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Configuration-mixed-shell-model relative-alpha-decay-rate calculations for spherical doubly odd nuclei (^{212}At and ^{212}At^m)

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A brief review of the development of α-decay-rate theory is given. Then in this paper are presented relative α-decay-rate theoretical calculations for doubly odd spherical nuclei for ^{212}At and ^{212}At^m α decay to excited states of ^{208}Bi using mixed-configuration-shell-model parent and daughter wave functions and α particle approximations. Results are presented for no mixing and for Kim and Rasmussen, Ma and True, and Kuo and Herling wave functions for both the parent and daughter nucleus. The results are compared with the experimental data. There is a great sensitivity to configuration mixing, and no set of wave functions is wholly satisfactory. Stripping and pickup experiments testing the wave functions are reviewed and difficulties noted. It is concluded that α-decay rates provide a stringent test for the effective shell-model neutron-proton interaction in the lead region. Further theoretical and experimental work is needed.

[NUCLEAR STRUCTURE α-decay-rate theory, ^{212}At and ^{212}At^m. Calculation of rates for odd-odd spherical nuclei in modified zero-size approximation with shell-model configuration mixing.]

I. INTRODUCTION

Within the past several years successful theoretical calculations of α-decay rates of nuclei near the doubly-closed shell in the ^{208}Pb region, based on shell-model wave functions of the nucleons outside the closed shell, have been carried out.[1-4] In particular, theoretical studies of the odd-odd nuclei are very important because the Pauli principle permits all possible components of the residual force to be operative for the nonidentical nucleons. The case of nuclei with one proton and neutron outside closed shells was investigated by de-Shalit[5] using a zero-range force. Calculations for specific odd-odd nuclei have been made by several workers for finite range forces in which central exchange forces are included.[6-8] With the assumption that the α-particle wave function is of Gaussian form, the reduced width for the ground-state transitions of even-even nuclei was formulated[2,3] in cases where the angular momentum of the parent, daughter nuclei, and the outgoing α-particles are equal to zero. Calculations, involving numerical integration, have been made by Rasmussen[9] to account for the inclusion of the angular momentum effects in the penetration factor P. Later, a point-α-particle, or δ-function approximation method, for α-decay was also carried out.[4] It was concluded that this method greatly simplified formulas and gave reliable agreement with other theories, although systematic overestimation of contributions from larger l values was noted and a simple approximate correction method introduced. Numerical calculations of relative α intensity have been made for finite-size Gaussian α particles using similar theoretical methods extended to even-even spheroidally deformed nuclei.[10] Similar theoretical methods were also applied in the deformed region.[11,12]

An application of the coupled-channel formalism[9] has been made to include electromagnetic and nuclear force coupling in the deformed region[13,14] and for vibrational states.[15]

These theories have enjoyed success mainly with a relative α-decay rate. It has been difficult to find a reasonable set of parameters giving the correct absolute decay rate, the calculated decay constants usually being low by at least an order of magnitude compared with the measured values.

Recently, a new theory which has the advantage of calculating the absolute α-decay rate (developed along the lines of Feshbach's[16] "unified theory of nuclear reactions") has been proposed by Harada and Rauscher[17] in which the nuclear-radius parameter is not used explicitly. Similar theoretical methods[18,19] have been developed to calculate absolute α decay. The calculations have been performed both in the zero-size α-particle and finite-size α-particle approximations. However, the theoretical α widths turn out to be smaller than the experimental ones. It was suggested that the difficulties might be cleared up by studying the finite-size effect with Woods-Saxon potential shell-model wave functions and with more sophisticated internal motion wave functions for the α particle.[20] Regarding the absolute α-decay rate problem, encouraging recent developments have come from the work of Fliessbach and collaborators[21,22] through a renormalization from more careful attention to Pauli principle effects.

Not only has Fliessbach shown that even-even decay rates are in satisfactory agreement,[21,22] but the theory has been successfully applied to odd-mass nuclei[23] (^{211}Bi and ^{211}Po) and to long-range α particles from ^{212}Po* excited states. Fortunately for the various earlier versions of the theory the relative α rate predictions from the renormalized theory are not greatly different in the odd-mass cases, but there are some substantial changes in ^{212}Po*, where configuration mixing and partial cancellation of terms are significant.

