Recoil momenta of fragments from relativistic nuclear heavy ions

Naohiko Masuda
*Department of Physics, University of Marburg, D-355 Marburg, Germany**
and Department of Physics, University of Leuven, B-3030 Heverlee, Belgium

Fumiyo Uchiyama†
National Laboratory for High Energy Physics, Oho-Machi, Tsukuba-Gun, Ibaraki, Japan

(Received 7 September 1976)

It is shown that the recoil momenta of isotopes produced by fragmentation of relativistic ^{12}C and ^{16}O projectiles can be understood from kinematics and a two-fragment approximation.

[NUCLEAR REACTIONS Recoil momenta of fragments from relativistic ^{12}C and ^{16}O projectiles. Kinematics and two-fragment model. Linear dependence on interfragment binding energy and target independence.]

In another work[1] we present an analysis of partial cross sections[2] and of momentum distributions[3] for the isotopes produced by fragmentation of ^{12}C and ^{16}O beams at 2.1 GeV/nucleon using a double-nova model.[4] The correlation between partial cross sections and di-fragment binding energies was clearly seen, and the momentum width distribution of fragments was also understood with di-fragment decay of nova. We here extend the model to calculate the recoil momenta of fragments[3] which appear in the momentum distribution as the shifts of the centers of Gaussian distributions in the projectile rest frame.

The model is a two step process: first the production of nova, and then the decay of the nova into fragments. Upon collision, the beam nuclei B and the target nuclei A acquire their internal energies characteristic to the individual nucleus and become novas with masses m_B^* and m_A^*, respectively. The nova B^*, whose decay products we are concerned with, is considered as a superposition of many two-fragment states. It decays directly into various pairs of nuclei C_i and D_i $(i=1,2,\ldots)$. As we assume that no quantum number is exchanged in the formation of the novas, baryon and charge quantum numbers are conserved in the transition from B to B^*.

We denote the momentum of B by $\vec{P}_B$ and that of A^* by $\vec{q}\,'$ in the laboratory frame. The energy-momentum conservation in the laboratory frame can be written as

$$(m_B^2+P_B^2)^{1/2}+m_A = (m_A^{*2}+q'^2)^{1/2}+[m_B^{*2}+(\vec{P}_B-\vec{q})^2]^{1/2}. \quad (1)$$

It is more convenient to write it in the beam rest frame for treatment of beam fragmentation reaction. Denoting the corresponding momentum of A by $\vec{P}_A$ and that of B^* by $\vec{q}$, we get

$$m_B+(m_A^2+P_A^2)^{1/2} = [m_A^{*2}+(\vec{P}_A-\vec{q})^2]^{1/2}+(m_B^{*2}+q^2)^{1/2}. \quad (2)$$

For small recoil momentum and recoil angle, the longitudinal component q_B (measured along the beam direction) of recoil momentum of B^* is related to the excitation energy $E=m_B^*-m_B$ of B^* by

$$q_B=\frac{(m_A^2+P_A^2)^{1/2}}{P_A}\times\left[m_B^*-m_B+\frac{m_A}{(m_A^2+P_A^2)^{1/2}}(m_A^*-m_A)\right], \quad (3)$$

where the second order term in q is neglected. The momentum of the target A in the beam rest frame can be expressed in terms of the incident kinetic energy per nucleon T in the laboratory frame and a nucleon mass m_N as $P_A=(T^2+2m_NT)^{1/2}$. n_A where n_A is the nucleon number of the nucleus A.

We now apply Eq. (3) for the case where a nucleus C is observed as a fragment from B^*. For this process to occur, the excitation energy $E=m_B^*-m_B$ of B^* has to be greater than the di-fragment binding energy δ_C, which is the binding energy between C and $(B-C)$ nuclei to form the nucleus B. For instance, in the case of ^{16}O beam, if $C={}^4$He, d, ^{6}Li, etc., then $(B-C)={}^{12}$C, ^{14}N, ^{10}B, etc., respectively. Let us denote the recoil momentum of the nova B^* which decays into C and $(B-C)$ as q_B^C, then we obtain the following expression

