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Spreading widths of giant resonances in ^{12}C and ^{16}O

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The $E2$ and $E4$ resonances in ^{12}C and ^{16}O are calculated within the core-coupling–random-phase approximation.

[NUCLEAR STRUCTURE ^{12}C; ^{16}O; calculated electric giant resonance energies and $B(EL)$ values. Core-coupling RPA.]

I. INTRODUCTION

A large amount of experimental information concerning higher multipole (mainly $E2$) resonances has appeared recently. In ^{16}O it is now commonly accepted[1-5] that there exists a broad giant $E2$ resonance centered at about 21 MeV and distributed over a range of ~10 MeV. In ^{12}C the existence of a localized GQR (giant quadrupole resonance) has stirred much controversy. Whilst some authors[6] locate a concentration of $E2$ strength at about 27 MeV within a range of ~4 MeV exhausting $\gtrsim 30\%$ of the $E2$, $T=0$ energy-weighted sum rate limit, there are more recent measurements[7,8] which have found only 7–17% of such a limit in the 26- to 30-MeV region.

On the other hand, most of the 1p-1h shell model calculations[9-13] are able to explain the positions of the giant $E2$ states. The inclusion of the coupling of the particles to the continuum[11] permits the calculation of the escape width Δ (due to the escape of one nucleon) which turns out to be much too small with respect to the experimental one, e.g., in ^{16}O, $\Delta\sim 1$ MeV.

It is then natural to think that the main part of the observed width of these resonances is due to the broadening (spreading width) caused by the coupling of 1p-1h excitations with more complex configurations, like, e.g., the 2p-2h states. The two existing attempts[14] on ^{16}O so far find strong fragmentation of the giant $E2$ states. Hoshino and Arima[14] find practically no strength below 20 MeV. Knüpfer and Huber[14] find a reasonable qualitative agreement. But their main strength is clustered in two areas around 15.5 and 23 MeV disagreeing quantitatively with the experiment.

Here the $E2$ and $E4$ resonances in ^{12}C and ^{16}O are studied within a new method which we call the core-coupling–random-phase approximation. This method is a special type of 2p-2h random-phase approximation (RPA) calculation which has two important advantages compared to the conventional one: (1) We get more physical insight into the structure of the multipole resonances; (2) we avoid the diagonalization of very large matrices. We can therefore deal almost equally easily with light and heavy nuclei.

The purpose of the present paper is threefold: (1) We develop first the basic equations of the core-coupling–random-phase approximation. This is done in the second section. (2) The new formalism is applied afterwards in Sec. III to the electric giant multipole resonances in light nuclei in order to explain the large broadening of the GQR in these nuclei. (3) Comparing the different experimental results of ^{16}O one notices an obvious discrepancy in the sum rule strength of the GQR between the (p,γ_0) data[1] and the results of (^{3}He, ^{3}He′) and (α,α') experiments.[2-4] The explanation of this puzzle is given in Secs. IV and V. In addition, the experiments show qualitative differences in the structure of the quadrupole resonances in ^{16}O and ^{12}C. Also this problem is discussed in the last two sections.

II. THEORY

The core-coupling–RPA wave functions are written as follows:

$$|\Psi\rangle=\sum_{mi}(X_{mi}\chi_m^\dagger\chi_i-Y_{mi}\chi_i^\dagger\chi_m)|\phi_0\rangle, \tag{1}$$

where the operator $\chi_m^\dagger$ defined in the core-coupling approach as

$$\chi_m^\dagger=C_m a_m^\dagger+\sum_{\alpha\nu}C_{\alpha\nu}^{(m)}B_\alpha^\dagger a_\nu^\dagger \tag{2}$$

creates when acting on the correlated ground state

$|\phi_0\rangle$ a single-particle state which contains admixtures of 2p-1h character. We call such a state a quasiparticle state. The operator $\chi_i^\dagger$ creates a quasiparticle below the Fermi surface $(i<F)$ while $a_m^\dagger$ $(m>F)$ and $a_\nu^\dagger$ $(\nu>F)$ are the usual single-particle creation operators, respectively. $B_\alpha^\dagger$ represents the creation operator of a core excited state, which is here described in the 1p-1h RPA as follows:

$$B_\alpha^\dagger = \sum_{p_1h_1} [\xi_{p_1h_1}^{(\alpha)} (a_{p_1}^\dagger a_{h_1})_{J_\alpha} - \eta_{p_1h_1}^{(\alpha)} (a_{h_1}^\dagger a_{p_1})_{J_\alpha}]. \tag{3}$$

