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Analysis of the (p,t) reaction on ^{158}Dy†

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The level structure of ^{156}Dy has been investigated with the (p,t) reaction at 29.9-MeV incident proton energy. Angular distributions were measured for transitions to 34 levels up to an excitation energy of 2.25 MeV in ^{156}Dy. The ground-state Q value for this reaction was determined to be $Q_0 = -7.535 \pm 0.015$ MeV. Optical-model parameters were obtained self-consistently in the coupled-channels approach, and reaction calculations using these parameters demonstrated a very large destructive interference between direct and indirect transfer to the 2^+ member of the ground-state band. The anomalously strong transition to the $J^{\pi} = 0^+$ state at 674 keV is interpreted in the model of Van Rij and Kahana, leading to suggestions as to the structure of this state. The strong $L = 2$ transition to the $J^{\pi} = 2^+$ state at 890 keV is also discussed. Further evidence is given on the structure of the $J^{\pi} = 2^+$ state at 1520 keV, and the $J^{\pi} = 4^+$ state at 1627.3 keV which has been suggested to belong to the "super" band in ^{156}Dy.

[NUCLEAR REACTIONS ^{158}Dy(p,t), $E_L = 29.9$ MeV, measured $\sigma(\theta)$, Q_0; CCBA and DWBA analysis; deduced E_x, L, J^{π} of ^{156}Dy levels.
NUCLEAR STRUCTURE Model of Van Rij and Kahana applied to anomalous 0^+ state; evidence for γ vibration built on these states, and on nature of "super" band.]

I. INTRODUCTION

Recent experimental evidence[1–4] demonstrating backbending in the "β-vibrational" bands of ^{154}Gd and ^{156}Dy has been interpreted in terms of a simple band-crossing model involving three bands. The structure of the third or "super" band is as yet unknown, although candidates for its 4^+, 6^+, 8^+, and 10^+ members (below the backbend at $J^{\pi} = 12^+$) have recently been proposed.[5,6] The present experiment was designed to investigate whether or not the (p,t) reaction could tell us anything about the low-spin members of the super band. During its course, we noted several interesting effects which will also be discussed below. First of all, it was found that both the "β-" and γ-vibrational states of ^{156}Dy were very strongly populated in ^{158}Dy$(p,t)^{156}$Dy. This effect, which will be discussed in terms of current theories[7–10] of the microscopic structure of "anomalous" 0^+ states, casts doubt on the β-vibrational nature of the 0^+ state at 674 keV in ^{156}Dy, and we will accordingly use quotation marks on "β vibration" when referring to this state. Secondly, a surprisingly strong $J^{\pi} = 2^+$ level at 1520 keV has been interpreted as a γ vibration built on the "β-vibration" bandhead, i.e., as a "β-γ" vibration.[11] Further evidence for this interpretation will be presented. Finally, suggestions for the structure of the super band will be proposed on the basis of this and other experiments.

II. EXPERIMENTAL METHOD

The experiment was carried out using a 29.9-MeV proton beam from the Princeton University cyclotron. The target was a 2-mm high by 5-mm wide spot of Dy oxide, enriched to >99% ^{158}Dy (natural abundance 0.09%), deposited directly onto a 60-μg/cm^2 C backing by a high-resolution mass spectrograph.[12] The total target thickness was 150 μg/cm^2, and its angle relative to the incident beam was kept fixed at 30°. The reaction tritons were detected in the focal plane of a quadrupole-dipole-dipole-dipole (Q3D) spectrometer (effective aperture 11.11 msr) with a 60-cm long position-sensitive proportional counter,[13] backed by a plastic scintillator, which also served to identify the tritons. A representative spectrum, taken at 10° (lab), is shown in Fig. 1. Because of the large dispersion of the Q3D magnet, it was necessary to take two exposures at different field settings for each angle, so that Fig. 1 is actually a composite of two spectra. The observed energy resolution is to some extent a function of position on the focal plane (due to nonlinearities in the position-sensitive counter) but the overall resolution was about 12 keV full width at half maximum (FWHM).

Excitation energies (Table I) were determined by direct comparison to (p,t) Q values for the Cd isotopes,[14] and also from the known excitation energies[1-4] of strong groups in ^{156}Dy. They are estimated to be accurate to ±5 keV or ±0.3% (whichever is greater) for groups with total strength ≳1% of the ground-state (gs) strength. The gs Q value for ^{158}Dy$(p,t)^{156}$Dy has also been determined to be -7.535 ± 0.015 MeV in this experiment.

Differential cross sections were measured at 5° intervals from 10° to 60° (lab), and the resulting angular distributions for all states up to $E_x = 2.25$ MeV are shown in Figs. 2–8. A 2-mm thick NaI(Tl) monitor counter, set at 30° to the beam axis, was used to check against target deterioration and/or beam-position fluctuations, but no significant effects were observed. The absolute cross sections were determined from measurements of proton elastic scattering at 10° and 15° (where it is ≃85% and 80% of Rutherford, respectively), and are believed to be accurate to better than ±25%. Total intensities were obtained by summing the measured cross sections multipled by the sine of the c.m. angle, and these are presented in Table I along with the ratio of the measured intensity to that of the gs transition.

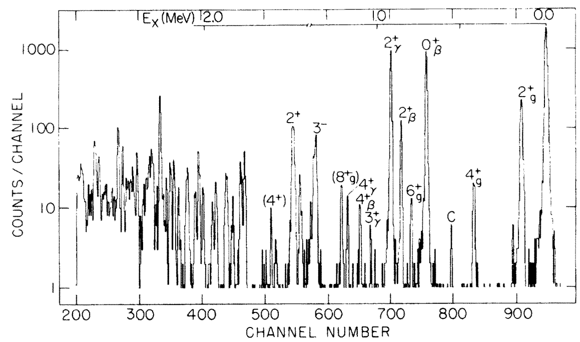

FIG. 1. Typical spectrum for the ^{158}Dy$(p,t)^{156}$Dy reaction at $E_p = 29.9$ MeV and $\theta_{\rm lab} = 10°$. This is a composite of two spectra taken at different magnetic field settings; the transition between the two regions is indicated on the upper excitation energy scale. The strong groups below 1.7-MeV excitation energy are labeled with their spectroscopic assignments from previous work, except for the $2^+,2$ state at 1520 keV and the $(4^+,4)$ state at 1635 keV which are assigned in the present experiment. Note the weak transition to the 3^+_γ state at 1022 keV, which is observed only at 10°. The opening of the pairing gap is also quite apparent in this spectrum.

TABLE I. Properties of levels in ^{156}Dy populated in this work. $\sum\sigma(\theta)\sin\theta\,\Delta\theta$ is the total intensity in μb obtained by summing over the observed angles. Σ_R is the ratio to gs intensity.

