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Prediction of ternary fission rates for element 126[†]

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Ternary and binary fission barriers and their relative WKB penetrability have been calculated for the isotopes $184 \leq N \leq 228$ of element 126, considering both $Z = 114$ or $Z = 126$ as magic numbers. In all cases the predicted relative rates Γ_T/Γ_B of ternary to binary low-energy fission events are substantially higher than those known for actinides. Thus their observation could serve as a test for the superheavy element 126.

[RADIOACTIVITY, FISSION $^{310,\ldots,354}126$(sf); calculated binary and ternary barriers, predicted relative $T_{1/2}$.]

The evidence for superheavy elements reported recently by Gentry *et al.*[1] is based on the observed energy spectrum of proton-induced x-ray emission from giant halo formations in certain micas. The evidence can best be confirmed by the observation of other predicted characteristic properties of superheavy elements. One of the characteristics, the probability of ternary fission, is the subject of the present note.

For nuclei around $^{298}114$ ($Z=110$–122) we have previously predicted[2,3] exceptionally high spontaneous ternary fission rates $\Gamma_T/\hbar$ (as compared to their spontaneous binary fission rates $\Gamma_B/\hbar$). In Hartree-Fock calculations Kolb[4] has recently found a tendency towards tripartition in $^{298}114$. For the longest-lived superheavy candidates (close to $^{294}110$, Refs. 5 and 6) we have predicted[3] that the relative counting rate Γ_T/Γ_B should be as high as 10^{-2}. This value is by several orders of magnitude larger than any number reported for low-energy actinide fission into three fragments with comparable mass (10^{-6} to 10^{-10} or even smaller, see Ref. 7).

As the recent experimental evidence[1] for superheavy elements is strongest for the element 126 we give in the following the corresponding estimates of Γ_T/Γ_B for $Z=126$ isotopes. The method of calculation is the same as in Ref. 3: For the binary and ternary fission barriers the shell energy is calculated for two sequences of shapes consisting of two or three spheroidal parts, respectively. For a plot of the shape parametrization and a description of the semiempirical calculation of the shell energy see Refs. 2 and 8. The liquid-drop energy is determined for Lawrence's shape parametrization.[9] For the small deformations under consideration differences in liquid-drop energy between binary and ternary deformations are negligible according to Refs. 10 and 11. For the calculated barriers the ratio Γ_T/Γ_B is obtained from the one-dimensional barrier-penetration formula of Nilsson *et al.*,[6] assuming parabolic barrier shapes.

The parameters entering into the calculation are the liquid-drop model parameters of Myers and Swiatecki,[12] two adjusted parameters in the shell energy,[8] the extrapolated magic numbers[13] $Z=114$ and 164 and $N=184$, 228, and 308 taken from Sobiczewski *et al.*,[14] and the inertia parameter $0.054A^{5/3}\,\hbar^2\,\mathrm{MeV}^{-1}$ of Nilsson *et al.*[6] The penetration energy is taken to be 0.5 MeV as usual (e.g., Refs. 5, 6, and 14).

In Table I, column 3 the resulting values of Γ_T/Γ_B are given for the isotopes of element 126 between the magic neutron numbers $N=184$ and $N=228$ (generally in steps of $\Delta N=4$). The calculations yield values between 10^{-4} and 10^{-2} for the relative fission probability Γ_T/Γ_B of isotopes close to the β-stability line. Towards the magic isotopes $N=184$ and $N=228$ (far off β stability) we obtain almost equal probability for binary and ternary fission. The deformation at the ternary saddle point corresponds to light fragment masses between 18 and 50. However, accurate predictions about the masses and charges of the ternary fragments would require more refined (multidimensional) penetrability calculations.

In an attempt to remove the discrepancy between the experimental evidence[1] for very long-lived nuclei with charge number $Z=126$, and the calculated short lifetimes[5,6] of such nuclei, we have also assumed $Z=126$ to be a magic number. Table I, column 4 gives the values of Γ_T/Γ_B which result after replacing the magic proton number 114 by 126 (maintaining the magic neutron numbers). In comparison to column 3, the relative ternary fission rates Γ_T/Γ_B are smaller for the doubly magic isotopes $^{310}126$ and $^{354}126$, and larger for the isotopes close to the β-stability line. In some cases (e.g., $^{326}126$) the estimates are less accurate than

TABLE I. Calculated relative rates Γ_T/Γ_B for ternary to binary fission of $Z=126$ isotopes.

