

Intermediate structure studies of ^{234}U cross sections*G. D. James,[†] J. W. T. Dabbs, J. A. Harvey, N. W. Hill, and R. H. Schindler[†]*Oak Ridge National Laboratory, Oak Ridge, Tennessee 37830*

(Received 16 November 1976)

Neutron induced fission and total cross sections of ^{234}U have been measured over the neutron energy range from a few eV to 8.9 MeV. Neutron and fission widths for 118 cross section resonances below 1500 eV have been determined and give a class I level spacing of 10.6 ± 0.5 eV and an *s*-wave strength function of $(0.86 \pm 0.11) \times 10^{-4}$. These fine structure resonances form two narrow intermediate structure resonances in the subthreshold fission cross section of ^{234}U . Parameters for the Lorentzian energy dependence of the mean fission width are deduced by maximum likelihood analysis on the assumption that, relative to this mean, the observed fission widths have a Porter-Thomas distribution. Two large fission widths measured for resonances at 1092.5 and 1134 eV prompted a likelihood ratio test which indicates the presence of a narrow intermediate structure resonances at 580 and at 1227 eV. The class II level spacing derived from the observation of seven intermediate structure resonances below 14 keV is 2.1 ± 0.3 keV. Broad structures in the fission cross section at 310, 550, and 770 keV are assumed to be due to β -vibrational levels in the second minimum of the Strutinsky potential. Fluctuations due to the presence of class II resonances are strongly evident for each of these vibrational levels. It is shown that the fluctuations near 310 keV are consistent with parameters deduced from the low energy data and this allows parameters for the double humped fission barrier potential to be obtained.

NUCLEAR REACTIONS $^{234}\text{U}(n, f)$, $E=3$ eV–8.9 MeV; measured $\sigma_f(E)$.
 $^{234}\text{U}(n, \text{total})$ $E=20$ –1500 eV; measured $\sigma_t(E)$. Determined D_I, D_{II}, S_0 , fission widths and neutron widths (118 levels below 1500 eV). Deduced E_0 , width (two class II levels), barrier heights V_A, V_B .

I. INTRODUCTION

A measurement of the subthreshold fission cross section of ^{234}U by James and Rae¹ showed the existence of a narrow intermediate structure resonance at an incident neutron energy of about 640 eV. It was later shown^{2,3} that the fission widths of the fine structure levels which comprise this resonance fluctuate with a Porter-Thomas⁴ distribution about a mean value which has a Lorentzian energy dependence. Data above 50 keV showed a distinct break in the fission cross section at about 325 keV⁵ and it has been suggested⁶ that this may arise from a β -vibrational level in the second fission potential barrier minimum,⁷ which is damped among the nuclear excitations associated with the secondary shape to give the narrow intermediate structures observed at low energy. With the neutron energy resolution at present available, the existence of narrow intermediate structure resonances, associated with a mode of β vibration, is expected to produce strong fission cross section fluctuations near 325 keV and such fluctuations have been shown to exist.⁸

This paper presents the results of (1) a high resolution measurement of the fission cross section of ^{234}U over the energy range 3 eV to 8.9 MeV and (2) a measurement of the total cross section of ^{234}U to provide the neutron widths of all fine structure resonances below 1500 eV. These measure-

ments were carried out by neutron time-of-flight experiments at the Oak Ridge electron linear accelerator (ORELA) and have the threefold aim of (1) determining the fission widths of all fine structure resonances below 1500 eV and thus enable the energy dependence of these widths to be studied in detail, (2) improving our knowledge of the class II resonances in the low keV energy range, and (3) measuring with improved statistical accuracy the fluctuations observed near 325 keV. All these aims have been achieved and the results have been analyzed to give the parameters of the double humped fission potential barrier on the assumption that a β -vibrational level near 325 keV is damped among the class II levels to give their fission widths. The experimental details, including the methods of determining the cross sections, are presented in Sec. II and the methods of determining the neutron and fission widths of the fine structure levels below 1500 eV are described in Sec. III. The problem of estimating the parameters of a Lorentzian curve which governs the energy dependence of the mean class I fission width near a class II resonance is solved in Sec. IV by a maximum likelihood analysis in which the contribution of each fission width to the likelihood is weighted by a function of the fission width error. Also, the likelihood ratio test is used to show that the fission widths below 1500 eV arise from two narrow intermediate structure resonances. Some average

properties of the class II levels for which the class I levels are not fully resolved are presented in Sec. V. Finally, in Sec. VI, an analysis of the fission cross section fluctuations near 310 keV is carried out to establish the parameters of the double humped fission potential barrier and the consistency of these parameters with the properties of the narrow intermediate structure resonances observed at low energy.

II. EXPERIMENTAL DETAILS

Both the fission and total cross sections of ^{234}U were measured by neutron time-of-flight experiments using the ORELA pulsed neutron source. Experimental details up to the derivation of the cross sections are given separately for these two cross sections in the following subsections.

A. Fission cross section

In the fission cross section experiment, fission fragments from ^{234}U were detected in a multiplate ionization chamber containing 181 mg ^{234}U on eight circular deposits of 7.6 cm diameter on 0.0127 cm thick aluminum. An additional plate containing 20.5 mg ^{235}U over a 7.6 cm diameter was also housed in the chamber and used to determine the neutron spectrum as described below. The separation between the deposits and the electron collector plates (blank) was 3 mm and the chamber was filled with methane at a pressure of 190 Torr. The ionization chamber was placed at an average distance of 20.14 m from the neutron source and four time-of-flight experiments with neutron burst widths of 3, 5, 8, and 28 ns were performed. Each experiment lasted about 6 days at a pulse repetition frequency of 800 Hz. An overlap filter comprising 29 g ^{10}B and 171 g S over a 16.5 cm diameter was used to prevent the overlap of low energy neutrons from the previous pulse. Electron collection current pulses from each of the five sections of the ionization chamber⁹ were amplified in a fast amplifier,⁹ sent through to a constant fraction discriminator, and used as stop pulses in a time digitizer to define the neutron time of flight. For each section of the chamber, a data acquisition computer provided disk storage for 20 000 timing channels with the following widths: 5000×2 ns, 2500×4 ns, 2500×8 ns, and 2000 each of 16, 32, 64, 128, and 256 ns. These data were transferred to magnetic tape once a day and at the end of the experiment the results from the four ^{234}U sections of the chamber were summed after inspection. It was found that the 3 ns data showed no more detailed structure than the 5 ns data and these two experimental runs were summed and used to produce the fission

cross section above 100 keV. Below 100 keV the data were derived from the sum of these two runs and the data taken with a pulse width of 28 ns. The data taken with a pulse width of 8 ns had a slightly different channel width structure and were not used in the present analysis.

In the time available to set up the equipment, it was not possible to determine the optimum values of gas pressure and fission plate spacing. As a result, the discriminator settings on the ^{234}U and ^{235}U sections of the chamber were not set at the same value of pulse height. The ratio of fission counts from the two sections of the chamber could not, therefore, be related to the quantities of fissile material in the chamber and the fission cross section data were normalized to previous measurements.^{1,5} This was done in two ways which are found to agree well. Fission cross section data below 100 keV were obtained using a neutron flux, proportional to dE/E^x , expressed as a power x of the neutron energy E . The fission cross section $\sigma_f(E)$ calculated for a timing channel of width Δt at energy E may then be written

$$\sigma_f(E) = CK_x F(E)/\Delta t T(E)E^{1.5-x}. \quad (1)$$

Here $F(E)$ is the fission count observed during the experiment in the timing channel at energy E , $T(E)$ is the transmission of the ^{10}B overlap filter¹⁰ at energy E , K_x is described below, and C is a normalization constant related to the length of the experimental run, the efficiency of fission fragment detection, the quantity of fissionable material in the chamber, and the intensity of the neutron flux. The value of x is determined as that required to ensure that the ratio of the unnormalized ^{235}U fission cross section $[\sigma_f(E)/C]$, deduced from the above equation, to selected published values of the cross section remains constant and independent of neutron energy. In this calculation average values of the ^{235}U fission cross section are used. These are taken from the evaluation of Sowerby¹¹ above 100 eV and from the data of de Saussure¹² below 100 eV. It is found that over the energy range 3 eV to 100 keV, three values of x are required to describe the neutron flux and the energy range to which they apply are given in Table I together with values of K_x . The K_x are constants required for continuous flux values at the boundaries of the energy ranges.

TABLE I. The neutron flux parameter x .

Energy range (eV)	x	K_x
$3 < E < 122.16$	0.91887	1.4866
$122.16 < E < 7794.3$	0.8703	1.8774
$7794.3 < E < 100 \text{ keV}$	0.73402	6.3669

Below 100 keV, the ^{234}U fission cross section is derived from Eq. (1) using the constants K_x and x given in Table I and a value of C deduced by normalizing the data such that the area under the resonance at 5.16 eV is equal to that given by James and Rae,¹ 1.250 ± 0.085 b eV. This measurement is in close agreement with that of Leonard and Odegarden.¹³

Above 100 keV, the ^{234}U fission cross section is deduced by taking the ratio of the ^{234}U to ^{235}U fission fragment yield per timing channel and multiplying by a parametric fit to the evaluated ^{235}U fission cross section.¹¹ This parametric fit is illustrated in Fig. 1. It consists of a cubic energy dependence of the form

$$\sigma_f(E) = B_0 + B_1E + B_2E^2 + B_3E^3, \quad (2)$$

in each of four energy ranges. Values of the coefficients B_i for each energy range are given in Table II. The resultant ^{234}U fission cross section is normalized to the data of Lamphere⁵ over the energy range 1.003 MeV to 1.3 MeV ($\sum \sigma_f dE = 350.5$ b keV); Lamphere quotes an overall error of $\pm 5\%$. It is found that the normalization constant used above 100 keV agrees with that derived for the low energy data to 6% which is within the error of the low energy data of James and Rae.