There has been but little theoretical α-decay rate work on odd-odd nuclei. It is generally recognized that the decay may be grouped into three categories: favored, in which the orbitals and coupling of odd protons and odd neutrons remain unchanged; once hindered, in which the orbitals of one odd nucleon remain unchanged and the other changes; and twice hindered, in which the orbitals of both odd nucleons change.

Favored decay should proceed at a rate comparable to ground decay of even-even neighbors. Twice-hindered α decay will not be considered in this paper; finite-size α theoretical calculations of the twice-hindered α decay of the ^{210}Bi isomers have been made by Tuggle.[24] We are concerned here with once-hindered decay of odd-odd nuclei. With the extensive configuration mixing and with the substantial angular momentum algebra the finite-size α theory involves rather formidable calculations. Thus, we have chosen to explore the problem first with zero-size α theory. Eventually, Fliessbach's renormalized theory should be applied.

Recently, Shihab-Eldin, Jardine and Rasmussen,[25] (hereafter denoted SJR) have extended the zero-size approximation method to calculate α-decay rates for odd-odd nuclei using graphical presentation techniques. Relative ^{210}At decay rates were found to be in qualitative agreement with experimentally measured values. The discrepancies were ascribed to mixing in daughter and parent wave functions, but shell-model theoretical ^{206}Bi wave functions were not available to test the approach in more detail. In this paper, we apply the same techniques to the α decay of ^{212}At and ^{212}At^m to levels in ^{208}Bi using mixed wave functions which have been calculated by shell-model theory. The structure of the low-lying states of ^{208}Bi has been calculated in terms of mixed 1p-1h shell-model configurations by different authors.[26-28] The structure of ^{212}At has not been calculated yet, but we assume as a first approximation that the structure of ^{212}At is the same as ^{210}Bi plus two protons in the $h_{9/2}$ orbital coupled to zero.[27-30]

II. OUTLINE OF THE THEORY

By definition of the reduced width γ_L^2, the Lth partial decay rates are given by[4]

$$\lambda_L = \frac{2\gamma_L^2}{\hbar} P_L, \tag{1}$$

where the penetration factor, P_L, is given by

$$P_L = \left.\frac{\rho}{G_L^2 + F_L^2}\right|_{r=R_0},$$

and where G_L and F_L are the irregular and regular Coulomb functions, respectively; $\rho = kr$, where k denotes the wave number of the α particle in the asymptotic region. For mixed parent and daughter wave functions, the Lth partial decay rate could be written as

$$\lambda_L = \frac{2}{\hbar}\left(\sum_{\mu\nu} \alpha_\mu \beta_\nu \gamma_L^{\mu\nu}\right)^2 P_L, \tag{2}$$

where α_μ and β_ν are the amplitudes of the different configurations of the parent and daughter nuclei. The important parent and daughter shell-model configurations for states of interest (low lying) will be briefly discussed in this section. In general, the transitions between ^{212}At and ^{208}Bi configurations can be classified into three groups. Symbolically, using SJR notation, the first group can be written as shown in Fig. 1. The transition overlap for this kind of transition can be written in graphical representation as

$$\left[\binom{3}{2}\binom{1}{1}\binom{2j_4+1}{1}\right]^{1/2} \sum_{J_P(\text{even})} b_{9/2\ 9/2\ J_P}{}^3 \times \langle\text{diagram of Fig. 2}\rangle. \tag{3}$$

This can be reduced by similar techniques to SJR and substituted into γ_L to give

$$\gamma_L = C\left[\binom{3}{2}\binom{1}{1}\binom{2j_4+1}{1}\right]^{1/2} \sum_{\substack{J_P=0\\(\text{even})}}^{9} \sum_{\substack{J_N=|j_3-j_4|\\(\text{odd})}}^{j_3+j_4} (-1)^{j_3+j_4+J_N} b_{9/2\ 9/2\ J_P}{}^3 R_1R_2R_3R_4 (\hat{J}_P^i \hat{J}_N^i \hat{J}_d \hat{J}_\alpha \hat{J}_N)^{1/2} \begin{Bmatrix} j_4 & j_3 & J_N \\ J_N^i & j_4 & 0 \end{Bmatrix}$$

$$\times \begin{Bmatrix} J_P^d(=\tfrac{9}{2}) & J_P & J_P^i(=\tfrac{9}{2}) \\ J_N^d(=j_4) & J_N & J_N^i(=j_3) \\ J_d & J_\alpha & J_i \end{Bmatrix} G(J_P, J_N, J_\alpha)$$

