He; and $\pi^\pm$ - ^{4}He data of the Dubna-Torino[12] and CERN[11] groups. We concentrate on the importance of binding corrections by examining the different choices of energy ω_o and ω_{3B}, (2.8) and (2.11), on the likelihood of these experiments measuring the spin distribution in ^{3}He, and the importance of averaging over nucleon motion (2.5) vs. (2.7). In evaluating the Fermi motion integral in (2.5) only the energy dependence of the πN T matrix is varied, the momenta variables are evaluated for p = 0. This procedure is excellent for π-deuteron scattering[22], and within the model calculation of Siciliano and Walker[5], gives results similar to the exact form (2.3).

In order to calculate charge exchange reactions (via Eq. (3.7)) and "pure" nuclear total cross sections, the Coulomb interaction is included via a simple amplitude addition. For the intermediate energies and light nuclei considered here, we have verified that the results differ only slightly from the exact[23] procedure.

A. π - ^{4}He Scattering

Spin form factors do not enter into the lowest order optical potential for ^{4}He, and in lieu of evidence to the contrary, we take neutron and proton matter distributions as both equal to the nuclear charge density[24] with finite proton size[25] removed; the only needed form factor is thus

$$F(q) = [1-(a^2q^2)^6]\exp[-b^2q^2](1+q^2/x) \ ,$$

$$a=0.316 \text{ fm}, \ b=0.681 \text{ fm}, \ x=18.2 \text{ fm}^{-2} \ . \tag{3.1}$$