		Γ_T/Γ_B		
Mass number A	Neutron number N	Magic nos. $Z=114$ $N=184,228$	Magic nos. $Z=126$ $N=184,228$	Magic nos. $Z=126$ $N=184,210$
310	184	9×10^{-1}	4×10^{-2}	
314	188	6×10^{-1}	5×10^{-2}	
318	192	2×10^{-1}	8×10^{-2}	
322	196	1×10^{-1}	4×10^{-2}	
326	200	1×10^{-2}	2×10^{-3}	3×10^{-2}
328	202			3×10^{-3}
330	204	2×10^{-2}	1×10^{-2}	4×10^{-3}
332	206			6×10^{-4}
334	208	3×10^{-3}	6×10^{-1}	3×10^{-4}
336[a]	210	1×10^{-2}	9×10^{-2}	1×10^{-3}
338	212	3×10^{-3}	5×10^{-1}	1×10^{-4}
340	214			2×10^{-4}
342	216	1×10^{-4}	3×10^{-1}	2×10^{-4}
344	218			7×10^{-4}
346	220	4×10^{-3}	3×10^{-1}	1×10^{-3}
350	224	2×10^{-2}	5×10^{-2}	
354	228	9×10^{-1}	2×10^{-1}	

[a] Valley of β stability according to Green's formula (Ref. 15).

on the average because the saddle point is located in a flat region of the potential energy surface.

We have further considered a doubly magic isotope of element 126 that is located on the line of β stability. This would explain the observed stability[1] of that element. In this case (column 5 of Table I) the sequence of magic numbers is taken to be $Z=126$, 164 for the protons and $N==184$, 210, 308 for the neutrons. The extrapolation $N=210$ results from Green's formula[15] for the valley of β stability. The resulting relative ternary fission rates in this hypothetical island of stability (column 5) vary within the same limits as those obtained by assuming the conventional island around $^{298}114$ (column 3).

It should be stressed that calculated *absolute* spontaneous-fission half-lives may be in error by many orders of magnitude (10^6 according to Nilsson *et al.*,[6] 10^{10} according to Fiset and Nix[5]). This is due to inaccuracies in the parameters of the liquid-drop and shell energy, in the inertia parameter, and in the energy assumed for the assaults on and penetration through the barrier. For our predicted *relative* rates Γ_T/Γ_B, however, many of the uncertainties enter into both the binary and ternary value in the same way, and mainly compensate one another. For instance, errors of as much as 50% in the barrier heights or of 30% in the deformations correspond to about one order of magnitude in Γ_T/Γ_B. The ratio Γ_T/Γ_B is particularly insensitive to the values of the penetration energy (0.5 MeV) and of the frequency ($1\ \text{MeV}/2\pi\hbar$) of assaults on the barrier assumed in the penetrability formula.[6] Therefore, the estimates of Table I should roughly be valid not only for spontaneous but also for induced fission at low energies.

Strictly speaking, our estimates of Γ_T/Γ_B in Table I refer only to the penetration probabilities along certain paths through the barrier, leading to binary or ternary constriction. The unknown dynamics of the nuclear motion after the barrier penetration could lead to discrepancies between the initial fragmentation after tunneling (which is predicted by Γ_T/Γ_B) and the final one at scission (which would be measured). However, barrier penetration calculations in binary fission (e.g., Ref. 16) at least qualitatively reproduce the experimental mass yield distribution. Therefore, the uncertainties due to the dynamics should not affect our results by many orders of magnitude. Moreover, dynamical effects in the competition between binary and ternary fission may be reduced by the structure of the potential energy surface: According to recent results of Möller and Nix[11] there are separate valleys at large deformations which lead to binary or ternary fission. [The potential energy surface at small deformations, however (Fig. 15 of Ref. 11), and their conclusions about binary and ternary saddle points of superheavy nuclei are far from reality due to the omission of shell effects in their calculation.]

According to our predictions the relative rates Γ_T/Γ_B of $Z=126$ isotopes should be of the order of 10^0 to 10^{-4} depending on the isotope mass and on the assumed magic numbers entering into the calculation. As the (controversial) actinide rates of Γ_T/Γ_B are at least two, possibly 10 orders of magnitude smaller, the recent evidence for the presence of $Z=126$ inclusions in certain micas can possibly be verified by searching for spontaneous (or induced low-energy) ternary fission events in the mica or in samples extracted from it.

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