Reduced transition probabilities of vibrational states in $^{154-160}$Gd and $^{176-180}$Hf

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The vibrational states of ^{154,156,158,160}Gd and ^{176,178,180}Hf have been studied via Coulomb excitation. Thin high-purity targets were prepared in an isotope separator. Coulomb excitation of these nuclei was studied by the scattering of 11–17 MeV α particles. The particles were detected in an Enge split-pole spectrograph. The reduced transition probabilities $B(E\lambda;0^+_{\text{g.s.}} \rightarrow I^\pi K)$, were obtained for $I^\pi K=2^+2$ states in each nucleus, and for $I^\pi K=2^+0$ and $I^\pi=3^-$ states in several nuclei. The $K^\pi=2^+$ levels are reasonably constant in energy and $B(E2)$ strength while the $K^\pi=0^+$ states change rapidly in energy and excitation strength with neutron number. In both the softly deformed Gd nuclei and the more strongly deformed Hf nuclei, the lightest nucleus has the lowest energy and largest $B(E2)$ for an $I^\pi K=2^+0$ state, despite the difference in the sizes of the deformations at these lightest masses.

[NUCLEAR REACTIONS $^{154-160}$Gd(α,α'), $^{176-180}$Hf(α,α'), $E=11$–17 MeV; measured σ, deduced $B(E\lambda)$ and $M(E\lambda)$. Enriched targets.]

I. INTRODUCTION

It has been pointed out in a review[1] of the relative transition probabilities from vibrational states in gadolinium, hafnium, and neighboring nuclei that a microscopic approach is needed to explain the decay properties of these levels except in the very middle of the deformed region ($150 \lesssim A \lesssim 190$). In any microscopic calculation, the $B(E\lambda)$ strengths from the vibrational levels provide a significant test of the particular approach. The stable even-even Gd and Hf isotopes are excellent candidates for testing microscopic calculations of deformed nuclei because (a) β- and γ-type vibrational bands occur in many of these isotopes; (b) other, $K^\pi=0^+$, bands are reported in several of these; (c) the region of deformation they span is large since with increasing neutron number N, the Gd nuclei go from ^{152}Gd at the onset of deformation with $N=88$ to the well-deformed ^{160}Gd, and the Hf nuclei go from the well-deformed ^{174}Hf to the more softly deformed ^{180}Hf. These same isotopes are also of interest because recent theoretical calculations[2-4] of ground-state deformations predict the Gd nuclei to have large positive hexadecapole deformation parameters (β_4) and the Hf nuclei to have small negative values. A later paper will present $E2$ and $E4$ moments from which one may extract these charge deformation parameters for most of the nuclei studied here.

In this paper we present the results of a series of measurements of the absolute $B(E\lambda)$ strengths for $I^\pi K=2^+2$ and 2^+0, and $I^\pi=3^-$ vibrational states in $^{154-160}$Gd and $^{176-180}$Hf as deduced from Coulomb excitation via the (α,α') reaction. Systematic trends for the energies and collective strengths of these states are presented. The $I^\pi K=2^+2$ and 2^+0 states in Gd and Hf show remarkable similarities with increasing neutron number despite the fact that the deformation is increasing in Gd and decreasing in Hf with N. Comparison with other reported measurements and with theoretical predictions of $B(E\lambda)$ strengths and level energies are made.

II. EXPERIMENTAL PROCEDURE

α particles were obtained in the EN tandem Van deGraaff accelerator at the Oak Ridge National Laboratory. A 20-cm-long position-sensitive gas-flow proportional counter, mounted in the focal plane of an Enge split-pole magnetic spectrograph, was used to detect the elastically and inelastically scattered α particles. Most of the targets were studied by observing the scattered particles at two laboratory angles, 150° and 90°. The efficiency and linearity calibrations of the detector-spectrograph system were made by measuring the yields and positions of the two main α groups in the decay of ^{244}Cm as a function of magnetic field strength. This detector was found to be uniformly efficient ($\leq 1\%$ average variation) across most of its length.

The targets were thin (~20–25 $\mu g/cm^2$ in most cases) very pure (>99%) isotopes of Gd and Hf material deposited on thin (75–100 $\mu g/cm^2$) carbon or nickel foils at the ORNL 180° sector isotope separator. The ratios of the elastic peak heights to average backgrounds above the $I^\pi = 4^+$ ground-band peaks were typically better than 10^4. The ratios of the elastic peak heights to the valley depths between the elastic and 2^+ ground-band peaks were typically better than 500. Peak widths at half-maximum were usually 18 to 25 keV. Beam energies were chosen by extrapolating the results of recent Coulomb-nuclear interference measurements on the excitations of the first 2^+ and 4^+ states.[5,6] We used higher beam energies when detecting particles scattered into 90° so that about the same distances of closest approach as for the 150° experiments would be achieved. However, few studies have been made concerning Coulomb-nuclear excitation interferences for vibrational states. Early studies[7] of Sm nuclei indicated that the nuclear excitation onset is the same for vibrational 2^+ and 3^- states as for the ground-band 2^+ state. A very recent study[8] of Os nuclei seems to yield the same conclusion for $I^\pi K = 2^+2$ states.