The aim of the present high energy measurement is to establish and analyze structure in the fission cross section near vibration levels which appear at 310, 550, and 770 keV. Fission counts per channel as a function of neutron energy for the summed 3 and 5 ns experiments are shown in Fig. 2 for both ^{234}U and ^{235}U . It will be seen that the ^{234}U data show considerably more structure than the ^{235}U data. Structure in the ^{235}U data is due partly to structure in the ^{235}U fission cross section and partly to structure in the neutron spectrum. The latter is correctly allowed for in the method used to calculate the ^{234}U results at high energy. The former can be accounted for only by using a known ^{235}U fission cross section measured at the same resolution as the present experiment. This is not available and thus the ^{234}U fission cross section calculated will be in error to some extent at certain energies. It is clear however that the structure near 310 keV, which is dis-

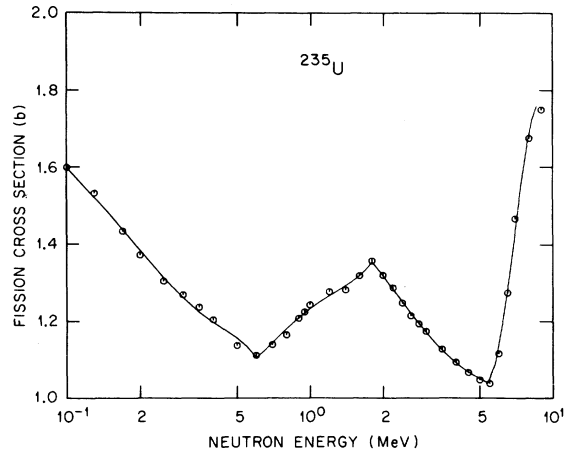


FIG. 1. Parametric fit to the evaluated ^{235}U fission cross section. See Table II for coefficients.

cussed more fully in Sec. VI, is due entirely to the ^{234}U fission cross section.

B. Total cross section

In the total cross section experiments, neutron pulses of 28 ns duration were produced by ORELA at a repetition rate of 800 Hz, moderated in a 2.86 cm thick slab of water, and detected in a ^6Li glass detector of thickness 1.27 cm and diameter 11.4 cm placed at 78 m from the source. The sample of 99.865% ^{234}U weighing 5.69 g in the form of $^{234}\text{U}_3\text{O}_8$ had a thickness of 6.45×10^{-1} atom $^{234}\text{U}/\text{b}$ and was cooled to liquid nitrogen temperature. It was alternately inserted and removed from the beam to measure its transmission. A ^{10}B filter was used to prevent the overlap of neutrons from one pulse to the next and enabled the time-independent neutron background to be measured by a gate set at long neutron time of flight. The time-dependent background is well represented by a single exponential component (17.6 μs) which was measured using two thicknesses of polyethylene. The first thickness served to scatter essentially all neutrons from the beam, leaving the exponentially time-dependent 2.23 MeV γ ray background from neutron capture in the accelerator target water

TABLE II. Coefficients in the parametric fit to the ^{235}U fission cross section.

Energy range (MeV)	B_0	B_1	B_2	B_3
0.1–0.6	1.939 97	-4.164 18	8.047 58	-5.712 24
0.6–1.8	0.625 471	1.185 60	-0.761 525	0.182 049
1.8–5.5	1.917 76	-0.430 258	0.074 1278	-0.004 55841
5.5–9.0	7.759 16	-3.457 06	0.556 992	-2.745 95

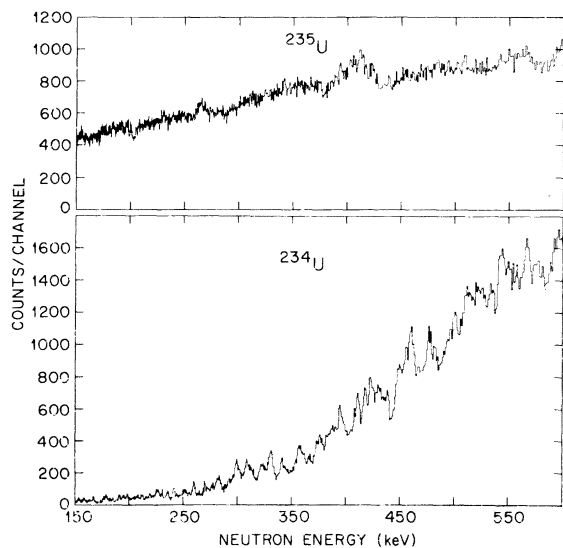


FIG. 2. Fission counts per channel versus neutron energy for ^{234}U and ^{235}U . These data cover a neutron energy range from 150 to 650 keV and show that the structure in the calculated ^{234}U fission cross section near 300 keV is not caused by structure in the ^{235}U cross section.

moderator. A second thickness of polyethylene then allowed extrapolation of the γ ray intensity to zero thickness. Other time-dependent backgrounds are unimportant.

III. PARAMETERS OF CLASS I LEVELS BELOW 1500 eV

A cumulative plot of 310 fine structure levels observed below 7 keV is shown in Fig. 3. It will

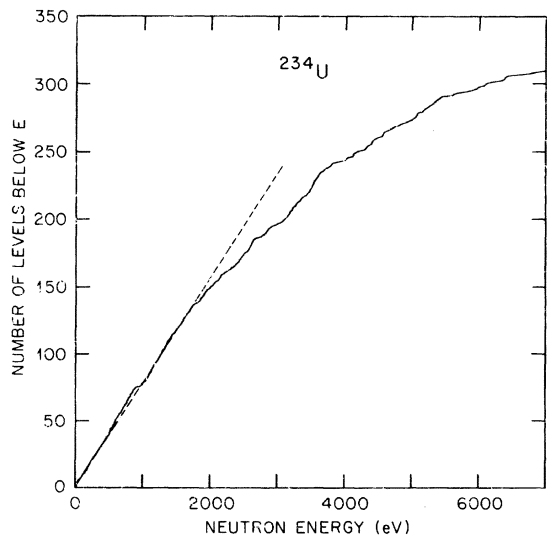


FIG. 3. Cumulative sum of observed fission resonances below 7000 eV. The average spacing of observed levels below 1700 eV is 12.8 ± 0.6 eV. The average spacing corrected for missed levels for an assumed Porter-Thomas distribution of Γ_n^0 is $D_1 = 10.6 \pm 0.5$ eV.

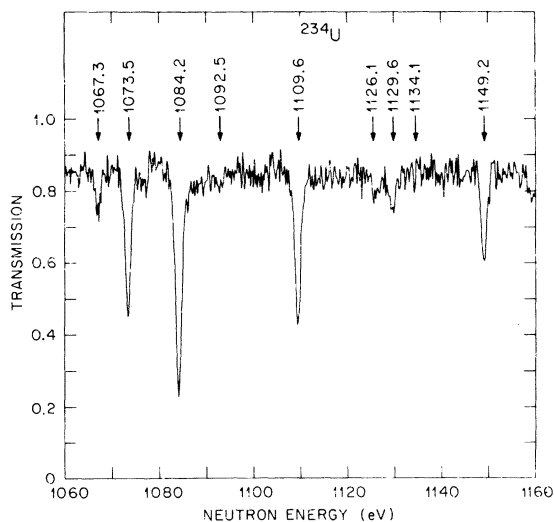


FIG. 4. The transmission of $^{234}\text{U}_9\text{O}_8$ in a region where there probably is a second narrow intermediate structure resonance. Two resonances which dominate the fission cross section in this energy range, at 1092.5 and 1134.1 eV, are not visible in the transmission data because of their small values of Γ_n .

be seen that levels begin to be missed at about 1700 eV. For the 140 levels below this energy the level spacing is $D_1(\text{observed}) = 12.8 \pm 0.6$ eV.

The transmission experiment covers the neutron energy range down to about 20 eV. Between 20 and 1500 eV, all but four of the levels seen in the fission cross section were also visible in the trans-

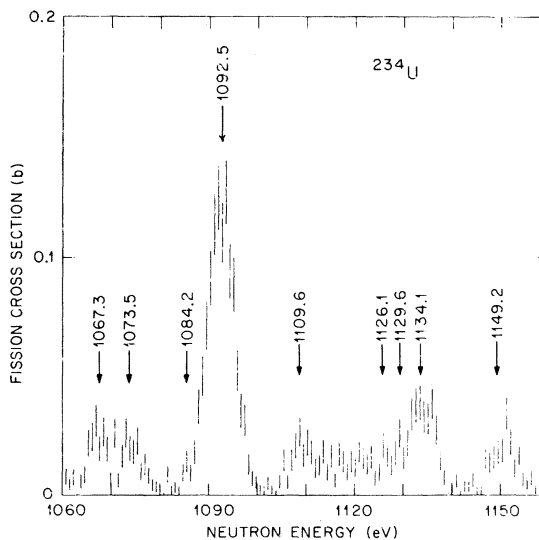


FIG. 5. The fission cross section of ^{234}U over the energy range 1060 to 1160 eV showing two dominating levels at 1092.5 and 1134.1 eV. These levels have low values of Γ_n and suggest the presence of a narrow intermediate structure resonance.

