PHYSICAL REVIEW C VOLUME 15, NUMBER 6 JUNE 1977

# Relativistic nuclear fluid dynamics*

Anthony A. Amsden, Francis H. Harlow, and J. Rayford Nix
*Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545*
(Received 17 January 1977)

By use of finite-difference methods we solve in three spatial dimensions the classical relativistic equations of motion for the collision of two heavy nuclei. These equations express the conservation of nucleon number, momentum, and energy, for a specified nuclear equation of state. For $^{20}Ne + ^{238}U$ at laboratory bombarding energies per nucleon of 250 MeV and 2.1 GeV, we calculate the time evolution of the matter distribution for several impact parameters. Nearly central collisions deform and compress the system enormously, whereas in peripheral collisions the projectile is fragmented into a portion that proceeds roughly straight ahead at its original velocity and other portion that deposits its energy in the target. For a given impact parameter we construct from the velocity vectors at some large time the energy and angular distributions for the expanding matter. An integration of these results over impact parameter then gives us the double differential cross section $d^2\sigma/dEd\Omega$. For the 250-MeV case we compare calculated and experimental results in the form of proton energy spectra for four laboratory angles ranging from 30° to 120°. The calculations reproduce correctly the experimental slopes at each angle, as well as the overall decrease in the experimental cross section when going from forward to backward angles. However, at 30° the calculated values are only one-half the experimental ones, whereas at 120° they are twice as large. These comparisons, together with comparisons of calculations done by other workers, suggest that heavy nuclei are partially transparent to each other during collisions at high energy, but that the process is not solely a superposition of individual nucleon-nucleon collisions. Instead, coherent collective-field effects play some role.

[NUCLEAR REACTIONS High-energy heavy-ion $^{20}Ne + ^{238}U$, $E_{bom}/20 = 250$ MeV, 2.1 GeV. Calculated $d^2\sigma/dEd\Omega$ for outgoing matter and compared with experimental data. Relativistic nuclear fluid dynamics, hydrodynamic model, nuclear equation of state, particle-in-cell finite-difference computing method.]

## I. INTRODUCTION

As part of a recent surge of interest in heavy-ion physics, many groups are now studying what happens when two heavy nuclei collide at high energies. Interest in this field stems from the possibility that during collisions at high energy, heavy nuclei may become compressed to more than their normal density. If this occurs it will permit us to learn about the nuclear equation of state, the fundamental relationship specifying how the pressure (or alternatively the energy per particle) depends upon density and internal energy. At present we have only two pieces of experimental information concerning this important quantity: the equilibrium values of the density and energy per particle. Even the nuclear compressibility coefficient is unknown.

The expected complexity of the nuclear equation of state gives rise to some tantalizing possibilities. For example, compression of nuclei may result in density isomers, or quasistable states existing at other than normal nuclear density.[1-11] These states could arise from any of several mechanisms involving the nucleon-nucleon potential: tensor or many-body components,[1] an attractive region inside the hard core,[2-4] or a restoration of chiral symmetry leading to almost massless nucleons in deeply bound states.[5-9] Increased nuclear density could also lead to the formation of pion condensates, or states containing a large number of bound pions.[9,12-20] Nuclear phase transitions of this type could alter substantially the properties of nuclear matter.

As we discuss in Sec. II, several different theoretical approaches are being pursued in the study of high-energy heavy-ion collisions. Our present investigation is based on one of these possible approaches: relativistic nuclear fluid dynamics. We describe our fluid-dynamics model in Sec. III and apply it in Sec. IV to the reaction $^{20}Ne + ^{238}U$ at laboratory bombarding energies per nucleon of 250 MeV and 2.1 GeV. In particular, we use it to calculate the time evolution of the matter distribution for several impact parameters, as well as the double differential cross section $d^2\sigma/dEd\Omega$. For the 250-MeV case our calculated results are compared with experimental data in the form of proton energy spectra at various angles. Our conclusions are presented in Sec. V.

## II. POSSIBLE APPROACHES

How should the collision of two heavy nuclei at high energies be described theoretically? In answering this question it is useful to begin with the ultimate— a quantal relativistic time-dependent many-body theory that includes all hadronic degrees of freedom— and see what approximations must be made in order to arrive at something tractable. Starting with such an ultimate theory, one must first specify which degrees of freedom are to be treated explicitly (for example, only the nucleons) and which implicitly. The further approximations that are made depend upon whether certain physical quantities are large or small. These include the bombarding energy, the masses of the constituent particles, the interaction strengths and correlations between particles, and the number of degrees of freedom.

Some of the major approaches obtained in this way are shown in Fig. 1. For example, if the bombarding energy per nucleon is small compared to the masses of the constituent particles, one may give up relativity and employ a quantal nonrelativistic theory based upon a given nucleon-nucleon interaction. (We use units in which the light speed $c=1$ and speak of energies and masses interchangeably.) This leads to a time-dependent many-body Schrödinger equation, for which a variety of additional approximations are available. If the interaction is small and the bombarding energy is large, one may use straight-line trajectories propagating through a nucleus whose nucleons are stationary. Such a nonrelativistic eikonal (Glauber) approximation is especially suitable for describing small-angle scattering.[21-24] (A relativistic eikonal approximation could also be used.) Alternatively, one may approximate the

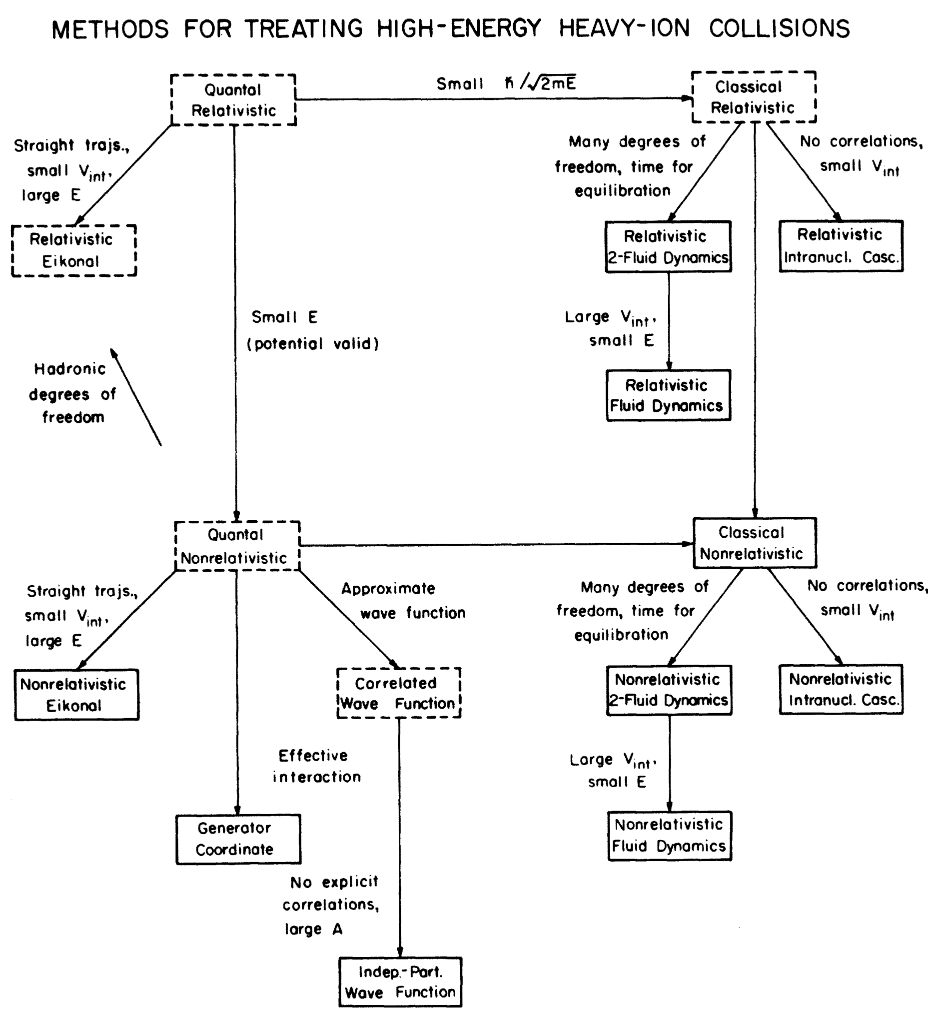


FIG. 1. Relationship among various methods for treating high-energy heavy-ion collisions. The solid boxes denote methods that are currently tractable, and the dashed boxes denote methods that are not.