$$q_B^C=\frac{1}{v}[\delta_c+Q_c+(1-v^2)^{1/2}\cdot\bar{E}_A], \quad (4)$$

where Q_c is the total kinetic energy of the decay fragments C and $(B-C)$ and $v=P_A/(m_A^2+P_A^2)^{1/2}$. We have replaced $m_A^*-m_A$ with the average excitation energy $\bar{E}_A$ of the target nova A^*. The third term in Eq. (4) is the contribution from the target nova. The velocity v is the velocity of the target A in the beam rest frame, and it is also equal to the velocity v_B of the beam nucleus in the laboratory frame. Thus, the incident ^{16}O or ^{12}C beam velocity $v_B = 0.95$ in the laboratory frame[1,2] means $v = 0.95$ for the target in the beam rest frame. The kinematical factor $(1-v^2)^{1/2}/v$ in Eq. (4) implies no target dependence of recoil momenta of beam fragments in the asymptotic energy region, which theoretically confirms the experimental observation.[3] The energy transfer function (excitation energy spectrum) of nova of carbon has been extracted from the experimental data[5] and can be fitted with the form $\rho(E) = \beta \exp(-\beta/E)/E^2$, which has the maximum at $\frac{1}{2}\beta$. It was also shown that the magnitude of $\frac{1}{2}\beta$ is of the order of the energy required to excite a nucleus into its lower lying states.[1] We here assume that these features of $\rho(E)$ remain the same for any nucleus as long as the nova is produced within a small angle. Using the excitation energy spectrum $\rho(E)$ with a cutoff at $E = m_\pi$, we obtain $\overline{E}_A = 15$ MeV. Thus, the contribution from the target term $(1-v^2)^{1/2} \cdot \overline{E}_A/v$ at $v_B = v = 0.95$ is about 5 MeV/c.

Taking into account the mass dependence of recoil momentum of each fragment, we obtain the desired recoil momentum of fragment C in terms of q_B^C as

$$-\langle P_{11}^C \rangle = \frac{m_C}{m_B} q_B^C = \frac{m_C}{m_B}\frac{1}{v}[\delta_c + Q_c + (1-v^2)^{1/2} \cdot \overline{E}_A]. \qquad (5)$$

Since the total kinetic energy Q_c of fragments C and $(B-C)$ can be obtained from the data on the momentum width distribution σ_c through

$$Q_c = 3\left(\frac{\langle P_{11}{}^2\rangle}{2m_C} + \frac{\langle P_{11}{}^2\rangle}{2m_{B\text{-}C}}\right) = \frac{3}{2}\frac{m_B \sigma_C{}^2}{m_C(m_B - m_C)}, \qquad (6)$$

we can compute the recoil momentum of each fragment in terms of di-fragment binding energy δ_c and the experimental data on σ_C from Eq. (5).

However, since the experimental data on $\langle P_{11}^C\rangle$ have large errors, it is more useful to see the general behavior of $-\langle P_{11}^C\rangle$ as a function of σ_c by fixing Q_c to the average value $\overline{Q}_c$. In order to obtain the average total kinetic energy $\overline{Q}_c$, we utilize the results of the analysis of momentum distribution. The momentum distribution is Gaussian with a width σ_c, which can approximately be expressed in terms of its mass m_C and projectile mass m_B as

$$\sigma_c{}^2 = \alpha \frac{m_C(m_B - m_C)}{m_B}, \qquad (7)$$

where $\alpha(^{16}O) = 7.7$ MeV. This value is obtained by fitting Eq. (7) to the momentum width distribution. Using Eqs. (6) and (7), we get

$$\overline{Q}_c = \tfrac{3}{2}\alpha = 11.55 \text{ MeV}. \qquad (8)$$

From Eqs. (5) and (8), we obtain a linear relationship between $-\langle P_{11}^C\rangle$ and δ_c as

$$-\langle P_{11}^C\rangle = \frac{m_C}{m_B}\frac{1}{v}[\delta_c + 11.55 + 15(1-v^2)^{1/2}]. \qquad (9)$$

In Fig. 1, we plot the experimental data of recoil momenta of fragments from ^{16}O beam with experimental errors against di-fragment binding energy. As is seen from our earlier work, the two body fragmentation model of the beam nucleus for smaller mass fragment $m_C \leq \frac{1}{2}m_B$ is not as good as for the case of larger mass fragment because the multiple production of smaller fragment is in most cases possible. (See Ref. 9 of our earlier work.) For this reason we divide the data into two cases: one with the data with fragment mass number larger than 8, and the other with those with mass number less than or equal to 8. For the former, we assume an average mass number $\overline{m}_C = 12$, and for the latter we assume $\overline{m}_C = 6$. With these assumptions we get

$$-\langle P_{11}^C\rangle \simeq 0.79\delta_c + 12.87 \quad \text{for } \overline{m}_c = 12 \qquad (10a)$$

and

$$-\langle P_{11}^C\rangle \simeq 0.39\delta_c + 6.43 \quad \text{for } \overline{m}_c = 6. \qquad (10b)$$