The amplitudes X and Y of the 2p-2h RPA state are determined by the following equations:

$$\begin{pmatrix} P & Q \\ Q^* & P^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = E \begin{pmatrix} X \\ -Y \end{pmatrix} \tag{4}$$

with

$$\begin{aligned} P_{mi;nk} &= (\epsilon_m - \epsilon_k)\delta_{m;n}\delta_{i;k} + \langle km|V|ni\rangle, \\ \langle km|V|ni\rangle &= C_m C_n \langle j_k^{-1} j_m JM|V|j_n j_i^{-1} JM\rangle \\ &\quad + C_m \sum_{\alpha,\nu_1} \sum_{p_1,h_1} C_{\alpha,\nu_1}^{(n)} [\xi_{p_1h_1}^{(\alpha)} - (-1)^{j_{h_1}+j_{p_1}-J_\alpha} \eta_{p_1h_1}^{(\alpha)}] \langle j_k^{-1} j_m; JM|V|j_{p_1} j_{h_1}^{-1}(J_\alpha) j_{\nu_1}(j_n) j_i^{-1}; JM\rangle \\ &\quad + C_n \sum_{\alpha,\nu_2} \sum_{p_2h_2} C_{\alpha,\nu_2}^{(m)} [(-1)^{j_{h_2}+j_{p_2}-J_\alpha} \xi_{p_2h_2}^{(\alpha)} - \eta_{p_2h_2}^{(\alpha)}] (-1)^{j_{\nu_2}+j_k+J_\alpha-J} \\ &\qquad \times \langle j_{p_2} j_{h_2}^{-1}(J_\alpha) j_{\nu_2}(j_m) j_k^{-1} JM|V|j_n j_i^{-1} JM\rangle + \langle 2p2h|V|2p2h\rangle, \end{aligned} \tag{5}$$

$$Q_{mi,nk} = \langle mn|V|ik\rangle = C_m C_n \langle j_m j_n JM|V|j_i^{-1} j_k^{-1} JM\rangle. \tag{6}$$

The interaction which connects two 2p-2h configurations with each other produces only slight shifts in the energy of the resonances but does not influence the strength distribution. We therefore have neglected these terms. The antisymmetrization between the core-excited states $B_\alpha^\dagger$ and the additional particle $a_m^\dagger$ has been neglected in order to simplify the numerical treatment. This neglected effect between the particle states will influence slightly the distribution of the single-particle strength in the 2p-1h problem. The most serious corrections in this respect are expected for the $2s_{1/2}$ particle state. Fortunately this level does not contribute $(2^+, 3^-)$ or contributes only slightly (2_1^-) to the core states which are the most important ones in our approach. Therefore the inclusion of the antisymmetrization may change somewhat the quantitative picture of the splitting but will certainly not modify the qualitative behavior. On the other hand, the Pauli principle is taken into account exactly in the following 2p-2h calculation (antisymmetrization of the hole states). In the present calculation the core coupling is only taken into account in the particle states, whereas the holes are described as pure single-hole states. Otherwise we would get contributions which would account also for 3p-3h admixtures.

III. NUMERICAL DETAILS

To calculate the quasiparticle states we first have to solve the equation of motion of the quasiparticle propagator according to the method outlined in Ref. 16. Equation (2) shows that the number of 2p-1h admixtures increases with the number of core states α which are to be coupled with the single-particle degrees of freedom ν. We have observed that for the splitting of the giant multipole resonances it is enough to consider those 2p-1h admixtures coming from the most collective low-lying (1p-1h) RPA states of the core. Therefore we need to take into account in (2) only a few states α. In the ^{12}C case we consider the 2^+ (4.44 MeV) and 3^- (9.64 MeV) as states α. In ^{16}O it is enough to take into account the 3^- (6.13 MeV) state, although for the sake of checking our method we also include in the present calculation two other states, namely, 1^- (13.10 MeV) and 2^- (12.53 MeV). It turns out that the influence of these two additional states is of minor importance for the splitting of the isoscalar GQR.