time-dependent many-body wave function in some way. At the highest level of approximation one could attempt to take into account correlations between the particles by use of a correlated wave function.[25] However, in practice it is necessary to introduce an effective interaction, at which stage an approximate generator-coordinate wave function may be used.[26-28] If in addition there are many degrees of freedom and no explicit correlations, one may use an independent-particle wave function, such as a time-dependent Hartree-Fock wave function.[29-32]

At the opposite extreme, if the bombarding energy and particle masses are sufficiently large, then the de Broglie wave length is small and one may give up quantum mechanics and employ a classical relativistic theory. If there are no correlations and the interaction is small, then a relativistic intranuclear cascade model with a given nucleon-nucleon cross section may be used.[33-36] Alternatively, if there are many degrees of freedom and sufficient time for local equilibration, then a relativistic fluid-dynamics description in terms of a given nuclear equation of state becomes appropriate. Because of the expected interpenetration of the target and projectile upon contact, it is necessary in general to employ two fluids that represent the target and projectile separately.[37] However, if the interaction is large or the bombarding energy is small there is little interpenetration, and conventional relativistic fluid dynamics is appropriate.[10,11,16-19,38-47]

For intermediate bombarding energies and particle masses, one may give up both quantum mechanics and relativity and employ a classical nonrelativistic theory. In this case it is possible to solve directly the classical many-body equations of motion with a given nucleon-nucleon potential.[48-55] The approximations discussed in connection with a classical relativistic theory could, of course, also be made here. This would lead either to a nonrelativistic intranuclear cascade model (which is equivalent to the Boltzmann equation[56]) or to nonrelativistic fluid dynamics.[57-63]

Finally, some specialized models have been developed for treating certain aspects of the problem, such as projectile fragmentation,[64-69] the relative frequency of various types of emitted particles,[70] and their energy and angular distributions.[71]

For the collision of two heavy nuclei at high energies, none of the above sets of approximations is entirely appropriate. It is therefore important to pursue simultaneously several different approaches, including especially classical many-body calculations with a given nucleon-nucleon potential, intranuclear cascade calculations with a given nucleon-nucleon cross section, and relativistic fluid dynamics with a given nuclear equation of state. Each approach has its own relative merits, but we are pursuing the last one because it deals directly rather than indirectly with the quantity of primary interest. With this approach we should be able to learn about the nuclear equation of state irrespective of the complexity of the underlying hadronic interactions that give rise to it.

## III. FLUID-DYNAMICS MODEL

### A. Previous work

As early as 1955 Belenkij and Landau used a fluid-dynamics model to describe collisions between two nucleons, a nucleon and a nucleus, and two nuclei.[38] In 1959 Glassgold, Heckrotte, and Watson considered the moderately weak shock waves that could be formed when a high-energy proton or pion passes through a nucleus.[57] They also pointed out that the angular distribution of the nucleons ejected after the shock wave passes through the nuclear surface could be used to determine the nuclear compressibility coefficient. However, these were ideas before their times, and they remained largely unnoticed until 1973.

Then the increasing availability of heavy-ion projectiles revived interest in the subject. Chapline *et al.*,[39,40] Wong and Welton,[58] Greiner and his associates,[10,11,16,17,41,42,59,60] Sobel *et al.*,[61] Sano and his associates,[18,19,45-47,63] and others[43,44,62] considered various aspects of high-energy nuclear fluid dynamics.

The previous theoretical discussions are based on a variety of simplifying assumptions. In most cases the shock waves are idealized as propagating in semi-infinite nuclear matter as pure cones, and the treatment is often nonrelativistic. In some calculations that have been performed for a finite system, a restricted parametrization in terms of overlapping spheroids is used to describe the nuclear shape and density.[11,47,59,60,63] The passage of a shock wave through the nuclear surface usually is assumed to eject nucleons at a definite angle straight ahead of the shock front; the width in the angular distribution of ejected nucleons then is attributed entirely to their Fermi motion.

However, the true situation is more complicated than this. Because of the finite size of nuclei, the shock waves that are generated are curved rather than conical. At higher bombarding energies relativistic effects become important. The target and projectile are deformed and compressed by the impact into crescents of revolution whose shape and density are not describable in terms of simple functions. The passage of the shock waves through the nuclear surface is followed by rarefaction waves and an overall expansion of the system. The expanding matter explodes into space in a distribution of angles that is moderately broad.

Although these points have been taken into account in a numerical solution of the relativistic equations of fluid dynamics, the earlier treatment was restricted to head-on collisions.[44] It is our purpose here to extend this study to off-center collisions in a fully three-dimensional calculation, in order that the predictions of nuclear fluid dynamics may be confronted with experimental data.

### B. Expected range of applicability

As we saw in Sec. II, the validity of fluid dynamics is based on (1) a large number of degrees of freedom, (2) sufficient time for local equilibration, and (3) either a large interaction or a small bombarding energy. How well are these three conditions satisfied for typical high-energy heavy-ion collisions, such as $^{20}Ne + ^{238}U$ at bombarding energies per nucleon of 250 MeV and 2.1 GeV?

The first condition is satisfied moderately well, because 258 is large compared to unity. In addition, if pions are produced and taken into account, this will increase the number of degrees of freedom even further.

The second condition is satisfied less well, but perhaps well enough. At relativistic energies the time required for the collision to take place is roughly the nuclear diameter divided by the light speed, or about $5\times10^{-23}$ s. The exchange of a pion between two adjacent nucleons requires about $5\times10^{-24}$ s, or one-tenth the collision time. Therefore, some degree of local equilibration should be achieved during the collision. This suggests that the introduction of a nuclear equation of state should be a fair approximation.

The third condition is more subtle. If the process were a superposition of individual nucleon-nucleon collisions[39,61] with interaction cross section $\sigma$, then at normal nuclear density $n_0$ the mean free path $\lambda_{col}$ between collisions would be

$$\lambda_{col} = 1/(n_0\sigma) \approx 1/[(0.15/\text{ fm}^3)(40\text{ mb})] \approx 1.7\text{ fm}.$$

The mean free path for stopping a nucleon would then be

$$\lambda_{stop} \approx \lambda_{col} T/\Delta T\,,$$

where $T$ is the initial kinetic energy of the nucleon and $\Delta T$ is its average energy loss per collision. The energy loss per collision is about 125 MeV at low energies and increases to about 500 MeV at high energies.[72] The stopping mean free path is therefore about 3 fm for a bombarding energy per nucleon of 250 MeV and is substantially larger for a bombarding energy per nucleon of 2.1 GeV. Thus, on the basis of such arguments,[39,61] nuclear fluid dynamics should be fair at the lower bombarding energy and poor at the higher bombarding energy.

However, these estimates neglect the possibility of coherent collective-field effects,[57] as might be caused, for example, if a pion condensate were produced in the collision.[16-19] Such collective effects could in principle decrease the mean free path substantially and result in the propagation of collisionless shocks. We therefore take the point of view that the applicability of nuclear fluid dynamics to high-energy heavy-ion collisions is an open question that ultimately must be decided experimentally.

### C. Equations of motion

Our equations of motion are simple because we neglect nuclear viscosity, surface energy, Coulomb energy, and single-particle effects. These energies are small compared to the kinetic energies involved at high bombarding energies, but their neglect nevertheless precludes an accurate description of the coalescence of matter into clusters following the demolition of the system. We also neglect the production of additional particles and the associated radiative loss of energy from the system; this approximation becomes increasingly serious as the bombarding energy increases. Such effects could be taken into account by including transport terms in the equations of motion.[73]

The covariant relativistic hydrodynamic equations that we solve express the conservation of nucleon number, momentum, and energy,[74] for a specified nuclear equation of state. For our purposes these equations are written conveniently as

$$\frac{\partial N}{\partial t} + \nabla\cdot(\vec{v}N) = 0\,, \tag{1a}$$

$$\frac{\partial \vec{M}}{\partial t} + \nabla\cdot(\vec{v}\vec{M}) = -\nabla p\,, \tag{1b}$$

and

$$\frac{\partial E}{\partial t} + \nabla\cdot(\vec{v}E) = -\nabla\cdot(\vec{v}p)\,, \tag{1c}$$

where $N$, $\vec{M}$, and $E$ are, respectively, the nucleon number density, momentum density, and energy density (including rest energy) in the laboratory reference frame. The velocity of matter relative to the laboratory frame is denoted by $\vec{v}$, and $p$ is the pressure in the rest frame. The three laboratory-frame quantities are related to rest-frame quantities by

$$N = \gamma n\,, \tag{2a}$$

$$\vec{M} = \gamma^2(\epsilon + p)\vec{v}\,, \tag{2b}$$

and

$$E = \gamma^2(\epsilon + p) - p\,, \tag{2c}$$

where $n$ and $\epsilon$ are, respectively, the nucleon number density and energy density in the rest frame and $\gamma = (1 - v^2)^{-1/2}$, with the velocity measured in units of the light speed.