III. DATA ANALYSIS

Figures 1 and 2 show typical α-particle spectra from our measurements. Experimental ratios of inelastic-to-elastic differential scattering cross sections, $d\sigma_{2^+ \text{or} 3^-}/d\sigma_{el} \equiv R_{exp}$, were found by normalizing the areas of the peaks due to inelastic scattering to the elastic peak, after fully resolving the latter from the $2^+_{g.s.}$ peak. Then the R_{exp} values were compared with values (R_{calc}) calculated with the semiclassical Coulomb-excitation code of Winther and deBoer,[9] modified[10] to include $E1$, $E3$, and $E4$ excitations, with the matrix elements $\langle 0^+_{g.s.} \| M(E\lambda) \| I^\pi \rangle$ as free parameters. Baktash *et al.*[11] reported differences of ~3% (at θ_{lab} ~170°) between semiclassical and quantum-mechanical calculations of R_{calc} for $I^\pi K = 2^+2$ states. This difference is in the treatment of higher-order excitation processes, such as $0 \to 2 \to 2'$. Since these are smaller at 90°, where our best data were obtained, we chose to use solely the semiclassical analysis. Reduced matrix elements connecting the $0^+_{g.s.}$ through $6^+_{g.s.}$ states with the 0^+ and 2^+ members of excited $K^\pi = 0^+$ bands and 2^+ and 4^+ members of the $K^\pi = 2^+$ bands were used in the calculations. Interband and intraband matrix element relations[12] were deduced from the adiabatic rotor-vibrator (collective) model predictions[13] although γ-ray branching ratio data, corrected for $M1$ admixtures,[1,14] were used for interband relations when they existed. Branching ratios of γ rays from vibrational to rotational states in most nuclei in the rare-earth region deviate significantly[1] from this model because of the rotation-vibration mixing. In ^{156}Gd, for $\theta = 90°$ and $E_\alpha = 15.3$ MeV, use of the collective model relations rather than the measured relations would decrease R_{calc} for the $I^\pi K = 2^+2$ state by 0.3% and for $I^\pi K = 2^+0$ states by about 6%. This reflects the stronger mixing with the ground band by the $K^\pi = 0^+$ band than by the $K^\pi = 2^+$ band. Baktash *et al.*[11] examined the

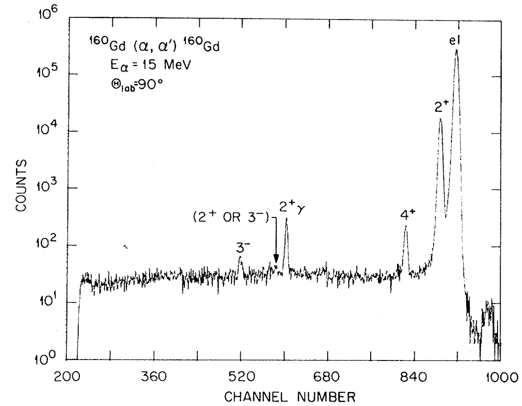

FIG. 1. Spectrum of elastically and inelastically scattered 15-MeV α particles from ^{160}Gd at a lab angle of 90°.

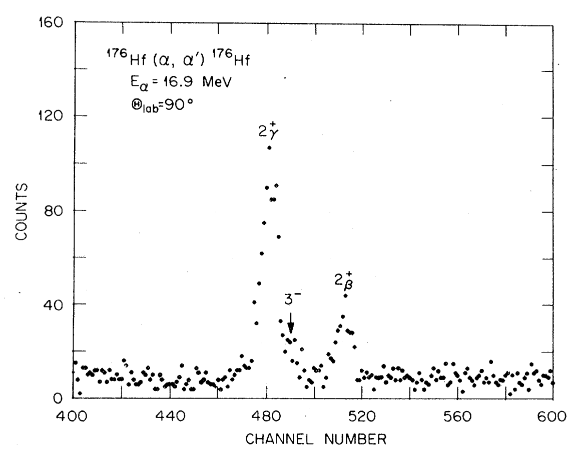

FIG. 2. Spectrum of inelastically scattered 16.9-MeV α particles from ^{176}Hf at a lab angle of 90°. Excitations of the $I^\pi K = 2^+2$, 2^+0, and 3^-2 states at 1341, 1227, and 1313 keV are observed.