This Work	E_x (keV) Ref. 6	Ref. 30	J^π, K [a]	$\sum\sigma(\theta)\sin\theta\,\Delta\theta$ (μb)	Σ_R (%)
0.0	0.0	0.0	0^+0_g	605	100
138	137.8	138	2^+0_g	141	23.3
404	404.1	403	4^+0_g	10	1.6
674	675.4	674	0^+0_β	142	23.5
770	770.3	768	6^+0_g	4.6	0.8
829	828.5	828	2^+0_β	21	3.5
891	891.0	890	2^+2_γ	141	23.3
1088	1088.3	1087	4^+0_β	4.1	0.7
1166	1168.6	1165	4^+2_γ	10	1.6
1208	1215.6		(8^+0_g)	5.2	0.9
1371	1369.0	1367	3^-1	39	6.5
1385			(3^-)	16	2.7
1408		1404	(3^-)	3.9	0.6
1483			(3^-)	4.9	0.8
1520		1523	2^+2	46	7.7
1610		1609	(0^+0)	2.0	0.3
1635			(4^+4)	3.2	0.5
1778			(3^-)	10	1.7
1798		1794	4^+	23	3.8
1844			(5^-)	14	2.4
1874				8.0	1.3
1884				4.6	0.8
1934		1927	(3^-)	6.1	1.0
1956		1948	(3^-)	3.4	0.6
2009				5.2	0.9
2032			2^+	11	1.8
2052				4.9	0.8
2094		2086	(5^-)	22	3.6
2103				2.2	0.4
2146		2135	(5^-)	12	2.0
2174				10	1.7
2193			4^+	26	4.3
2217			(0^+0)	8.0	1.3
2250			2^+	49	8.1

[a] Spectroscopic assignments for all states below 1375 keV are taken from the literature (see Ref. 6, for example).

III. ANALYSIS AND RESULTS

A. Coupled-channels Born-approximation analysis of ground-state band transitions

It is well known from previous (p,t) studied on rare-earth nuclei at lower incident proton energy [see, e.g., Refs. 15–17] that angular distributions for a given L value can show quite variable shapes. In the present experiment, at 30 MeV, this shape variability is less pronounced and seems to be associated mainly with the members of the ground-state band (gsb) (see Figs. 2–4), as one would expect if it were due to the influence of two-step processes.[18] Therefore, we decided to compute the gs and first 2^+ angular distributions in the CCBA formalism, using the computer code CHUCK.[19]

One of the first problems encountered in the course of distorted-wave Born-approximation (DWBA) or coupled-channels Born-approximation

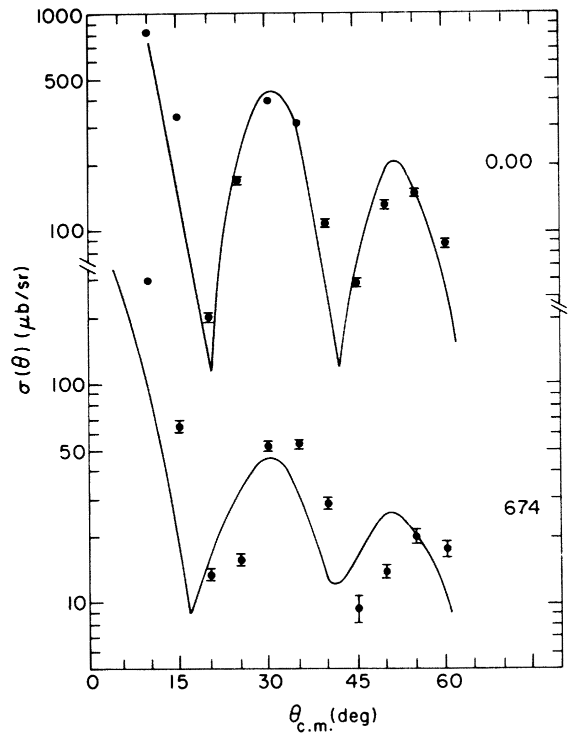

FIG. 2. Angular distributions for unambiguous $L=0$ transitions, labeled with the excitation energy of the final state in keV. The curves are the CCBA (gs) and DWBA calculations discussed in the text.

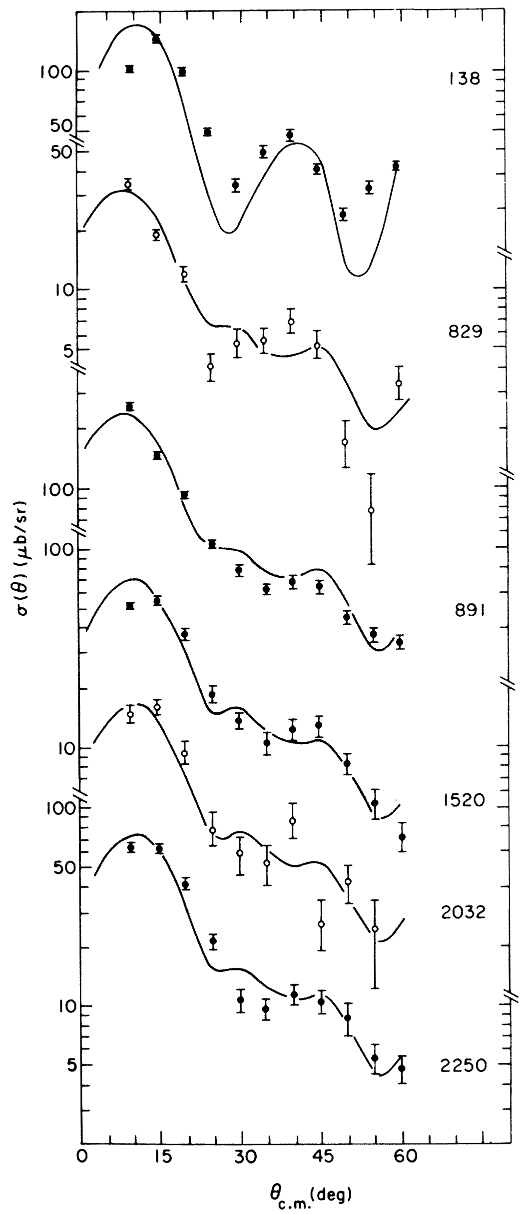

FIG. 3. Angular distributions for $L=2$ transitions. The curves are the CCBA (2_g^+, 138 keV) and DWBA calculations discussed in the text.

(CCBA) analyses on deformed systems has been the selection of appropriate optical-model parameters to represent the elastic channels. It is found (see, e.g., Refs. 17, 20, and 21) that the best-fit parameters for well-deformed targets are substantially different than those for their closest spherical neighbors, presumably due to two-step effects in the elastic scattering. Therefore, the optical-model parameters should in principle be determined self-consistently by fitting elastic scattering data in the CCBA. This has not been done in previous work due to the expense involved in programming and running automatic coupled-channels search codes. However, we have found that only a few manual iterations are necessary to achieve very good fits to the data, provided only that one starts with parameters appropriate for neighboring spherical targets.

In the present case, we analyzed 19-MeV ^{176}Yb(p,p) data[17] and 20-MeV ^{182}W(t,t) data[21] found in the literature. The "data sets" to be fitted (Fig. 9) were actually obtained from optical-model calculations using parameters given in Refs. 17 and 21, which represented the experimental data with $\chi^2=1$. Coupled-channels calculations were then performed for each case, starting with parameters derived for neighboring spherical systems and making small changes to improve the fit.