In Figs. 1a and b we present a comparison

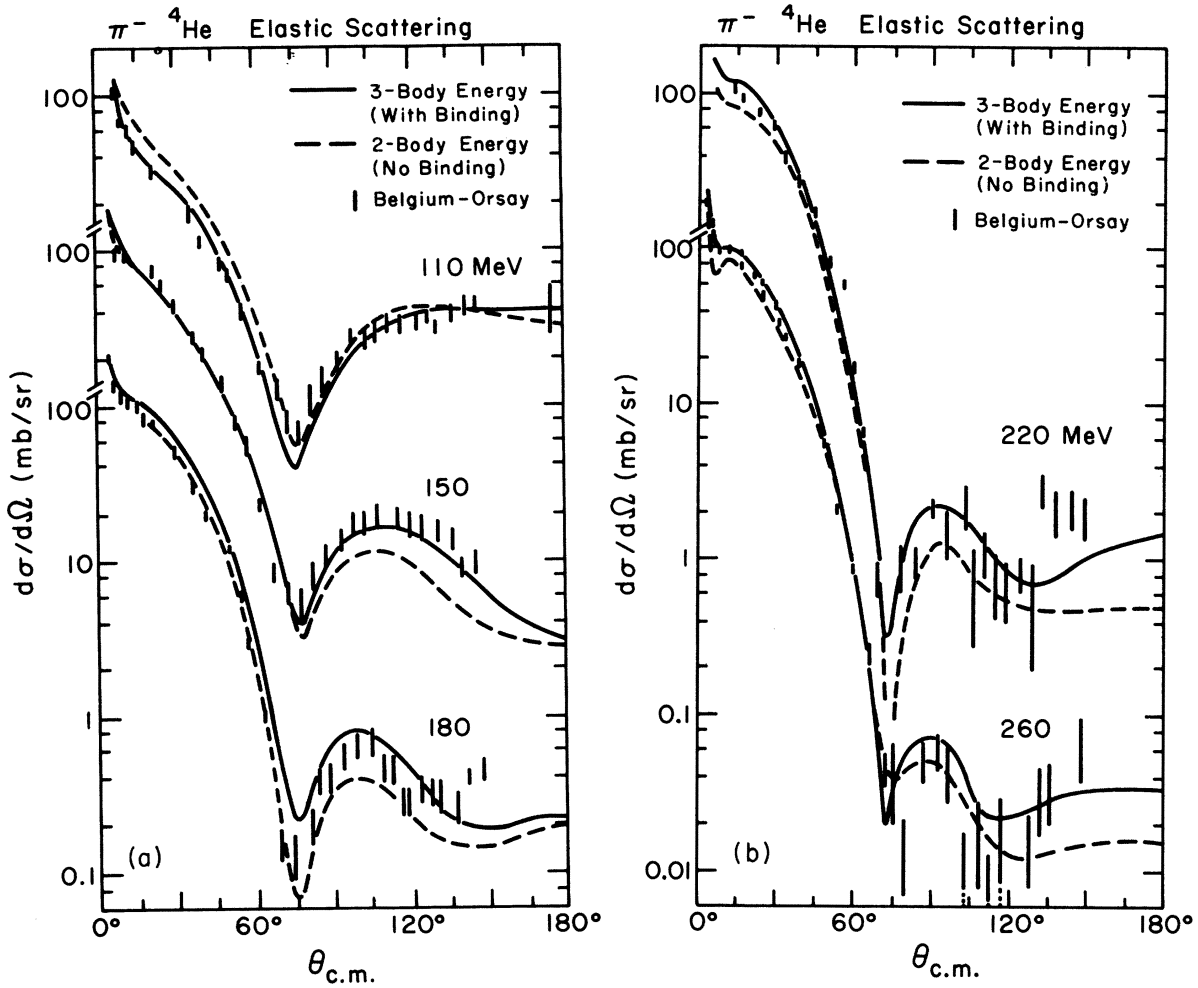


FIG. 1 (a) and (b), π^--^{4}He elastic scattering differential cross sections calculated at different pion lab kinetic energies. The 3-body πN subenergy ω_{3B}, includes, nuclear binding effects (solid curves), the 2-body subenergy does not (dashed curves). The data are from the CERN group, Ref. 11. For $T_\pi < 180$ MeV a value $E_B = -5$ MeV is used, for $T_\pi \geq 180$ MeV a value of 0 is used.

of the data from the CERN group and our theory (2.1), (2.5), using the 3-body energy ω_{3B} or the 2-body energy ω_o. As found earlier for the more sensitive scattering of low energy pions[13], the 3-body choice of subenergy (which includes binding effects) provides a better fit to data than the 2-body choice. The only parameter in all these calculations is the magnitude of E_B - and it hardly varies at all, from a value of $\sim$ 5 MeV for energies below resonance, to a value of $\sim$ 0 MeV above resonance. In Fig. 2 we see the relatively low sensitivity at these energies to the exact value of the "binding energy" and for comparison, the somewhat greater sensitivity[4] to the inclusion of internal nucleon motion.

These results once more indicate that the present state of experiment and theory definitely require account of nucleon binding, recoil, and internal motion,[4,5,8,9] and that a three body approach is a workable way to include them. In addition, the small value for the effective binding energy E_B compared to what might be expected on the basis of an independent particle shell model, indicates the need for higher order corrections (active core) which decrease the binding of an interacting nucleon.

The covariant calculation of Celenza et al.[8], which produces comparable agreement with data, displays sensitivity to internal nucleon motion with the same characteristics as shown in Fig. 2, but of slightly greater magnitude. This small

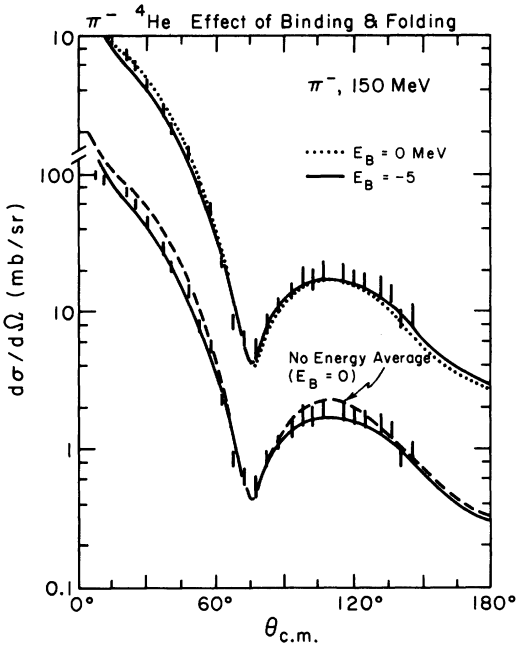


FIG. 2 The sensitivity of π^--^{4}He elastic scattering at 150 MeV to the value of the binding energy E_B and to the inclusion of nucleon Fermi motion. The solid curves include Fermi motion and E_B = -5 MeV. The dotted curve includes Fermi motion and E_B = 0 MeV. The dashed curve is calculated with E_B = 0 MeV and no Fermi averaging.