In the actual calculations the density dependent Migdal δ-function force with parameters fitted[15] in the ^{208}Pb region to electromagnetic properties is used. The interpolation radius R and the total strength C were readjusted to reproduce the low-lying 2^+ state in the ^{12}C case and the low-lying 3^- state and its $B(E3)$ value in the ^{16}O case. These values are shown in Table I. The single-particle

TABLE I. Interpolation radii R and strength parameters C of the p-h interaction.

	R (fm)	$r_0 = R/A^{1/3}$	C (fm^3 MeV)
^{12}C	2.44	1.06	42.5
^{16}O 1p-1h	2.80	1.11	43.0
^{16}O 1p-1h + 2p-2h	2.80	1.11	40.0

TABLE II. Single-particle energies (MeV).

	^{12}C		^{16}O	
Orbit	Proton	Neutron	Proton	Neutron
$1s_{1/2}$	−34.80	−38.80	−40.00	−42.00
$1p_{3/2}$	−14.80	−17.00	−18.43	−21.80
$1p_{1/2}$	−1.94	−4.95	−12.10	−15.70
$1d_{5/2}$	1.60	−0.88	−0.60	−4.15
$2s_{1/2}$	1.15	−1.55	−0.10	−3.27
$1d_{3/2}$	5.84	3.00	4.54	1.57
$1f_{7/2}$	15.00	11.00	15.50	12.50
$1f_{5/2}$	21.50	19.60	18.90	15.50
$2p_{3/2}$	19.20	17.00	16.00	14.50
$2p_{1/2}$	20.50	18.80	19.50	16.20
$1g_{9/2}$			27.60	27.00

energies are written down in Table II. They were taken from experiment when available and all other energies and wave functions were taken from a Woods-Saxon calculation.

IV. RESULTS AND DISCUSSION

Figure 1 shows the distribution of the single-particle strength and the main structure of the quasiparticle states $\frac{7}{2}^-$ and $\frac{5}{2}^-$ of ^{17}F created by $\chi_m^\dagger$ in putting a proton on said orbits of ^{16}O core. Owing to the $(3^-, 1^-, 2^-)$ coupling a large part of the $f_{7/2}$ strength is fragmented by shifting from its initial location in an interval of about 10 MeV. Mainly three new states appear at energies 4.8