$$\times F[l_1(=5), l_2(=5), j_1(=\tfrac{9}{2}), j_2(=\tfrac{9}{2}), J_P] F(l_3, l_4, j_3, j_4, J_N), \qquad (4)$$

where

$$C = \left(\frac{\hbar R_0}{2M}\right)^{1/2} \left(\frac{4\pi S_\alpha{}^3}{3}\right)^{3/2},$$

$$G(J_P, J_N, J_\alpha) = (2J_P+1)^{1/2} (J_\alpha J_P 00 \,|\, J_N 0),$$

$$F(l_1, l_2, j_1, j_2, J) = (-1)^{l_1} \{[1-0.013 l_1(l_1+1)][1-0.013 l_2(l_2+1)]\}^{1/2} (2j_1+1)^{1/2} (J j_1 0 - \tfrac{1}{2} | j_2 - \tfrac{1}{2}),$$

and where R_i is the value of the nucleon radial wave function evaluated at R_0 for nucleon i.

The second group can be written as shown in Fig. 3. Proceeding as before we find

$$\gamma_L = C\left[\binom{2}{2}\binom{1}{1}\binom{2j_4+1}{1}\right]^{1/2} \sum_{\substack{J_N=|j_3-j_4|\\(\text{odd})}}^{j_3+j_4} (-1)^{j_3+j_4+J_N} R_1R_2R_3R_4 (\hat{J}_P^i \hat{J}_N^i \hat{J}_d \hat{J}_\alpha \hat{J}_N)^{1/2} \begin{Bmatrix} j_4 & j_3 & J_N \\ J_N^i(=j_3) & j_4 & 0 \end{Bmatrix}$$

$$\times \begin{Bmatrix} J_P^d(=j_1) & 0 & J_P^i(=j_1) \\ J_N^d(=j_4) & J_N & J_N^i(=j_3) \\ J_d & J_\alpha & J_i \end{Bmatrix} G(0, J_N, J_\alpha)$$

$$\times F[l_1, l_2(=5), j_1, j_2(=\tfrac{9}{2}), 0] F(l_3, l_4, j_3, j_4, J_N). \qquad (5)$$

Finally, the third group (doubly hindered) is symbolically written as shown in Fig. 4 which leads to

$$\gamma_L = C\left[\binom{2}{1}\binom{1}{1}^2\binom{2j_4+1}{1}\right]^{1/2}$$

$$\times \sum_{\substack{J_P=|j_1-j_2|\\(\text{even})\\(\text{odd})}}^{j_1+j_2} \sum_{\substack{J_N=|j_3-j_4|\\(\text{odd})\\(\text{even})}}^{j_3+j_4} (-1)^{j_1+j_2+j_3+j_4+J_P+J_N} R_1R_2R_3R_4 (\hat{J}_P^i \hat{J}_N^i \hat{J}_d \hat{J}_\alpha \hat{J}_P \hat{J}_N)^{1/2} \begin{Bmatrix} j_2 & j_1 & J_P \\ J_P^i(=j_1) & j_2 & 0 \end{Bmatrix}$$

$$\times \begin{Bmatrix} j_4 & j_3 & J_N \\ J_N^i(=j_3) & j_4 & 0 \end{Bmatrix} \begin{Bmatrix} J_P^d(=j_2) & J_P & J_P^i(=j_1) \\ J_N^d(=j_4) & J_N & J_N^i(=j_3) \\ J_d & J_\alpha & J_i \end{Bmatrix} G(J_P, J_N, J_\alpha)$$

$$\times F(l_1, l_2, j_1, j_2, J_P) F(l_3, l_4, j_3, j_4, J_N). \qquad (6)$$

III. RESULTS AND DISCUSSIONS

We used the formulation given in the previous section to calculate the relative α-decay rates from ^{212}At and ^{212}At^m ($J^\pi = 1^-$ and 9^-, respectively) to the 5_1^+, 4_1^+, 6_1^+, 4_2^+, 5_2^+, 3_1^+, 7_1^+, 5_3^+, 2_1^+, 3_2^+, 4_3^+, 4_4^+, 3_3^+, and 6_2^+ states of ^{208}Bi. The radial nucleon wave functions were taken from Blomqvist and Wahlborn.[31] The eigenfunctions obtained by Kim and Rasmussen (KR),[26,29] Ma and True (MT),[28] and Kuo and Herling (KH)[27] were used for these states. For compactness we do not tabulate the wave functions