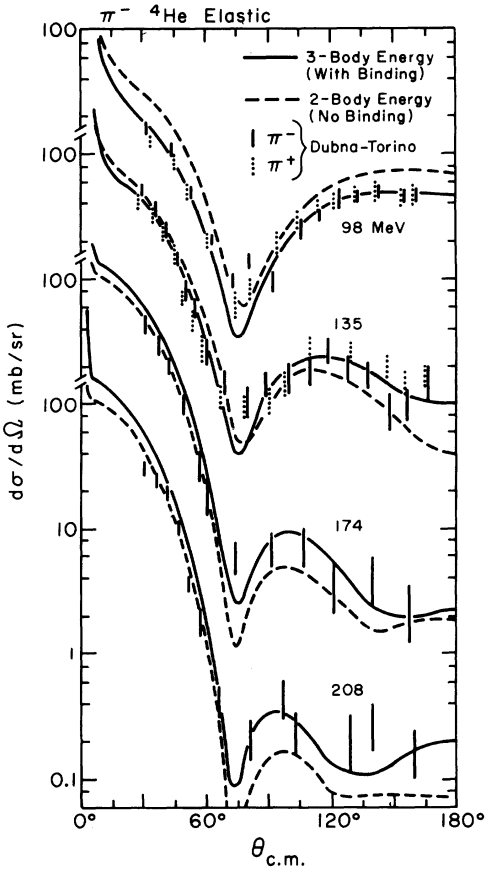


FIG. 3 Same meaning of curves as in Fig. 1. The data are now the combined π^+-^{4}He and π^--^{4}He results of the Dubna-Torino group, Ref. 12.

difference is probably due to different nuclear wave functions or details in the folding procedure, and is consistent with the model calculations of Siciliano and Walker[5]. It is also interesting to note, in examining Figs. 1A and B, that Ref. 5 indicates that better agreement at back angles for the higher energies could be obtained by including intermediate nuclear excitation into the bound T matrix, i.e. as core excitation.

The Dubna-Torino group[12] has measured both π^+ and π^- -^{4}He scattering, a rather courageous task since the final results must be self-consistent (calculations indicate that at these energies $d\sigma/d\Omega$ for π^+ and π^- differ only at small angles). As we see in Fig. 3, the error bars do overlap at most angles and the theory, particularly with ω_{3B}, provides rather good agreement with the combined data (as much as can be expected with these error bars). The precise level of agreement for π-^{4}He is relevant for three reasons: first, the actual values of these differential cross sections in the minimum region has been questioned recently;[26] second, this same Dubna-Torino group also measured the very interesting $\pi^\pm$ elastic scattering from ^{3}He, where the π^+ and π^- data cannot be combined ($I \neq 0$) to compare with an even more complicated theory; and third, agreement for

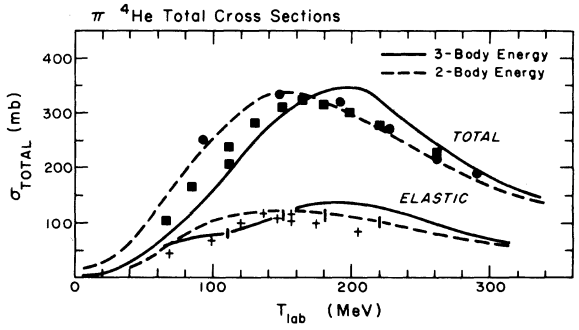


FIG. 4 π-^{4}He total and elastic cross sections calculated with the 3-body subenergy including binding (solid curves) and the 2-body subenergy without binding (dashed curve). The data, ■ |, +, and ●, are from Refs. 11, 12, and 27, respectively. There are no adjustable parameters used in these calculations.

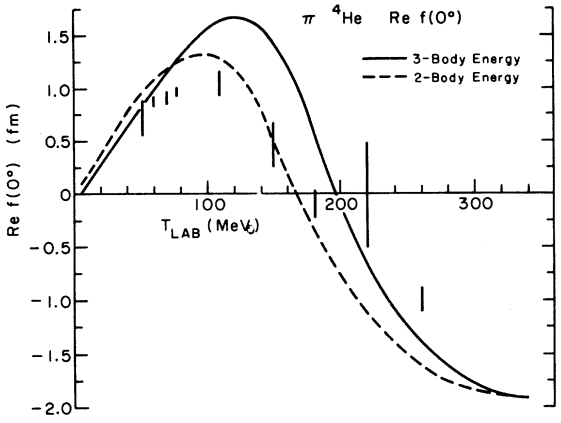


FIG. 5 The real part of the π-^{4}He strong interaction scattering amplitude compared with the data of Ref. 11. Same meaning of curves as in Figs. 1 and 4.

π-^{4}He is important since it provides a test of the reaction theory which can then be used to investigate ^{3}He. In section B we examine π-^{3}He data.

In Fig. 4 we compare the total and elastic cross sections predicted by the optical model (2.5) with the data of Wilkin et al.[27], Binon et al.[11], and Shcherbakov et al.[12] In Fig. 5 we compare the theory with the real part of the strong interaction forward scattering amplitude as deduced by Binon et al. There are no adjustable parameters in our calculation, the same value of E_B in Eq. (2.11) was used as for the differential cross sections.