Some details of the CCBA analysis should now be discussed. First of all, only the 0^+ and 2^+ members of the gsb were included in the calculations, since preliminary investigations showed that coupling to the 4^+ state has little effect on the elastic scattering (although it does modify the inelastic-scattering predictions). The deformation param-

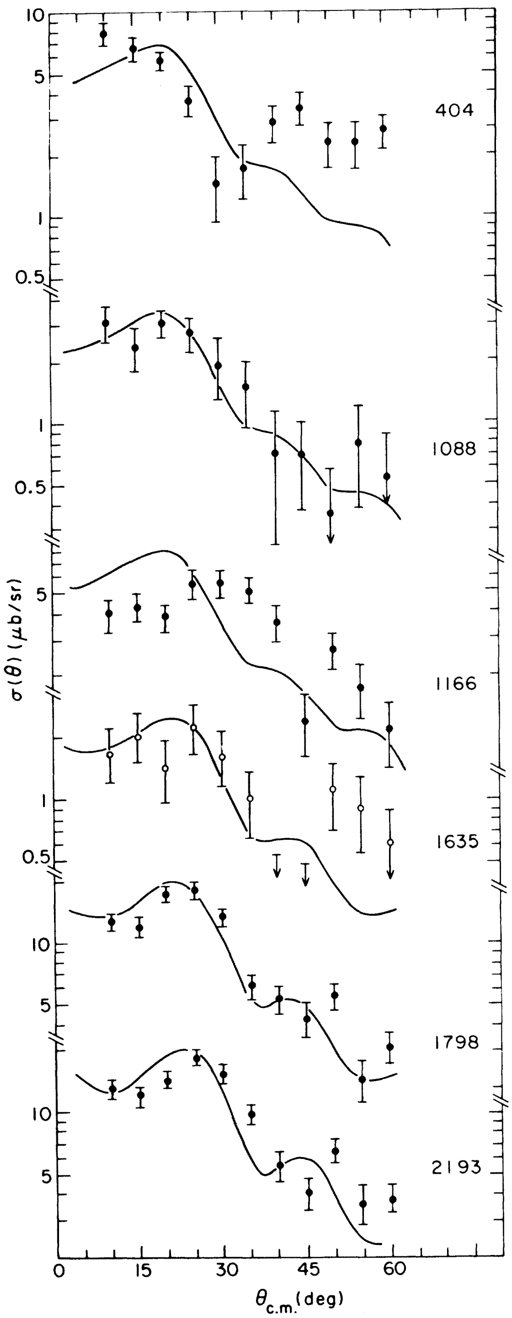

FIG. 4. Angular distributions for $L=4$ transitions. The curves are the DWBA calculations discussed in the text. Note the poor agreement for the transitions to the 4_g^+ (404 keV) and 4_γ^+ (1166 keV) states.

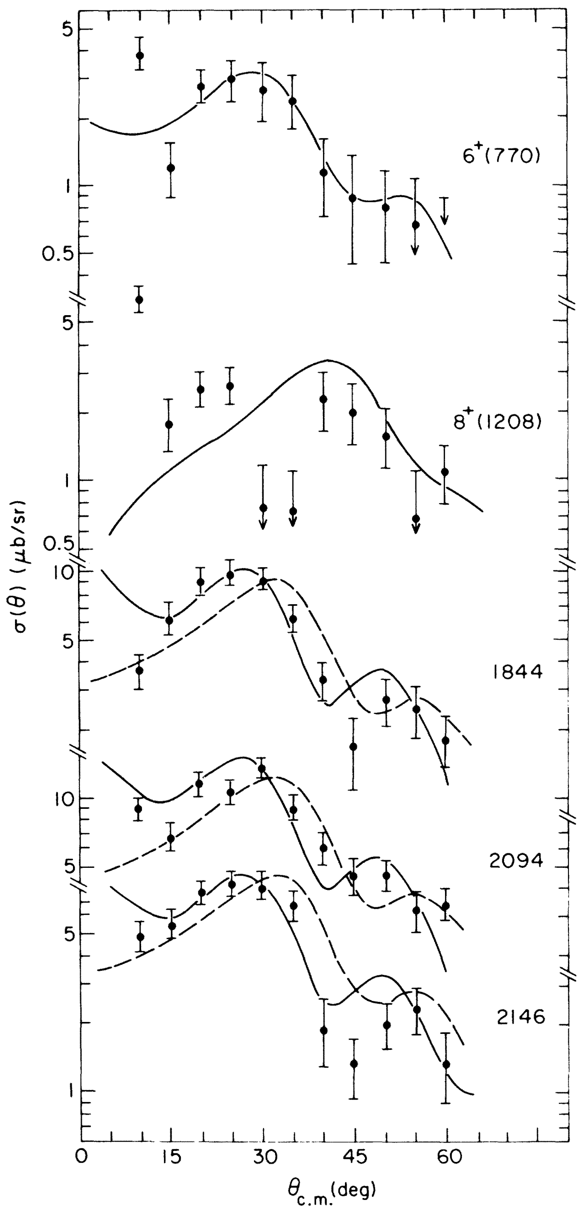

FIG. 5. Angular distributions for transitions to high-spin states. The curves are the DWBA calculations discussed in the text. Note the rather good agreement with experiment for the 6^+ state, as compared with the transition to the 8_g^+ state. The experimental data for the 1844-, 2094-, and 2146-keV states are compared with $L=5$ (solid curves) and $L=6$ (dashed curves) predictions.

eters were taken from the literature,[22] and an option of CHUCK was selected which computes the optical-model potentials as $L=0$ projections of a deformed well.

The results shown in Fig. 9, achieved after three–five attempts, represent the experimental data sets equally as well as do the initial optical-model calculations. It should also be noted that these CCBA calculations do a good job of reproducing the ^{176}Yb(p,p') data[17] to the 2^+ member of the gsb, if the 4^+ state is included in the calculation. The associated optical-model parameters (Table II) are much closer to those appropriate