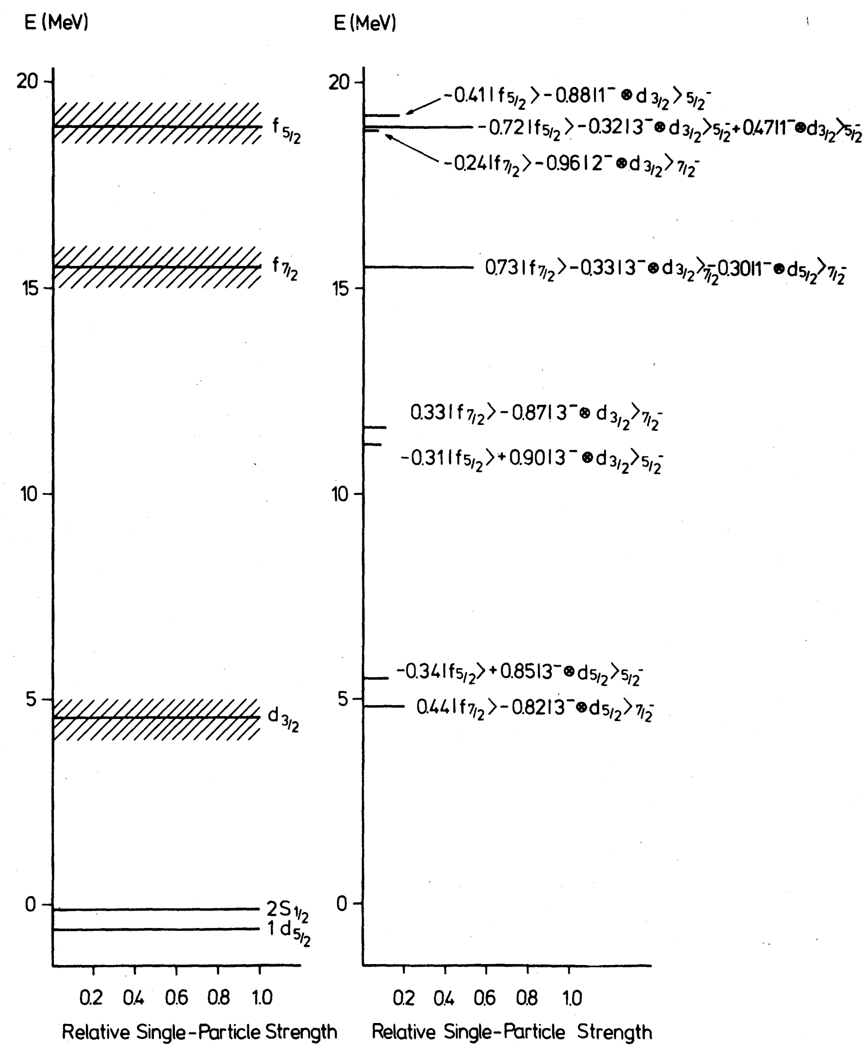

FIG. 1. The left side of this figure shows the unperturbed single-particle position for the last proton in ^{17}F. The right-hand side gives the splitting of the $f_{7/2}$ and $f_{5/2}$ strength due to the admixture of the collective 3^- and the 2^- and 1^- states indicated in the text.

MeV (19.8%), 11.6 MeV (11%), and 18.9 MeV (6%). The $f_{5/2}$ single-particle strength is also distributed in four states at energies 5.5 MeV (11.8%), 11.2 MeV (9.4%), 18.9 MeV (52.4%), and 19.2 MeV (16.7%). These states, except the one at 19.2 MeV, have $(3^- \times 1d_{3/2})$ and $(3^- \times 1d_{5/2})$ as dominant contributions.

The $E2$ and $E4$ states in ^{12}C obtained in our calculations are presented in Fig. 2. In the upper part the 1p-1h calculations show a strong isoscalar $E2$ resonance at 25 MeV and an isovector one at 31.4 MeV. Both resonances have as main particle-hole (p-h) components the pairs $1p_{1/2}$-$1p_{3/2}^{-1}$ and $1f_{7/2}$-$1p_{3/2}^{-1}$. Owing to the $(2^+, 3^-)$ coupling, the $1f_{7/2}$ state strongly fragments by shifting up more than 50% of its single-particle strength in an interval $\gtrsim$ 15 MeV. As shown in the lower part of the figure, the inclusion of the 2p-2h states strongly splits the isoscalar GQR.

We also observe two concentrations of $E2$ strength between 30 and 40 MeV and between 45 and 55 MeV exhausting, respectively, 32 and 25% of the isovector plus isoscalar energy-weighted sum rule (IV+IS EWSR). In the 15- to 26-MeV region we find that all the states, except the narrow one at 15 MeV, are purely isoscalar and deplete 13% of the IS EWSR. Two other main narrow $E2$ peaks at 24 and 30.7 MeV jump out. Whereas the first one contains many p-h configurations, none of which is dominant, the second peak has a large component of the $(f_{7/2}$-$p_{3/2}^{-1})$ configuration where the quasiparticle $f_{7/2}$ state contains 35% of the single-particle strength. Consequently the $E2$ peak at 30.7 MeV depends strongly on the "unperturbed" energy $\epsilon(f_{7/2})$ which is not known experimentally.