We see in these figures that the 3-body choice of energy which provides better fits to the differential cross sections, appears to shift the energy downward by too great an amount when used in the total cross section calculation (i.e. the peaks shift upward). This same conclusion has been reached independently by Celenza et al.[8] although they do not exhibit the total cross sections. The agreement with the deduced Ref(0°) is not quite as good as obtained with C and O[4]. At present, the reason for these discrepancies in σ_{TOT} and Ref(0°) is not known. Although this does seem to indicate the importance of corrections to the impulse approximation, there are several effects, such as nucleon-core interactions, Pauli suppression, true pion absorption, different nuclear wave functions, off-shell effects, and higher order scattering processess,[9,28] all of which can affect the calculated total cross sections by the amount needed for agreement without any dramatic influence on $d\sigma/d\Omega$; one must perform complete calculations with realistic models before significance is ascribed to detailed fits to total cross sections. In particular, the low energy (< 100 MeV) values for σ_{TOT} and Ref(0°) are sensitive to the input πN phase shifts and will change with the different low energy input now being calculated.[19]

B. π - ^{3}He Scattering

To apply the optical potential to scattering from ^{3}He we need to know the form factors for the matter and spin distribution of both neutrons and protons within the nucleus. Our previous analysis,[10] based on the extensive work of Schiff and Gibson,[29] indicates that it is a good approximation to express these four functions in terms of the charge (F_c) and magnetic (F_m) form factors of ^{3}He and ^{3}H:

$$F^p_{matter}(q) = F_c(^3\mathrm{He})/f^p_c \ ,$$

$$F^n_{matter}(q) = F_c(^3\mathrm{H})/f^p_c \qquad (3.2)$$

$$F^p_{spin} = [F_m(^3\mathrm{He}) - \tfrac{4}{3}F_c(^3\mathrm{He}) + \tfrac{1}{3}F_c(^3\mathrm{H})]\xi \qquad (3.3)$$

$$F^n_{spin} = [F_m(^3\mathrm{He}) + \frac{4\mu_p}{3\mu_n}F_c(^3\mathrm{He}) - \frac{\mu_p}{3\mu_n}F_c(^3\mathrm{H})]\xi \qquad (3.4)$$

where $\xi = \mu_n/2(\mu_p + 2\mu_n)f^p_c$, $\mu_{p,n}$ is the proton or neutron static magnetic moment, and f^p_c is the proton charge form factor.[25] For ^{3}He we use the charge and magnetic form factors determined by McCarthy et al.,[30] and for ^{3}H we use the actual data points of Collard et al.[30]

At present there is some uncertainty in the relationship of the magnetic and charge radii for the trinucleon system which makes an investigation of these nuclei with a pion probe of high value. Specifically, the best fit rms radii obey the relation

$$R^{He}_m(1.95\pm.11\mathrm{fm}) > R^{He}_c(1.88\pm.05)$$

$$> R^H_c(1.70\pm.05) = R^H_m(1.70\pm.05) \ . \qquad (3.5)$$

The "large" magnetic size of ^{3}He differs from the value 1.74 ± .10 fm measured earlier by Collard et al.[30], and from the simple model of ^{3}He as two protons with their spins paired off to zero and a more bound unpaired neutron.[29,10] Although the simple model is surely not complete, a detailed wave function analysis[31], which is consistent with $R^{He}_m > R^{He}_c$, also predicts $R^H_m > R^H_c$ —— which is not consistent with the best values. In either case then, there are uncertainties which are probably related to the difficulties in making and extrapolating the electron scattering measurement near q = 0, and using pions

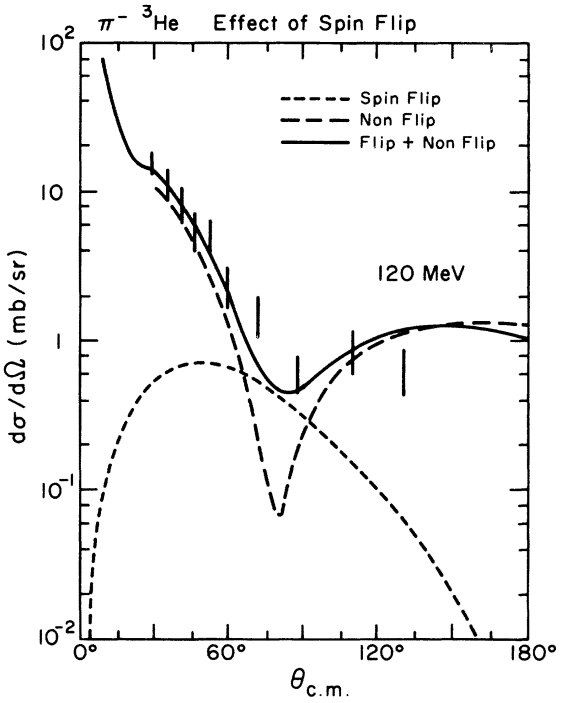


FIG. 6 π^- elastic scattering from an unpolarized ^{3}He nucleus. The spin flip (short dashed) and non-flip (long dashed) contributions and their sum (solid curve) are shown separately. The spin flip contribution is sensitive to the nuclear spin form factor. The data are from Ref. 12. Spin flip is less important for π^+-^{3}He scattering.

to help extend or confirm our knowledge of these form factors is of value.