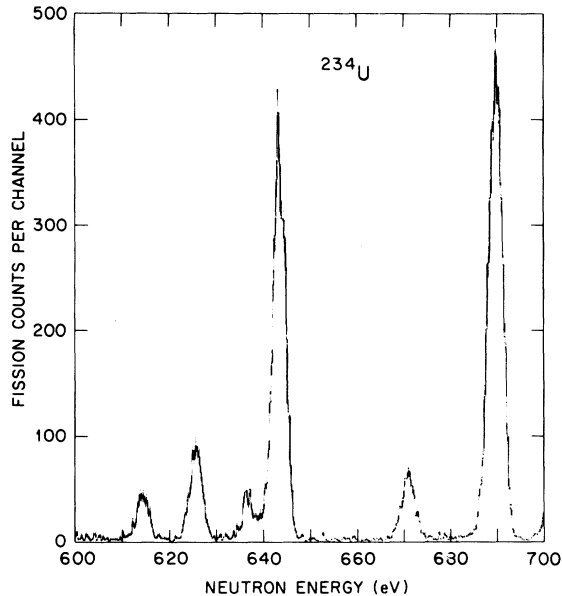


FIG. 6. Fission counts per timing channel for ^{234}U as a function of neutron energy over the range 600 to 700 eV. This illustrates the quality of the data near a peak in the energy dependence of the fission widths.

mission data. Closer examination of these data revealed three of these levels but the fourth, at 1134 eV, could not be seen in transmission because of its small neutron width. The position of this level is shown in Fig. 4 which gives the transmission data over the energy range 1060 to 1160 eV. A corresponding illustration of the fission cross section is given in Fig. 5 which also shows

a prominent resonance at 1092.5 eV which would have been missed in the transmission data.

The quality of the fission cross section data near a peak in the energy dependence of the fission widths is shown in Fig. 6 which covers the energy range 600 to 700 eV. A corresponding plot of transmission is given in Fig. 7. Here the solid line is a fit to the data given by the Harvey-Atta area analysis program¹⁴ using an effective temperature of 120.1°K in the calculation of Doppler broadening. All 118 resonances observed below 1500 eV have been analyzed by the Harvey-Atta area analysis method to obtain the neutron widths Γ_n . This is done iteratively so that the full width Γ assumed in the analysis is equal to the sum of Γ_n , an assumed constant value of the radiation width $\Gamma_\gamma = 40$ meV and the fission width Γ_f . The fission widths are derived from the area under a fission cross section resonance, $\frac{1}{2}\pi\sigma_0\Gamma_f$, which entails a Γ -dependent wing correction. Only for a few resonances is the fission width large enough to necessitate more than two iterations. The measured values of resonance energy, neutron widths, and fission widths for 118 levels below 1500 eV are given in Table III. Reduced neutron widths Γ_n^0 are shown for 118 resonances below 1500 eV in Fig. 8. Only one measured value lies below $\Gamma_n^0 = 0.02$ meV. Following the method of analysis described by Porter and Thomas,⁴ the number of degrees of freedom ν of the χ^2 distribution of these reduced neutron widths is $\nu = 1.11 \pm 0.2$ if the cutoff at which the efficiency for detecting levels is 0.5 is taken to be $\chi_{1/2} = 0.04$. Alternatively, by assum-

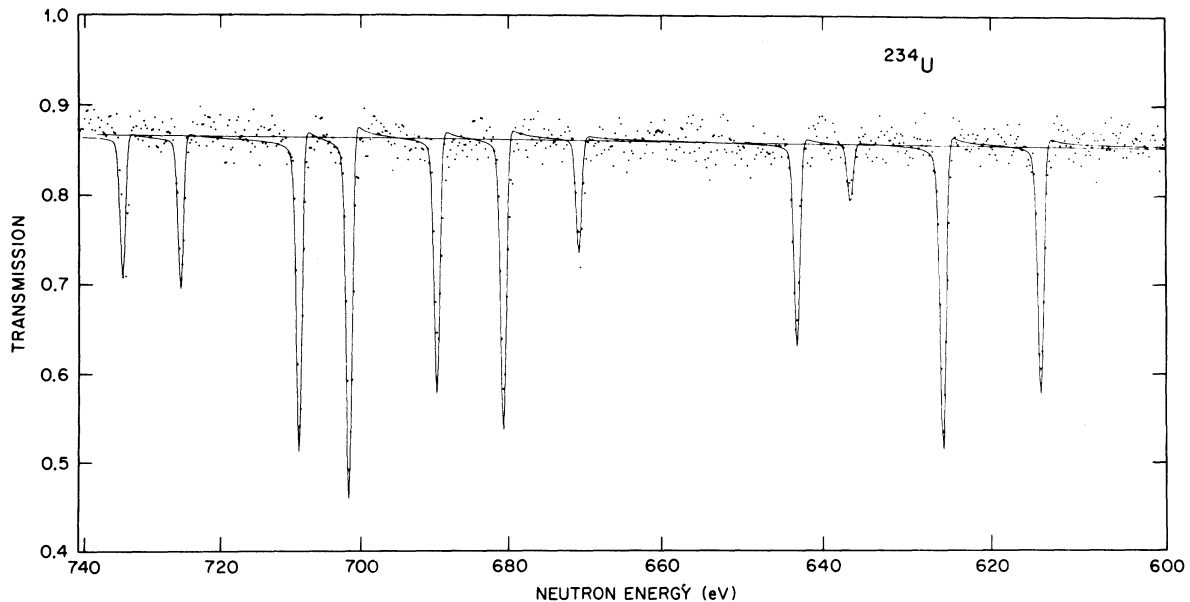


FIG. 7. An example of the results obtained by area analysis of the ^{234}U transmission data near 640 eV using the Atta-Harvey code. The small transmission dip is at 637 eV.

TABLE III. Resonance parameters for ^{234}U .