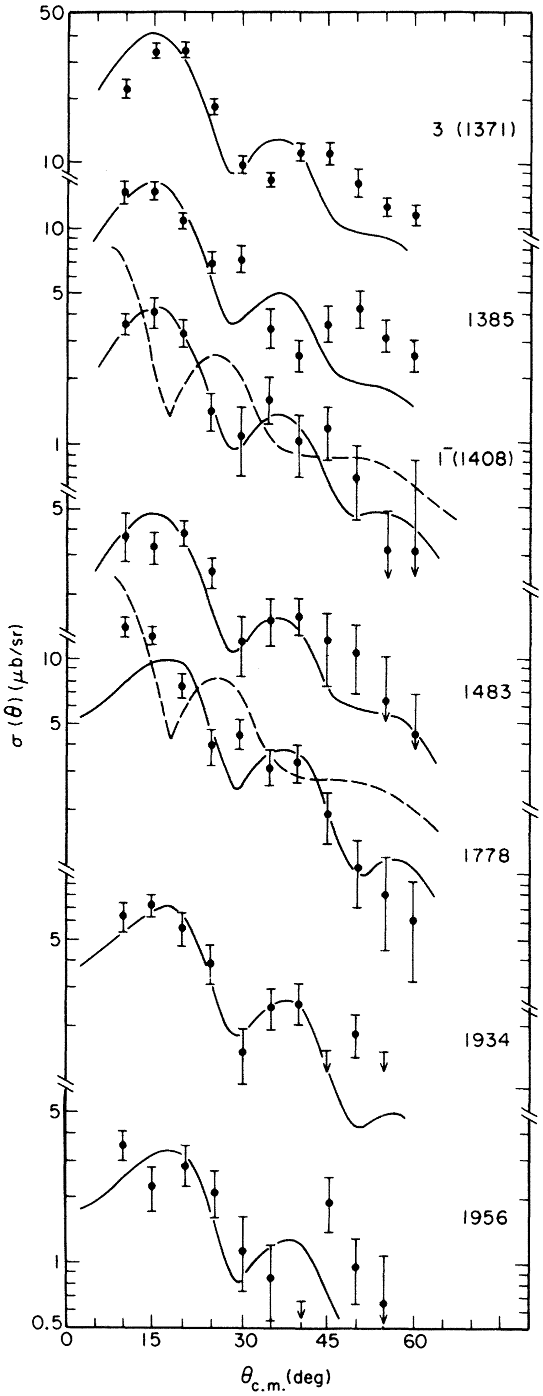

FIG. 6. Angular distributions for transitions to known and probable odd-parity states. All solid curves are $L = 3$ DWBA calculations. The experimental data for the transitions to the previously assigned 1^- state at 1408 keV and the 1778-keV state (which has a similar angular distribution) are also compared with $L = 1$ calculations (dashed curves). It is seen that $L = 3$ predictions also give the best description of these transitions.

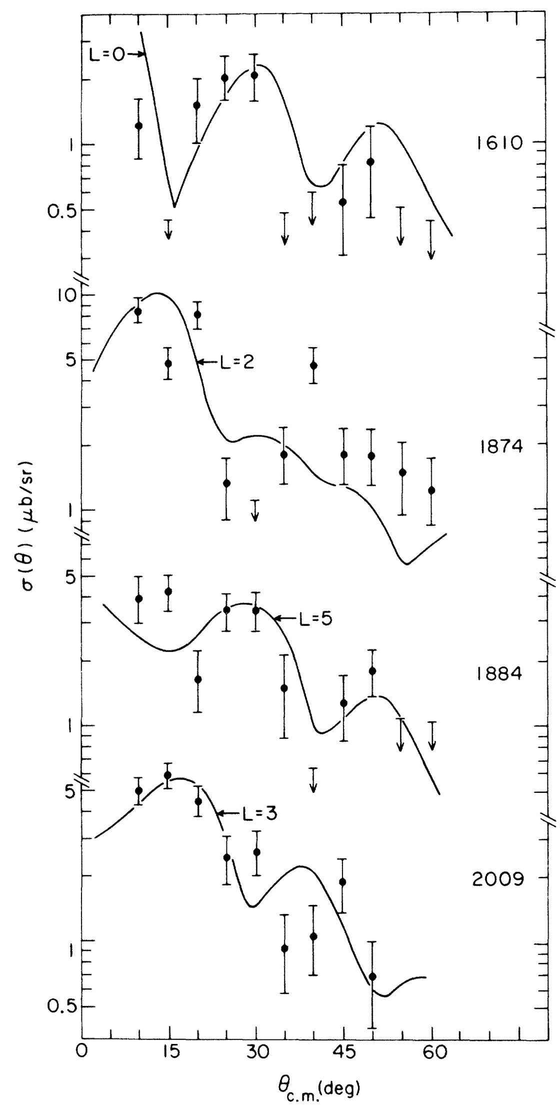

FIG. 7. Angular distributions for transitions to some of the remaining states below 2.25 MeV of excitation energy, compared with DWBA calculations with the indicated L values. Note that while the $L = 0$ prediction for the transition to the 1610-keV state gives a rather good description of its angular distribution, the $J^\pi = 0^+$ suggestion is very heavily dependent on a single datum point at 15°.

for neighboring spherical targets, and have the advantage of being self-consistently determined in the CCBA. They were used directly to represent p and t scattering on the Dy isotopes, save only for the requisite change in deformation.

Realistic (p, t) form factors were generated using the Bohr-Mottelson adiabatic hypothesis to construct microscopic wave functions. The intrinsic basis consisted of Nilsson-model states whose occupation probabilities were determined

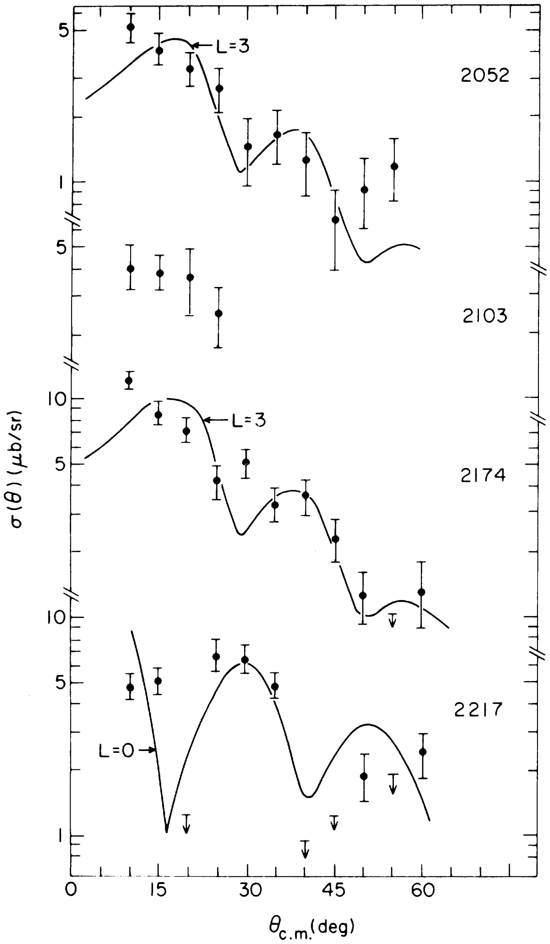

FIG. 8. Angular distributions for transitions to the remaining states below $E_x = 2.25$ MeV. See also Fig. 7 caption for a discussion of the $L = 0$ "assignment" to the 2217-keV state. Note that angular distributions for states above $E_x = 2.25$ MeV were not obtained because of the high level density in this region.

from a BCS calculation. The parameters of the Nilsson model ($\mu = 0.42$; $\kappa = 0.0637$; $\epsilon = 0.242$) were selected to best reproduce the single-neutron level scheme of Ogle *et al.*[23] near $N = 90$. The pairing strength $G_0 = 30$ MeV was estimated from