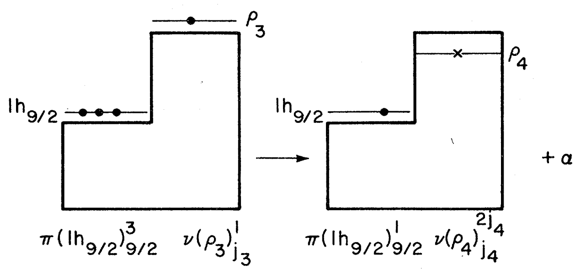

FIG. 1. Representation of the first group of transitions.

here, but they are given as an appendix to the LBL report No. LBL-5071 and are available on request from the authors.

In Tables I and II are given the calculated α-decay rates of the two isomeric states of ^{212}At to the first 14 states of ^{208}Bi. The last column gives the experimental percentages of α branching as measured by Reeder.[32] The theoretical relative intensities are normalized to the most intense α group in each case.

For all sets of wave functions the radii were varied from 7.0 to 9.5 fm to search for the best fits between the experimental and the calculated values. However, we only tabulate results for R_0 equal to 8.0 fm for all cases, except for KR and KH wave functions for the 1^- where we give the results for $R_0 = 7.5$ fm. Figure 5 plots some theoretical relative intensities for various assumed channel radii R_0, and some idea of the general fit and the dependence of results on the assumed channel radius can be gained. The dramatic disagreement for 4_4^+ and 3_2^+ decay, for which no set of wave functions gives agreement, is evident, and this matter is discussed later in this section.

Table III gives angular momentum mixing ratios to the lowest doublet in ^{208}Bi. For the 9^- decay $L\alpha$ values higher than 7 are not shown, as they are successively at least an order of magnitude down from $L\alpha = 7$. There are no experimental data, but it should be feasible to test the predictions for 4^+ decay by α-γ angular correlations.

At the outset we note that the unmixed wave functions give serious disagreement for the 1^- decay, although the 9^- decay is fairly well reproduced. The method of normalizing results to the experimentally most abundant group perhaps exaggerates the discrepancies with pure configurations and with MT wave functions in Table I. From spin selection rules we note that the decay to ground 5^+ can only occur through a single $L\alpha$ value ($L\alpha = 5$), whereas decay to 4^+ can go by $L\alpha = 3$ or $L\alpha = 5$. The results in Tables I and III show that both pure and MT cases give very small $L\alpha = 5$ decay to both members of the ground-state doublet. Both KR and KH wave functions give satisfactory agreement for decay from 1^- to the ground doublet, and they suggest that $L\alpha = 5$ is predominant over $L\alpha = 3$ decay. However, inspection of the top two rows of Table II shows that KR and KH wave functions give the least satisfactory fit, and that MT wave functions are best. Table III shows in all cases the prediction of dominant $L\alpha = 5$ decay from the 9^- state to the ground doublet.

It is worth noting that of the two initial states 1^- and 9^- and the two final states 4^+ and 5^+ the configuration mixing is never over 9% except in one case. That is, the 1^- state of Ma and True has only 60% of the dominant configuration $h_{9/2}g_{9/2}$ and 21% of $f_{7/2}g_{9/2}$, more than an order of magnitude greater admixture of the latter configuration than in the other author's wave functions. The 9^- state is quite pure for the results of all the authors. These facts raise some question about the MT 1^- wave function.

Let us turn from consideration of the ground doublet population pattern to consider the pattern for the $h_{9/2}f_{5/2}^{-1}$ sextet. We should give little weight to the magnitude, since our method of normalization in Tables I and II will bias results where the dominant group was not matched by theory. Consider first the rather pure 9^- state decay to the sextet (Table II). Even the pure daughter case gives reasonable agreement through a slight overemphasis on the high-spin (6 and 7) members. With KR wave functions the fit is worse, the 5_2^+ being underestimated and the 6_1^+ overestimated. The MT and KH wave functions give the best fit. Now consider