effect of the static moment of the $I^{\pi}K=2^{+}2$ state on the analysis at a backward angle of ~170°. A variation of the static moment for ^{160}Gd from zero to a rotational model value increases the calculated cross section ($E_{\alpha}=12$ MeV) of this state by ~11% at 150°, whereas for the same separation distance at 90°, the effect is ~6%. In our calculations rotational model values are assumed for the static moments of the 2^{+} states. Some recent experimental data[15] indicate the validity of this assumption at least for $I^{\pi}K=2^{+}2$ states. In general, much more experimental effort is needed to determine signs and magnitudes of static moments for $I^{\pi}=2^{+}$ and 3^{-} states of excited bands.

Strong mixing of the ground and excited $K=0$ bands can affect the interband transition matrix elements to the point that their signs may differ from the collective model predictions. Not only are the magnitudes of these matrix elements somewhat uncertain but also their signs. Kumar[16,17] pointed out that the signs should be taken either from experiment or a microscopic theory. Lacking experimental or theoretically reliable signs, we have used those deduced from the collective model. For an estimate of the importance of this, we considered the $4_{g.s.}\to 2\beta$ element for ^{156}Gd ($\theta=150°$ and $E_{\alpha}=15.3$ MeV). It is strongly affected by bandmixing (see Sec. IV C). A change in the sign of the matrix elements connecting the $4^{+}_{g.s.}$ state with the $I^{\pi}K=2^{+}0$ vibrational states was made. These sign changes have a negligible (~1%) effect on R_{calc} for these $I=2^{+}$ states.

TABLE I. Experimental absolute $B(E\lambda)$ strengths for $^{154-160}$Gd and $^{176-180}$Hf from (α,α') Coulomb excitation studies.

Nucleus	E (keV)	$I^{\pi}K$	$B(E\lambda; 0^{+}\mathrm{g.s.}\to I^{\pi}K)$ [a] (e^2b^{λ})	$B(E\lambda)/B(E\lambda)_{s.p.}$ [b]	Other measurements Value	Other measurements Reference
^{154}Gd	123	2^{+}g.s.	3.85(8)	157(3)		
	815	$2^{+}0$	0.015(4)	0.6(2)		
	996	$2^{+}2$	0.143(11)	5.9(5)		
^{156}Gd	89	2^{+}g.s.	4.57(5)	183(2)		
	1129	$2^{+}0$	0.013(4)	0.5(2)		
	1154	$2^{+}2$	0.120(4)	4.8(2)		
	1258	$2^{+}0$	<0.008	<0.3		
	1276	3^{-}	0.16(4)	16(4)		
^{158}Gd	80	2^{+}g.s.	4.97(5)	196(2)		
	1187	$2^{+}2$	0.090(10)	3.6(4)	0.106(15)	11
	1260	$2^{+}0$	<0.002	<0.08		
	1517	$2^{+}0$	<0.002	<0.08		
^{160}Gd	75	2^{+}g.s.	5.15(6)	200(2)		
	992	$2^{+}2$	0.101(3)	3.9(1)	0.104(4)	11
	1070	($2^{+}0$ or 3^{-}?)	0.002(2) e^2b^2 or 0.01(1) e^2b^3	0.08(8) or $(1\pm1)\times10^{-4}$		
	1289	(3^{-}?)	0.127(14)	11.9(13)		
^{176}Hf	88	2^{+}g.s.	5.19(6)	177(2)		
	1227	$2^{+}0$	0.031(3)	1.0(1)	0.025(5)	18
	1313	3^{-}?	0.093(29)	7.2(23)		
	1341	$2^{+}2$	0.119(8)	4.1(3)	0.075(6)	18
^{178}Hf	93	2^{+}g.s.	4.86(5)	163(2)		
	1175	$2^{+}2$	0.115(4)	3.9(1)	0.100(8)	19
	1277	$2^{+}0$	0.0018(7)	0.07(3)	≤0.002	19
	1323	$3^{-}2$	0.053(10)	4.0(8)		
	1496	$2^{+}0$	0.013(2)	0.44(7)	≤0.010	19
^{180}Hf	93	2^{+}g.s.	4.73(5)	157(2)		
	1201	$2^{+}2$	0.114(7)	3.8(3)	0.110(11)	20

[a] The uncertainties (bracketed) are one-standard-deviation values and represent variations in the last digits of the best values. For example, 0.143(11) may be written as 0.143 ± 0.011.