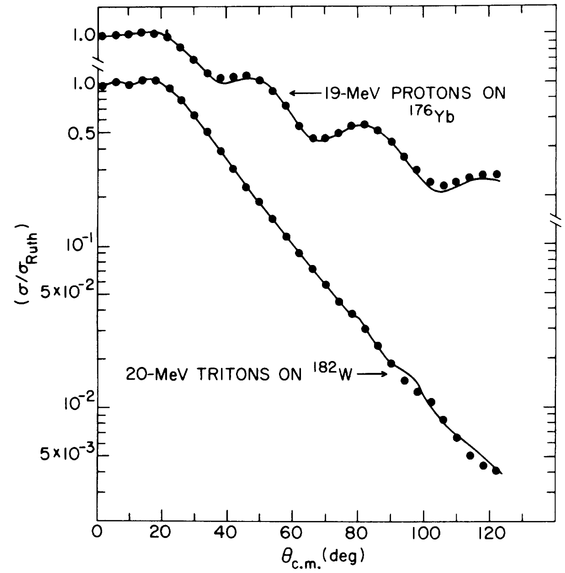

FIG. 9. Elastic-scattering predictions for proton and triton scattering from the CCBA calculations discussed in the text. The "data points" were obtained from optical-model calculations, with parameters obtained from the literature, which fitted the actual experimental data with $\chi^2 = 1$.

the work of Ascuitto and Sorensen[24] for the transitional Yb region. The corresponding neutron pairing gap $\Delta_n = 825$ keV is less than the value $\Delta_n = 1080$ keV computed from the Nilsson-Prior formula[25] for ^{158}Dy, but is consistent with that expected for heavier-mass (and more deformed) Dy isotopes as suggested in Ref. 24. The pairing interaction was allowed to extend with constant strength over the 40 neutron orbitals closest to the Fermi energy, except that the pairing strength between oblate and prolate orbitals was reduced to 5% of the prolate-prolate and oblate-oblate strength (as discussed in more detail below) so that the off-diagonal contributions of 11 oblate orbitals to the gs transition were substantially reduced.

Reaction form factors were computed from the microscopic wave functions by expanding each

TABLE II. Optical-model parameters used in the CCBA and DWBA calculations.

	V (MeV)	r_0 (fm)	r_C (fm)	a (fm)	W (MeV)	W_D (MeV)	r_I (fm)	a_I (fm)	P_{NL} [a] (fm)
Incident channel (p + ^{158}Dy) [b]	53.2	1.23	1.35	0.75	3.90	6.28	1.32	0.625	0.85
Exit channel (t + ^{156}Dy)	150.0	1.24	1.35	0.75	18.4	···	1.50	0.725	0.25

[a] Nonlocality correction parameter. See Ref. 19.

[b] A spin-orbit potential with $V_{so} = 6.20$ MeV, $r_{so} = 1.01$ fm, and $a_{so} = 0.75$ fm was also included in this channel.

TABLE III. Bound-state well and deformation parameters used in the CCBA and DWBA calculations.

	β_2	β_4	V (MeV)	r (fm)	a (fm)	λ_{so} (MeV)	P_{NL} [a] (fm)	R_{FR} [b] (fm)
Bound state			c	1.20	0.75	25.0	0.85	0.75
^{158}Dy	0.246	0.027						
^{156}Dy	0.224	0.023						

[a] Nonlocality correction parameter. See Ref. 19.
[b] Finite-range correction parameter. See Ref. 19.
[c] Adjusted to give $\frac{1}{2}$ the two-neutron separation energy.

Nilsson orbital in a spherical harmonic-oscillator basis (with $\hbar\omega_0 A^{1/3} = 41$ MeV), multiplying by the appropriate statistical factors and occupation probabilities,[10] and summing over all the orbitals. The result is a description of the total (p,t) transfer form factor in terms of a coherent sum of transfer form factors for spherical shell-model states, each of which was generated by binding neutrons in a Woods-Saxon well with the parameters listed in Table III, at an energy equal to $\frac{1}{2}$ the two-neutron separation energy.

The results of a CCBA analysis of transfer to the 0^+ and 2^+ members of the gsb (using the optical-model parameters and form factors discussed above) appear in Fig. 10. This calculation included the 0^+ and 2^+ states in both Dy isotopes, and all inelastic and transfer routes amongst them. All of the curves shown in Figs. 10 have the same overall normalization. The curves labelled DIRECT and INDIRECT are the results of calculations including either the direct-transfer routes only (essentially DWBA), or the two-step modes only, while the solid curves are the results of the complete CCBA calculation.

It can be seen by inspection of Fig. 10 that the CCBA does a much better job of accounting for both the oscillatory behavior of the 2^+ angular distribution and its strength relative to the gs transition, in agreement with previous results at lower incident proton energy.[16-18] On the other hand, the shape of the gs transition is not much different from that predicted by the DWBA (although the predicted 3° backward shift of the first minimum is apparently reflected in the experimental data). Both of these observations can be explained on the basis of the fact that the indirect modes have only ≈10% of the strength of the direct route for the gs transition, while the corresponding ratio for the 2^+ transition is 50–100%. It is apparent, then, that there is a very large destructive interference between direct and indirect transfer to the 2^+ member of the gsb.

The absolute normalization for the curves shown in Fig. 10, determined from the second maximum of the gs transition, turned out to be a factor of 5.85 (using the value $S^{1/2}D_0 = -1560$ recommended for CHUCK [19]). On the other hand, this normalization parameter is a factor of 2 *smaller* for the DWBA calculation, corresponding to the fact that the higher-order processes *reduce* the predicted

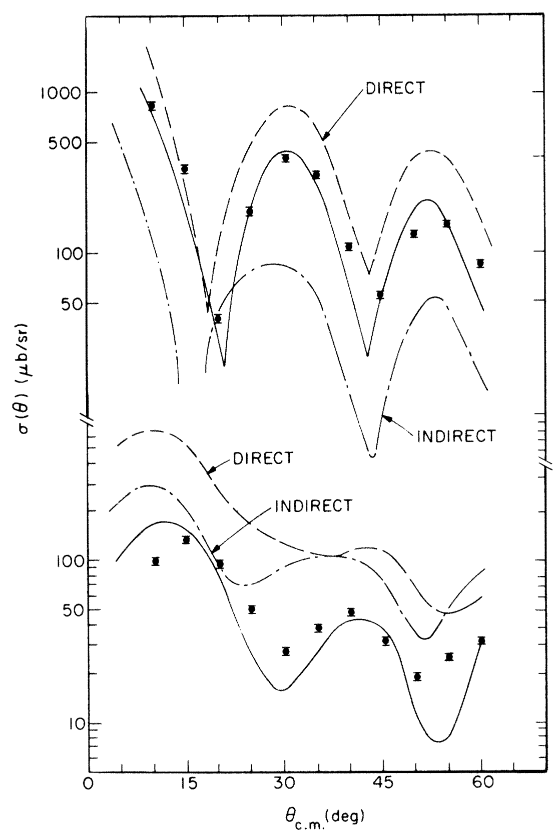

FIG. 10. Decomposition of the CCBA predictions for transitions to the first two states of the gsb into direct-transfer and indirect-transfer pieces. Note the extremely large destructive interference between these two modes which occurs in the transition to the 2^+_g level. It is apparent that the complete CCBA calculation (solid curve) does a much better job of accounting for the angular distribution in the transition to this state than either mode alone.

cross section. Therefore, the source of the discrepancy in the absolute cross sections is unexplained.