Figure 2 also shows the splitting of the strong 1p-1h $E4$ resonance produced by the inclusion of the 2p-2h states. We find that 2.8, 12.7, and 13.1% of the corresponding IV+IS EWSR is exhausted in the intervals 26–30, 30–40, and 45–55 MeV, respectively. In recent (α, α') experiments[8] a relatively broad peak at 29.2 MeV has been detected, whose angular distribution does not correspond to a pure $L=2$ state. According to our results, this peak may be a mixture of $E2$ and $E4$ states.

In Fig. 3 the 2^+ and 4^+ states of ^{16}O are shown. The 1p-1h calculations present a giant $E2$, $T=0$ resonance at 23.1 MeV and a giant $E2$, $T=1$ state at 43.1 MeV. We have already seen above that the coupling of the $(3^-, 1^-, 2^-)$ core states causes a strong fragmentation in the single-particle resonances $\frac{7}{2}^-$ and $\frac{5}{2}^-$. Therefore we expect that the inclusion of the corresponding 2p-2h states will produce a strong splitting of the giant 1p-1h GQR. This is seen in the lower part of the figure and is examined and compared with various recent experiments in Table II. Agreement is found everywhere except with the $^{15}N(p, \gamma_0)$ experiment. This is probably due to the fact that this reaction does not permit direct excitation in the $f_{7/2}$ channel, owing to the initial $p_{1/2}$ hole in ^{15}N.

The 2^+ levels at 6.92, 11.52, and 13.1 MeV, which carry an appreciable part of the sum rule strength, are not described in our calculation, because they have important admixtures of np-nh ($n>2$) excitations. These higher configurations are presumably also responsible for a larger fragmentation of the isoscalar $E2$ resonance in the interval between 16 and 26 MeV compared with the present results. The three strong theoretical levels in this interval exhaust 55% of the IS EWSR which is the upper limit of the experimental sum rule strength. Even so, we are aware of the fact that the admixture of higher np-nh configurations which are responsible for also the 2^+ levels at 6.92, 11.52, and 13.1 MeV (which exhaust ~25% of the sum rule) reduces the sum rule strength in the region between 16 and 26 MeV. This would further improve the agreement with the experimental data (see Table III).

The position of the isovector GQR remains unchanged at 43 MeV, but its $B(E2)$ value is slightly reduced. We also observe that ~8% of the IS EWSR lies over the 30- to 35-MeV region and 33% of the IV+IS EWSR is exhausted between 30 and 40 MeV.

On the right hand side of the figure, we observe that the giant 1p-1h isoscalar $E4$ resonance at 25.5 MeV is very much fragmented, keeping the main strength at 21.3 MeV. We find that ~28% of the IS EWSR lies between 16 and 28 MeV.

In order to investigate the dependence of our results on the number of core states, we have used a larger basis including the following three additional core states, namely, 2^- at 12.97 MeV, 1^- at 12.44 MeV, and 3^- at 13.25 MeV. These states are known to be fairly well described by the 1p-1h RPA. From Fig. 4 one notices that the strength distribution in the isoscalar GQR region remains

TABLE III. Comparison of $E2$ strength in percentages of the isoscalar EWSR with various recent experiments in ^{16}O.

E_1–E_2 (MeV)	S_{exp} (%)	Reaction	S_{th}(%)
16–26	40^{+20}_{-10}	(α, α') [a]	55
17–28	75	$(^3He, ^3He')$ [b]	63
15–28	67 ± 25	(α, α') [c]	83
20–30	30	(p, γ_0) [d]	62

[a]See Ref. 4. [c]See Ref. 3.
[b]See Ref. 2. [d]See Ref. 1.