[b] $B(E\lambda)_{s.p.}=(2\lambda+1)/4\pi\,[3/(\lambda+3)]^2(0.12A^{1/3})^{2\lambda}\,e^2b^{\lambda}$ for $I_i=0$ and $I_f=\lambda$.

IV. RESULTS AND DISCUSSION

A. Experimental results

Table I gives energies and absolute $B(E2)$ and $B(E3)$ values for $I^\pi = 2^+$, 3^- states in $^{154-160}$Gd and $^{176-180}$Hf. The $E2$ and $E4$ reduced matrix elements for these nuclei are being prepared for publication, and will be compared with other results then. The $B(E2)$ values for the $2^+_{\text{g.s.}}$ states do verify that the heavier Gd nuclei are more deformed than the lighter ones. The falling energies of the $2^+_{\text{g.s.}}$ states and the rising $B(E2)$ values are signatures of increasing deformation. In contrast, the heavier Hf nuclei are less deformed than the lighter ones. Previous measurements[18-20] of some of the vibrational $B(E2)$ values are also given in Table I. Uncertainties related to effects not discussed in Sec. III (e.g., those from unincorporated quantum-mechanical corrections) are not included in our quoted error limits.

1. ^{154}Gd

Because of relatively poor target quality, only the low-lying $I^\pi K = 2^+0$ and 2^+2 states at 816 and 996 keV, respectively, could be identified; the $I^\pi = 3^-$ state at 1353 keV[21] was not observed. Although the $B(E2)$ strength of the 2^+0 state is the largest compared with similar states in the Gd nuclei, it may be less than a single particle unit (s.p.u.). There is, however, strong mixing[22] between this excited $K^\pi = 0^+$ band and the ground band.

2. ^{156}Gd

This nucleus has three $I^\pi = 2^+$ states[23] within 130 keV of each other. However, only two, the $K=2$ state at 1154 keV and $K=0$ state at 1129 keV, were found to have significant $B(E2)$ strength.

3. ^{158}Gd

Only the $I^\pi = 2^+2$ state at 1187 keV is seen with measurable collective strength; upper limits are placed on the absolute $B(E2)$ strengths of the two $I^\pi K = 2^+0$ states observed at 1260 and 1517 keV in decay studies.[24]

4. ^{160}Gd

We observed strongly excited states at 992 and 1289 keV, and a weakly excited state at 1070 keV. All three of these states correspond to levels seen at or near these energies in other studies.[11,21,25-27] A state at 1070 keV has previously been assigned $I^\pi K = 2^+0$ (Ref. 21), 3^-1 (Ref. 21), $3^+(?)$ (Ref. 25), and $4^+(?)$ (Ref. 26). However, the large direct excitation probabilities with α particles restricts the possible spin parity assignments to 2^+ or 3^-. We cannot choose between these two. For the same reason, we favor the $I^\pi = 3^-$ assignment[21] rather than $I^\pi = 2^-$ (Ref. 26) for the state at 1289 keV.

5. ^{176}Hf

Well established $I^\pi K = 2^+2$, 2^+0, and 3^-2 states at 1341, 1227, and 1313 keV, respectively, are definitely excited, but there is no evidence for collectivity in the proposed[28] second $I^\pi K = 2^+0$ state at 1379 keV. Our $B(E2)$ value for the 1227-keV state is in agreement with the recent (α,α') measurement of Hammer, Ejiri, and Hagemann[18] but for the 1341-keV state their $B(E2)$ value is two-thirds of ours. Our result is the average of data taken at 90° and 150°, which were found to be in good agreement.

6. ^{178}Hf

Two $I^\pi K = 2^+0$ states[29] have been observed here, one at 1277 keV and the other at 1496 keV. The latter has a $B(E2)$ value more typical of a vibrational state. Earlier $(\alpha, \alpha'\gamma)$ Coulomb excitation results[19] for these states and the $I^\pi = 2^+$ state at 1175 keV are in good agreement with our more accurate $B(E2)$ values.

7. ^{180}Hf

Our study has shown that only the $I^\pi K = 2^+2$ state at 1201 keV is collective, thus supporting the work of Varnell, Hamilton, and Robinson.[20]

B. Collective strength and energy level trends

The trends of the $B(E2)$ strengths (in single particle units) and of the energies of $I^\pi = 2^+$ vibrational states are shown in Fig. 3 for $^{154-160}$Gd. Figure 4 shows these systematics for $^{174-180}$Hf, with the data for ^{174}Hf taken from Ref. 30. In both the Gd and Hf nuclei the $I^\pi K = 2^+2$ states are nearly constant in energy and decrease slowly in collectivity as N increases. On the other hand, the most collective $I^\pi K = 2^+0$ states change rapidly in energy. The 2^+2 and 2^+0 states with the largest $B(E2)$ values are in the most neutron deficient isotopes of Gd and Hf. The 2^+0 state then lies the lowest in energy. That the same trends are seen in both Gd and Hf nuclei is surprising in that for increasing N, the Gd nuclei are more deformed but the Hf nuclei are less deformed.