B. $L = 0$ and $L = 2$ transitions to excited bands

Only one unambigious $L=0$ transition to an excited state of ^{156}Dy, viz., that to the 674-keV "β-vibrational" bandhead, has been observed in the present experiment, although two weak transitions to states at 1610 and 2217 kev may also be $L=0$. The 674-keV transition is extremely strong, amounting to over 23% of the gs strength, thus continuing the systematics in this mass region (Table IV). Several authors[7–10,24,26,27] have attempted to account for these unusually strong $L=0$ transitions, which also occur in actinide nuclei[28,29] whose excited 0^+ states were also initially thought to be β vibrations. Recently, Ragnarsson and Broglia[10] have termed such states "pairing isomers."

We have used the model of Van Rij and Kahana[8] to compute the transition to the 674-keV state. In this model, which is similar to those later proposed by Bes, Broglia, and Nilsson[9] and by Ragnarsson and Broglia,[10] the pairing force between two neutrons in oblate and prolate orbitals is assumed to be weaker than the corresponding prolate-prolate or oblate-oblate strengths. (Ragnarsson and Broglia[10] consider a pairing-quadrupole force so that the pairing strength between two orbitals is a continuous function of their single-particle quadrupole moments.) Strong excited 0^+ states are then expected to appear in mass regions where the distribution of oblate and prolate levels in the vicinity of the Fermi surface is inhomogeneous, i.e., in the actinides and near $N=90$.

The details of the form-factor computation have been discussed in the previous section, and the results of a calculation using this model are shown in Fig. 2 where the normalizations of the gs and excited 0^+ transitions are the same. No indirect modes are considered in the transition to the excited 0^+ state because form factors for such transfers were not available. It can be seen that the magnitude of the 674-keV transition is reasonably well reproduced by the calculation, although its predicted angular distribution is in only fair agreement with the experimental data. In this case, the phase of the direct transfer to the excited state is such that the interference with two-step routes is expected to be constructive,[24] which would tend to improve the agreement with experiment both as to the magnitude and the shape of the angular distribution.

In contrast to the marked scarcity of $L=0$ transitions, several $L=2$ transitions to excited 2^+ states were observed in this experiment (Fig. 3). Because complete microscopic-model wave functions for these states were not available (except for the 2 member of the gsb), the curves shown in Fig. 3 are microscopic two-nucleon transfer calculations assuming a $(2f_{7/2})^2$ configuration (each curve is independently normalized to the experimental data). It can be seen that direct-transfer calculations for the most part give a reasonably good account of the angular distributions of these 2^+ states. However, comparison of the 829- and 891-keV transitions, for example, demonstrates the variability discussed above.

C. Other transitions

The transitions to 4^+ states observed in the present work (Fig. 4) are all quite weak, and the observed variability in shape is larger than for the $L=2$ transitions (compare, for example, the transitions to the known 4^+ states at 1088 and 1166 keV. The curves shown in Fig. 4 are all two-nucleon transfer calculations assuming a $(2f_{7/2})^2$ configuration. The probable effect of two-step processes on the transfer to the 4^+ member of the gsb is apparent, although CCBA calculations were not made for this transition due to the lack of appropriate microscopic model form factors.

Transitions to higher-spin states are shown in Fig. 5. The angular distribution for the known 6^+ state at 770 keV is surprisingly well reproduced by the DWBA calculation for a $(2f_{7/2})^2$ configuration. The fit to the 8^+ state at 1208 keV (assuming a $(1i_{13/2})^2$ congiguration) is not as good. In both cases, however, there is apparently more cross section observed than predicted to these high-spin states at forward angles. The angular distributions of the three remaining states in Fig. 5 seem to be best fitted by $L=5$ calculations (solid curve, for which a $2f_{7/2}\otimes 1i_{13/2}$ configuration is assumed), although they are also compared with the $L=6$ predictions (dashed curves). In view of the marked variability of the 4^+ transitions, however, these (5^-) assignments must be considered to be tentative.

The angular distribution for the transition to the known[6,30] 3^- state at 1371 keV is compared with an $L=3$ calculation (assuming a $2f_{7/2}\otimes 1i_{13/2}$ configuration) in Fig. 6. The agreement with experiment is only fair. Angular distributions for transitions to six other states given tentative $J^\pi=3^-$ assignments are also shown in Fig. 6. Two of the angular distributions, for the states at 1408 and 1778 keV, are also compared with $L=1$ calculations (assuming a $1h_{11/2}\otimes 1i_{13/2}$ configuration). The 1408-keV state has previously been assigned[30] $J^\pi=1^-$, but its angular distribution in the (p,t)

reaction (Fig. 6) suggests a tentative $J^\pi = 3^-$ assignment instead. There is, in fact, no compelling evidence for the population of any 1^- state in the present experiment, in agreement with previous work on the Gd isotopes.[16]

Angular distributions for the remaining states seen in this work are shown in Figs. 7 and 8, compared with DWBA calculations assuming the various configurations discussed above (except for $L=0$, for which a $2f_{7/2}{}^2$ configuration was used). In none of these cases is the agreement with experiment good enough to yield a definite J^π assignment, but the suggested L values do give the best fit to the experimental data. The very tentative $J^\pi = 0^+$ assignments to the states at 1610 and 2217 keV depend heavily on the apparent minima observed at 15°–20° (Figs. 7 and 8), which appear at only one angle in each case.

IV. DISCUSSION

A. Anomalous $L = 0$ and $L = 2$ transitions

The bandhead of the "β vibration" in ^{158}Dy, at 674 keV, is very strongly populated in the (p, t) reaction, as might have been expected from the systematics of previously reported two-neutron transfer data on $N = 92$–98 targets[15–17,31] (Table IV). We have shown that the model of Van Rij and Kahana,[8] which suggests that this anomalous 0^+ strength is due to a weakened pairing force between neutrons in prolate and oblate Nilsson orbitals, can quite readily account for the experimental data on ^{158}Dy$(p, t)^{156}$Dy. Therefore, it would seem that the "β-vibrational" nature of these states is called into question, since the model implies that they are constructed from a relatively small number of oblate Nilsson orbitals. Perhaps they are more properly termed "pairing isomers," as discussed by Ragnarsson and Broglia.[10]

Another anomaly in the present experiment data, which has not yet been mentioned, is the strong population of the 2_γ^+ bandhead, with an intensity equal to that of the 2^+ member of the gsb (Table I). To our knowledge, in no other case does the population of any excited 2^+ state exceed about 50% of the 2_g^+ strength in (p, t) reactions on deformed nuclei. A similar anomaly does occur in the (t, p) reaction on light W nuclides, but it is not reflected in the (p, t) data.[32] To some extent, the large $2_\gamma^+/2_g^+$ ratio may be a reflection of the very large suppression of the direct 2_g^+ cross section due to destructive interference with two-step transfer modes. But in that case, one might expect to see similar large ratios in other $N = 90$ = 90–96 nuclides. Preferential population of the 2_γ^+ excitation has in fact been observed, in far less severe form, in (p, t) reactions on Dy and Er isotopes at lower energies,[15] but no systematic effect of this type is observed for the Gd isotopes.[16] Therefore, it is probable that much of the explanation lies in the microscopic structure of the 2_γ^+ state. Unfortunately, microscope (p, t) form factors for γ-vibrational states in deformed nuclei are not readily available, so that we have only been able to compare the experimental data with single-component calculations which do not give a prediction for relative cross sections.