### D. Nuclear equation of state

Ultimately we will vary the equation of state in an attempt to determine it from comparisons with experimental data, but for our initial studies we are using one derived from theory. It is obtained from a Thomas-Fermi treatment of the effective two-nucleon interaction that consists of an attractive Yukawa function multiplied by a quadratic momentum-dependent term.[75] This leads to a rest-frame energy per nucleon $\epsilon/n$ of the form

$$\epsilon/n = m_0 + a(n/n_0)^{2/3} - b(n/n_0) + c(n/n_0)^{5/3} + I/n\,, \tag{3}$$

where $m_0$ is the nucleon rest mass, $n_0 = 3/(4\pi r_0^3)$ is the equilibrium value of $n$, and $I/n$ is the rest-frame internal (heat) energy per nucleon. For the specific choices[75] of 1.2049 fm for $r_0$ and $-15.677$ MeV for the nonrelativistic energy per nucleon at equilibrium (excluding rest energy), the values of the three constants that appear are $a = 19.88$ MeV, $b = 69.02$ MeV, and $c = 33.46$ MeV. The resulting value of the nuclear compressibility coefficient is $K = 9n_0^2 \partial^2(\epsilon/n)/\partial n^2|_0 = 294.8$ MeV.

The pressure $p$ is obtained from the relationship $p = n^2 \partial(\epsilon/n)/\partial n|_S$, with differentiation at constant entropy $S$. The relationship between $I/n$ and the nuclear temperature is taken from a nonrelativistic Fermi-gas model for the thermal motion of the nucleons relative to the hydrodynamic flow, for which $n^2\partial(I/n)/\partial n|_S = \frac{2}{3}I$. This is the exact result for a nonrelativistic Fermi-gas model, instead of being true only to second order in the nuclear temperature, as is often implied. The pressure is given finally by

$$\begin{aligned} p &= [\tfrac{2}{3}a(n/n_0)^{5/3} - b(n/n_0)^2 + \tfrac{5}{3}c(n/n_0)^{8/3}]n_0 + \tfrac{2}{3}I \\ &= [-\tfrac{2}{3}m_0(n/n_0) - \tfrac{1}{3}b(n/n_0)^2 + c(n/n_0)^{8/3}]n_0 + \tfrac{2}{3}\epsilon\,. \end{aligned} \tag{4}$$

In a heavy-ion collision the pressure is positive during the initial compression stage and negative during the later expansion stage, at which time the driving forces attempt to form the matter into small clusters of near-equilibrium density. These clusters would be physically meaningful if precisely calculated, but their representation in this calculation is precluded because we are neglecting the surface energy, Coulomb energy, and single-particle corrections and because the finite-difference solution of the equations does not resolve them. Nevertheless, we know that these particles will cluster to zero pressure, and we accordingly set the pressure to zero when it would otherwise be negative.

### E. Maximum compression

We now consider the maximum compression that can be achieved in a fluid-dynamics model of high-energy heavy-ion collisions. (It should be borne in mind that for a system in which the target and projectile interpenetrate somewhat upon contact, the maximum compression is less than that calculated here.) This is determined by first integrating Eqs. (1) over an infinitesimal volume near the contact point in a head-on collision. This gives

$$v_s(N - N_0) - vN = 0\,, \tag{5a}$$

$$v_s M - vM = p\,, \tag{5b}$$

and

$$v_s(E - E_0) - vE = vp\,, \tag{5c}$$

where $v_s$ denotes the velocity of the shock. Quantities with the subscript 0 refer to the matter at equilibrium ahead of the shock, and quantities without a subscript refer to the compressed matter behind the shock.

Upon solving Eq. (5a) for the shock velocity, substituting this result into Eqs. (5b) and (5c), and using Eqs. (2), we obtain

$$v_s = \gamma v n/(\gamma n - n_0)\,, \tag{6a}$$

$$\gamma^2(\epsilon + p)v^2 n_0 = p(\gamma n - n_0)\,, \tag{6b}$$

and

$$[\gamma^2(\epsilon + p) - p]n_0 - \gamma\epsilon_0 n = p(\gamma n - n_0)\,. \tag{6c}$$

From Eq. (4) we write $p$ in the form

$$p = \tfrac{2}{3}\epsilon + f(n)\,,$$

with

$$f(n) = [-\tfrac{2}{3}m_0(n/n_0) - \tfrac{1}{3}b(n/n_0)^2 + c(n/n_0)^{8/3}]n_0\,,$$

and substitute this result into Eqs. (6b) and (6c). Elimination of $\epsilon$ between these two equations then yields

$$[2\gamma n + (3 - 5\gamma^2)n_0]\epsilon_0 n + 3(n - \gamma n_0)n_0 f(n) = 0\,. \tag{7}$$

For a given bombarding energy and equation of state, this expression may be solved iteratively to obtain the maximum rest-frame compression $n/n_0$. For example, with our equation of state, the maximum rest-frame compression is 3.1 when the bombarding energy per nucleon is 250 MeV and is 7.4 when the bombarding energy per nucleon is 2.1 GeV. The maximum compression would be less if the equation of state were stiffer than the one we have used, and would be more if the equation of state were softer.

For a system in which the energy per particle is independent of density and the pressure is related to the internal energy density by $p=\frac{2}{3}I$, we are able to obtain an explicit expression for the maximum compression. In this case $f(n)=-\frac{2}{3}m_0n$ and $\epsilon_0=m_0n_0$. The solution to Eq. (7) is then

$$n/n_0=\tfrac{1}{2}(5\gamma^2-2\gamma-3)/(\gamma-1)\,.$$

In the limit of small velocities this reduces to the value 4, which is the usual nonrelativistic limit for this equation of state. However, in the ultra-relativistic limit it reduces to $\frac{5}{2}\gamma$. Therefore, contrary to the misconception created by many nonrelativistic treatments[59-61,63] and by an incorrect relativistic treatment,[43] the maximum compression in a fluid-dynamics model does *not* approach a constant value with increasing bombarding energy. Instead, the maximum rest-frame compression goes to $\infty$ as $\frac{5}{2}\gamma$, and the maximum laboratory-frame compression goes to $\infty$ as $\frac{5}{2}\gamma^2$.

Even in a head-on collision in a fluid-dynamics model this large compression is achieved only for an infinitesimal time in an infinitesimal volume near the contact point. As time proceeds, the maximum compression is reduced substantially