In a macroscopic sense, the increase of a vibrational energy and corresponding decrease in its $B(E2)$ strength indicate an increasing stiffness to this vibration since the level energy E and the $B(E2)$ value are related in the collective model to

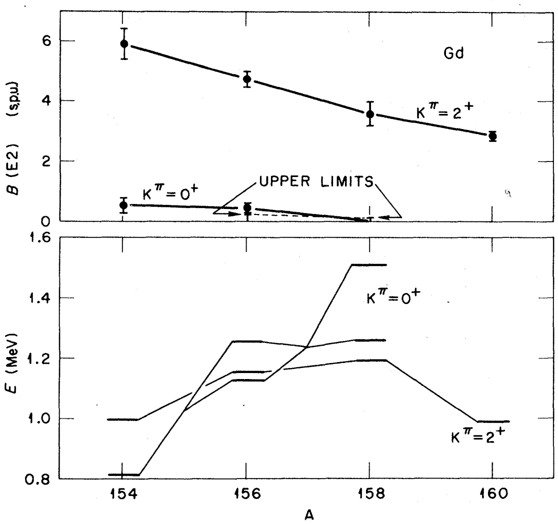

FIG. 3. Absolute $B(E2)$ strength (in single particle units) and level energy systematics for $^{154-156}$Gd for 2^+ states of bands with K values given in the figure.

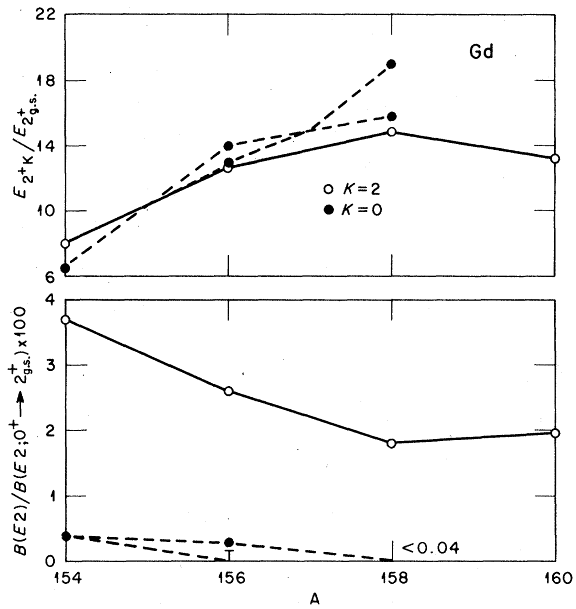

FIG. 5. Ratios of level energies and absolute $B(E2)$ strengths of vibrational states to those of the $2^+_{g.s.}$ states in $^{154-160}$Gd. The $B(E2)$ ratios for $K^\pi = 0^+$ states in ^{158,160}Gd and the 1258-keV state in ^{156}Gd are extremely small and are omitted.

the vibrational stiffness parameter C according to

$$E \propto \left(\frac{C}{B}\right)^{1/2} \quad \text{and} \quad B(E2) \propto (BC)^{-1/2},$$

where B is a mass parameter, which includes moment-of-inertia effects. Figures 5 and 6 show plots of ratios of energies and $B(E2)$ values of the vibrational states to the corresponding values for the ground-band 2^+ state. Such ratios should minimize the ground-state deformation effects (by near cancellation of the mass parameter) and emphasize the stiffness as a function of neutron number. Our data presented in this manner show the similarity of the Gd and Hf nuclei with the Gd nuclei being slightly more sensitive to the neutron number than the Hf nuclei.

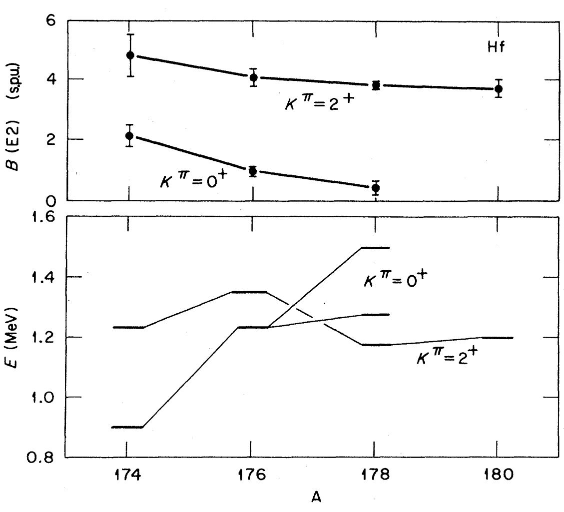

FIG. 4. Absolute $B(E2)$ strength (in single particle units) and level energy systematics for $^{174-180}$Hf for 2^+ states of bands with K values given in the figure. The data for ^{174}Hf are taken from Ref. 30.