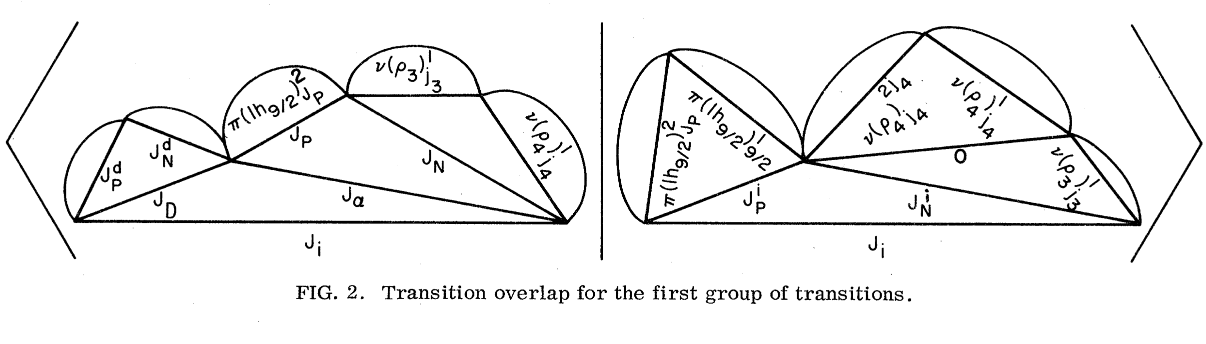

FIG. 2. Transition overlap for the first group of transitions.

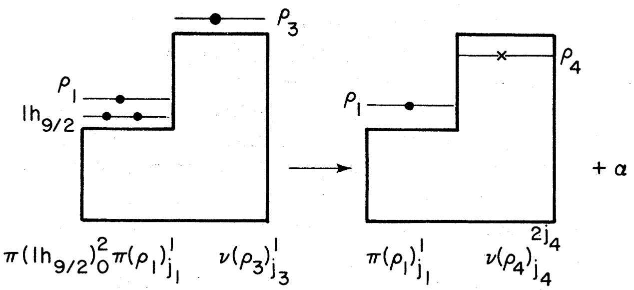

FIG. 3. Representation of the second group of transitions.

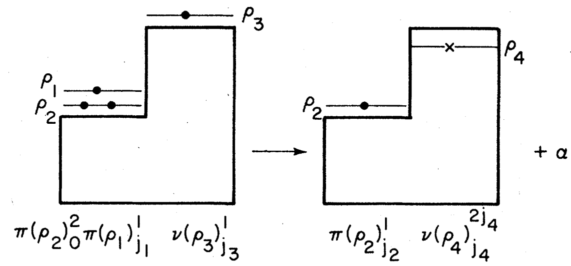

FIG. 4. Representation of the third group of transitions.

the 1^- decay to the sextet (Table I), especially relative to the 4_2^+ intensity. All cases overestimated the 2_1^+ and the data are poor. The pure case underestimates the 3_1^+. The KR case overestimates the 3_1^+ and is over the limit on the 5_2^+. The MT and KH cases underestimate the 6_1^+ and the 3_1^+. Probably the KR case gives the best fit here.

The experimental data probably do not warrant much comparison for the higher multiplets. Note that the 3_2^+ and 4_4^+ should receive no decay at all with the pure configurations, since these states have an $f_{7/2}$ proton. Indeed no decay is seen to these states from the 9^-. All cases fail to give enough intensity for the decay from the 1^-. Three of the four members of the $h_{9/2}p_{3/2}^{-1}$ quartet are reported populated by decay from the 1^-. The MT case again has the same difficulty with the $5_3^+:4_3^+$ ratio as for the $5_1^+:4_1^+$ ratio, namely, underestimation of the 5^+ (pure $L\alpha=5$) transition. The KR and KH cases also underestimate the ratio but not so badly, and the pure case overestimates the ratio. All cases give too much 3_3^+ relative to 5_3^+, with KR the worst. The MT case is much over the limit on decay to the 6_2^+ state. Both states of the $f_{7/2}p_{1/2}^{-1}$ doublet are greatly underestimated for 1^- decay in all cases.

On the basis of α-decay calculations it is not possible to point to clear superiority of one or another of the shell-model wave functions tested. The KR wave functions work the best for 1^- decay, but the MT are best for 9^- decay. With this uncertainty it

TABLE I. Comparison between the calculated relative α-decay intensities of ^{212}At and the experimental results.