TABLE IV. (p,t) strengths (relative to ground state) of excited 0^+ states for nuclides with neutron number $86 \le N \le 98$.

N	86	88	90	92	94	96	98
Gd[a]	0.124	0.761	0.132	0.079	0.193		
Dy[b]			0.235	0.09	0.16	0.13	
Er[c]						0.07	0.02
Yb[d]						(0.12)	0.02

[a] From Ref. 16.
[b] From Ref. 15, except $N = 90$ from present experiment.
[c] From Ref. 15.
[d] From Ref. 17.

B. "β-γ" band

In a previous publication[11] reporting on some aspects of the present experiment, we have suggested that the $J^\pi = 2^+$ state at 1520 keV is the bandhead of a $K = 2$ "β-γ" band. Such a state is predicted for the two-phonon excitations of an axially symmetric rotor according to the theory of Davydov.[13] In its simplest form (neglecting coupling of the β and γ vibration) this theory gives for the excitation energy of a multiphonon state:

$$E(IKn_\gamma n_\beta) = \hbar\omega_\gamma(2n_\gamma + \tfrac{1}{2}K) + A[I(I+1) - \tfrac{3}{4}K^2] + \hbar\omega_\beta n_\beta,$$

where I is the spin of the state, K is the K value of the band, n_γ and n_β are the number of vibrational quanta, and A is the rotational energy derived from $E(2_g^+)$. The "β-γ" bandhead is then the $|2201\rangle$ excitation, predicted to occur at 1565 keV in ^{156}Dy in excellent agreement with our $J^\pi = 2^+$ state at 1520 keV. Note that this state is called a β-γ vibration even though $n_\gamma = 0$ because the strong rotation-vibration coupling term in the Davydov Hamiltonian forces the γ vibration to be present in an excited state as soon as $K \ne 0$. A similar situation occurs for the γ vibration, described by $|I200\rangle$. In comparison, the $(K=0)$ $|0010\rangle$ excitation, which we have previously suggested[11] to be the (0^+) state at 1610 keV, occurs at twice the γ-vibrational energy and is therefore also a two-phonon excitation.

One potentially troublesome defect of this simple theory is the neglect of the coupling of β and γ vibrations. Davydov also presents expressions for the energy taking into account this coupling, but these equations are difficult to solve in the case where the parameter $\mu = [E(2^+_g)/E(0^+_\beta)]^{1/2}$ is greater than $\frac{1}{3}$. For ^{156}Dy, $\mu = 0.45$. If, however, the so-called "β vibration" is in fact a pairing isomer, as suggested in the previous section, and not the $K = 0$ projection of a quadrupole vibration, the coupling problem does not arise. In this case, the 1520-keV state is more properly described as a γ vibration built on the pairing isomer.

It is also possible to understand the strong (7.7% of gs) transition to the $J^\pi = 2^+$ state at 1520 keV in the light of the discussion in the previous paragraph. Although this would be a two-phonon transition in the terminology of the collective model, we propose that it is nevertheless a two-quasiparticle transition since the final state consists of a $K = 2$ recoupling of neutron holes in the (predominantly oblate) Nilsson orbitals making up the pairing isomer. To first order, then, one expects the approximate relation:

$$\frac{\Sigma(1520)}{\Sigma(674)} \simeq \Sigma(890)\,,$$

where $\Sigma(E)$ is the ratio of the intensity of a transition to that of the gs transition. Using the intensities listed in Table I, we find $\Sigma(1520) = 5.5\%$ in reasonable agreement with experiment.

Additional evidence on the nature of the 1520-keV state is to be found in a comparison to the results of a (p, t) investigation of the Gd isotopes.[16] In this experiment, the only known 2^+ state excited with a measurable intensity (other than those of the gs, "β," or γ band) was the state at 1531 keV in ^{154}Gd, an isotone of ^{156}Dy ($N = 90$). The predicted (p, t) intensity of this state on the basis of the model discussed above is $\Sigma(1531) = 1.3\%$, compared with the observed value $\Sigma = 4\%$, and the predicted excitation energy is 1677 keV. Although the agreement with this simple theory is only fair, these two states are clearly related to one another. This is important, since the 1531-keV state in ^{154}Gd has been definitely shown to be the bandhead of a $K = 2$ band from an investigation of the β^- decay of ^{154}Eu.[34] Furthermore, the electromagnetic decay of this state is dominated by transitions to the "β band," as would be expected for a state of the type discussed above. Taken together, then, the experimental data on these states in ^{154}Gd and ^{156}Dy give strong evidence for their interpretation as γ vibrations built on the pairing isomer.

C. Relationship to the "super" band

At the time of submission of our earlier publication, the Louvain-la-Neuve group[5] had already suggested possible candidates for the 6^+, 8^+, and 10^+ members of the super band in ^{156}Dy from their studies of the ^{159}Tb$(p, 4n\gamma)^{156}$Dy reaction. We pointed out[11] that the 1635-keV (4^+) state and the 1520-keV 2^+ state seem to form a natural extension of this rotational band, with nearly the rigid-body moment of inertia. Since that time, a state at 1627.3 keV has been assigned $J^\pi = 4^+$ and associated with the super band by the same group,[6] and it is highly probable that it is the same as our 1635-keV state. They do not, however, find evidence for the decay of a $J^\pi = 2^+$, 1520-keV state. Nevertheless, it is tempting to speculate that the 1520-keV state is in fact the bandhead of the super band since this would account in a natural way for several observations. First of all, a γ-vibrational band built on the pairing isomer might be expected to have a relatively large moment of inertia (although perhaps not quite as large as the near-rigid value observed) since its intrinsic state is described by holes in oblate Nilsson orbitals, thus increasing the prolateness of the system. Secondly, the weak interaction with the gs and "β" bands[3] might be due to the $\Delta K = 2$ nature of the coupling. Finally, the confinement of the "super" band phenomenon to only two nuclides, ^{156}Dy and ^{154}Gd, is explained by the fact that only these two systems have "β" and γ bandheads which lie low enough to put the $K = 2$ "β-γ" state at a reasonably low excitation energy.

Unfortunately for this interpretation, there exists some experimental evidence that the 1520- and 1635-keV states may be unrelated to one another. The analogous levels in ^{154}Gd are at 1531.3 and 1646.0 keV, respectively. The former state has been discussed in detail above and is clearly the bandhead of a $K = 2$ band. The latter state is populated in the electron-capture decay of the 22.5 h $J^\pi = 7^-$ isomer in ^{154}Tb,[35] and has been assigned $J^\pi, K = 4^+, 4$. Therefore, despite the fact that the intensity of the transition to the 1635-keV state in ^{158}Dy$(p, t)^{156}$Dy is just what would be expected for the 4^+ member of a rotational band built on the 1520-keV state, it is likely that these two states are not related. Furthermore, it seems probable that the 1627.3-keV $J^\pi = 4^+$ state in ^{156}Dy reported by the Louvain-la-Neuve group[6] is in fact the bandhead of a $K = 4$ band. Additional evidence on these two points may be obtainable from a study of the β^+ decay of the known high-spin isomer of ^{156}Ho.[36]

V. CONCLUSION

We have investigated the ^{158}Dy$(p,t)^{156}$Dy reaction at 29.9-MeV incident proton energy, and obtained angular distributions for transitions to 34 levels up to an excitation energy of 2.25 MeV in ^{156}Dy. The ground-state Q value for this reaction has been measured to be -7.535 ± 0.015 MeV.