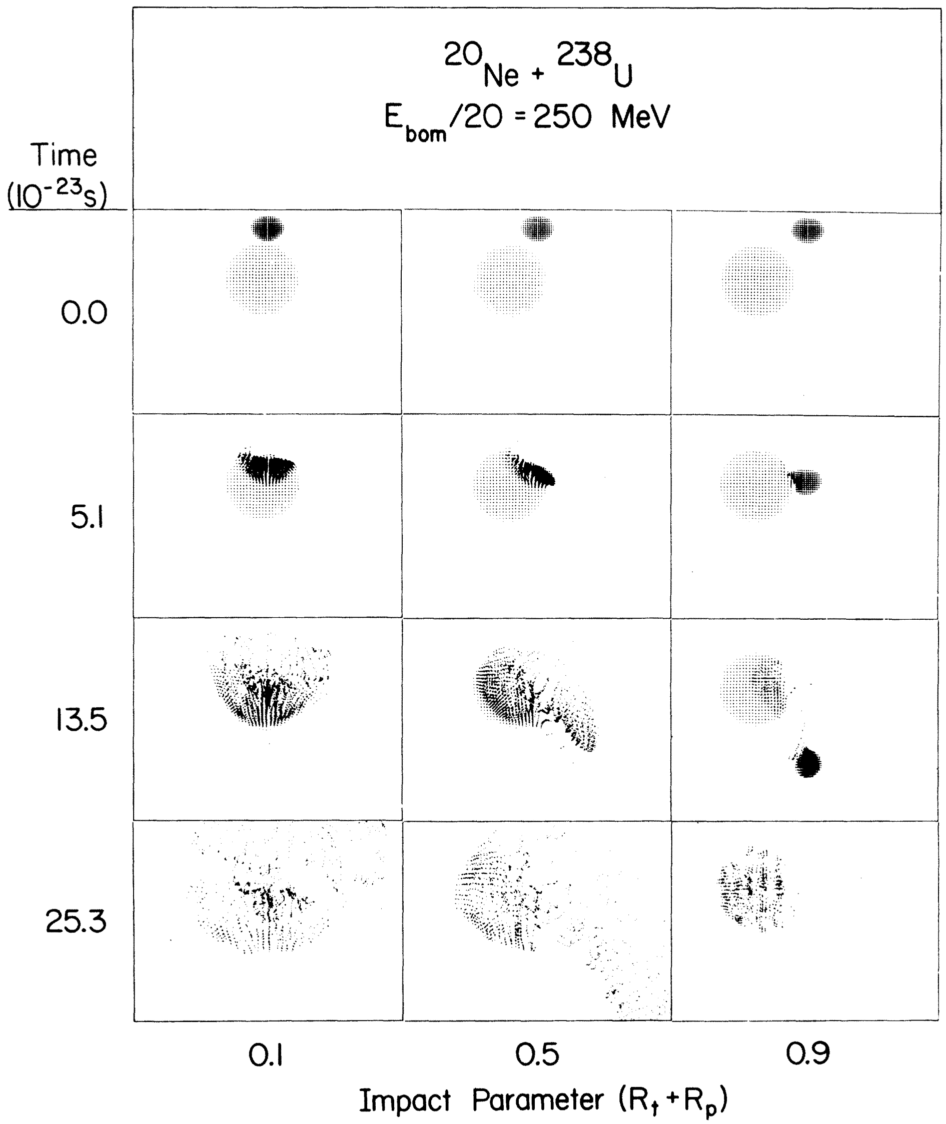


FIG. 2. Calculated time evolution of the matter distribution in the collision of $^{20}$Ne with $^{238}$U for three different impact parameters. The impact parameter is measured in units of the sum of the target and projectile radii, $R_t+R_p=1.2049(A_t^{1/3}+A_p^{1/3})$ fm. The laboratory bombarding energy per nucleon is 250 MeV.

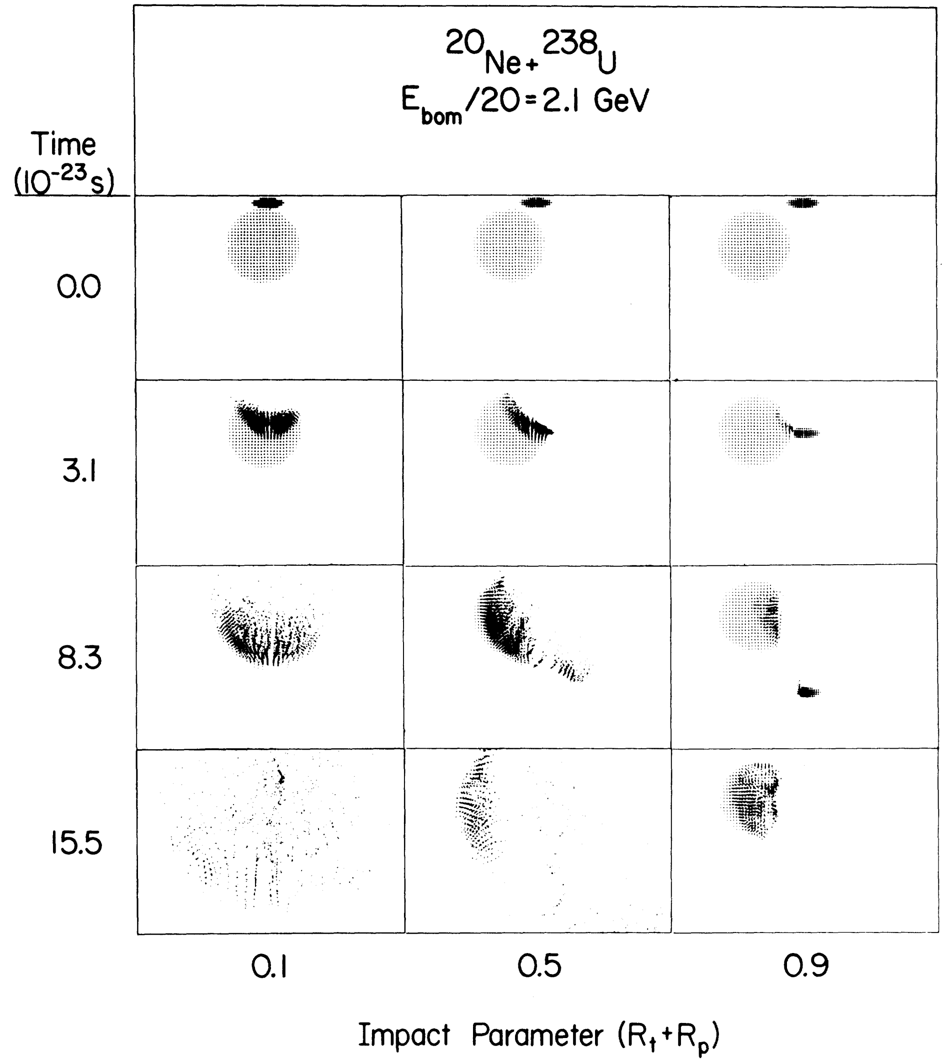


FIG. 3. Calculated time evolution of the matter distribution in the collision of $^{20}Ne$ with $^{238}U$ for three different impact parameters. The laboratory bombarding energy per nucleon is 2.1 GeV.

because of the rarefaction from the trailing surface of the projectile and the divergence of the curved shock waves. The incorporation of such effects requires an accurate numerical solution of the equations of motion, to which we now turn our attention.

### F. Numerical solution

For given initial conditions the equations of motion are solved as functions of time for the nucleon number density, momentum density, energy density, pressure, and velocity throughout the system. This is done in three spatial dimensions by means of a relativistic generalization of a standard particle-in-cell finite-difference computing method.[76] This technique is applicable to supersonic flow and combines some of the advantages of both Eulerian and Lagrangian methods. To facilitate comparisons with experimental results, the calculations are performed in the laboratory reference frame.

Briefly, our computational method utilizes a fixed rectangular mesh of Eulerian cells through which the fluid moves. The fluid is represented by a set of discrete Lagrangian particles, which conserves automatically the nucleon number. From finite-difference representations of Eqs. (1), the values of $N$, $\vec{M}$, and $E$ for each Eulerian cell are calculated at later times in terms of preceding values. Then, by means of a partial algebraic reduction followed by the iterative solution of a transcendental equation in one unknown, Eqs. (2) and (4) are solved to yield the values of $n$, $\vec{v}$, $\epsilon$, and $p$

throughout the mesh. Then the values of $I$ can be found from Eq. (3).

## IV. CALCULATED RESULTS

### A. Time evolution of the matter distribution

We have solved the equations of motion for the reaction $^{20}Ne + ^{238}U$ at laboratory bombarding energies per nucleon of 250 MeV and 2.1 GeV, for five different impact parameters. Some examples are shown in Fig. 2. Each column presents a side view of the matter distribution evolving in time for a given value of the impact parameter. The initial frame in each case shows a $^{238}U$ target bombarded from above by a $^{20}Ne$ projectile whose energy per nucleon is 250 MeV. The projectile, which is Lorentz contracted in the incident direction, is represented by heavy points, and the target is represented by light points. These points, or computational particles, are aligned so that in the direction perpendicular to the page only a single point is visible initially. However, as the impulse resulting from the collision propagates throughout the system this alignment is destroyed and additional particles come into view.

The characteristic features of the time evolution vary systematically with impact parameter. Nearly central collisions deform and compress the system enormously. At the other extreme, in peripheral collisions the projectile is fragmented into a portion that proceeds roughly straight ahead at its original velocity and another portion that deposits its energy in the target. This disturbs the target much less violently than in nearly central collisions, and its deformation and compression are therefore much less.