Daughter states J^π	Main configuration	Parent spin = 1^- Theoretical relative intensities for various wave functions Pure parent and daughter	Kim and Rasmussen[a]	Ma and True[b]	Kuo and Herling[c]	Exp. (%) Reeder[d]
5_1^+	$h_{9/2}p_{1/2}^{-1}$	80.9	80.9	80.9	80.9	80.900 ± 0.80
4_1^+	$h_{9/2}p_{1/2}^{-1}$	242.4	16.8	333.4	32.3	17.00 ± 0.50
6_1^+	$h_{9/2}f_{5/2}^{-1}$	1.31	0.23	0.24	0.15	0.26 ± 0.06
4_2^+	$h_{9/2}f_{5/2}^{-1}$	3.11	0.62	3.03	0.86	0.63 ± 0.06
5_2^+	$h_{9/2}f_{5/2}^{-1}$	0.02	0.55	1.2	0.34	<0.4
3_1^+	$h_{9/2}f_{5/2}^{-1}$	0.26	1.00	1.4	0.01	0.50 ± 0.08
7_1^+	$h_{9/2}f_{5/2}^{-1}$	0.09	0.10	0.15	0.13	<0.1
5_3^+	$h_{9/2}p_{3/2}^{-1}$	0.13	0.03	0.07	0.09	0.26 ± 0.03
2_1^+	$h_{9/2}f_{5/2}^{-1}$	0.70	0.14	0.59	0.59	0.04 ± 0.03
3_2^+	$f_{7/2}p_{1/2}^{-1}$	0.0	0.6$E-5$	0.004	0.013	0.12 ± 0.03
4_3^+	$h_{9/2}p_{3/2}^{-1}$	0.005	0.03	0.80	0.12	0.06 ± 0.02
4_4^+	$f_{7/2}p_{1/2}^{-1}$	0.0	0.6$E-4$	0.001	0.0003	0.05 ± 0.02
3_3^+	$h_{9/2}p_{3/2}^{-1}$	0.18	0.10	0.17	0.15	0.15 ± 0.01
6_2^+	$h_{9/2}p_{3/2}^{-1}$	0.014	0.001	0.08	0.006	<0.02

[a]References 26 and 29.
[b]Reference 28.
[c]Reference 27.
[d]Reference 32.

TABLE II. Comparison between the calculated relative α-decay intensities of $^{212}At^m$ and the experimental results.

Daughter states J^π	Main configuration	Parent spin = 9^- Theoretical relative intensities for various wave functions: Pure parent and daughter	Kim and Rasmussen[a]	Ma and True[b]	Kuo and Herling[c]	Exp. (%) Reeder[d]
5_1^+	$h_{9/2}p_{1/2}^{-1}$	39.4	19.3	31.6	42.2	29.00 ± 0.30
4_1^+	$h_{9/2}p_{1/2}^{-1}$	67.3	67.3	67.3	67.3	67.30 ± 1.00
6_1^+	$h_{9/2}f_{5/2}^{-1}$	0.85	1.65	0.97	0.99	0.65 ± 0.07
4_2^+	$h_{9/2}f_{5/2}^{-1}$	0.07	0.16	0.18	0.15	0.11 ± 0.03
5_2^+	$h_{9/2}f_{5/2}^{-1}$	0.34	0.12	0.83	0.47	0.53 ± 0.04
3_1^+	$h_{9/2}f_{5/2}^{-1}$	0.11	0.07	0.12	0.12	<0.40
7_1^+	$h_{9/2}f_{5/2}^{-1}$	0.77	0.62	0.78	0.67	0.61 ± 0.11
5_3^+	$h_{9/2}p_{3/2}^{-1}$	0.15	0.18	0.11	0.17	0.26 ± 0.13
2_1^+	$h_{9/2}f_{9/2}$	0.003	0.003	0.001	0.004	<0.02
3_2^+	$f_{7/2}p_{1/2}$	0.0	$0.6E-5$	$0.4E-4$	$0.3E-4$	<0.02
4_3^+	$h_{9/2}p_{3/2}$	0.02	0.01	0.04	0.05	0.16 ± 0.1
4_4^+	$f_{1/2}p_{1/2}$	0.0	$0.3E-4$	$0.3E-3$	$0.1E-3$	<0.1
3_3^+	$h_{9/2}p_{3/2}$	0.0002	$0.7E-3$	$0.1E-3$	$0.1E-3$	<0.03
6_2^+	$h_{9/2}p_{3/2}$	0.22	0.20	0.27	0.28	0.19 ± 0.12

[a]References 26 and 29.
[b]Reference 28.
[c]Reference 27.
[d]Reference 32.

