Pion double charge exchange on ^{4}He†

W. R. Gibbs, B. F. Gibson, A. T. Hess,* and G. J. Stephenson, Jr.
Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545
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We have examined the reaction $^4\mathrm{He}(\pi^\pm,\pi^\mp)4(^p_n)$ in a model in which there occur two single π-N charge-exchange scatterings with intermediate off-shell pion propagation between scatterings. Separable π-nucleon t matrices having off-shell form factors were used with fully antisymmetrized nuclear wave functions. Spin-flip of the struck nucleons was included. Both angular distributions and total cross sections were calculated and compared with available data.

[NUCLEAR REACTIONS $^4\mathrm{He}(\pi^-,\pi^+)$; $E=0$–500 MeV calculated $\sigma(E)$; $E=140$ MeV calculated $\sigma(\theta=0°, E_f)$; $E_f=176$ MeV calculated $\sigma(\theta=0°, E)$; $E=485$ MeV calculated $\sigma(E_f)$.]

I. INTRODUCTION

Pion-^{4}He experiments have been performed[1,2] which have searched for a final four-neutron bound state. While no evidence for the tetraneutron was found, data were obtained for the double-charge-exchange (DCX) reaction $^4\mathrm{He}(\pi^-,\pi^+)4n$. These data consist of a measurement of forward positive pions having an energy of 176 MeV while the beam π^- energy is varied in 6 MeV steps,[1] and a measurement of the π^+ energy spectrum at 20° for an incident 140 MeV π^- beam.[2] Also, experiments have been performed[3,4] in which total cross section data were obtained for the double-charge-exchange reaction of incident π^+ on ^{4}He at π^+ energies of 100 and 486 MeV.

There has been some difficulty in obtaining agreement between theory and experimental results. Becker and Schmit[5] have reported a calculation in which they considered the reaction to proceed by a two-step pion-nucleon process entirely in the P_{33} π-N channel. While their calculation reproduces the Gilly *et al.* data[1] well, the Kaufman, Perez-Mendez, and Sperinde data[2] is reproduced only in shape, the theoretical prediction being too large by a factor of 2300. This lack of agreement is presumably due to the treatment of the antisymmetry of the nuclear wave functions, the problems involved in a complete treatment of the five-body phase space, and approximation of the effective π-nucleon t matrix. A recent publication[6] by Germond and Wilkin considers the contribution to the double-charge-exchange process arising from the π-π scattering amplitude in which the incident pion scatters from a pion in the clouds which surround the nucleons. However, this work does not include Pauli correlations or a complete treatment of the five-particle final phase space. In the present work, we report our attempt to make a realistic estimate of the contribution to the double-charge-exchange reaction arising from two π-N charge-exchange scatterings, which provides an obvious competition to the mechanism of Ref. 6.

II. FORMALISM

We have used a double-scattering amplitude obtained from two single-scattering π-N charge-exchange amplitudes with intermediate off-shell propagation[7]:

$$F(\vec{k},\vec{k}') = \frac{4\pi}{(2\pi)^3}\int d\vec{p}\, f_2(\vec{p},\vec{k}')\,\frac{e^{i\vec{p}\cdot\vec{r}_{21}}e^{i\vec{k}\cdot\vec{r}_1}e^{-i\vec{k}'\cdot\vec{r}_2}f_1(\vec{k},\vec{p})}{p^2-k^2-i\eta}\,, \tag{1}$$

where $\vec{r}_2$ and $\vec{r}_1$ denote the position vectors of the two struck nucleons and $\vec{r}_{21}=\vec{r}_2-\vec{r}_1$. Separable s- and p-wave amplitudes having off-shell form factors were used:

$$f_i(\vec{p},\vec{q}) = \frac{2\pi}{ik}\sum_{lm} f_i^l(p,q)Y^*_{lm}(\hat{p})Y_{lm}(\hat{q})\,, \tag{2}$$

where

$$f_i^l(p,q) = \lambda_i^l(\omega)v_l(p)v_l(q)\,. \tag{3}$$

The $\lambda_i^l(\omega)$ is a function of the π-N phase shifts which can be written

$$\lambda_i^l(\omega) = \{\exp[2i\delta_l(\omega)]-1\}k/\kappa\,, \tag{4}$$

where $\omega=(\kappa^2+\mu^2)^{1/2}$ is the pion energy in the π-N center of mass frame and the ratio k/κ transforms the π-N amplitude to the laboratory frame. For the pion-nucleon phase shifts, we have used the McKinley parametrization.[8] The functions $v_l(q) = (q/k)^l(k^2+\alpha_l^2)/(q^2+\alpha_l^2)$ describe the off-shell extension of the t matrix and go to unity on shell ($q\to k$) as they should. The parameters α_l have

been estimated previously from fits to π-deuteron absorption[9] to be approximately $\alpha_0 = 500$ MeV/c, $\alpha_1 = 300$ MeV/c. Factor of 2 variations of α_1 produced only 30% variations of the cross sections.