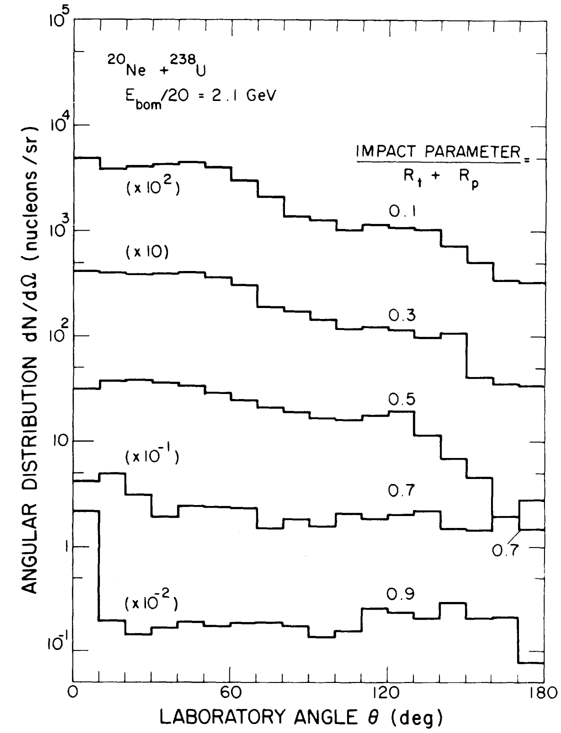


FIG. 5. Angular distributions for five different impact parameters at a bombarding energy per nucleon of 2.1 GeV.

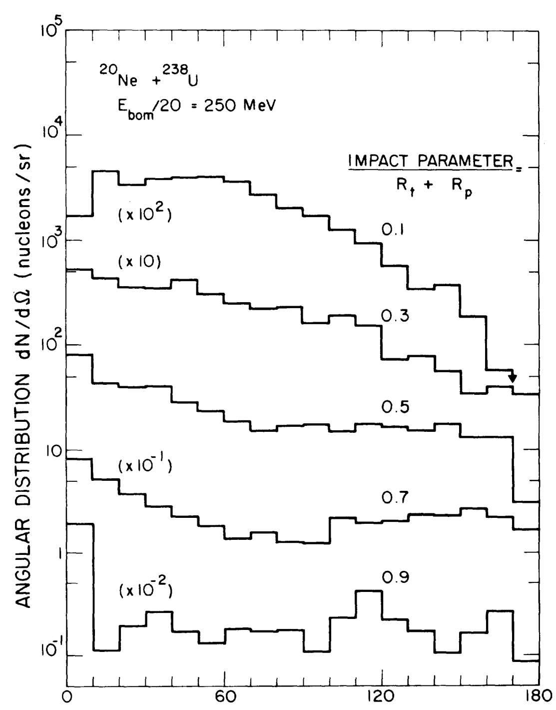


FIG. 4. Angular distributions for five different impact parameters at a bombarding energy per nucleon of 250 MeV.

The analogous solutions for a bombarding energy per nucleon of 2.1 GeV are shown in Fig. 3. Because of the higher bombarding energy, the Lorentz contraction of the projectile is more extreme and the entire process occurs more rapidly than before. The remaining features of the collision are qualitatively similar to those at the lower bombarding energy.

### B. Angular distributions

For a given impact parameter we construct from the velocity vectors at some large time the energy and angular distributions for the expanding matter. The small amount of matter that already has passed through the top and side boundaries of the computational mesh is also included. For five different impact parameters we show in Figs. 4 and 5 the resulting angular distributions, integrated over the energy of the outgoing matter. For central and in-

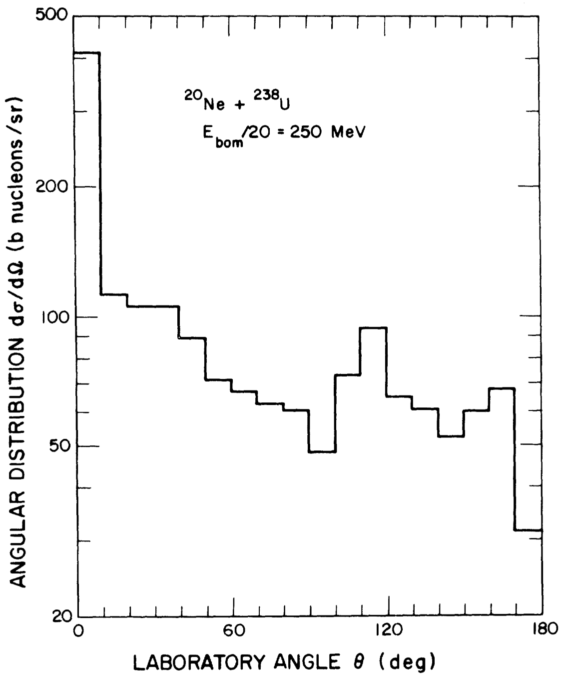


FIG. 6. Angular distribution at a bombarding energy per nucleon of 250 MeV.

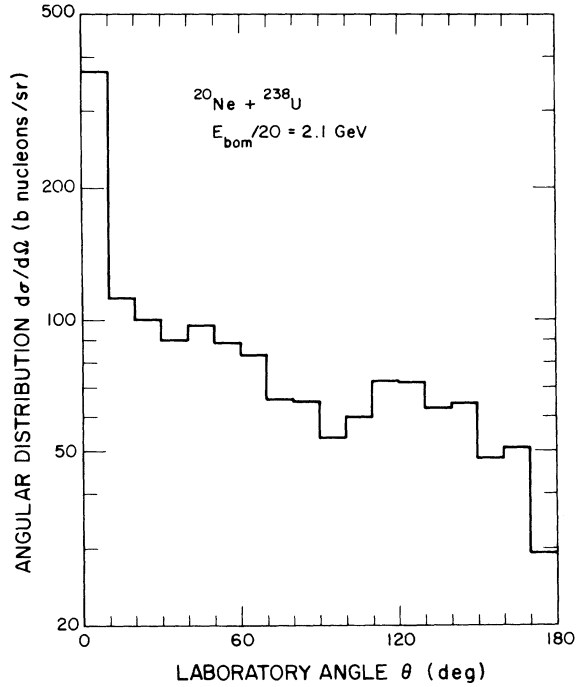


FIG. 7. Angular distribution at a bombarding energy per nucleon of 2.1 GeV.

termediate collisions the angular distributions are concentrated in a forward cone and decrease with increasing angle. For grazing collisions the angular distributions contain large peaks nearly straight ahead arising from projectile fragmentation and are roughly constant at larger angles. However, there are small peaks in these angular distributions at about 120°, especially for a bombarding energy per nucleon of 250 MeV.

To obtain the total angular distributions for the outgoing matter, we integrate these results over impact parameter by use of a trapezoidal approximation, taking into account the linear weighting with impact parameter. As shown in Figs. 6 and 7, the angular distributions are peaked in the forward direction and decrease with increasing angle, except for small peaks near 120°.

In Figs. 8 and 9 these angular distributions are decomposed into three energy intervals of the outgoing matter. The intervals used here are chosen to display the fact that the cross sections scale approximately with bombarding energy, rather than for comparisons with experimental results. The major contributions to the cross section come from particles in the low-energy intervals, for which the angular distributions vary only slowly with angle. For the higher-energy intervals, the angular distributions decrease more rapidly with increasing angle. For the highest-energy interval in Fig. 8, the total high-energy component may be ob-

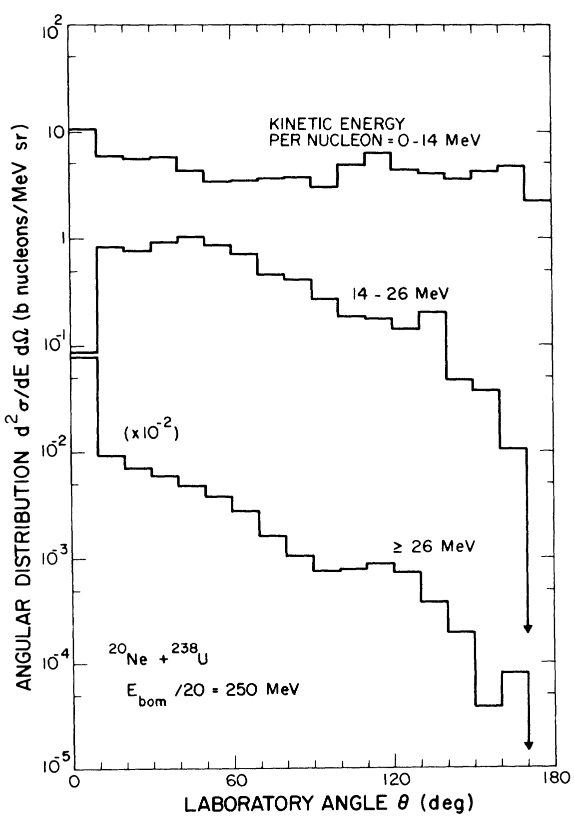


FIG. 8. Angular distributions for three different outgoing-energy intervals at a bombarding energy per nucleon of 250 MeV.