E_0 (eV)	Γ_n (meV)	Γ_f^a (meV)	E_0 (eV)	Γ_n (meV)	Γ_f^a (meV)
5.16	3.920 ± 0.020	0.018 22 ± 0.000 91	671.2	3.46 ± 0.48	1.50 ± 0.22
22.74	0.0180 ± 0.0018	0.0125 ± 0.0044	681.1	11.50 ± 0.76	0.0140 ± 0.0047
23.42	0.160 ± 0.020	0.003 27 ± 0.000 88	690.0	10.30 ± 0.85	5.06 ± 0.42
31.13	7.20 ± 0.79	0.0086 ± 0.0011	702.1	16.14 ± 0.95	0.334 ± 0.026
45.61	0.430 ± 0.031	0.0667 ± 0.0068	709.1	14.30 ± 1.0	-0.0038 ± 0.0022
48.56	8.73 ± 0.47	0.0000 ± 0.0010	726.1	5.27 ± 0.58	4.87 ± 0.54
77.38	10.28 ± 0.93	0.004 44 ± 0.000 53	734.6	5.27 ± 0.68	0.045 ± 0.017
94.29	41.7 ± 2.3	0.0335 ± 0.0026	757.1	90.3 ± 4.8	0.045 ± 0.0083
106.13	4.05 ± 0.28	0.0911 ± 0.0080	764.7	265 ± 103	0.0307 ± 0.0041
111.06	18.5 ± 1.4	0.308 ± 0.028	766.7	69.3 ± 3.0	0.0119 ± 0.0033
146.25	13.39 ± 0.74	0.0288 ± 0.0023	780.0	3.3 ± 0.62	0.270 ± 0.067
152.16	17.88 ± 0.75	0.0216 ± 0.0016	788.3	39.0 ± 1.9	0.186 ± 0.024
176.18	39.3 ± 1.4	0.0896 ± 0.0057	813.3	25.8 ± 1.4	0.54 ± 0.16
182.49	47.1 ± 1.6	0.114 ± 0.0071	814.8	180.0 ± 7.6	0.160 ± 0.048
187.52	54.2 ± 1.8	0.0175 ± 0.0012	822.5	7.98 ± 0.80	0.303 ± 0.038
194.35	0.526 ± 0.068	-0.0042 ± 0.0048	846.1	3.27 ± 0.69	0.061 ± 0.023
208.4	5.51 ± 0.38	0.0381 ± 0.0038	855.8	11.70 ± 0.92	0.696 ± 0.066
220.9	0.341 ± 0.048	0.008 91 ± 0.005 1	859.7	15.40 ± 1.04	0.256 ± 0.077
226.7	1.59 ± 0.16	0.0397 ± 0.0061	882.2	19.40 ± 1.10	0.207 ± 0.018
237.8	5.09 ± 0.21	0.0130 ± 0.0018	890.1	33.10 ± 1.59	0.089 ± 0.0097
254.3	1.48 ± 0.12	0.676 ± 0.066	957.8	6.49 ± 0.84	0.043 ± 0.015
258.3	6.03 ± 0.26	0.189 ± 0.013	974.5	159.4 ± 7.2	0.235 ± 0.014
276.5	5.77 ± 0.24	0.0688 ± 0.0055	982.3	3.97 ± 0.99	0.365 ± 0.097
290.7	37.2 ± 1.5	0.177 ± 0.0098	1010.3	74.6 ± 3.6	0.0227 ± 0.0034
307.5	16.40 ± 0.72	0.150 ± 0.0106	1037.7	64.3 ± 3.0	0.115 ± 0.011
322.6	22.60 ± 0.95	0.0211 ± 0.0020	1051.4	10.7 ± 1.1	0.108 ± 0.016
331.4	1.05 ± 0.16	0.0823 ± 0.019	1067.3	6.75 ± 0.94	0.121 ± 0.024
349.7	35.90 ± 1.9	0.165 ± 0.013	1073.5	42.5 ± 2.5	0.0446 ± 0.0073
359.1	40.30 ± 2.1	0.0070 ± 0.0010	1084.2	113.8 ± 6.8	0.0047 ± 0.0022
364.0	0.90 ± 0.17	0.326 ± 0.069	1092.5	1.43 ± 0.86	5.8 -2.2, +8.7
387.6	0.79 ± 0.24	1.80 ± 0.54	1109.6	49.5 ± 3.2	0.051 ± 0.0080
391.0	15.80 ± 0.82	0.728 ± 0.053	1126.1	6.0 ± 1.2	0.0070 ± 0.0036
412.6	8.10 ± 0.54	0.274 ± 0.024	1129.6	8.9 ± 1.4	0.0204 ± 0.0080
436.3	5.14 ± 0.36	0.0102 ± 0.0023	1134.0	1.5 ± 1.5	1.1 -0.54, +4000
440.9	2.44 ± 0.27	1.10 ± 0.13	1149.2	22.6 ± 1.7	0.0694 ± 0.012
455.3	18.94 ± 0.93	4.15 ± 0.20	1159.4	7.4 ± 1.2	-0.0092 ± 0.0074
464.2	14.79 ± 0.63	0.491 ± 0.024	1167.1	21.8 ± 1.8	0.0551 ± 0.0097
465.5	6.15 ± 0.39	0.187 ± 0.065	1183.6	11.1 ± 1.3	0.0052 ± 0.0024
489.0	63.3 ± 3.7	0.838 ± 0.050	1195.6	8.8 ± 1.2	0.472 ± 0.085
504.1	8.85 ± 0.52	0.306 ± 0.029	1216.5	24.3 ± 1.8	0.079 ± 0.017
511.0	13.10 ± 0.75	0.597 ± 0.042	1222.4	29.6 ± 1.8	0.390 ± 0.030
515.9	7.73 ± 0.46	8.72 ± 0.53	1231.4	138.0 ± 7.3	0.263 ± 0.020
518.9	3.02 ± 0.33	4.00 ± 0.47	1248.0	31.0 ± 2.3	0.0622 ± 0.0095
526.2	3.11 ± 0.31	2.22 ± 0.23	1255.6	36.4 ± 2.2	0.166 ± 0.018
574.4	68.5 ± 3.5	0.0364 ± 0.0061	1272.9	40.4 ± 2.3	0.000 ± 0.001
555.7	46.3 ± 2.4	3.22 ± 0.17	1278.8	4.5 ± 1.3	0.318 ± 0.099
560.9	8.76 ± 0.50	5.05 ± 0.29	1288.7	4.4 ± 1.2	0.047 ± 0.020
574.4	26.3 ± 1.1	1.15 ± 0.052	1295.3	11.9 ± 1.5	-0.0035 ± 0.0028
582.4	8.83 ± 0.56	6.97 ± 0.45	1325.6	56.5 ± 3.2	0.100 ± 0.011
585.6	6.76 ± 0.60	2.51 ± 0.27	1327.8	26.8 ± 2.3	0.261 ± 0.029
594.1	2.84 ± 0.43	0.467 ± 0.090	1342.7	65.9 ± 3.7	0.092 ± 0.010
614.6	8.17 ± 0.56	0.407 ± 0.040	1354.2	4.2 ± 1.4	0.018 ± 0.011
625.9	11.10 ± 0.71	0.690 ± 0.052	1360.2	38.1 ± 2.7	0.0075 ± 0.0030
637.0	1.68 ± 0.39	1.84 ± 0.43	1363.9	5.8 ± 1.5	0.0042 ± 0.0027
643.5	7.02 ± 0.54	4.97 ± 0.39	1376.5	13.2 ± 1.8	0.083 ± 0.016

TABLE III. (Continued)

E_0 (eV)	Γ_n (meV)	Γ_f^a (meV)	E_0 (eV)	Γ_n (meV)	Γ_f^a (meV)
1396.4	71.4 ± 3.9	0.0954 ± 0.0095	1439.3	110.0 ± 5.6	0.0445 ± 0.0081
1410.0	126.0 ± 9.8	0.068 ± 0.014	1468.1	82.8 ± 4.9	0.0171 ± 0.0036
1411.5	24.8 ± 2.0	0.118 ± 0.025	1486.4	10.6 ± 1.8	0.060 ± 0.015
1436.2	29.7 ± 2.3	0.055 ± 0.011	1492.2	41.1 ± 2.8	0.0115 ± 0.0036

^a Negative values of Γ_f arise by statistical chance when, at an energy where a resonance is indicated by the total cross section, the fission cross section is lower than the "background value" derived from extended regions of the data which are free from resonances.

ing that the data obey a Porter-Thomas distribution and that all levels with Γ_n^0 above a chosen value are observed, it is possible to correct the level spacing and strength function for the numbers of levels with Γ_n^0 below the chosen value. It is found that the corrected number of levels below 1.5 keV is 141 giving $D_I = 10.6 \pm 0.5$ eV and the corrected strength function is $S_0 = \langle \Gamma_n^0 \rangle / D_I = (0.86 \pm 0.11) \times 10^{-4}$. Identical corrected values were obtained for two selected values of the cutoff, $\Gamma_n^0 = 0.1$ and $\Gamma_n^0 = 0.2$ meV.

IV. ENERGY DEPENDENCE OF FISSION WIDTHS

It has been shown by both Weigmann¹¹ and Lynn^{16,17} that the fission width $\Gamma_{I(f)}$ of resonances

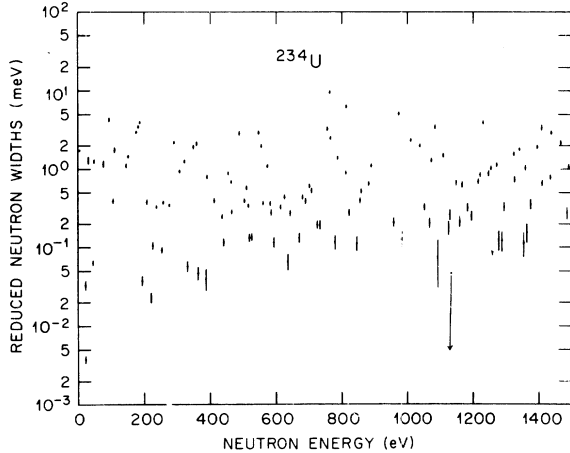


FIG. 8. Reduced neutron widths for the 118 levels observed in the cross sections of ^{234}U below 1.5 keV. The level at 1134.1 eV is not observed in the transmission data and has been assigned an upper limit to its reduced neutron width equal to half the value deduced for the level at 1092.5 eV which has the lowest measured value in this neighborhood. For a 50% observational efficiency at $\Gamma_n^0 = 0.04$ eV, the number of degrees of freedom for the χ^2 distribution followed by these data is $\nu = 1.11 \pm 0.2$. An assumed Porter-Thomas distribution of these data gives the corrected values $D_I = 10.6 \pm 0.5$ eV and $S_0 = (0.86 \pm 0.11) \times 10^{-4}$ both using data with $\Gamma_n^0 > 0.1$ meV and data with $\Gamma_n^0 > 0.2$ meV.

that arise by mixing of the dense class I states of the first potential barrier minimum with a class II state, are expected in the case of "weak" coupling to have a Lorentzian energy dependence of the form

$$\bar{\Gamma}_{I(f)}(E) = K / [(E - E_{II})^2 + \frac{1}{4}W^2], \quad (3)$$

superimposed on a Porter-Thomas⁴ distribution provided $K \ll D_I W^2 / 4\pi$. Here E_{II} is the class II resonance energy, W is the full width of the distribution at half maximum and gives the average coupling width $\Gamma_{II(c)}$ between class I and class II levels, and K is a constant related to the class II fission width $\Gamma_{II(f)}$ by the equation $K = D_I \Gamma_{II(c)} \Gamma_{II(f)} / 2\pi$.