We have calculated π^+- and π^--^{3}He scattering for most energies measured by the Dubna-Torino group. Although the π^+ cross sections differ considerably from the π^- ones for this isospin nonzero nucleus[6], we concentrate on the π^- cross sections since the interesting spin effects are largest here (the π^- can have a resonant interaction with the unpaired neutron).

As in the ^{4}He case, we find that the 3-body choice of energy ω_{3B}, with the same binding energy as for ^{4}He, provides better overall agreement with these intermediate energy data than ω_o(no binding).

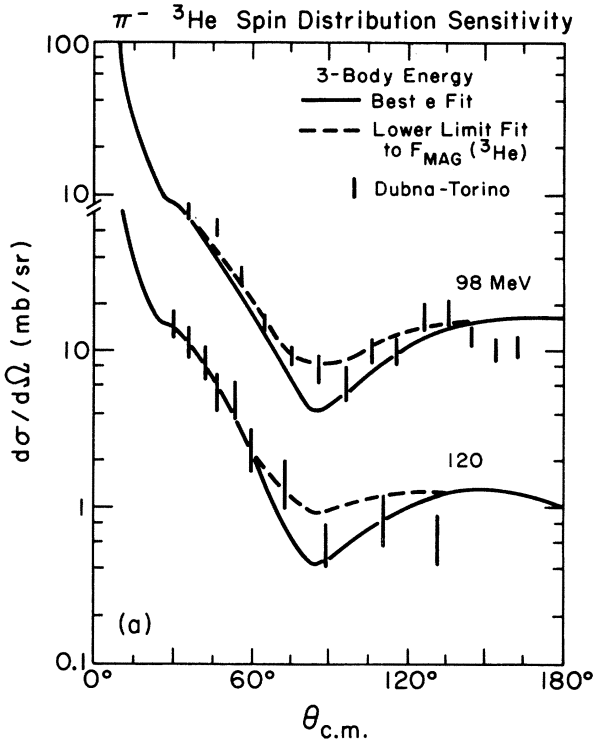
In Fig. 6 we see π^--^{3}He scattering at 120 MeV using $|E_B|$ = 5 MeV. The deep minimum in the non-spin-flip scattering is filled in rather completely by the spin-flip scattering. The amount of "fill" is determined predominately by the size of the neutron distribution $F^n_{spin}(q)$, which in turn is directly related to the magnetic form factor of ^{3}He, (3.4). Thus π^--^{3}He scattering provides a means of determining some properties of this important magnetic moment form factor.

Gibson[31] has indicated the need for caution in using pions, as opposed to electrons, to learn about single particle properties of nuclei, in particular by filling of minima. Pions, however, have a good deal in their favor for this problem: (1) The small values of momentum transfer at which pions investigate the magnetic form factor are not readily accessible to electrons. (2) Since $\sim \frac{1}{3}$ of the nucleons in ^{3}He contribute to the spin flip scattering which is present with exactly the same behavior in the dominant single scattering term, it is not a small second order effect of the same size as many other small effects. (In fact, at these energies our calculations have shown the spin flip scattering to be much larger than corrections to the theory of second order in nuclear density.) (3) The technique to investigate the spin distribution is somewhat self-consistent, if good agreement with the π-^{4}He data is obtained (as it seems to be), and if the theory agrees with the π-^{3}He data in the angular region where spin effects are small, then one could isolate the spin contribution.

In Figs. 7a and b is a comparison of the Dubna-Torino π^--^{3}He data with the theory. The solid curves employ the electron scattering best fit parameters to the ^{3}He magnetic form factor (R_m^{He} = 1.95 fm) and the dashed curves employ a smaller size consistent with the electron data (R_m^{He} = 1.84 fm). The larger size calculations generally lie on the other side of the best fit calculation. At 98 MeV (Fig. 7a) the smaller size for the distribution provides a somewhat better fit, at 120 MeV both sizes appear to provide equally good fits, at the higher energies the best electron scattering fit seems preferred. Of course the error bars are rather large (the corresponding ^{4}He data agreed fairly well only after combining π^+ and π^- results), the precise depths of the minima have been questioned[26], and one should question the reliability of the first order optical potential, even with binding corrections, for as detailed a comparison as is required here. Yet, these comparisons do show the high level of sensitivity to 0.1 fm changes in spin distributions, and appear promising for better tests in the future.

In Fig. 8 we show the $\pi^\pm$-^{3}He total and elastic cross sections calculated with the three body energy and the data of the Dubna-Torino group. Although on the basis of the ^{4}He calculation we expect the calculated peak position to be at too high an energy, up to $T_\pi \simeq 160$ MeV the agreement is (fortuitously?) rather good.

