Comments

The Comments section is for short papers which comment on papers previously published in **The Physical Review.** *Manuscripts intended for this section must be accompanied by a brief abstract for information retrieval purposes and a keyword abstract.*

Can charge-density waves occur in finite nuclei?

A. W. Overhauser
Department of Physics, Purdue University, West Lafayette, Indiana 47907
(Received 7 July 1976)

Recent observations by Cooper *et al.* of the charge distributions in ^{166}Er and ^{176}Yb bring to mind an old idea that the ground state of finite nuclear matter may exhibit spatially periodic mass-density and charge-density modulation. A feasible experiment employing a polarized ^{165}Ho target is proposed. A spatially periodic charge modulation could most easily be detected by Bragg scattering of high-energy electrons.

[NUCLEAR STRUCTURE Charge distribution; collective instabilities; electron scattering.]

The purpose of this comment is to propose a specific experiment addressing the question suggested in the title. Charge-density contours obtained by Cooper *et al.*[1] for ^{166}Er and ^{176}Yb have indicated unexpected charge maxima within the nuclei. These electron-scattering experiments were interpreted with a spherical-harmonic model for the charge density $\rho(r,\theta)$. Such a model might not provide a convenient description of a nucleus with a spatially periodic charge distribution. The reasons for considering periodic density modulations are strictly appropriate to infinite nuclear matter. But it is (of course) uncertain at what size a finite nucleus might exhibit such phenomena.

A Fermi-surface instability in nuclear matter can lead to a ground state with a mass- and charge-density wave modulation[2]

$$\rho=\bar{\rho}\,(1+\gamma\cos qz)\,, \tag{1}$$

where $q\approx 2k_F$, the diameter of the Fermi surface. In the Hartree-Fock approximation attractive interactions favor instabilities of this type, whereas repulsive interactions favor spin-density-wave instabilities.[3] Early estimates[2] of the magnitude of γ, the fractional modulation, were based on an oversimplified, effective nucleon interaction[4] and were excessively large. Estimates[5] based on more realistic potentials led to much smaller values.

Subsequent work[6] has shown that particle-particle correlations (previously neglected) play an important role in stabilizing a state with a density modulation. (On the other hand spin-density or isospin-density waves would tend to be suppressed.) This stabilizing effect depends on most of the low-lying excited (one-particle) states being below the energy gap caused by the density modulation. Such a requirement is optimized by having a one-dimensional density wave, as in Eq. (1), rather than a three-dimensional one, as originally[2] supposed. Needless to say attempts to predict γ would be futile; we merely *conjecture* that a nonzero value may not be automatically excluded. A further observation[6] is that a one-dimensional density wave can enhance the one-particle density of states near the Fermi surface, which would magnify the pairing force.

However unlikely it may seem, we shall consider finite nuclei with a modulated mass (and charge) density, say for $A\gtrsim 150$. Ordinarily a deformed nucleus is supposed to have a charge density given (approximately) by

$$\rho_0(r,\theta)=\bar{\rho}\left[1+\exp\left(\frac{r-R(\theta)}{t}\right)\right]^{-1}, \tag{2}$$

where $R(\theta)\cong R_0[1+\beta Y_{20}(\theta)]$ and t is the surface thickness parameter. Instead we shall assume that

$$\rho(r,\theta)\cong\rho_0(r,\theta)(1+\gamma\cos qz)\,, \tag{3}$$

where $z=r\cos\theta$ is the axis of the spheroidal nucleus. Since $q\approx 2k_F\approx 2.5\ \mathrm{fm}^{-1}$, $2R(0)$ is long enough to contain 5, 6, or 7 wavelengths ($\lambda\sim 2.5$ fm) of a density modulation, depending on the magnitude of the deformation β. A schematic illustration of a deformed nucleus with charge modulation is shown in Fig. 1. It may be noted that if n is the number of density maxima, the nucleus will have a large 2^{2n-2} pole electric moment. This would allow direct γ transitions from, for example, a $2n-2$ rotational level to the ground state.