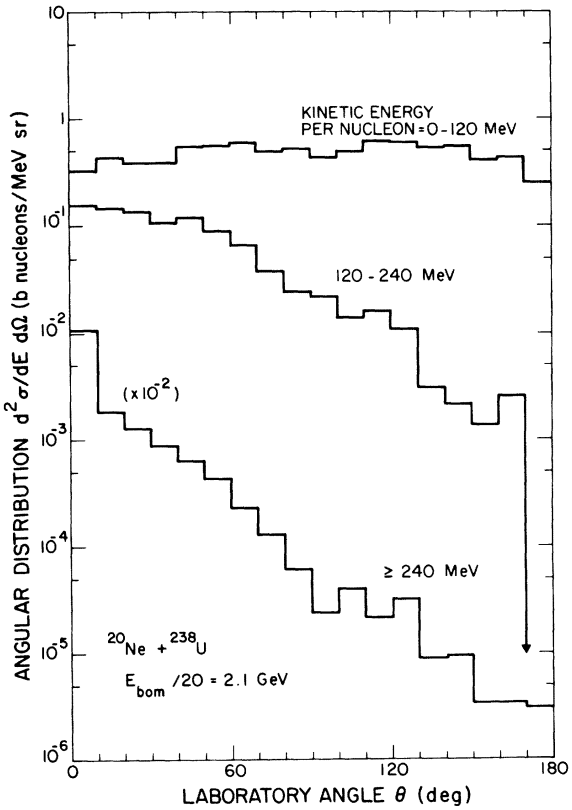


FIG. 9. Angular distributions for three different outgoing-energy intervals at a bombarding energy per nucleon of 2.1 GeV.

tained by integrating from 26 to 60 MeV. For the highest-energy interval in Fig. 9, the total high-energy component may be obtained by integrating from 240 to 540 MeV. Therefore, the heights themselves are not meaningful for the highest-energy intervals. In experiments that determine only out-going particles with energy per nucleon below about 150 MeV, the more rapid variation of the angular distributions that is observed at lower bombarding energies[70,71] is a consequence of such an approximate scaling of the results with bombarding energy.

### C. Energy spectra

The calculated energy spectra for the outgoing matter at fixed laboratory angles are shown in Figs. 10 and 11. The energy intervals are again chosen to display the approximate scaling with bombarding energy. The total high-energy components may be obtained by integrating the values given in the last intervals up to the end points of the graphs. Therefore, the heights themselves are not meaningful for the last intervals.

At forward angles the spectra contain very-low-energy peaks associated with the forward motion of the target and high-energy peaks associated with projectile fragmentation. For larger angles the energy spectra decrease with increasing energy. The rate of decrease is more rapid at backward angles than at intermediate angles.

### D. Comparison with experimental data

Much experimental data now exist for collisions between heavy nuclei at high energies.[42,70,71,77-98] For our comparisons we use the recent data of Gutbrod *et al.* for protons emitted from the reaction $^{20}Ne + ^{238}U$ at a bombarding energy per nucleon of 250 MeV.[70] Under the assumption that the protons are distributed uniformly throughout the entire matter, we calculate the proton distribution by multiplying the matter distribution by 102/258. The contributions to the experimental results from emitted particles heavier than protons are not included; this does not affect the conclusions to be drawn below. (However, to facilitate more satisfactory comparisons of this type in the future, ex-

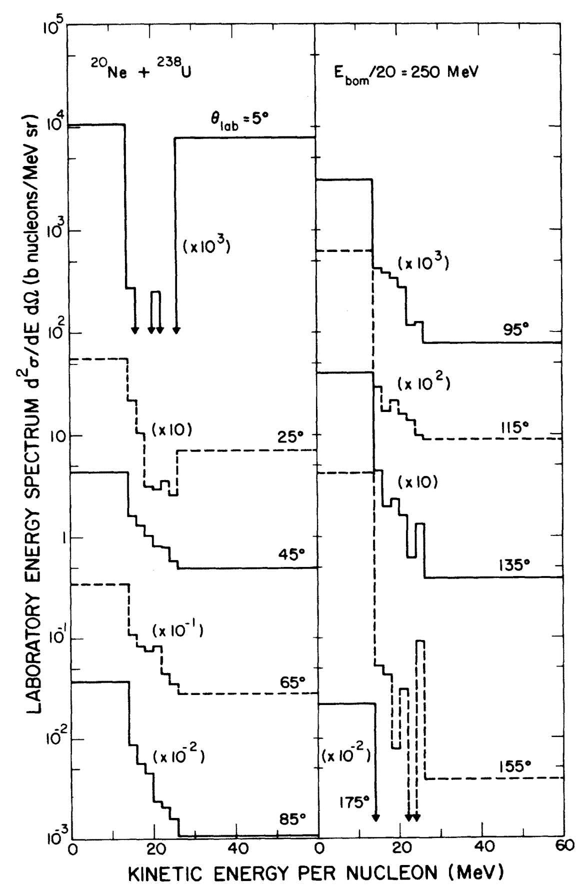


FIG. 10. Energy spectra for 10 different angles at a bombarding energy per nucleon of 250 MeV.

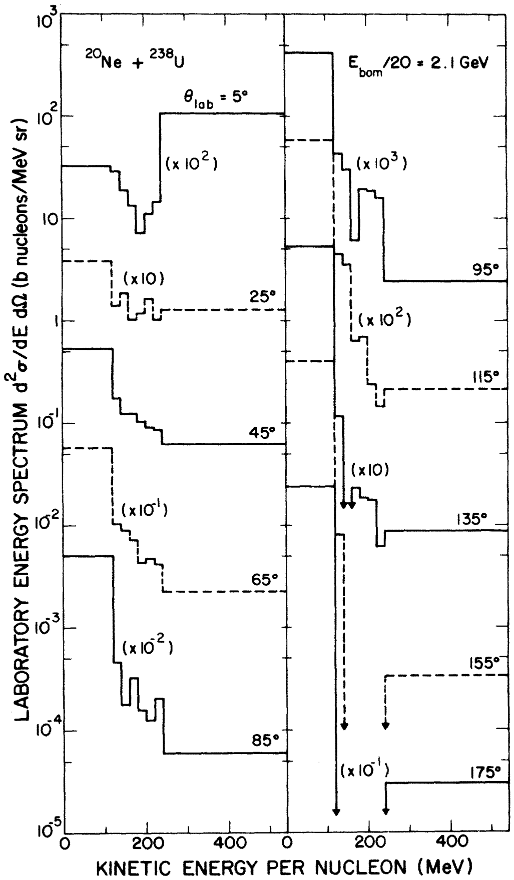


FIG. 11. Energy spectra for 10 different angles at a bombarding energy per nucleon of 2.1 GeV.

perimentalists should construct from their data all-inclusive distributions of both free and bound outgoing protons, by use of a coalescence model[70] or other model.)

Our comparison is presented in Fig. 12 in the form of proton energy spectra for four laboratory angles ranging from 30° to 120°. At each angle the calculated and experimental slopes are in approximate agreement. In addition, the calculations reproduce the overall decrease in the experimental cross section when going from forward to backward angles. However, the detailed dependence upon angle is significantly incorrect: At 30° the calculated values are only one-half the experimental ones, whereas at 120° they are twice as large.

This discrepancy suggests that in collisions at high energy, heavy nuclei are partially transparent to each other. In other words, upon impact the target and projectile do not maintain an interface but instead interpenetrate somewhat. Such interpenetration is the result of a finite cross section and momentum transfer for each collision between the individual nucleons comprising the system. However, both intranuclear cascade calculations[34] and classical many-body calculations[53] that have been performed in related studies yield cross sections that lie below the experimental results at practically all energies and angles. This implies that the process is not solely a superposition of individual nucleon-nucleon collisions, but that instead coherent collective-field effects play some role.

