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Bremsstrahlung in the nuclear fireball model*

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The recently proposed nuclear fireball model is used to calculate the low-energy ($\lesssim 10$ MeV) bremsstrahlung from relativistic heavy ion collisions. For low-energy photons with wavelengths larger than nuclear dimensions and frequencies smaller than the inverse collision time the bremsstrahlung is nearly independent of details of the collision process. The number of photons produced per collision depends strongly on the impact parameter. Production cross sections are generally strongly peaked in the forward direction with a broad hump in the backward direction, and appear to be large enough to be seen experimentally.

[NUCLEAR REACTIONS Relativistic heavy ions, calculated bremsstrahlung, $E \lesssim 10$ MeV.]

In a recent paper[1] a nuclear fireball model was used to calculate the proton inclusive spectra from relativistic heavy ion collisions. The essential ingredients of the model are: geometry, to calculate the number of nucleons in the fireball; kinematics, to calculate the velocity of the fireball and the energy deposited in it; and thermodynamics, to describe the decay of the fireball. At 400 MeV/nucleon the model with a single fireball reproduces the gross features of the proton energy and angular distribution. At 2100 MeV/nucleon it is necessary to assume two fireballs with their relative velocities determined by a transparency parameter.

Without making any further assumptions it is possible to calculate the low-energy bremsstrahlung radiated during the direct collision process. The fireball model does not specify how the system evolves from projectile plus target to fireball(s) plus fragments. For relativistic heavy ion collisions the inverse collision time is of the order of $c/2R$, where R is a nuclear radius. To a photon with frequency $\omega \ll c/2R$ the collision will appear as instantaneous. Thus we assume that the acceleration of charge which produces the bremsstrahlung can be treated as a δ function. All that need be specified then are the incoming and outgoing currents. This limits us to calculating bremsstrahlung photons with energy less than 10 MeV. This assumption is equivalent to a long-wavelength approximation so that all nuclear form factors which might appear can be set equal to one. The use of classical electrodynamics is justified since all photon energies will be negligible compared to nucleon and pion masses. Note that the decay of the fireball(s) will not contribute substantially to the bremsstrahlung since it is a thermodynamic expansion.

It should be noted that bremsstrahlung has been calculated[2,3] and observed[4] for the (basically) Coulomb scattering of nonrelativistic heavy ions. The acceleration of charge due to the nuclear Coulomb field is small compared to accelerations during the direct collision process and hence will be ignored.

The bremsstrahlung calculation proceeds in the standard way.[5] For one-fireball production the number of photons per unit energy per unit solid angle is

$$\frac{d^2N}{dE\,d\Omega}=\frac{\alpha}{4\pi^2}\frac{\sin^2\theta}{E}\left[\frac{F_P(b)Z_Pv_P}{1-v_P\cos\theta}-\frac{(F_P(b)Z_P+F_T(b)Z_T)v_F(b)}{1-v_F(b)\cos\theta}\right]^2, \tag{1}$$

and for two-fireball production it is

$$\frac{d^2N}{dE\,d\Omega}=\frac{\alpha}{4\pi^2}\frac{\sin^2\theta}{E}\left[\frac{F_P(b)Z_Pv_P}{1-v_P\cos\theta}-\frac{F_P(b)Z_Pv_{PF}(b)}{1-v_{PF}(b)\cos\theta}-\frac{F_T(b)Z_Tv_{TF}(b)}{1-v_{TF}(b)\cos\theta}\right]^2. \tag{2}$$

The cross section is obtained by integrating over all impact parameters b:

$$\frac{d^2\sigma}{dE\,d\Omega}=2\pi\int_0^{R_P+R_T}\frac{d^2N}{dE\,d\Omega}b\,db\,. \tag{3}$$

Here α is the fine structure constant; θ is the angle from the incident direction in the lab; the P, T, and