In Fig. 9 we show some calculated polarizations of the recoiling nucleus in π^--^{3}He elastic scattering which indicate the sensitivity to the magnetic form factor of ^{3}He and to the inclusion of binding effects. The polarizations peak at $\sim$80°, and only at higher energies where diffractive struc-



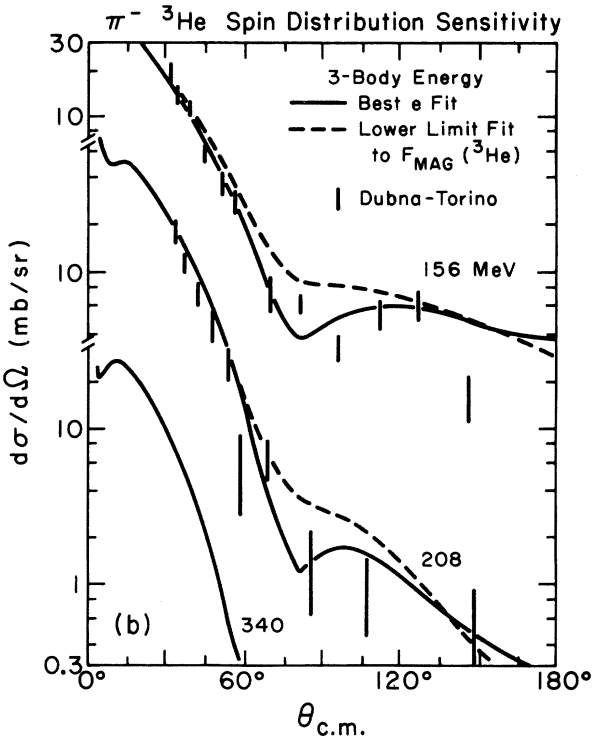


FIG. 7 (a) and (b), Differential cross sections calculated with ω_{3B} for elastic pion scattering from an unpolarized ^{3}He nucleus. The solid curves are obtained with the best fit electron scattering ^{3}He magnetic form factor (R_m = 1.95fm) and the dashed curves with the lower limit electron fit (R_m = 1.84fm). No adjustable parameters are used here. The data are from Ref. 12.

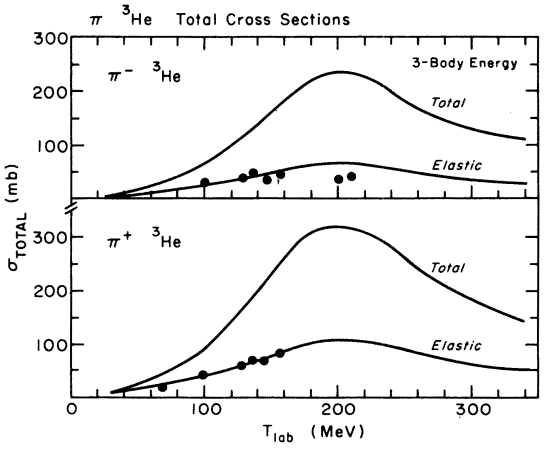


FIG. 8 π^--^{3}He total and elastic cross sections calculated with ω_{3B} and the data of Ref. 12. The $\pi^\pm$ cross sections differ for a non-zero isospin nucleus.

ture begins to enter, do they show moderate sensitivity to these size and off-shell effects.

C. π - ^{3}He Single Charge Exchange

We calculate single charge exchange scattering to an analog state by observing that in the absence of the Coulomb force, the amplitude for isovector-isospinor scattering has the form:

$$T = T_0 + T_1 \vec{t}\cdot\vec{\tau} \qquad (3.6)$$

By taking matrix elements between different charge states of the nucleus and pion we obtain the simple relation among scattering amplitudes:

$$T(\pi^-+{}^3\mathrm{He} \to \pi^0+{}^3\mathrm{H}) = [T(\pi^++{}^3\mathrm{He} \to \pi^++{}^3\mathrm{He}) - T(\pi^-+{}^3\mathrm{He} \to \pi^-+{}^3\mathrm{He})]/\sqrt{2} \quad . \qquad (3.7)$$

This relation ignores Coulomb distortion and multiple charge exchange scattering, effects which Spencer and Miller, and Hess and Eisenberg[32] show to be quite small for carbon, yet which Franco[32] indicates may be important at lower energies. It thus should be consistent in validity with the