Two methods have been suggested to determine the parameters K , E_{II} , and W in Eq. (3). One is a least squares method¹⁸ applied to the cumulative sum of the fission widths. As the number of fission widths increases, this sum approaches normality and the results obtained improve. In the other method, use is made of the assumption that the fission widths have a χ^2 distribution with one degree of freedom to establish maximum likelihood equations. This method of analysis was first used by Werg *et al.*¹⁹ but was developed independently for the analysis of the present data by James and Evans²⁰ who extended the analysis to use a weighted likelihood and to decide, using the likelihood ratio test,²¹ whether the ^{234}U fission widths below 1500 eV in fact represent two narrow intermediate structure resonances. The results of this analysis are presented here. The maximum likelihood method of analysis was selected in preference to the least squares method for the following reasons: It applies to data which do not have a normal distribution, it allows the variance of the deduced parameters to be specified, and, finally, it can be extended to decide whether the data are derived from more than one narrow intermediate structure resonance.

The fission widths measured for 118 resonances in ^{234}U are displayed in Fig. 9 together with three curves which illustrate the energy dependence of

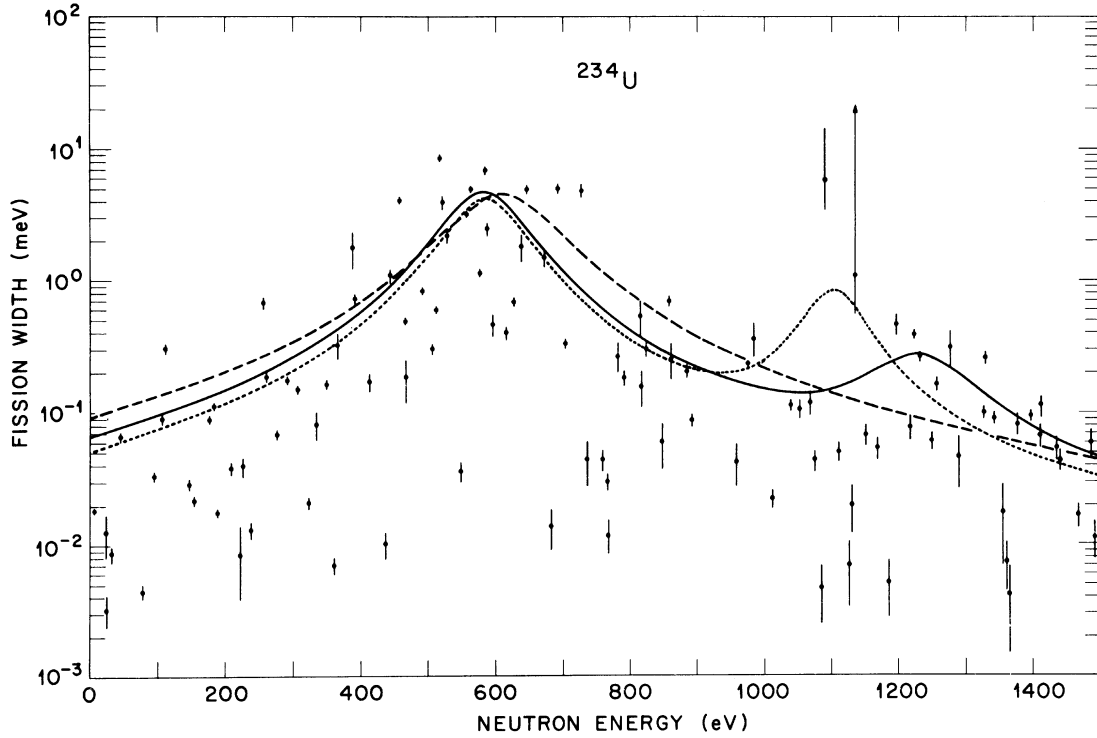


FIG. 9. Energy dependence of class I fission widths for resonances in the fission cross section of ^{234}U below 1500 eV. The curves represent the energy dependence of the mean fission widths defined by Lorentzian functions with the parameters given in Table IV. The solid line is the preferred fit. It shows the sum of two noninterfering Lorentzian curves with parameters deduced by a weighted maximum likelihood method. The dashed curve represents a single Lorentzian derived by weighted maximum likelihood. The dotted curve shows the sum of two Lorentzians derived from an analysis in which the contribution from each fission width to the likelihood is not weighted.

the average fission width as deduced by three methods of analyzing the data. It is possible that the large values of Γ_f at 1092.5 and 1134 eV arise because of a second narrow intermediate structure level at about this energy. This hypothesis has been tested for an energy dependence of the average fission width defined as the sum of two noninterfering levels by the equation

$$\bar{\Gamma}_{\text{I}(f)}(E) = K_1 / [(E - E_{\text{II1}})^2 + \frac{1}{4}W_1^2] + K_2 / [(E - E_{\text{II2}})^2 + \frac{1}{4}W_2^2]. \quad (4)$$

For a Porter-Thomas distribution of widths Γ about an average width $\langle\Gamma\rangle$, the probability distribution function is given by

$$f(\Gamma) = (1/\sqrt{2\pi})\Gamma^{-1/2}\langle\Gamma\rangle^{-1/2}e^{-\Gamma/\langle\Gamma\rangle}$$

and the likelihood L is the product of these functions for all the observed widths and is given by $L = \prod_i f(\Gamma_i)$. The maximum likelihood analysis has been applied by maximizing $\ln L = \sum_i \ln f(\Gamma_i)$ to determine the three parameters K , E_{II} , and W of a single class II level when $\langle\Gamma\rangle$ is given by $\bar{\Gamma}_{\text{I}(f)}(E)$ of Eq. (3) and also to determine the six parameters K_1 , E_{II1} , W_1 , K_2 , E_{II2} , and W_2 of two

class II levels when $\langle\Gamma\rangle$ is given by $\bar{\Gamma}_{\text{I}(f)}(E)$ of Eq. (4). The results obtained are listed in Table IV and the average fission width defined by Eq. (4) is illustrated by the dotted curve in Fig. 9. Whether one or two class II levels represents the better fit to the data can be tested by the value of the likelihood ratio $\lambda = L_1/L_2$. Here L_1 and L_2 are the likelihoods obtained for a three and a six parameter fit, respectively. The null hypothesis is that the data derived from a curve defined by three parameters. It can be rejected if λ , which must lie in the range 0 to 1, is close to zero. It is found that $\lambda = e^{-14.4}$ and, on the basis of this analysis, we can conclude that the data are derived from two class II levels.

It will be seen from Fig. 9 that the properties of the second class II level are strongly dependent on the two large fission widths at 1092.5 and 1134 eV. These two widths have large errors and it seems unreasonable to accept the results of an analysis which does not take these errors into account. The analysis has therefore been repeated by maximizing a quantity $\ln L' = \sum W_i \ln f(\Gamma_i)$ in which the logarithm of the probability distribution func-

TABLE IV. Lorentzian fit to ^{234}U fission width data.

Method	Number of Class II levels	E (eV)	W (eV)	K (eV ² meV)	ν^a
Maximum likelihood	1	645 ± 26	137 ⁺²³ ₋₁₉	48 460 ⁺⁸⁸⁰⁰ ₋₃₈₀₀	0.98 ± 0.11
Maximum likelihood	2	582 ± 17	64 ± 14	17 200 ⁺³¹⁰⁰ ₋₂₄₀₀	1.16 ± 0.11
Weighted maximum likelihood	1	1101 ± 21	52 ± 15	2100 ⁺⁷⁸⁰ ₋₅₇₀	
Weighted maximum likelihood	1	608 ± 18	89 ± 14	36 090 ± 4500	1.05 ± 0.11
Weighted maximum likelihood	2	580 ± 16	68 ± 10	22 140 ⁺³⁴⁰⁰ ₋₂₇₅₀	1.08 ± 0.11
Least squares on cumulative sum	1	1227 ± 65	87 ⁺¹⁷ ₋₁₃	1700 ⁺¹²⁰⁰ ₋₆₅₀	
Least squares on cumulative sum	1	596	103	34 088	1.14 ± 0.11

^a ν is the number of degrees of freedom for a χ^2 distribution describing the spread of fission widths about the energy-dependent mean value.

tion is weighted by the inverse of the squared fractional error in the fission widths given by $W_i = (\Gamma_i / d\Gamma_i)^2$. The minimum value found for L' is subsequently renormalized to obtain the likelihood $L = 118L' / \sum W_i$. This analysis is conveniently referred to as weighted maximum likelihood. However, it is not a standard procedure. The likelihood ratio obtained is $\lambda = e^{-5.41}$ which again enables us to state that the data are derived from two class II levels. The significance level at which the null hypothesis is rejected is given by Wilks's theorem²¹ which states that $-2 \ln \lambda$ has a χ^2 distribution with, in this example, three degrees of freedom. It is found that $-2 \ln \lambda = 10.82$ giving a significance level of 1.27%. The parameters obtained by the weighted maximum likelihood analysis are presented in Table IV and the energy dependence of the average fission width defined by these parameters is illustrated in Fig. 9 by the dashed line for a single class II level and by a solid line for the two class II levels. It will be seen now that the two large fission widths at 1092.5 and 1134 eV are largely ignored in the analysis and that the second class II level arises from the group of relatively large fission widths in the energy range above about 1200 eV.

For each parameter, Table IV also gives approximate values of the standard deviations in these parameters. These are obtained by determining the change in each parameter required to reduce $\ln L$ by 0.5.