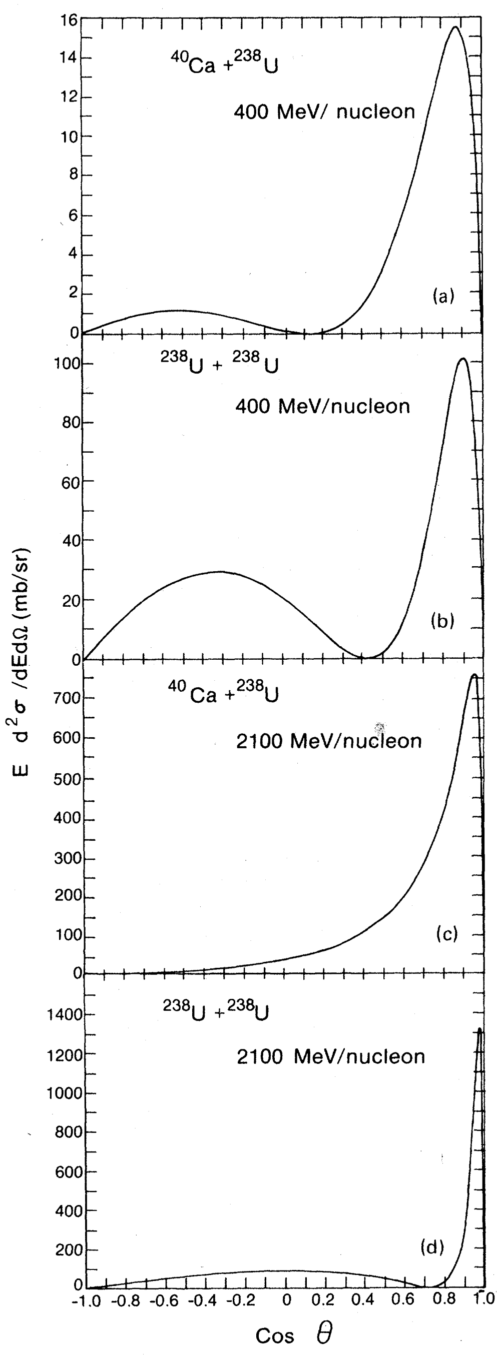

FIG. 1. Bremsstrahlung cross sections as a function of angle: (a) and (b) assume one fireball, (c) and (d) assume two fireballs with a transparency of 75%.

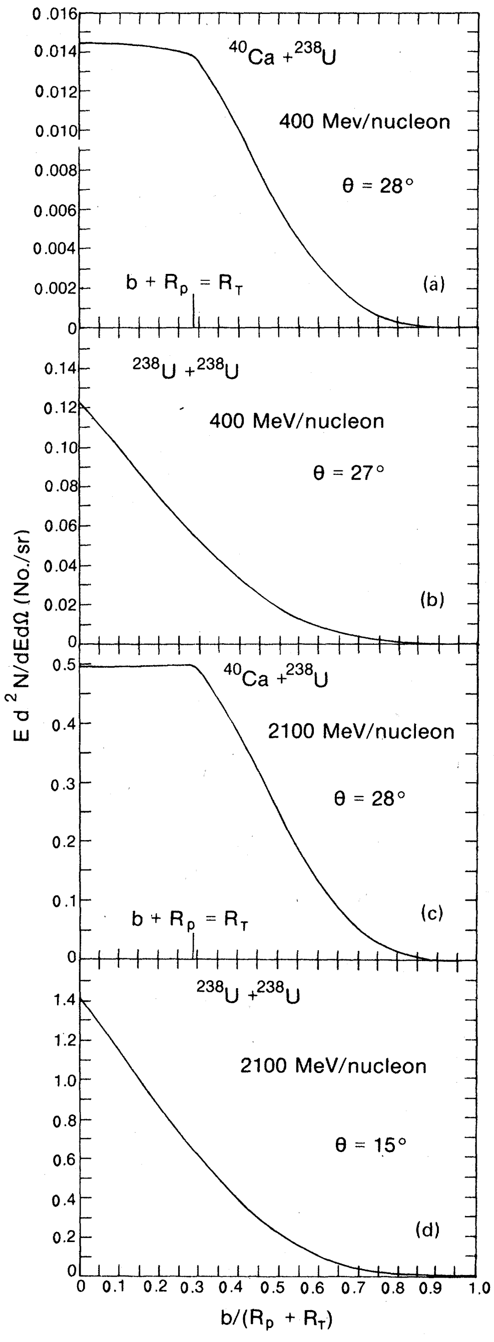

FIG. 2. Number of radiated photons as a function of impact parameter: (a) and (b) assume one fireball, (c) and (d) assume two fireballs.