Holmium has a single isotope, 165, a spin of $\frac{7}{2}$, and a large quadrupolar deformation $\beta \sim 0.3$. The metal is magnetic and generates a hyperfine field of $\sim 9\times 10^6$ Oe.[7] Consequently highly polarized low-temperature targets are available.[8] A spatially periodic charge modulation, as posed in Eq. (3), would be detectable in an elastic Bragg diffraction experiment. Bragg reflection of high-energy electrons would occur for momentum transfers corresponding to q, i.e., ~ 0.5 GeV/c.

Bragg scattering from a laminar structure, illustrated in Fig. 1, can occur only if the intrinsic nuclear axis lies in the scattering plane and bisects the angle between the incident and scattered beam. Such a condition cannot be perfectly achieved even if the nuclei are 100% polarized. Consider Fig. 2. If one assumes that the nuclear spin $\vec{I}$ arises primarily from the last nucleon, the intrinsic axis will precess about $\vec{I}$, as shown in Fig. 2, at an angle $\sim 28°$ (the minimum projection of $\vec{I}$ on the intrinsic axis[9]). Furthermore the spin $\vec{I}$ will precess about the hyperfine field $\vec{H}$ at an angle of 28° (if the spin is 100% polarized). It is clear (from Fig. 2) that the most probable orientation of the intrinsic axis, under these assumptions, is along the hyperfine field $\vec{H}$. The broad angular distribution of the intrinsic axis in the laboratory frame, even under such ideal conditions, will reduce any Bragg scattering compared with what would be obtained if the intrinsic axis itself could be perfectly aligned.

The optimum experimental configuration to search for this effect is identical to that already employed by Uhrhane, McCarthy, and Yearian[10] on a polarized holmium target. In their experiment $\vec{H}$ was in the scattering plane and bisected the angle between the incident and scattered beam. However their data extend only to momentum transfers of 280 MeV/c, which is about half the value at which a Bragg reflection might occur. High-energy resolution should also be employed to eliminate inelastic scattering.

The experimental configuration of Cooper *et al.*[1] is not suitable for finding laminar structure, since they were able to determine spherical-harmonic components of charge density up to only $l=4$. A nucleus the size of ^{165}Ho, which could have six laminar layers, would manifest a charge-density wave structure in harmonic components near $l=10$.

Extension of the work of Uhrhane *et al.* to higher momentum transfer (and with high-energy resolution) should be attempted even though the Hartree-Fock calculations of Bertozzi and Negele[11] do not provide any evidence for charge-density wave laminations. There are two reasons: It has been shown[6] that particle-particle correlations (in this context, two-nucleon–two-hole excitations) can significantly enhance density-wave instabilities for which all particles are modulated in phase. Such effects are not included in Hartree-Fock approximations. Finally, in a finite nucleus the fact that one-particle excitation energies are not infinitesimal would endow the usual (unmodulated) state with local stability even if a finite-amplitude modulated state with lower energy did exist. Consequently an iterative Hartree-Fock calculation could converge to the unmodulated state, even though it were not the ground state.

The possibility that heavy nuclei possess charge-density wave modulations is, naturally, a very

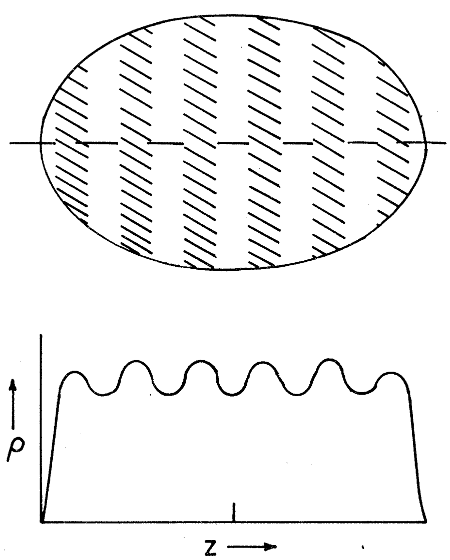

FIG. 1. Schematic illustration of a deformed nucleus with a spatially periodic charge modulation.