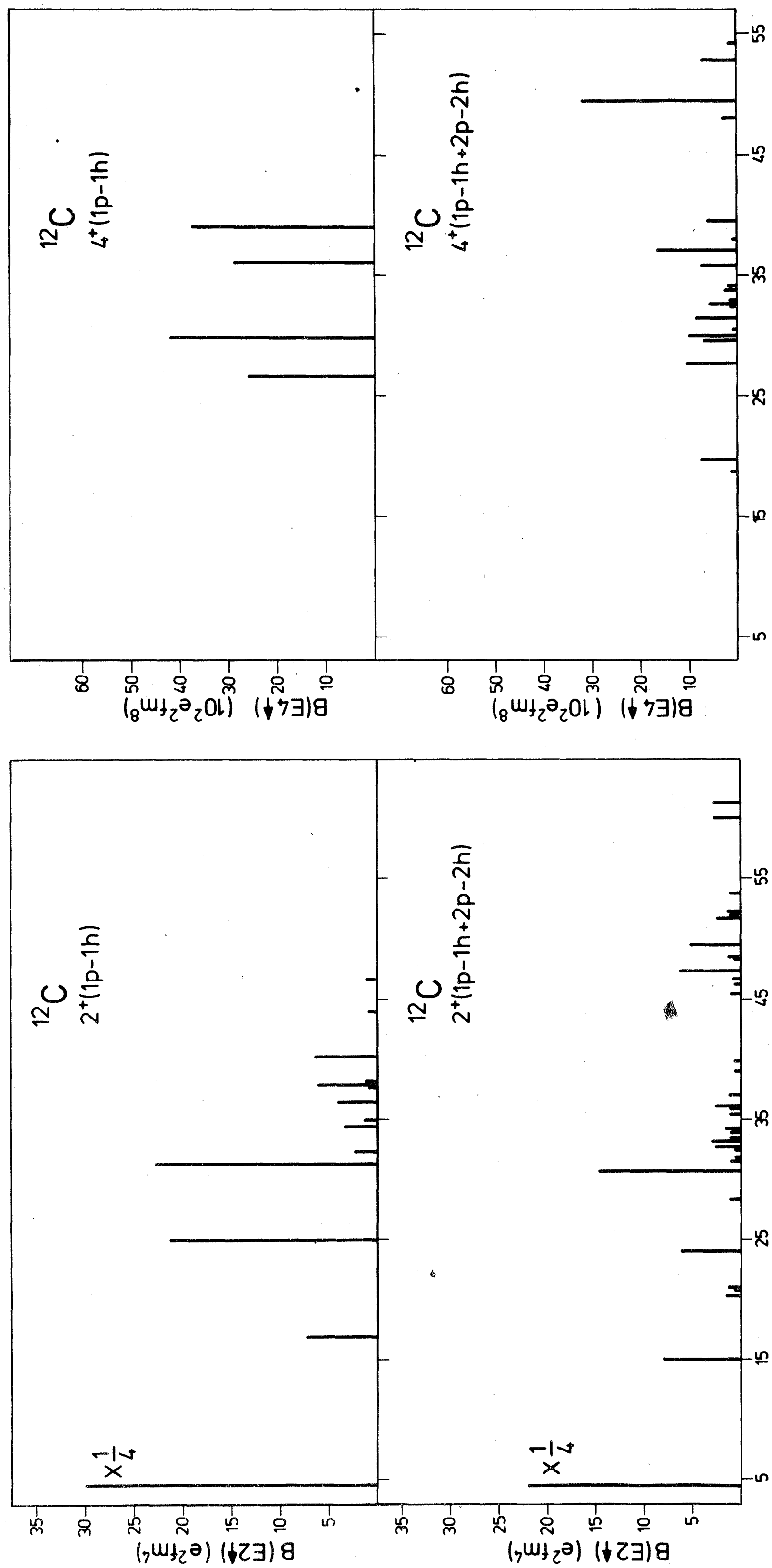

FIG. 2. Giant quadrupole and hexadecapole resonances in ^{12}C calculated in the 1p-1h and in the core-coupling (1p-1h+2p-2h) random-phase approximation. The core excited states 2^+ and 3^- are taken into account.