## V. CONCLUSION

The study of what happens when two heavy nuclei collide at high energies holds great promise for the future. The present comparisons suggest that nuclei are partially transparent to each other at high energy, but that collective effects should lead to some increase in nuclear density. We are therefore in a unique position to learn about the nuclear equation of state, and such new phenomena as density isomers and pion condensates may be discovered.

However, because of the finite target and projectile interpenetration, a pure fluid-dynamics description of high-energy heavy-ion collisions appears to be inadequate. The theoretical description of such collisions will probably require the development of hybrid models that take into account the dual particle-collective nature of the process.

One such possibility that we are currently pur-

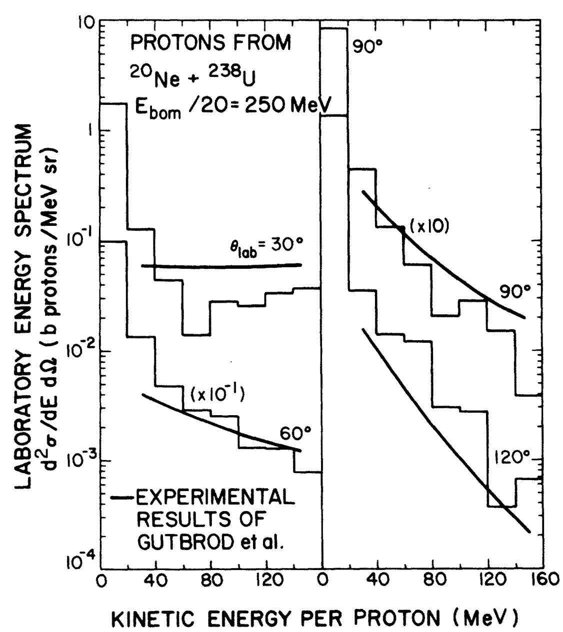


FIG. 12. Comparison of calculated and experimental proton energy spectra for $^{20}$Ne incident on $^{238}$U at a bombarding energy per nucleon of 250 MeV. The histograms give the calculated results, and the heavy curves give the experimental results (Ref. 70).

suing in collaboration with Goldhaber is a two-fluid generalization of the present calculations.[37] In this two-fluid model coupled relativistic hydrodynamic equations are solved for separate target and projectile nuclear fluids. The terms in the equations that couple the two nuclear fluids are obtained from the cross section and momentum transfer for each individual nucleon-nucleon collision.

It is equally important that other approaches, especially intranuclear-cascade calculations that treat the projectile nucleons simultaneously and classical many-body calculations with more realistic nucleon-nucleon potentials, be pursued simultaneously. Only by comparing the predictions of several different approaches with experimental data can we make the optimum advancement in our overall understanding of the interaction of heavy nuclei at high energies.

We are grateful to G. F. Bertsch, J. N. Ginocchio, A. S. Goldhaber, H. H. Gutbrod, E. C. Halbert, S. E. Koonin, W. D. Myers, J. W. Negele, and A. M. Poskanzer for discussions concerning this work.

---

*This work was supported by the U. S. Energy Research and Development Administration.

[1]E. Feenberg and H. Primakoff, Phys. Rev. 70, 980 (1946).
[2]A. R. Bodmer, Phys. Rev. D 4, 1601 (1971).
[3]A. R. Bodmer, in *Proceedings of the International Symposium on Present Status and Novel Developments in the Nuclear Many-Body Problem, Rome, Italy, 1972*, edited by F. Calogero and C. Ciofi Degli Atti (Editrice Compositori, Bologna, 1973), Vol. 2, p. 505.
[4]Y. Ne'eman, in *Proceedings of the International Astronomical Union Symposium No. 53 on Physics of Dense Matter, Boulder, Colorado, 1972*, edited by C. J. Hansen (Reidel, Dordrecht/Boston, 1974), p. 111.
[5]T. D. Lee and G. C. Wick, Phys. Rev. D 9, 2291 (1974).
[6]T. D. Lee, Rev. Mod. Phys. 47, 267 (1975).
[7]A. K. Kerman and L. D. Miller, in Proceedings of the 2nd High Energy Heavy Ion Summer Study, Berkeley, California, 1974, edited by L. S. Schroeder [Lawrence Berkeley Laboratory Report No. LBL-3675, 1975 (unpublished)], p. 73.
[8]E. M. Nyman and M. Rho, Nucl. Phys. A268, 408 (1976).
[9]J. M. Irvine, Rep. Prog. Phys. 38, 1385 (1975).
[10]J. Hofmann, H. Stöcker, U. Heinz, W. Scheid, and W. Greiner, Phys. Rev. Lett. 36, 88 (1976).
[11]H. Stöcker, J. Hofmann, W. Scheid, and W. Greiner, in Proceedings of the International Workshop IV on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1976 [Technische Hochschule Darmstadt Report No. AED-Conf-76-015-000, 1976 (unpublished)], p. 89.
[12]A. B. Migdal, Zh. Eksp. Teor. Fiz. 63, 1993 (1972) [Sov. Phys.-JETP 36, 1052 (1973)].
[13]A. B. Migdal, N. A. Kirichenko, and G. A. Sorokin, Phys. Lett. 50B, 411 (1974).
[14]D. K. Campbell, R. F. Dashen, and J. T. Manassah, Phys. Rev. D 12, 979 (1975).
[15]D. K. Campbell, R. F. Dashen, and J. T. Manassah, Phys. Rev. D 12, 1010 (1975).
[16]V. Ruck, M. Gyulassy, and W. Greiner, Z. Phys. A277, 391 (1976).
[17]M. Gyulassy, V. Ruck, and W. Greiner, in Proceedings of the International Workshop IV on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1976 (see Ref. 11), p. 115.
[18]Y. Kitazoe and M. Sano, Nuovo Cimento Lett. 14, 400 (1975).
[19]Y. Kitazoe and M. Sano, Prog. Theor. Phys. 54, 1574 (1975).
[20]R. F. Sawyer, Nucl. Phys. A271, 235 (1976).
[21]C. W. Wong and S. K. Young, Phys. Rev. C 12, 1301 (1975).
[22]J. Hüfner, K. Schäfer, and B. Schürmann, Phys. Rev. C 12, 1888 (1975).
[23]A. Abul-Magd and J. Hüfner, Z. Phys. A277, 379 (1976).
[24]D. I. Julius and A. S. Rinat, Nucl. Phys. A268, 477 (1976).
[25]S. E. Koonin and J. W. Negele (private communication).
[26]J. J. Griffin and J. A. Wheeler, Phys. Rev. 108, 311 (1957).
[27]P. Bonche and B. Giraud, Phys. Rev. Lett. 28, 1720 (1972).
[28]B. Giraud and B. Grammaticos, Nucl. Phys. A233, 373 (1974).
[29]S. E. Koonin, Phys. Lett. 61B, 227 (1976).
[30]S. E. Koonin, K. T. R. Davies, V. Maruhn-Rezwani, H. Feldmeier, S. J. Krieger, and J. W. Negele, Phys. Rev. C 15, 1359 (1977).
[31]R. Y. Cusson, R. K. Smith, and J. A. Maruhn, Phys. Rev. Lett. 36, 1166 (1976).
[32]J. Błocki and H. Flocard, Lawrence Berkeley Laboratory Report No. LBL-5016, 1976 (unpublished).
[33]H. W. Bertini, R. T. Santoro, and O. W. Hermann, Phys. Rev. C 14, 590 (1976).
[34]J. N. Ginocchio (unpublished).
[35]M. Danos and R. K. Smith (unpublished).
[36]Z. Fraenkel *et al.* (unpublished).
[37]A. A. Amsden, A. S. Goldhaber, F. H. Harlow, and J. R. Nix (unpublished).
[38]S. Z. Belenkij and L. D. Landau, Usp. Fiz. Nauk 56, 309 (1955); Nuovo Cimento Suppl. 3, 15 (1956); in *Collected Papers of L. D. Landau*, edited by D. ter Haar (Gordon and Breach, New York, 1967), 2nd printing, paper No. 88, p. 665.
[39]G. F. Chapline, M. H. Johnson, E. Teller, and M. S. Weiss, Phys. Rev. D 8, 4302 (1973).
[40]G. F. Chapline, in Proceedings of the 2nd High Energy Heavy Ion Summer Study, Berkeley, California, 1974 (see Ref. 7), p. 51.
[41]J. Hofmann, H. Stöcker, W. Scheid, and W. Greiner, in Proceedings of the XIII International Winter Meeting on Nuclear Physics, Bormio, Italy, 1975 [University of Milan report, 1975 (unpublished)], p. 79.
[42]H. G. Baumgardt, J. U. Schott, Y. Sakamoto, E. Schopper, H. Stöcker, J. Hofmann, W. Scheid, and W. Greiner, Z. Phys. A273, 359 (1975).
[43]A. Y. Abul-Magd, Phys. Rev. C 12, 343 (1975).
[44]A. A. Amsden, G. F. Bertsch, F. H. Harlow, and J. R. Nix, Phys. Rev. Lett. 35, 905 (1975).