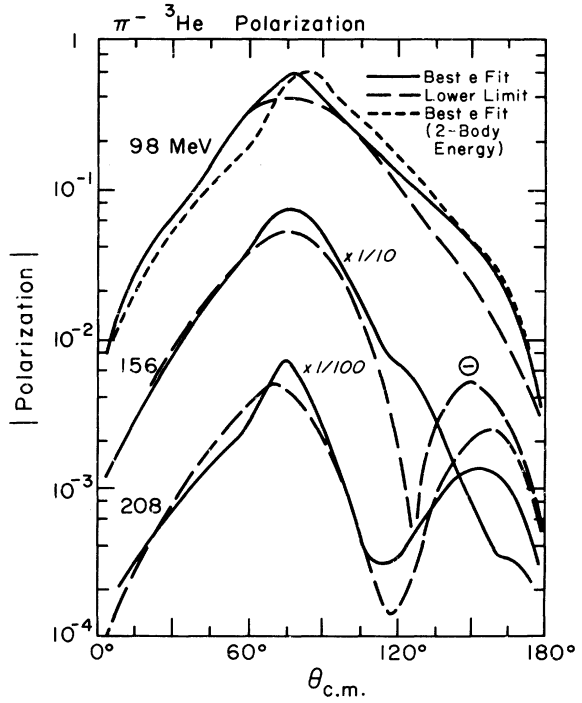


FIG. 9 The polarization of the recoiling nucleus in π^--^{3}He elastic scattering calculated with the 3-body energy. The solid curves are obtained with the best fit magnetic form factor and the dashed curves with the lower limit electron fit. The polarizations are all positive except for a sign change at $\sim$ 120° for the 156 MeV curve.

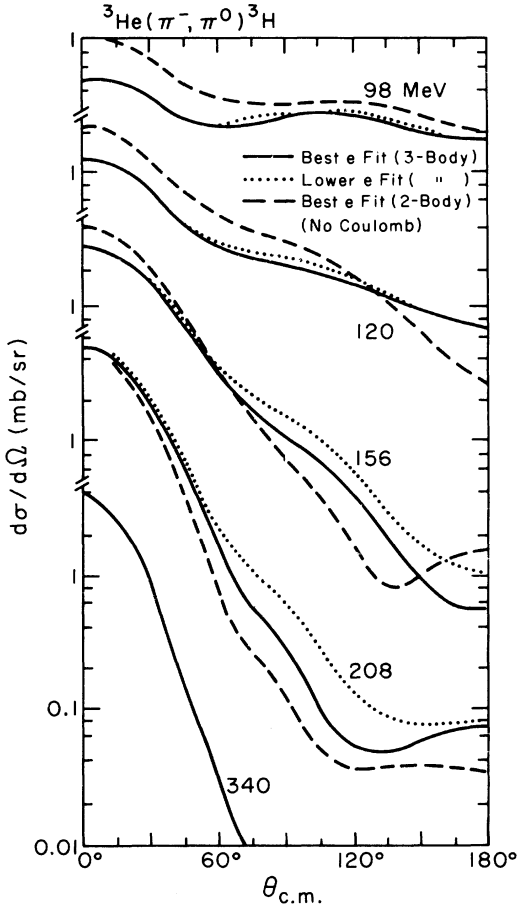


FIG. 10 Differential cross sections for the charge exchange reaction ^{3}He(π^-,π^0)^{3}H calculated with an improved theoretical optical potential. The solid and dotted curves are calculated with the πN subenergy ω_{3B}(binding) and, respectively, with the best fit and lower limit fit to the ^{3}He magnetic moment form factor. The dashed curve is calculated with the best electron magnetic moment fit and with ω_o (no binding).

first order optical potential used for elastic scattering.

In Fig. 10 we show differential cross sections for the isospin flip reaction, ^{3}He(π^-,π^0)^{3}H or ^{3}H(π^-,π^0)^{3}He. Again we exhibit the sensitivity to uncertainty in magnetic form factor (solid vs. dotted curves) and to the inclusion of binding (solid vs. dashed curve). Although these reactions do display significant variation (a factor of $\sim$3 at 120° for the higher energies) the spin flip contribution masks the diffraction-like structures[33] of the non-flip scattering. There, consequently, is required rather precise data over a sizeable angular region to determine these structure or reaction effects (a conclusion also reached by Hess and Gibson[7]).