Table IV also shows, for comparison, the parameters obtained for a single class II level by the least squares analysis of the cumulative sum of the fission widths as described by Lane *et al.*¹⁸ These parameters are remarkably close to those obtained from a maximum likelihood analysis in which two parameters, E and W , are free and K is fixed by

the normalization equation $\sum \Gamma_{fi} = \sum \bar{\Gamma}_{fi}$ rather than by the likelihood equation $1/n \sum \Gamma_{fi} / \bar{\Gamma}_{fi} = 1$ where n is the number of levels.

Thus we conclude from the likelihood ratio test that the fission widths of ^{234}U below 1500 eV are probably derived from two narrow intermediate structure resonances which, in this analysis, have been assumed to be noninterfering. The preferred fit, illustrated by the solid line in Fig. 9, was derived by a weighted maximum likelihood procedure. When analyzed according to the formalism for "weak" coupling, the preferred parameters from Table IV give $\Gamma_{\text{II}(c)}(580 \text{ eV}) = W_1 = 68 \pm 10 \text{ eV}$, $\Gamma_{\text{II}(c)}(1227 \text{ eV}) = W_2 = 87^{+17}_{-13} \text{ eV}$, $\Gamma_{\text{II}(f)}(580 \text{ eV}) = 193 \pm 40 \text{ meV}$, and $\Gamma_{\text{II}(f)}(1227 \text{ eV}) = 12^{+8}_{-5} \text{ meV}$. For a single class II level the data are probably best fit by the parameters in case three of Table IV. These give the results $\Gamma_{\text{II}(c)}(608 \text{ eV}) = 89 \pm 14 \text{ eV}$ and $\Gamma_{\text{II}(f)} = 240 \pm 50 \text{ meV}$.

V. CLASS II LEVELS UP TO 20 keV

It is possible to obtain approximate values for the fission widths of narrow intermediate structure resonances on the assumption that they are equal to the sum of class I fission widths for levels within the structure and that each of these levels has a neutron width equal to the mean neutron width derived from the strength function. Thus we take the area under a fission cross section class II level to be

$$\begin{aligned} \frac{\pi}{2} \sum_{\lambda_{\text{I}}} \sigma_{0\lambda_{\text{I}}} \Gamma_{\text{I}(f)\lambda_{\text{I}}} &= \frac{\pi}{2} \sigma_{0\lambda_{\text{II}}} \sum_{\lambda_{\text{I}}} \Gamma_{\text{I}(f)\lambda_{\text{I}}} \\ &= \frac{\pi}{2} \sigma_{0\lambda_{\text{II}}} \Gamma_{\text{II}(f)\lambda_{\text{II}}} \end{aligned}$$

Fission widths for the seven class II levels observed up to 13.08 keV together with their reson-

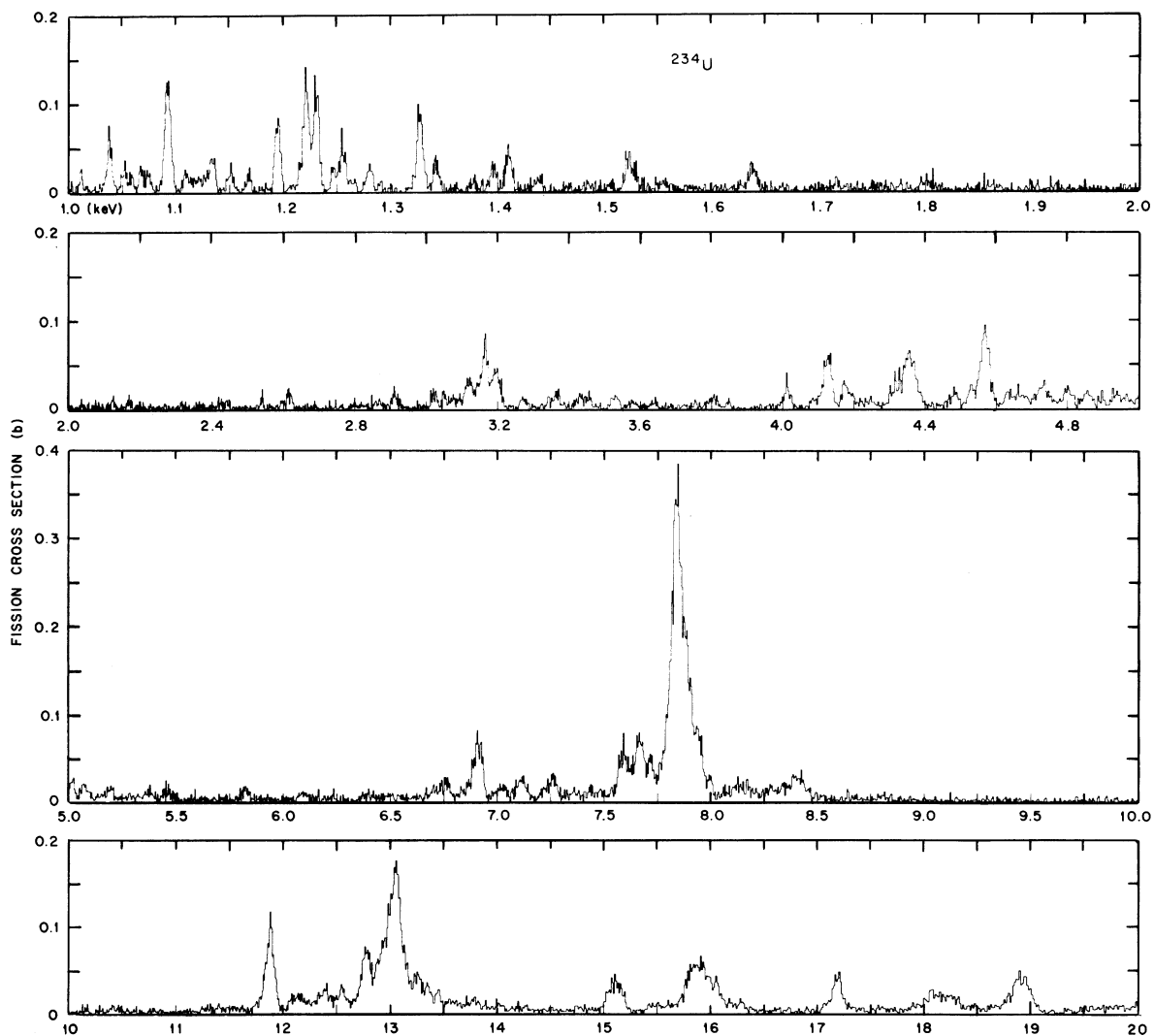
TABLE V. Parameters of class II levels up to 14 keV.

Resonance energy E_{II} (eV)	Fission Resonance Area (b eV)	Approximate class II fission widths $\Gamma_{II(f)}$ (eV)
580	137	0.0478
1227	7.39	0.0107
3100	8.51	0.0284
4575	20.82	0.0979
7845	66.55	0.4954
11886	10.67	0.1159
13076	22.70	0.2681

ance energies are listed in Table V. Over this energy range the average class II fission width is $\langle \Gamma_{II(f)} \rangle = 0.152$ eV. If the level at 1227 eV is discarded, the fission width for the level at 580 eV estimated in this way becomes $\Gamma_{II(f)}(580 \text{ eV})$

$= 0.050$ eV and the average fission width for six levels below 14 keV becomes $\langle \Gamma_{II(f)} \rangle = 0.176$ eV.

Figure 10 illustrates the fission cross section between 1 and 20 keV. In the absence of further evidence such as the Lorentzian distribution of fission widths used below 1.5 keV, and without the adoption of an arbitrary criterion of demarcation, it is difficult to know which peaks in the fission cross section should be regarded as class II levels and which are statistical fluctuations of class I levels within a narrow intermediate structure group. An upper limit to the class II level spacing is about 2.1 keV obtained by considering the seven levels below 14 keV listed in Table V where all the levels from 4 to 5 keV are taken to be part of one class II level at 4575 eV. By taking all the prominent cross section peaks below 20 keV, including five peaks between 4 and 4.6 keV, to be

FIG. 10. The ^{234}U fission cross section between 1 and 20 keV.

class II levels, D_{II} spacing is reduced to about 1 keV. There are several other isolated peaks in the cross section, such as that at 2600 eV which, if they are class II levels, will reduce the class II level spacing even further. We have chosen to use the values obtained below 14 keV and given in Table V.

VI. FLUCTUATIONS NEAR FISSION THRESHOLD

Figure 11 illustrates the ^{234}U fission cross section between 20 keV and 1.6 MeV. The cross section is rich in structure and shows evidence of plateaus at 310, 550, and 770 keV which may be attributable to β -vibrational levels. A careful check has been made to ensure that this structure does not arise artificially from the slight differences between the four region cubic representation of the ^{235}U fission cross section and the latest data available.²⁵ The plateaus listed are confirmed by a recent measurement by Behrens and Carlson²⁶

whose data show a pronounced peak at 770 keV. Only the structure near 310 keV has been analyzed and it is shown in more detail in Fig. 12. By subtracting away the rising trend of the fission cross section, it is found that the remaining cross section has a peak height of 0.0725 b and a full width at half maximum of 50 keV. Of the two solid lines running through the data in Fig. 12, one is a running sum over 20 timing channels and the other is a guideline indicating the 7 fluctuation peaks that lie between 280 and 350 keV. Average values of the height, width, and spacing of these fluctuations are given in Table VI. They are in good agreement with the structure parameters derived previously⁸ from data of poorer statistical accuracy.