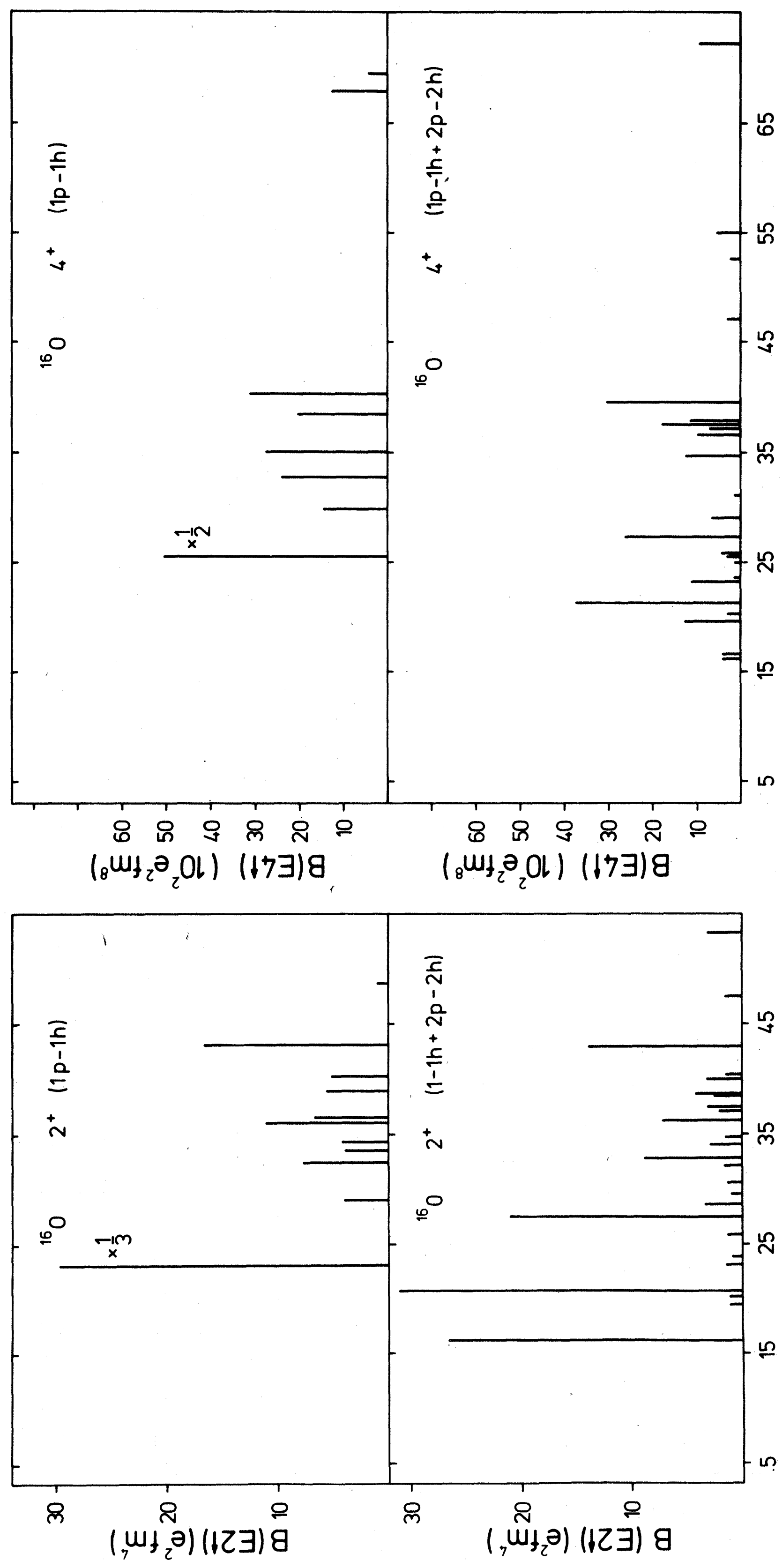

FIG. 3. Giant quadrupole and giant hexadecapole resonances in ^{16}O calculated in the 1p-1h and in the core-coupling (1p-1h+2p-2h) random-phase approximation. The core excited states 3_1^-, 1_1^-, and 2_1^- are taken into account.