[45]Y. Kitazoe and M. Sano, Prog. Theor. Phys. 54, 922 (1975).
[46]Y. Kitazoe and M. Sano, Nuovo Cimento Lett. 14, 407 (1975).
[47]Y. Kitazoe, K. Matsuoka, and M. Sano, Prog. Theor. Phys. 56, 860 (1976).
[48]A. R. Bodmer and C. N. Panos, in Proceedings of the Symposium on Macroscopic Features of Heavy-Ion Collisions, Argonne, Illinois, 1976 [Argonne National Laboratory Report No. ANL/PHY 76-2, 1976 (unpublished)], Vol. II, p. 463.
[49]A. R. Bodmer and C. N. Panos, Phys. Rev. C (to be published).
[50]J. P. Bondorf, P. J. Siemens, H. Feldmeier, S. Garpman, and E. C. Halbert, in Proceedings of the Symposium on Macroscopic Features of Heavy-Ion Collisions, Argonne, Illinois, 1976 (see Ref. 48), Vol. II, p. 483.
[51]J. P. Bondorf, P. J. Siemens, S. Garpman, and E. C. Halbert, Z. Phys. A279, 385 (1976).
[52]J. P. Bondorf, J. Phys. Suppl. 37, C5–195 (1976).
[53]E. C. Halbert (unpublished).
[54]L. Wilets, A. MacKellar, and G. A. Rinker, Jr., in Proceedings of the International Workshop IV on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1976 (see Ref. 11), p. 111.
[55]A. D. MacKellar and L. Wilets, in Proceedings of the Symposium on Macroscopic Features of Heavy-Ion Collisions, Argonne, Illinois, 1976 (see Ref. 48), Vol. II, p. 663.
[56]R. A. Malfliet (unpublished).
[57]A. E. Glassgold, W. Heckrotte, and K. M. Watson, Ann. Phys. (N.Y.) 6, 1 (1959).
[58]C. Y. Wong and T. A. Welton, Phys. Lett. 49B, 243 (1974).
[59]W. Scheid, H. Müller, and W. Greiner, Phys. Rev. Lett. 32, 741 (1974).
[60]W. Scheid, J. Hofmann, and W. Greiner, in Proceedings of the 2nd High Energy Heavy Ion Summer Study, Berkeley, California, 1974 (see Ref. 7), p. 1.
[61]M. I. Sobel, P. J. Siemens, J. P. Bondorf, and H. A. Bethe, Nucl. Phys. A251, 502 (1975).
[62]G. F. Bertsch, Phys. Rev. Lett. 34, 697 (1975).
[63]Y. Kitazoe, M. Sano, and H. Toki, Nuovo Cimento Lett. 13, 139 (1975).
[64]J. D. Bowman, W. J. Swiatecki, and C. F. Tsang, Lawrence Berkeley Laboratory Report No. LBL-2908, 1973 (unpublished).
[65]W. J. Swiatecki (unpublished).
[66]H. Feshbach and K. Huang, Phys. Lett. 47B, 300 (1973).
[67]A. S. Goldhaber, Phys. Lett. 53B, 306 (1974).
[68]R. Weiner and M. Weström, Phys. Rev. Lett. 34, 1523 (1975).
[69]A. Abul-Magd, J. Hüfner, and B. Schürmann, Phys. Lett. 60B, 327 (1976).
[70]H. H. Gutbrod, A. Sandoval, P. J. Johansen, A. M. Poskanzer, J. Gosset, W. G. Meyer, G. D. Westfall, and R. Stock, Phys. Rev. Lett. 37, 667 (1976).
[71]G. D. Westfall, J. Gosset, P. J. Johansen, A. M. Poskanzer, W. G. Meyer, H. H. Gutbrod, A. Sandoval, and R. Stock, Phys. Rev. Lett. 37, 1202 (1976).
[72]O. Benary, L. R. Price, and G. Alexander, Lawrence Berkeley Laboratory Report No. UCRL-20000 NN, 1970 (unpublished).
[73]B. R. Wienke, Los Alamos Scientific Laboratory Report No. LA-5392-MS, 1973 (unpublished).
[74]L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, London, 1959), Chap. XV, pp. 499–506.
[75]W. D. Myers and W. J. Swiatecki, Ann. Phys. (N.Y.) 55, 395 (1969).
[76]F. H. Harlow, A. A. Amsden, and J. R. Nix, J. Comp. Phys. 20, 119 (1976).
[77]I. Otterlund and R. Resman, Ark. Fys. 39, 265 (1969).
[78]R. Kullberg, I. Otterlund, and R. Resman, Phys. Scr. 5, 5 (1972).
[79]R. Kullberg and I. Otterlund, Z. Phys. 259, 245 (1973).
[80]B. Jakobsson, R. Kullberg, and I. Otterlund, Z. Phys. 268, 1 (1974).
[81]B. Jakobsson, R. Kullberg, and I. Otterlund, Z. Phys. A272, 159 (1975).
[82]J. D. Sullivan, P. B. Price, H. J. Crawford, and M. Whitehead, Phys. Rev. Lett. 30, 136 (1973).
[83]H. J. Crawford, P. B. Price, J. Stevenson, and L. W. Wilson, Phys. Rev. Lett. 34, 329 (1975).
[84]P. B. Price and J. Stevenson, Phys. Rev. Lett. 34, 409 (1975).
[85]K. Frankel and J. Stevenson, Phys. Rev. C 14, 1455 (1976).
[86]D. E. Greiner, P. J. Lindstrom, H. H. Heckman, B. Cork, and F. S. Bieser, Phys. Rev. Lett. 35, 152 (1975).
[87]P. J. Lindstrom, D. E. Greiner, H. H. Heckman, B. Cork, and F. S. Bieser, Phys. Rev. C (to be published).
[88]A. M. Poskanzer, R. G. Sextro, A. M. Zebelman, H. H. Gutbrod, A. Sandoval, and R. Stock, Phys. Rev. Lett. 35, 1701 (1975).
[89]H. H. Gutbrod, in Proceedings of the International Workshop IV on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1976 (see Ref. 11), p. 99.
[90]J. B. Carroll, G. Igo, V. Perez-Mendez, A. Sagle, H. Spinka, M. Gazzaly, J. Geaga, M. Nasser, J. McClelland, E. T. Whipple, and S. Evan (unpublished).
[91]R. Poe, S. Fung, B. Gorn, G. Kiernan, F. Liu, J. Lu, Y. Oh, J. Ozawa, G. Van Dalen, L. Schroeder, and H. Steiner (unpublished).
[92]J. O. Rasmussen *et al.* (unpublished).
[93]H. G. Baumgardt, E. Schopper, J. U. Schott, N. P. Kocherov, A. V. Voronow, I. D. Issinsky, and L. G. Makarov, in Proceedings of the International Workshop IV on Gross Properties of Nuclei and Nuclear Excitations (see Ref. 11), p. 105.
[94]V. K. Lukyanov, Yu. A. Panebratsev, and A. I. Titov, Joint Institute for Nuclear Research Report No. JINR-E2-9089, 1975 (unpublished).
[95]K. K. Gudima and V. D. Toneev, Joint Institute for Nuclear Research Report No. JINR-E4-9765, 1976 (unpublished).
[96]E. S. Basova, A. I. Bondarenko, K. G. Gulamov, U. G. Guliamov, Sh. Z. Nasyrov, L. N. Svechnikova, and G. M. Chernov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 24, 257 (1976) [JETP Lett. (to be published)].
[97]T. Siemiarczuk and P. Zieliński, Phys. Rev. C 13, 2439 (1976).
[98]T. Siemiarczuk, Phys. Rev. C (to be published).