In order to decide whether these results are consistent with fluctuations arising from the presence of class II levels, the fission cross section over the energy range 250 to 400 keV has been simulated by Monte Carlo techniques using the following

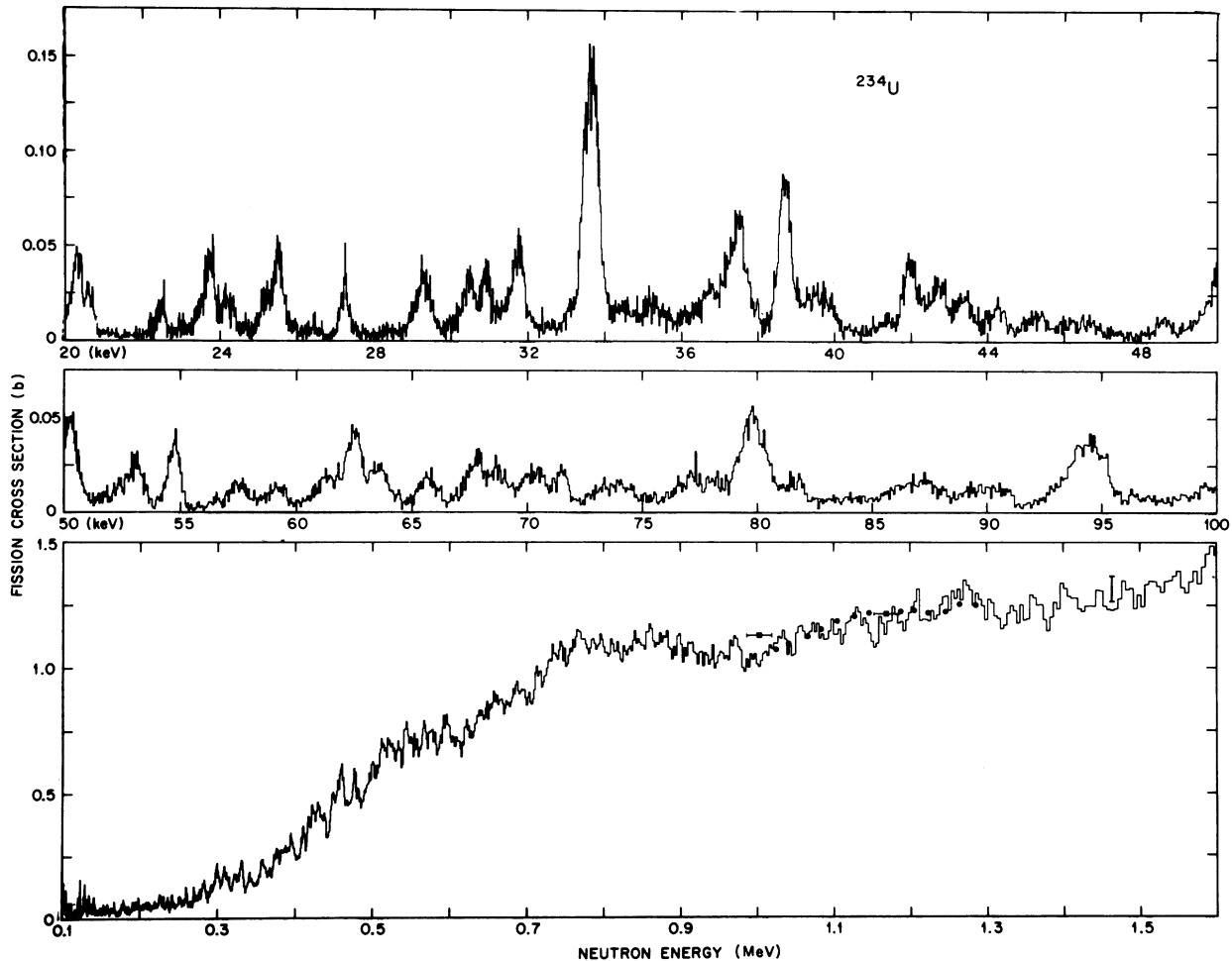


FIG. 11. The ^{234}U fission cross section between 20 keV and 1.6 MeV. Points shown in the region 1.0–1.3 MeV are adjusted data of Lamphere (Ref. 5) to which our data above 100 keV are normalized.

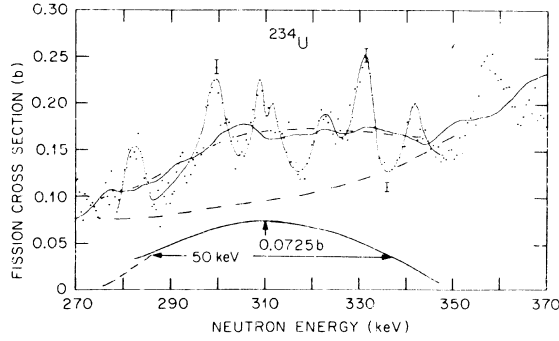


FIG. 12. The fission cross section of ^{234}U between 270 and 370 keV is shown by the full circles. Of the two solid lines through the data, one shows a running sum over 20 timing channels and the other is a guideline indicating fluctuations. The presumed contribution of the vibrational level is shown by the dashed lines and again by the diagram below the data.

equations which describe a vibrational level damped into compound class II levels which in turn are coupled to class I levels.

$$\Gamma_{\text{vib}(f)} = \frac{D_{\text{vib}}}{2\pi[1 + \exp(2\pi(V_B - E_{\text{II}})/\hbar\omega_B)]}, \quad (5)$$

$$\Gamma_{\text{II}(f)} = \frac{D_{\text{II}}W\Gamma_{\text{vib}(f)}}{2\pi[(E_{\text{II}} - E_{\text{vib}})^2 + \frac{1}{4}(W + \Gamma_{\text{vib}(f)})^2]}, \quad (6)$$

$$\Gamma_{\text{II}(c)} = \frac{D_{\text{II}}}{2\pi[1 + \exp(2\pi(V_A - E_{\text{II}})/\hbar\omega_A)]}, \quad (7)$$

$$\Gamma_{\text{I}(f)} = \frac{D_{\text{I}}\Gamma_{\text{II}(c)}\Gamma_{\text{II}(f)}}{2\pi[(E_{\text{II}} - E_{\text{I}})^2 + \frac{1}{4}(\Gamma_{\text{II}(c)} + \Gamma_{\text{II}(f)})^2]}, \quad (8)$$

$$\langle\sigma_f\rangle = \sum_J 2\pi^2\lambda^2 g \langle\Gamma_n\rangle \langle\Gamma_{\text{I}(f)}\rangle \times F/D_{\text{I}} \langle\Gamma_n + \Gamma_\gamma + \Gamma_{\text{I}(f)} + \sum \Gamma_{n'}\rangle. \quad (9)$$

In these equations V_A and V_B are the heights above the neutron binding energy of the intermediate and outer fission potential barriers; $\hbar\omega_A$ and $\hbar\omega_B$ are frequencies of inverted parabolic potentials representing these two barriers; E_{vib} , $\Gamma_{\text{vib}(f)}$, and D_{vib} are the resonance energy, fission width, and level spacing of the vibrational level; E_{II} , $\Gamma_{\text{II}(f)}$, and

D_{II} represent the energy, fission width, and spacing of class II levels, W is the damping width of the vibrational level into the class II levels; E_{I} , $\Gamma_{\text{I}(f)}$, and D_{I} represent the energy, fission width, and spacing of class I levels; $\Gamma_{\text{II}(c)}$ is a coupling width between class I and class II levels; $2\pi\lambda$ is the neutron wave length, g is the spin weighting factor, Γ_n and Γ_γ are the neutron and capture widths of class I resonances; $\Gamma_{n'}$ represents neutron inelastic scattering widths over channels that are open at 310 keV; and F is the fluctuation factor $\langle\Gamma_n\Gamma_f/\Gamma\rangle\langle\Gamma\rangle/\langle\Gamma_n\rangle\langle\Gamma_f\rangle$. Many of the quantities in Eq. (9) are spin-dependent but this is not shown explicitly. It is assumed that the fission strengths, $\Gamma_{\text{I}(f)}/D_{\text{I}}$, $\Gamma_{\text{II}(f)}/D_{\text{II}}$ and the capture width Γ_γ are spin-independent. Significant contributions to the fission cross section arise only for $J = \frac{1}{2}$, $\frac{3}{2}$, and $\frac{5}{2}$ for an assumed positive parity state. A list of the parameters used in the calculation is given in Table VII. Many of these are derived from the measured properties of the low energy class II levels. The value $V_B = 674$ keV is deduced by adjusting the average fission cross section at the peak of the vibrational level at 0.0725 b. Table VI gives the parameters of the simulated fluctuations derived after broadening the cross section with the resolution of the present experiment. It will be seen that they are in good agreement with the experimental values and this shows that the observed fluctuations, despite their wide spacing, can arise from the presence of class II levels of spacing ≤ 0.97 keV. However, the value of V_B derived in this simulation gives $\Gamma_{\text{II}(f)} = 0.00747$ eV at $E_{\text{II}} = 1$ keV in contrast to the average value of 0.152 eV for seven class II levels below 14 keV. The low energy fission widths are thus not derived predominantly by coupling through the vibrational level at 310 keV. For a direct coupling, we find from the equation

$$\Gamma_{\text{II}(f)} = D_{\text{II}}/2\pi[1 + \exp(2\pi(V_B - E_{\text{II}})/\hbar\omega_B)], \quad (10)$$

that the seven levels with average $\Gamma_{\text{II}(f)} = 0.152$ eV give $V_B = 684$ keV.