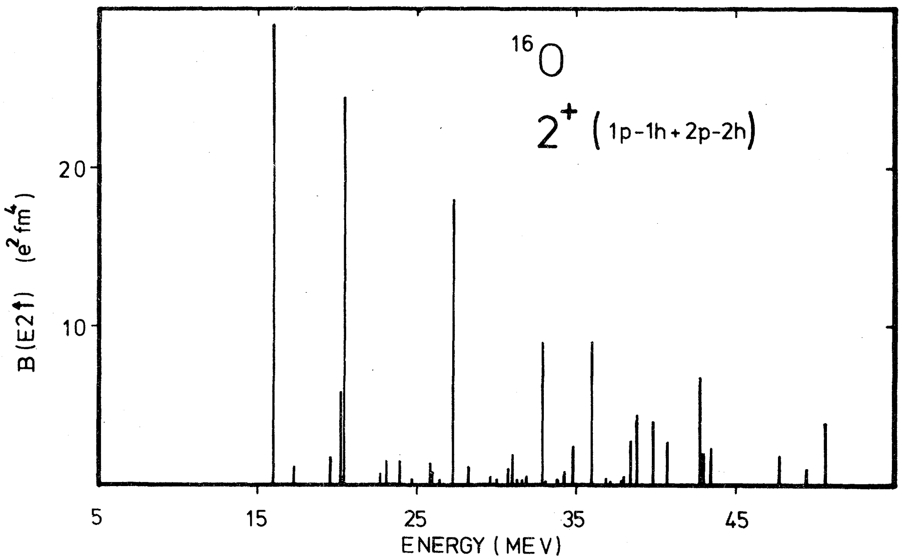

FIG. 4. Giant quadrupole resonances in ^{16}O calculated in the core-coupling (1p-1h+2p-2h) random-phase approximation. The core excited states 3_1^-, 1_1^-, 2_1^-, 3_2^-, 1_2^-, and 2_2^- discussed in the text are considered.

nearly unchanged, i.e., there are always three main peaks exhausting the same portion of the IS $E2$ EWSR strength. These high-lying core states influence only the strength distribution above 30 MeV. One can readily observe that the only appreciable change is the splitting of the IV $E2$ state at ~ 43 MeV into three pieces. The strength remains however in the same energy region. On the other hand, for a consistent 2p-2h RPA approach one should not include the low-lying positive parity states of ^{16}O because it is well known that these states are built up by higher np-nh configurations.

V. CONCLUSION

We have developed in the present work the core-coupling-random-phase approximation which is a special kind of a 2p-2h RPA calculation. One advantage of our approach compared with the conventional 2p-2h RPA is a more physical insight into the results of the calculation. In addition the method can also be used in heavy nuclei such as ^{208}Pb, because one can restrict, with physical arguments, the number of the core excited states.

The new approach has been applied to the giant multipole resonances in ^{12}C and ^{16}O. We have shown that the broadening of the GQR in ^{12}C and ^{16}O and very probably in all the light nuclei ($A \leq 30$) is due to the fragmentation of the $f_{7/2}$ resonance, since the narrow part of the GQR is mainly built up by the $f_{7/2} \times p_{3/2}^{-1}$ configuration. The other possible configurations contribute only to a broad background, since the theoretical widths of the $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ single-particle resonances are very large. In heavier nuclei ($A > 30$) this situation changes because other channels than the $f_{7/2}$ show also narrow width. In that case one obtains the normal situation of a coherent superposition of many 1p-1h configurations.

The present calculation gives also a natural explanation of the different experimental results in ^{16}O obtained with (p,γ_0) (Ref. 1) and (^{3}He, ^{3}He′) and (α,α').[2-4] In the first experiment the (narrow) $f_{7/2} \times p_{3/2}^{-1}$ configuration cannot be excited directly. Therefore, in this experiment mainly the broad background is excited via the $p_{1/2}$ hole. On the other hand, in the (^{3}He, ^{3}He′) and (α,α') experiments also the (narrow) $f_{7/2} \times p_{3/2}^{-1}$ component is excited.

In addition our calculation of the GQR in ^{12}C gives only a small fraction of the GQR-sum rule strength between 20–30 MeV, in qualitative agreement with experiment.

Finally, in both cases, ^{12}C and ^{16}O, an appreciable fraction of the $E4$-sum rule strength is concentrated in the same region as the GQR. This one should always bear in mind if one tries to deduce quantitatively $B(E2)$ strength from inelastic hadron scattering experiments and if one compares the sum rule fraction of the GQR obtained with different experimental methods.

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