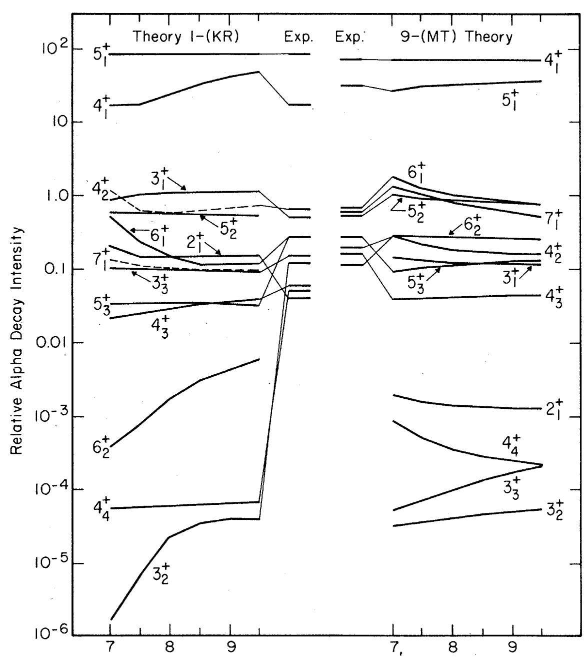

FIG. 5. Comparison of theoretical and experimental α-decay ratios as a function of assumed channel radius. On the left are results for the ^{212}At ground-state decay with Kim-Rasmussen wave functions. On the right are results for the $^{212}At^m$ isomeric state decay with Ma-True wave functions.

is worth reviewing what other tests can be made of the wave functions. Kim and Rasmussen tested their 1^- wave function for β decay of ^{210}Bi and were able to explain the anomalous features. They argued that the off-diagonal elements of the tensor force were essential to give the correct sign of configuration mixing of $h_{9/2}i_{11/2}$ into the dominant $h_{9/2}g_{9/2}$. Indeed, the KR wave function differs in this phase from MT, who used no tensor force, and from Kuo, who used core polarization as well as tensor. Perhaps this sign difference is significant in α decay as well as β decay, though the poorer agreement of KR for the 9^- decay precludes singling out this phase factor as significant.

We shall not attempt a comprehensive review of spectroscopic factors for nucleon transfer reactions into ^{208}Bi and ^{210}Bi, since Ma and True reviewed matters in 1973. Amid overall satisfactory agreement for strong transitions there are remaining puzzles about the weaker transitions. The

TABLE III. Angular momentum mixing ratios.

Transition	$L\alpha$ values	Wave functions: Pure	KR	MT	KH
$1^- \to 4^+$	5 : 3	0.17	21.5	$3.6E-4$	2.59
$9^- \to 4^+$	7 : 5	0.023	0.039	0.034	0.032
$9^- \to 5^+$	7 : 5	0.016	0.072	0.092	0.052

(^{3}He, d) studies on ^{207}Pb by Alford, Schiffer, and Schwartz[33] would seem to offer a sensitive tool to measure configuration mixing in the ^{208}Bi $h_{9/2}f_{5/2}^{-1}$ sextet, since the transitions vanish for no mixing. However, in Table VIII of Ma and True there is a factor of 3 to 6 underestimation of the spectroscopic factors to 4_2^+ and 5_2^+ states. The KR and KH wave functions have even slightly less of the needed mixing than those of MT. Again referring to Table III of Ma and True we see the spectroscopic factor of the 4_4^+ state (mainly $f_{7/2}p_{1/2}^{-1}$) is underestimated by a factor of ~120 for the ^{209}Bi(d, t) reaction, which must go by virtue of $h_{9/2}$ admixtures. This discrepancy ties in with our observation that α decay from 1^- to this 4_4^+ state is also greatly underestimated. This 4_4^+ state was also observed in the ^{209}Bi(p, d) studies of Crawley *et al.*,[34] but they did not report an intensity to facilitate quantitative comparisons. The nice agreement of spectroscopic factors and those with Kuo wave functions in their Table III mainly reflects the dominant shell-model configuration and is not a sensitive test of mixing.

There is thus a concurrence of evidence from nucleon transfer and α decay that none of the three shell-model calculations has enough configuration mixing.

For completeness we should note also the tests of ^{208}Bi by γ branching carried out by Ellegaard, Barnes, and Canada.[35] They compared KR and KH wave functions. For γ branching from most levels KR was best, but there remain significant discrepancies.

There is a clear need for new shell-model theoretical studies on ^{208}Bi with a view to getting the greater configuration mixing called for by experiments.

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