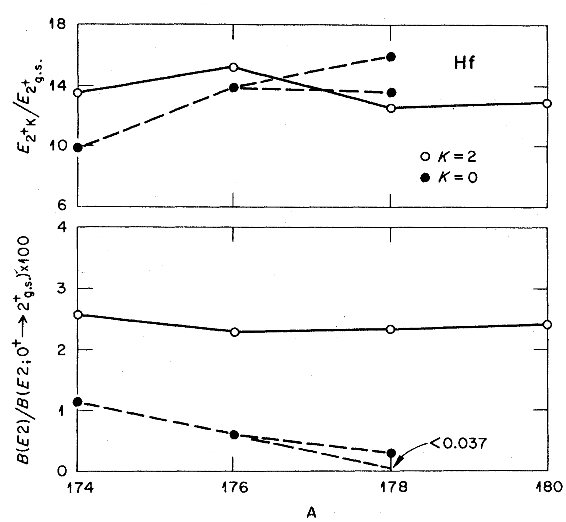

FIG. 6. Ratios of level energies and absolute $B(E2)$ strengths of vibrational states to those of the $2^+_{g.s.}$ states in $^{174-180}$Hf. The data for ^{174}Hf are taken from Ref. 30. The $B(E2)$ ratio for the 1277-keV $K^\pi = 0^+$ state in ^{178}Hf is extremely small and is omitted.

C. Interpretations of $I^\pi K = 2^+2$ and 2^+0 vibrational states

If collective, the $I^\pi = 2^+$ states may be interpreted as γ vibrational when $K=2$, or β vibrational when $K=0$. Our results show that the γ vibrational mode is present in each nucleus studied here. The $I^\pi K = 2^+0$ states in ^{154,156}Gd and $^{174-178}$Hf with the largest $B(E2)$ values do have strengths typical of β vibrational states. The $B(E2)$ values are only ~1 s.p.u. and one might incorrectly conclude that these 2^+ states are not members of a collective band. However, the collective $K^\pi = 0^+$ vibrational band may mix strongly with the $K^\pi = 0^+$ ground band. As a result of the mixing, the collective $E2$ strength is redistributed between the two $K=0$ bands. To first order, the $0^+_{g.s.} \to 2^+_\beta$ matrix element is decreased, the $2^+_{g.s.} \to 0^+_\beta$ and $4^+_{g.s.} \to 2^+_\beta$ elements are increased. Then, in a first-order perturbation treatment of this mixing,[22] the intrinsic value of $B(E2;\ 0^+_{g.s.} \to 2^+_\beta)$ will be multiplied by the factor $(1 - 6Z_\beta)^2$. The value of $B(E2;0^+_\beta \to 2^+_{g.s.})$ will be multiplied by $(1+6Z_\beta)^2$. For ^{154}Gd, where $Z_\beta \approx 0.05$ (Ref. 22), the measured $B(E2;0^+_{g.s.} \to 2^+_\beta)$ is thus 49% of the intrinsic value. The intrinsic $4^+_{g.s.} \to 2^+_\beta)$ element is multiplied by $(1+14Z_\beta)$. With $Z_\beta \approx 0.05$, the measured value of $B(E2;4^+_{g.s.} \to 2^+_\beta)$ is 289% of its intrinsic value. It is difficult then to conclude the collectivity of a $K=0$ band simply by measuring $B(E2;0^+_{g.s.} \to 2^+_\beta)$. The complementary measurement of $B(E2;0^+_\beta \to 2^+_{g.s.})$ can lead to a better assessment of the collectivity. Mixing of the γ and ground bands produces a smaller effect on $B(E2;0^+_{g.s.} \to 2^+_\gamma)$. Here, the factor has the form $(1 - Z_\gamma)^2$. For ^{154}Gd, $Z_\gamma \approx 0.09$ (Ref. 22), and the reduction is only 17%. The weaker $I^\pi K = 2^+0$ states are experimentally strikingly different from the stronger $I^\pi K = 2^+0$ states. Studies of the (p,t) reaction on Gd (Ref. 31) and Hf (Ref. 32) nuclei show that the 0^+ and 2^+ members of the band usually associated with the β vibration are more strongly excited than the second excited $K=0$ band members. Such is true for the (d,d') reaction[21] also. Directional correlation studies show that the $2^+0 \to 2^+_{g.s.}$ transitions from the second $K=0$ band, such as in ^{156}Gd (Ref. 33) and ^{178}Hf (Ref. 34), are mostly $M1$ in character. It is interesting that for ^{178}Hf the transition from the 1277-keV 2^+ state to the 2^+ ground state has an 80% $M1$ admixture[34] and that $E2$ γ-ray branching ratios from this state do not agree[1] with the collective model even with perturbational corrections. On the other hand, the transition from the more collective 2^+ state at 1496 keV to the 2^+ ground state has also a large (62%) $M1$ admixture[34] but there the $E2$ branching ratios do agree[1] when a perturbational correction is applied. Such $M1$ admixtures were originally thought[35] to be responsible for bandmixing anomalies in the β bands but this was subsequently ruled out (see Ref. 1). Other interpretations besides that of a collective β vibration have been given to such bands. A single two-quasiparticle amplitude may be dominant, possibly as in the cases of the $K^\pi = 0^+$ bandheads at 1168 keV in ^{156}Gd (Ref. 36) and at 1199 keV in ^{178}Hf (Ref. 34). The 1715-keV 0^+ state in ^{156}Gd may be a two-β phonon ($n_\beta = 2, n_\gamma = 0$) bandhead.[37] Also, theoretical calculations[38] of potential energy surfaces in this region indicate that it is possible to have a $K=0$ band built on a minimum corresponding to an oblate deformation, whereas the normal β vibrations occur about prolate shapes.