The coupling between class I and class II states

TABLE VI. Comparison of experimental and simulated structure.

	Experimental data	Simulated results ^a
Vibrational level width	50 keV	59 ± 4 keV
Average peak cross section	0.0725 b	0.0854 ± 0.0117 b
Average structure height	0.088 ± 0.011 b	0.081 ± 0.0053 b
Average structure width	4.6 ± 0.5 keV	3.68 ± 0.25 keV
Average structure spacing	10.0 ± 2.1 keV	9.7 ± 0.7 keV

^aFor $V_B = 590$ keV; a change to 674 keV was made to bring the average peak cross section into agreement with experiment, but the simulation was not redone, since the height, width, and spacing values would remain substantially unchanged.

TABLE VII. Parameters used in cross section simulation.

Parameter	Value	
D_{vib}	500 keV	Assumed spacing of vibrational levels of same spin and parity
V_B	674 keV	Derived by adjusting $\langle\sigma_{f\text{max}}\rangle = 0.0725$ b
$\hbar\omega_A$	1000 keV	Assumed
$\hbar\omega_B$	560 keV	Assumed
W	50 keV	Observed in this experiment
D_{II}	$(J = \frac{1}{2})$ 970 eV	From $D_{\text{II}} = 2100$ eV at low energy
	$(J = \frac{3}{2})$ 480 eV	
	$(J = \frac{5}{2})$ 320 eV	
V_A	101 keV	From $\langle\Gamma_{\text{II}(c)}\rangle = 115$ eV at low energy ^a
D_{I}	$(J = \frac{1}{2})$ 6.2 eV	From $D_{\text{I}} = 10.64$ eV at low energy
	$(J = \frac{3}{2})$ 3.2 eV	
	$(J = \frac{5}{2})$ 2.1 eV	
$\Sigma\Gamma_n$	$(J = \frac{1}{2})$ 0.107 eV	From the black nucleus barrier transmissions of Rae <i>et al.</i> (Ref. 23) for assumed 2^+ level at 44 keV and 4^+ level at 144 keV
	$(J = \frac{3}{2})$ 0.12 eV	
	$(J = \frac{5}{2})$ 0.089 eV	
Γ_γ	0.04 eV	Assumed
F	$(J = \frac{1}{2})$ 0.5	Calculations of Sowerby (Ref. 24)
	$(J = \frac{3}{2})$ 0.58	
	$(J = \frac{5}{2})$ 0.61	

^aThis value of V_A is derived from an early estimate of $\langle\Gamma_{\text{II}(c)}\rangle = 115$ eV made prior to the maximum likelihood analysis of the fission widths. The preferred value $\langle\Gamma_{\text{II}(c)}\rangle = 77$ eV corresponds to $V_A = 190$ keV. It has been shown that the simulated structure parameters of Table VI are insensitive to this change in V_A because penetration of the intermediate barrier affects only the structure width which is dominated at 310 keV by the energy resolution.

may also be interpreted as very weak coupling to a broad class II level described by the equation

$$\Gamma_{\text{I}(f)} = D_{\text{I}}\Gamma_{\text{II}(c)}\Gamma_{\text{II}(f)}/2\pi[(E_{\text{II}} - E_{\text{I}})^2 + \frac{1}{4}\Gamma_{\text{II}(f)}^2]. \quad (11)$$

Assuming two class II levels below 1.5 keV, this interpretation gives $\Gamma_{\text{II}(f)} = 77$ eV and $\Gamma_{\text{II}(c)} = 0.091$ eV corresponding to $V_A = 1307$ keV and $V_B = 106$ keV. This interpretation does not allow coupling through a vibrational level because the required value of $\Gamma_{\text{vib}(f)}$ would exceed the observed vibrational level width of 50 keV.

In the absence of coupling to the low energy resonances, the vibrational level at 315 keV may be interpreted as a p -wave vibrational resonance. In this case, a peak cross section of 0.0725 b gives an average class I fission width of 0.00294 eV. This may be interpreted in terms either of very weak coupling to a strong class II level $V_A > V_B$, or as an example of weak coupling $V_A < V_B$. For $V_A > V_B$, by averaging Eq. (11) we get $\Gamma_{\text{II}(c)}/D_{\text{II}} = \Gamma_{\text{I}(f)}/D_{\text{I}}$ which gives $V_A = 1240$ keV from Eq. (7). From Eqs. (5) and (6) taking $\Gamma_{\text{vib}(f)} = 50$ keV $\gg W$ and $D_{\text{vib}} = 500$ keV we get $V_B = 268$ keV. For weak coupling, $W \gg \Gamma_{\text{vib}}$ and averaged versions of

Eqs. (6) and (8) give $\Gamma_{\text{vib}(f)}/D_{\text{vib}} = \Gamma_{\text{I}(f)}/D_{\text{I}}$ from which, using Eq. (5), $V_B = 823$ keV. Also, $\Gamma_{\text{II}(c)}/D_{\text{II}} = W/D_{\text{vib}} = 50/500$ which, from Eq. (7) gives $V_A = 231$ keV. There are no reasons from the present measurements to prefer one of these interpretations over the others but it is noted that the very weak coupling values $V_A = 1240$ keV and $V_B = 268$ keV are in agreement with the systematic behavior over several isotopes derived by Back *et al.*²²

VII. CONCLUSION

The measurement and analysis of the ^{234}U neutron fission and total cross sections reported here have resulted in the derivation of neutron and fission widths for all the 118 fine structure resonances observed below 1.5 keV. When corrected for missing levels on the assumption of a Porter-Thomas distribution, these results give a fine structure level spacing $D_{\text{I}} = 10.6 \pm 0.5$ eV and a strength of $(0.86 \pm 0.11) \times 10^{-4}$. It is shown by an application of the likelihood ratio test that these low energy resonances comprise two narrow intermediate structure levels, one at 580 ± 16 eV and the other at 1227 ± 65 eV. The parameters of the presumed Lorentzian energy dependence of the

average fission width have been found on the assumption that there is no interference between these structure levels. This has been done both by the maximum likelihood method and by a method in which the contribution to the likelihood from each resonance is weighted inversely as the square of the fractional error in its fission width. It is believed that the weighted maximum likelihood method gives the better estimate of these parameters.

Between 1.5 and 14 keV the data reveal five narrow intermediate groups of fission resonances, only two of which had been previously observed. This reduces the class II level spacing at low energy to $D_{II} = 2.1 \pm 0.3$ keV. At 310 keV the fluctuations previously noted are confirmed and it is shown that strong fluctuations persist to at least 1 MeV. Broad structures, presumed to arise from β -vibrational levels, are present at 310, 550, and 770 keV. It is shown by cross section simulation, using parameters derived from the low energy data, that the fluctuations expected near 310 keV on the assumption of a two stage process involving a β -vibrational level have the same average properties as those observed. However, the value of outer barrier height $V_B = 674$ keV required to give the correct vibrational level contribution to the fission cross section at 310 keV provides, through a two stage process, little of the fission strength observed at low energy. It is clear that this fission strength could arise in two other ways. It can be readily accounted for by "weak" coupling of class I and class II levels

(without the intervention of a vibrational level) using Eq. (10) with $V_B = 684$ keV.

For "very weak" coupling of class I levels to a broad class II level, the barrier parameters derived from the low energy data are $V_A = 1307$ keV and $V_B = 106$ keV (again without the intervention of a vibrational level). In the presence of such coupling, the β -vibrational level at 310 keV can be interpreted as a p -wave resonance with barrier parameters $V_A = 1240$ keV and $V_B = 268$ keV. There are no reasons from the present measurement to prefer one interpretation over the others but it is noted that the last quoted set of barrier heights is in agreement with the systematic behavior over several isotopes derived by Back *et al.*²²

ACKNOWLEDGMENTS

The authors are grateful to Mr. J. G. Craven for helpful advice on the operation of the ORNL computer system, Dr. R. Gwin for the loan of a flight path, Dr. H. W. Schmitt and Dr. F. Plasil for the loan of the ²³⁴U foils and ionization chamber (designed by Dr. H. Rösler), and to the late Mr. A. D. Williams for assistance in setting up the cooled sample changer. We also thank Mr. H. A. Todd and his staff at ORELA for the smooth operation of the linac, and Mr. F. E. Gillespie for fission chamber assembly and filling. Dr. F. H. C. Marriott, Department of Biomathematics at Oxford kindly gave us advice on the use of the likelihood ratio test.

*Research sponsored by the U. S. Energy Research and Development Administration under contract with Union Carbide Corporation.

†On assignment from UKAEA, Atomic Energy Research Establishment, Harwell, England.

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