F subscripts refer to projectile, target, and fireball; $F_{P(T)}$ is the fraction of projectile (target) nucleons participating in the collision to form a fireball; Z is the charge number; and $v > 0$ refers to lab velocity ($c = 1$). It should be noted that the factorization into E^{-1} times a function of angle is characteristic of soft bremsstrahlung.

For the special case when projectile and target are identical we can write down analytic formulas for the above expressions[6]:

$$\frac{d^2N}{dE\,d\Omega}=\frac{Z^2\alpha}{4\pi^2}\frac{\sin^2\theta}{E}\left(\frac{v_P}{1-v_P\cos\theta}-\frac{v_{PF}}{1-v_{PF}\cos\theta}-\frac{v_{TF}}{1-v_{TF}\cos\theta}\right)^2\left[1+\left(\frac{3}{\sqrt{2}}-1\right)B\right]^2(1-B)^4\,, \tag{4}$$

$$\frac{d^2\sigma}{dE\,d\Omega}=\frac{(1.9+\sqrt{2})}{28\pi}\frac{Z^2\alpha R^2\sin^2\theta}{E}\left(\frac{v_P}{1-v_P\cos\theta}-\frac{v_{PF}}{1-v_{PF}\cos\theta}-\frac{v_{TF}}{1-v_{TF}\cos\theta}\right)^2, \tag{5}$$

where

$$v_{PF}=\frac{v_P[1+(1-v_P{}^2)^{1/2}](1+\eta)}{[1+(1-v_P{}^2)^{1/2}]^2+\eta v_P{}^2}\,,\qquad v_{TF}=v_{PF}(\eta\to-\eta)\,. \tag{6}$$

Here R is the nuclear radius, $B\equiv b/2R$, and η is the transparency. For one-fireball production $\eta=0$ so that $v_{PF}=v_{TF}$, and for two-fireball production $0<\eta<1$.

Figure 1 shows the double differential cross section for photon production for ^{40}Ca and ^{238}U on ^{238}U at 400 and 2100 MeV/nucleon. At the higher energy two fireballs have been assumed with $\eta=75\%$. The sharp peak in the forward direction, which is present in all the cross sections, is due to the denominators $(1-v\cos\theta)$ which arise from relativity. The broad hump in the backward direction, which is not even noticeable in one of the graphs, arises for the same reason. The dip in the central region is due to interference between the incoming and outgoing currents.

Figure 2 shows the number of photons produced per unit energy per unit solid angle as a function of impact parameter, at some fixed angle chosen to be at or near the maximum of the cross section. When the projectile is smaller than the target the curve is relatively flat out to a critical impact parameter R_T-R_P and then decays away. When projectile and target are identical there is no plateau, the curve just decays from its maximum at zero impact parameter. Notice that for identical projectile and target over 90% of the bremsstrahlung is produced in collisions with an impact parameter less than one-half its maximum value.

In this paper only the bremsstrahlung from the direct nucleus-nucleus collision has been considered. There are other sources which might interfere with the observation of this bremsstrahlung. In the low keV region there may be photons emitted during electronic transitions. In the high keV and low MeV region there may be photons emitted by the residual projectile and target nuclei (if any), making electromagnetic (EM) transitions after the direct collision. That portion of these and other processes which is isotropic in the lab can be accounted for when comparing with the calculations of direct bremsstrahlung, but the rest may or may not present difficulties.

Experimental confirmation of these calculations would be an independent verification of the validity of the nuclear fireball concept. On the other hand, the low-energy bremsstrahlung does not provide any information on the details of the collision process since it only depends on the incoming and outgoing currents. One should be able to obtain much more information about how the nuclear system evolves during the collision by examining the bremsstrahlung in the 10–140 MeV region.

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[6]I would like to thank W. J. Swiatecki for providing approximate formulas for the function F. See also J. D. Bowman, W. J. Swiatecki, and C.-F. Tsang, Report No. LBL-2908 (TID-4500-R61), 1973 (unpublished).