D. Application of a pairing-plus-quadrupole model to ^{154,156}Gd

For γ vibrations, the pairing-plus-quadrupole (PPQ) interaction calculations of Bes *et al.*[39] attempted to provide an in-depth view of this mode in the rare-earth region. However, $B(E2)$ strengths are overestimated by factors of 2 or 3. Much better agreement with experiment has been obtained either by the more exact PPQ calculations of Kumar for ^{150,152}Sm (Ref. 40), or by a variable moment of inertia approach[41] for ^{152}Sm. Gupta *et al.*[42,43] have extended the PPQ calculations to the more deformed ^{154,156}Gd. Tables II and III present our experimental values of the reduced $E2$ matrix elements $\langle 0^+_{g.s.} \| M(E2) \| I^\pi = 2^+, K = 0 \text{ or } 2\rangle$, which are square roots of the $B(E2)$ values of Table I. Experimental magnitudes of

$$\langle 2^+_{g.s.} \text{ or } 4^+_{g.s.} \| M(E2) \| I^\pi = 2^+, K=0 \text{ or } 2\rangle$$

were obtained by combining our $B(E2)$ values with $B(E2)$ ratio data[1] from γ-ray studies. These are compared with PPQ predictions as well as with predictions of the collective model, using the

TABLE II. $E2$ matrix elements in ^{154}Gd compared with pairing-plus-quadrupole and collective models.

		$M(E2;IK \to I'K')$ (eb)		
$IK \to I'K'$	$E_{I'K'}$ (keV)	Exp (magnitude)	PPQ[a]	Collective model[b]
$0g \to 2g$	123	1.96(2)	1.96	
$0g \to 2\gamma$	996	0.38(6)	0.37	
$0g \to 2\beta$	815	0.12(7)	0.14	
$2g \to 2\gamma$		0.46(9)	−0.54	−0.68(10)
$2g \to 2\beta$		0.35(18)	0.41	0.15(8)
$4g \to 2\gamma$		0.22(3)	−0.14	−0.10(2)
$4g \to 2\beta$		0.6(4)	−0.88	−0.20(10)

[a] Predictions of the pairing-plus-quadrupole model calculations of Gupta *et al.* (Refs. 42 and 43).

[b] These values are obtained using the collective model (Ref. 13) relations (Ref. 12) between $B(E2)$ values and our experimental $0g \to I'K'$ matrix elements. The signs result from assuming that PPQ theory has correctly predicted the signs of the $0g \to I'K'$ matrix elements.

TABLE III. Matrix elements in ^{156}Gd compared with pairing-plus-quadrupole and collective models.