Using the expansions of the scattering amplitudes given by Eqs. (2) and (3) along with the plane wave expansion of $e^{i\vec{p}\cdot\vec{r}_{21}}$ and integrating over the magnitude of p by contour methods, we obtain

$$F(\vec{k},\vec{k}') = \frac{4\pi}{(2\pi)^3}\left(\frac{2\pi}{ik}\right)\left(\frac{2\pi}{ik'}\right)\sum_{lm\,l'm'\lambda\mu}\lambda_2^l(\omega')Y_{lm}(\hat{k}')4\pi i^\lambda Y_{\lambda\mu}(\hat{r}_{21})\tfrac{1}{2}i\pi k^{l+l'+1}Z^\lambda_{ll'}(kr_{21})G^{lm}_{\lambda\mu\,l'm'}$$
$$\times e^{i\vec{k}\cdot\vec{r}_1}e^{-i\vec{k}'\cdot\vec{r}_2}\lambda_1^{l'}(\omega)Y^*_{l'm'}(\hat{k})\left(\frac{k'^2+\alpha_l^{\,2}}{k^2+\alpha_l^{\,2}}\right)\left(\frac{k}{k'}\right)^l. \tag{5}$$

In this expression

$$Z^\lambda_{ll'}(kr_{21}) = \frac{2}{i\pi k^{l+l'+1}}\int_0^\infty p^2dp\,p^{l+l'}\left(\frac{k^2+\alpha_l^{\,2}}{p^2+\alpha_l^{\,2}}\right)\left(\frac{k^2+\alpha_{l'}^{\,2}}{p^2+\alpha_{l'}^{\,2}}\right)\frac{j_\lambda(pr_{21})}{p^2-k^2-i\eta} \tag{6}$$

which, when evaluated, becomes

$$Z^\lambda_{ll'}(kr_{21}) = h^{(+)}_\lambda(kr_{21}) - \frac{1}{(\alpha_{l'}^{\,2}-\alpha_l^{\,2})k^{l+l'+1}}\left[(i\alpha_l)^{l+l'+1}(k^2+\alpha_{l'}^{\,2})h^{(+)}_\lambda(i\alpha_l r_{21}) - (i\alpha_{l'})^{l+l'+1}(k^2+\alpha_l^{\,2})h^{(+)}_\lambda(i\alpha_{l'}r_{21})\right]. \tag{7}$$

Also, in Eq. (5), the symbol $G^{lm}_{\lambda\mu\,l'm'}$ is

$$G^{lm}_{\lambda\mu\,l'm'} = \int d\hat{p}\;Y^*_{lm}(\hat{p})Y_{\lambda\mu}(\hat{p})Y_{l'm'}(\hat{p}). \tag{8}$$

The expectation value of the double-charge-exchange amplitude of Eq. (5) between the initial and final states must be computed. For the initial ^{4}He wave function we have utilized the product form consisting of a purely symmetric space wave function and an antisymmetric spin-isospin wave function. This completely antisymmetric wave function can be thus written

$$\psi_{\rm init} = [(\psi_{1s})^4/\sqrt{2}\,][\,|0000\rangle|1100) + |1100\rangle|0000)]. \tag{9}$$

The four indices in the spin and isospin functions defined in Eqs. (B2) and (B3) denote the spin or isospin of the coupled pairs of particles 1 and 2, 3 and 4, and the total spin or isospin (and projection) of the four-particle system.

For the final-state wave functions, we assume that the four nucleons can be described by a product of plane waves along with the appropriate spin and isospin functions

$$\psi_{\rm final} = e^{i\vec{k}_1\cdot\vec{r}_1}e^{i\vec{k}_2\cdot\vec{r}_2}e^{i\vec{k}_3\cdot\vec{r}_3}e^{i\vec{k}_4\cdot\vec{r}_4}|S'S''SM\rangle$$
$$\times|\tfrac{1}{2}\tfrac{1}{2})_1|\tfrac{1}{2}\tfrac{1}{2})_2|\tfrac{1}{2}\tfrac{1}{2})_3|\tfrac{1}{2}\tfrac{1}{2})_4. \tag{10}$$