Similarly for the fragments from ^{12}C beam, we divide the data into two classes, one with fragments with mass number larger than 6, and the

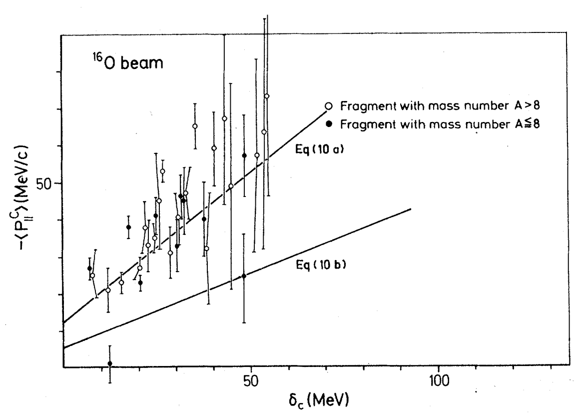

FIG. 1. Plot of the experimental data of recoil momentum of fragment C from ^{16}O against the di-fragment binding energy δ_c. The straight lines represent Eqs. (10).

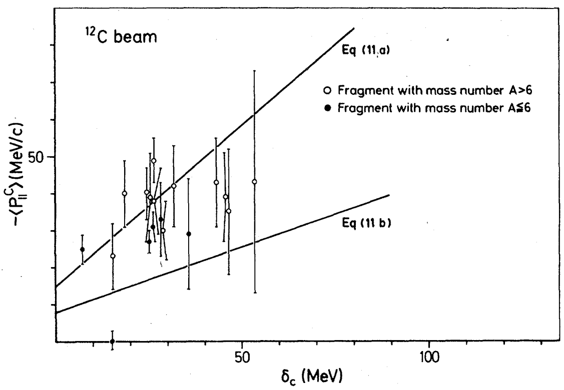

FIG. 2. The same as in Fig 1 with ^{12}C projectile. The straight lines represent Eqs. (11).

other with the rest of fragments. And assuming that $\bar{m}_c = 10$ for the former and $\bar{m}_c = 4$ for the latter, we obtain

$$-\langle P_{11}^C \rangle \simeq 0.88\delta_c + 14.30 \quad \text{for } \bar{m}_c = 10 \tag{11a}$$

and

$$-\langle P_{11}^C \rangle \simeq 0.35\delta_c + 7.72 \quad \text{for } \bar{m}_c = 4 . \tag{11b}$$

Even though we do not expect good fits to the data for smaller mass fragments with Eqs. (10.2) and (11.2) as is explained above, we nevertheless plot these theoretical predictions along with Eqs. (10a) and (11a) for larger mass fragments in Figs. 1 and 2.

Considering larger experimental errors, our fit to the data for the larger mass fragments from both ^{16}O and ^{12}C seems quite reasonable. As is expected, the fit to the data for the smaller mass fragments is not as good as that for the larger mass fragments.

In conclusion, the gross feature of the data of recoil momenta of isotopes produced by the fragmentation of relativistic ^{16}O and ^{12}C projectiles can be understood by only the kinematics and the di-fragment model. Together with the results of our earlier work one can draw a consistent picture of mechanism of fragmentation of relativistic nuclear heavy ions, namely, the formation of nova by diffraction and the structural effect of the nucleus in the decay process.

A possible connection of the beam fragmentation by diffraction with the cluster substructure of nuclei has already been explained in our earlier paper.[1]

One of the authors (F.U.) would like to thank the members of the Institute for Nuclear Study, The University of Tokyo, and of the Research Institute for Fundamental Physics, Kyoto University for their hospitality.

---

*Partially supported by the Deutsche Forschungsgemeinschaft.

†Research Fellow of the Japan Society for the Promotion of Science. A part of this work was done at Research Institute for Fundamental Physics, Kyoto University.

[1]N. Masuda and F. Uchiyama, LBL Report No. LBL-4263, 1975 (unpublished).

[2]P. J. Lindstrom, D. E. Greiner, H. H. Heckman, B. Cork, and F. S. Bieser, LBL Report No. LBL-3650, 1975 (unpublished).

[3]D. E. Greiner, P. J. Lindstrom, H. H. Heckman, B. Cork, and F. S. Bieser, Phys. Rev. Lett. 35, 152 (1975).

[4]M. Jacob and R. Slansky, Phys. Rev. D 5, 1847 (1972).

[5]J. Jaros, J. Papp, L. Schroeder, J. Staples, H. Steiner, and A. Wagner, LBL Report No. LBL-2115 (unpublished).