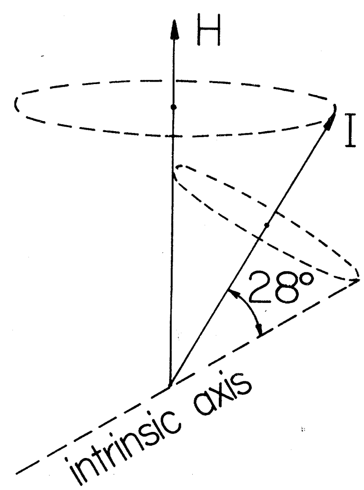

FIG. 2. Diagram showing the precession of the intrinsic nuclear axis about the nuclear spin $\vec{I}$ which in turn precesses about the hyperfine field $\vec{H}$.

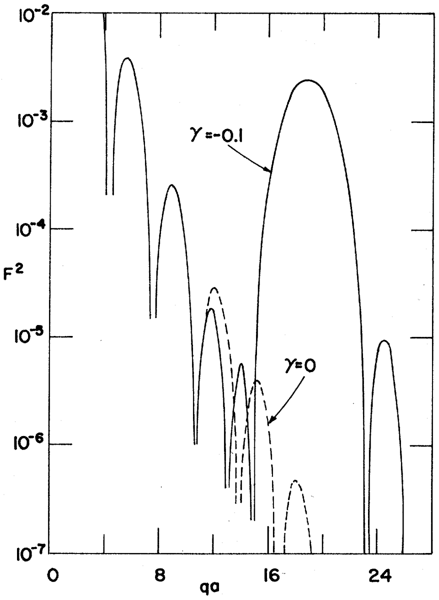

FIG. 3. Square of the form factor versus momentum transfer (in dimensionless units). Dashed curve is for an unmodulated density, solid curve is for 10% modulation. The curves are normalized to unity at $qa = 0$.

unlikely speculation. Nevertheless, the current feasibility of high-energy elastic electron scattering to probe for such structure may be of interest, and could lead to an experimental upper limit on the amplitude of such a modulation. Similar experiments with proton or pion scattering may also be feasible.

A qualitative indication of the effect of a density modulation on the scattering cross section is shown in Fig. 3. The Fourier transform F of $\rho(r, \theta)$, Eq. (3), was computed for momentum transfers parallel to the intrinsic axis. The wave vector of the density modulation was taken to be $6\pi/a$, where $a = R(0)$, and $t = a/12$. The solid curve of Fig. 3 is F^2 for a modulation amplitude $\gamma = -0.1$, corresponding to a density profile similar to Fig. 1. The dashed curve is the result for no modulation, and is indistinguishable from the solid curve when the momentum transfer is less than the value corresponding to $qa = 10$. No allowance has been made for the angular distribution of the intrinsic axis which, from Fig. 2, is significant even for 100% polarization. This effect would reduce the magnitude of the Bragg reflection by one or two orders of magnitude. Nevertheless, the sensitivity seems sufficient to permit a definite answer to the question proposed here.

---

[1]T. Cooper, W. Bertozzi, J. Heisenberg, S. Kowalski, W. Turchinetz, C. Williamson, L. Cardman, S. Fivozinsky, J. Lightbody, Jr., and S. Penner, Phys. Rev. C 13, 1083 (1976).

[2]A. W. Overhauser, Phys. Rev. Lett. 4, 415 (1960).

[3]A. W. Overhauser, Phys. Rev. Lett. 4, 462 (1960); Phys. Rev. 128, 1437 (1962).

[4]R. Karplus and K. M. Watson, Am. J. Phys. 25, 641 (1957).

[5]E. M. Henley and L. Wilets, Ann. Phys. (N.Y.) 14, 120 (1961).

[6]A. W. Overhauser, Phys. Rev. 167, 691 (1968).

[7]J. Kondo, J. Phys. Soc. Jpn. 16, 1690 (1961).

[8]R. Wagner, P. D. Miller, T. Tamura, and H. Marshak, Phys. Rev. 139, B29 (1965); T. R. Fisher, R. S. Safrata, E. G. Shelley, J. McCarthy, S. M. Austin, and R. C. Barrett, *ibid.* 157, 1149 (1967).

[9]$\cos^{-1}\{I/[I(I+1)]^{1/2}\} = 28°$ for $I = \frac{7}{2}$.

[10]F. J. Uhrhane, J. S. McCarthy, and M. R. Yearian, Phys. Rev. Lett. 26, 578 (1971).

[11]W. Bertozzi and J. Negele, Comments Nucl. Part. Phys. 6, 59 (1976).