In Fig. 11 we present the total cross section for the single charge exchange reaction calculated with the 3-body energy. The cross section is rather flat and is not very sensitive ($\lesssim$ 10%) to the energy choice or nuclear size variation. The small maximum is caused by the spin flip contribution filling in the non-flip minimum due to a resonance in the elastic channel.[6,33]

IV. Summary and Conclusions

We have presented many calculated curves and comparisons with data: π^--^{4}He elastic scattering and the data of the CERN group, $\pi^\pm$-^{4}He and π^--^{3}He elastic scattering and the data of the Dubna group, and $d\sigma/d\Omega$ and σ_{TOT} for the ^{3}He(π^-,π^0)^{3}H reaction. Our aim is to provide some perspective on these intermediate energy reactions from which to judge if we understand the basic physics and if so, to determine the avenues to proceed along towards more complete theories and towards more extensive or accurate experiments. To this end it is of value to compare the results of our theoretical calculation, which has provided rather good

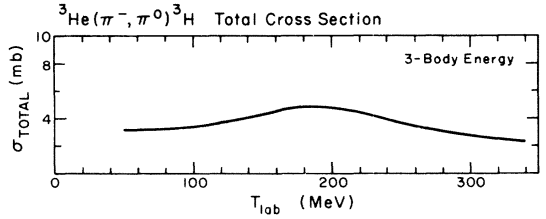


FIG. 11 The total cross section for the reaction $^3He(\pi^-,\pi^0)^3H$ calculated with the 3-body energy and best fit form factors.

but not excellent agreement with much of the data, to the work of others.

The essential ingredients of our theory are: (i) A first order, momentum space, optical potential used in a relativistic wave equation. This provides all orders of multiple scattering in which the nucleus remains in its ground state, but excludes virtual, excited intermediate states and nucleon-nucleon correlations. (ii) A πN T matrix whose on-shell properties are determined by up-to-date phase shifts and whose off-shell behavior incorporates the finite size of the πN interaction. (iii) A careful treatment of the relation between the πN amplitude in the πN c.m. frame and in the pi-nucleus c.m. frame, incorporating momentum conservation, on-shell Lorentz invariance and the "angle-transformation". (iv) A choice of the πN subenergy variable (3-body picture) which incorporates nucleon recoil, internal motion and in an average way, binding. (v) Realistic nuclear and nucleon form factors consistent with those measured by electron scattering.

The extensive, illuminating, and independent calculations of Mach[6], and Shcherbakov et al.[12] produce cross sections and conclusions concerning trends and sensitivities which are similar to ours, but the general level of agreement with data, is not quite as good. This appears to be due to their calculations being performed in coordinate space with Kisslinger-type and Laplacian potentials which, a priori, do not include: the finite size of the πN interaction, i.e. physical off-shell behavior (ii), the full effect of the amplitude transformation (iii), the average over internal nucleon motion, and the binding of the active nucleon (iv). It appears that all these effects, and possibly the difference in the on-shell amplitudes, are important for detailed agreement.

The Glauber approximation calculations of Sparrow[33] on π-^3He and $^3He(\pi^-,\pi^0)^3H$ scattering indicate the importance of spin flip in both the differential and total cross sections. Although this conclusion can also be deduced from earlier work on elastic scattering, our more realistic calculation does confirm this conclusion and, as might be hoped, agrees better with the data.

Gibson et al.[7] have applied Gibb's multiple scattering theory to the π-^4He elastic scattering at energies < 100 MeV and have verified the importance of points (iii) and (iv). In a similar calculation by Hess and Gibson[7], this theory is also applied to the single charge exchange reaction on ^3He. The physical content of the ^3He calculation is similar to ours in its inclusion of: all orders of multiple scattering (i), a finite size for the πN interaction (ii), and an effective, on-shell, "angle transformation" which should approximately include some of the important effects (iii). The main difference is in its use of a separate coupled channel for the charge-exchange state, a fixed nucleon type of approximation which does not implicitly include nucleon internal motion, recoil or binding (iv), the use of simpler, and presumably less realistic nuclear form factors (v), and its use of less recent phase shift tabulation.

Both groups' results for π^--^3He elastic scattering at 100 and 154 MeV are quite similar. The results for the charge exchange $d\sigma/d\Omega$ are similar in shape at all energies, yet except for the 98 MeV cross sections, ours tend to fall more rapidly with angle (by nearly an order of magnitude at 140°). The reasons for this difference are not clear; it may be the different scattering theories, or the different procedure for calculating charge exchange. The nuclear form factors used in both calculations however, do not differ enough at these energies to produce an order of magnitude effect. Both groups' results for the total charge exchange cross sections are also similar in shape (i.e. hardly any) but somewhat different in magnitude; this I suspect is due to the subtractions (3.7) necessary to obtain these cross sections which make them sensitive to our use of recent πN phase shifts, our inclusion of energy averaging and Ref. 7's use of a coupled channel mechanism.

Finally, the covariant calculation of π^--^4He scattering in the resonance region by Celenza et al.[8], as mentioned in previous sections, appears to include the same essential physical ingredients as our calculation and produces comparably good but not excellent agreement with the data. The message appears clear: it is time to use those theoretical tools which work well and incorporate additional higher order physics into our calculations in a consistent reliable manner. In particular, if we want to obtain significant, detailed fits to total cross sections or to differential cross sections at low energies, calculations which include points (ii) to (iv) above and the new physics are necessary.

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