$IK \to I'K'$	$E_{I'K'}$ (keV)	$M(E2; IK \to I'K')$ (eb) Exp (magnitude)	PPQ [a]	Collective model [b]
$0g \to 2g$	89	2.14(2)	−2.09	
$0g \to 2\gamma$	1154	0.35(3)	−0.38	
$0g \to 2\beta$	1129	0.12(7)	−0.14	
$0g \to 2o$	1258	<0.09	−0.08	
$2g \to 2\gamma$		0.42(3)	−0.47	−0.41(3)
$2g \to 2\beta$		0.28(2)	0.40	0.14(8)
$2g \to 2o$		<0.07 [c]	0.09	<0.11
$4g \to 2\gamma$		0.12(2)	−0.21	−0.09(1)
$4g \to 2\beta$		0.31(19)	−0.58	−0.18(11)
$4g \to 2o$		<0.34	−0.15	−<0.14

[a] Predictions of the pairing-plus-quadrupole model calculations of Gupta *et al.* (Refs. 42 and 43).

[b] These values are obtained using the collective model (Ref. 13) relations (Ref. 12) between $B(E2)$ values and our experimental $0g \to I'K'$ matrix elements. The signs result from assuming that PPQ theory has correctly predicted the signs of the $0g \to I'K'$ matrix elements.

[c] In Ref. 14 the branching ratios were taken from Ref. 1 and are not corrected for the 87.4% $M1$ admixture in the 2_g-2_0 transition. This result is corrected.

"Alaga rules"[12]; that is, ratios of the appropriate Clebsch-Gordan coefficients normalized to the measured $0^+_{g.s.} \to 2^+$ matrix elements. Overall agreement with PPQ theory is very good, even to predicting the small matrix element to the 2^+ state at 1278 keV in ^{156}Gd.

E. Results and discussion for 3^- states

Several $I^\pi = 3^-$ octupole vibrational states were observed and the measured $B(E3)$ strengths are presented in Table IV. Also presented are energies, proposed K values, and the predictions[44] of the microscopic calculations of Neergard and Vogel.[45,46]

Our experimental results verify several predictions of the theory. To be noted is the very good agreement between theory and experiment in both absolute strength and energy level position. The inclusion of the Coriolis interaction is responsible for this agreement, as also is the case for the actinides.[46] For example, in ^{156}Gd, unperturbed random-phase approximation calculations yield $B(E3)$ values for $K=0$, 1, 2, and 3 of 5.6, 6.1, 5.7, and 0.0 $e^2b^3 \times 10^{-2}$ with energies of 1420, 1370, 1570, and 1840 keV, respectively.[45] But, as is observed experimentally, the collectivity concentrates in the state with the lowest energy. In addition, the measurements seem to verify the prediction that the collectivity decreases with increasing mass.

TABLE IV. Comparisons of measurements and microscopic calculations of $B(E3; 0^+\text{g.s.} \to 3^-)$ for the one-octupole-phonon 3^- states. The label α is the K quantum number corresponding to the largest component of the Coriolis-coupled wave function since K is not a "good" quantum number for the mixed octupole states.

Nucleus	$I^\pi K$	Experiment E (keV)	Experiment $B(E3)$ ($e^2b^3 \times 10^{-2}$)	α	Theory [a] E (keV)	Theory [a] $B(E3)$ ($e^2b^3 \times 10^{-2}$)
^{156}Gd	$3^-(0)$	1276	16(4)	0	1848	1.1
				1	1309	15.2
				2	1664	1.0
				3	1818	0.0
^{160}Gd	$(3^-?)$	1289	12.7(14)	0	1855	0.0
				1	1607	0.9
				2	1434	11.3
				3	1882	0.0
^{176}Hf	3^-2	1313	9.3(29)	0	1962	0.1
				1	1642	0.9
				2	1294	8.1
				3	1890	4.9
^{178}Hf	3^-2	1323	5.3(1)	0	2117	0.5
				1	1549	0.0
				2	1411	9.8
				3	1806	4.4

[a] References 44 and 45.

V. CONCLUSION

We have used the (α, α') reaction to Coulomb-excited $I^\pi = 2^+$ and 3^- vibrational states in even-even $^{154-160}$Gd and $^{176-180}$Hf nuclei to obtain precise values of the absolute $B(E2)$ and $B(E3)$ strengths.

For both the Gd and Hf nuclei, the 2^+ states behave with marked similarity in trends with increasing mass of both level energy and $E2$ collectivity despite the opposite trend in ground state deformation. The $I^\pi = 2^+$ members of second $K=0$ excited bands have small, if any, $B(E2)$ strength.

A pairing-plus-quadrupole model applied to the $I^\pi = 2^+$ states in $^{154-156}$Gd works very well and suggests further applicability of this model to well-deformed nuclei. For octupole-vibrational states, the theory of Neergard and Vogel works well if the Coriolis interaction is included.

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