The observed variability in the shapes of angular distributions of known L value, which is particularly evident in the gsb transitions, led us to use the CCBA to investigate the importance of two-step processes. As a preliminary to this study, optical-model parameters for p and t scattering from deformed targets were obtained, self-consistently in the CCBA, by fitting experimental data sets from the literature. The resulting parameters were quite close to those appropriate for elastic scattering from neighboring spherical targets. Coupled-channels calculations using these parameters demonstrated very large destructive interference between direct and indirect transfer to the 2^+ member of the gsb in the (p,t) reaction.

As expected from previous (p,t) studies of the $90 \leq N \leq 96$ region, it was found that the 0^+ bandhead of the "β vibration" in ^{156}Dy is very strongly populated in ^{158}Dy$(p,t)^{156}$Dy. The experimental data on this reaction are quite readily accounted for by the model of Van Rij and Kahana, which suggests that the anomalous 0^+ states in this mass region are more properly termed "pairing isomers." It was also found that the $L=2$ transition to the 2^+_γ bandhead is very strong. The explanation for this anomaly probably lies in the microscopic structure of the γ vibration, for which no predictions are readily available.

Evidence has been given which suggests that the $J^\pi = 2^+$ state at 1520 keV in ^{156}Dy is the bandhead of a $K=2$ γ-vibrational band built on the pairing-isomeric state. Comparison to the analogous state at 1531.3 keV in ^{154}Gd was particularly useful here, since the electromagnetic decays of this level are known. The observation of such a mode provides additional evidence that the pairing isomer is almost completely decoupled from the superfluid ground state.

Finally, the available experimental evidence on the 1627.3-keV state in ^{156}Dy suggests that this state is the bandhead of a $K=4$ band, primarily on the basis of a comparison to the analogous 1646.0-keV level in ^{154}Gd. If so, and if we also accept the assignment of this level to the "super" band by the Louvain-la-Neuve group, then the origin of the "super" band is apparently in a $K^\pi = 4^+$ state of as yet unknown structure.

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[1] D. Ward, R. L. Graham, J. S. Geiger, and H. R. Andrews, Phys. Lett. 44B, 39 (1973).

[2] T. L. Khoo, F. M. Bernthal, J. S. Boyno, and R. A. Warner, Phys. Rev. Lett. 31, 1146 (1973).

[3] H. R. Andrews, D. Ward, R. L. Graham, and J. S. Geiger, Nucl. Phys. A219, 141 (1974).

[4] R. M. Lieder, H. Beuscher, W. F. Davidson, A. Neskakis, C. Mayer-Böricke, Y. El Masri, P. Monseu, J. Steyaert, and J. Vervier, Phys. Lett. 49B, 161 (1974).

[5] Y. El Masri, R. Janssens, C. Michel, P. Monseu, J. Steyaert, and J. Vervier, Z. Phys. A274, 113 (1975).

[6] Y. El Masri, J. M. Ferté, R. Janssens, C. Michel, P. Monseu, J. Steyaert, and J. Vervier, Nucl. Phys. A271, 133 (1976).

[7] R. E. Griffen, A. D. Jackson, and A. B. Volkov, Phys. Lett. 36B, 281 (1971).

[8] W. I. van Rij and S. H. Kahana, Phys. Rev. Lett. 28, 50 (1972).

[9] D. R. Bes, R. A. Broglia, and B. Nilsson, Phys. Lett. 40B, 338 (1972).

[10] I. Ragnarsson and R. A. Broglia, Nucl. Phys. A263, 315 (1976).

[11] J. J. Kolata and M. Oothoudt, Phys. Lett. 65B, 116 (1976).

[12] We are indebted to J. Lerner, Argonne National Laboratory, for fabricating this target.

[13] R. T. Kouzes, Ph.D. thesis, Princeton University, 1974 (unpublished).

[14] A. H. Wapstra and N. B. Gove, Nucl. Data A9, 265 (1971).

[15] J. V. Maher, J. J. Kolata, and R. W. Miller, Phys. Rev. C 6, 358 (1972).

[16] D. G. Fleming, G. Günther, G. Hagemann, B. Herskind, and P. O. Thom, Phys. Rev. C 8, 806 (1973).

[17] M. A. Oothoudt and N. M. Hintz, Nucl. Phys. A213, 221 (1973).

[18] R. J. Ascuitto, N. K. Glendenning, and B. Sorensen, Nucl. Phys. A183, 60 (1972).

[19] P. D. Kunz, University of Colorado (unpublished).

[20] F. D. Becchetti, Jr., and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

[21] E. R. Flynn, D. D. Armstrong, J. G. Beery, and A. G. Blair, Phys. Rev. 182, 1113 (1969).

[22] K. A. Erb, J. E. Holden, I. Y. Lee, J. X. Saladin, and T. K. Saylor, Phys. Rev. Lett. 29, 1010 (1972).

[23] W. Ogle, S. Wahlborn, R. Piepenbring, and S. Fred-

riksson, Rev. Mod. Phys. 43, 424 (1971).
[24]R. J. Ascuitto and B. Sorenson, Nucl. Phys. A190, 309 (1972).
[25]S. G. Nilsson and O. Prior, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 32, No. 16 (1961).
[26]S. K. Abdulvagabova, S. P. Ivanova, and N. I. Pyatov, Phys. Lett. 38B, 215 (1972).
[27]S. T. Belyaev and B. A. Rumiantsev, Phys. Lett. 30B, 444 (1969).
[28]J. V. Maher, J. R. Erskine, A. M. Friedman, J. P. Schiffer, and R. H. Siemssen, Phys. Rev. Lett. 25, 302 (1970).
[29]J. V. Maher, J. R. Erskine, A. M. Friedman, J. P. Schiffer, and R. H. Siemssen, Phys. Rev. C 5, 1380 (1972).
[30]T. Grotdal, K. Nybo, and T. Thorsteinsen, Nucl. Phys. A110, 385 (1968).
[31]M. Oothoudt, P. Vedelsby, and N. M. Hintz, Phys. Lett. 32B, 270 (1970).
[32]R. F. Casten and J. D. Garrett, Phys. Lett. 47B, 436 (1973); R. F. Casten, D. Burke, and O. Hansen, Nucl. Phys. A261, 445 (1976).
[33]A. Davydov, Nucl. Phys. 24, 682 (1961).
[34]L. C. Whitlock, J. H. Hamilton, and A. V. Ramayya, Phys. Rev. C 3, 313 (1971).
[35]L. L. Riedinger, D. C. Sousa, E. G. Funk, and J. W. Mihelich, Phys. Rev. C 4, 1352 (1971); Nucl. Phys. A238, 365 (1975).
[36]H. Schepers, Bonn University, Report No. BMBW-FB K 70-30 (unpublished).