Here S' is the spin of particle pair 1 and 2, S'' is the spin of particle pair 3 and 4, and S and M are the total spin and spin projection of the four-particle system. Since the double-charge-exchange scattering operator, Eq. (5), is symmetric and the initial-state wave function is purely antisymmetric, the appropriate antisymmetric components of the final-state wave function will be projected out. This procedure is described in detail in Appendix B, and results in two amplitudes, one of which corresponds to the initial $|1100)$ isospin state and the other corresponds to the $|0000)$ isospin state. We can represent these as

$$A(1100) = \frac{1}{\sqrt{12}}[2A(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4) + 2A(\vec{k}_2,\vec{k}_1,\vec{k}_3,\vec{k}_4) - A(\vec{k}_1,\vec{k}_4,\vec{k}_2,\vec{k}_3) - A(\vec{k}_1,\vec{k}_3,\vec{k}_2,\vec{k}_4)$$
$$- A(\vec{k}_2,\vec{k}_4,\vec{k}_3,\vec{k}_1) - A(\vec{k}_2,\vec{k}_3,\vec{k}_4,\vec{k}_1) - A(\vec{k}_3,\vec{k}_2,\vec{k}_1,\vec{k}_4) - A(\vec{k}_4,\vec{k}_2,\vec{k}_1,\vec{k}_3)$$
$$- A(\vec{k}_3,\vec{k}_1,\vec{k}_2,\vec{k}_4) - A(\vec{k}_4,\vec{k}_1,\vec{k}_2,\vec{k}_3) + 2A(\vec{k}_3,\vec{k}_4,\vec{k}_1,\vec{k}_2) + 2A(\vec{k}_4,\vec{k}_3,\vec{k}_2,\vec{k}_1)], \tag{11}$$

$$A(0000) = \tfrac{1}{2}[A(\vec{k}_1,\vec{k}_3,\vec{k}_2,\vec{k}_4) - A(\vec{k}_1,\vec{k}_4,\vec{k}_2,\vec{k}_3) + A(\vec{k}_2,\vec{k}_4,\vec{k}_3,\vec{k}_1) - A(\vec{k}_2,\vec{k}_3,\vec{k}_4,\vec{k}_1)$$
$$- A(\vec{k}_3,\vec{k}_2,\vec{k}_1,\vec{k}_4) + A(\vec{k}_4,\vec{k}_2,\vec{k}_1,\vec{k}_3) + A(\vec{k}_3,\vec{k}_1,\vec{k}_2,\vec{k}_4) - A(\vec{k}_4,\vec{k}_1,\vec{k}_2,\vec{k}_3)], \tag{12}$$

where the momenta in the first two positions correspond to those associated with the two nucleons on which the charge exchange occurs. It is apparent how this expression accounts for Pauli correlations. If there is little energy loss by the pion (or equivalently, little energy to be shared by the four outgoing nucleons), $\vec{k}_1$, $\vec{k}_2$, $\vec{k}_3$, and $\vec{k}_4$ will all be small and nearly equal. This implies that there will be strong cancellation among the individual terms of Eq. (11) and Eq. (12), and thus a reduction in the amplitudes due to the Pauli principle. If, on the other hand, we did not use proper antisymmetric wave functions, the terms in Eqs. (11) and (12) would add incoherently, giving no Pauli suppression. In Sec. III, we shall present results for both the Pauli and the non-Pauli cases. Since we are interested in the differential cross section for pion double charge exchange, it is necessary for us to perform the integral over the phase space of the four outgoing nucleons. The phase space integral can be written as

$$\int \delta\left(\sum_{i=1}^{4}\vec{k}_i+\vec{k}'-\vec{k}\right)\delta\left(\sum_{i=1}^{4}\frac{k_i^2}{2M}-\omega+\omega'+E_B\right)$$
$$\times\sigma(\vec{k}_1,\vec{k}_2,\vec{k}_3,\vec{k}_4)d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4\,, \qquad (13)$$

where ω and ω' are the initial and final pion energies and E_B is the binding energy which must be supplied to break up the target nucleus. In practice E_B was taken to be 28 MeV. The method used for evaluating the phase space integral by Monte Carlo techniques is discussed in detail in Appendix A.

The radial wave functions of the initial state in Eq. (9) were at first chosen to be simply a product of four 1S-state harmonic oscillator functions. However, such a Gaussian density for ^{4}He does not reproduce the measured charge form factor[10] for $q^2 \geq 6$ fm^{-2}. This is shown in Fig. 1, where the proton charge form factor[11] has been folded in to properly convert the body density to a charge density. We have therefore used a wave function which includes N-N correlations and which gives a more reasonable tail to the density than the Gaussian. This "exponential-hole" wave function can be written

$$\psi=\sqrt{2\rho}\;e^{\rho r_0}\frac{e^{-\rho r'}}{r'}\sqrt{2\sigma}\;\frac{e^{-\sigma s}}{s}\sqrt{2\rho}\;e^{\rho r_0}\frac{e^{-\rho r}}{r}, \quad r,r'\geq r_0\,; \qquad (14)$$
$$\psi=0, \quad r,r'<r_0\,,$$

where $r,r'>r_0$ and ρ, σ, and r_0 are parameters to be determined so as to fit the charge form factor and the rms radius. The coordinates r, r', and s are defined as follows: r is the magnitude of the relative coordinate between the two nucleons on which the charge exchange occurs, $r=|\vec{r}_2-\vec{r}_1|$; r' is the magnitude of the relative coordinates between the other two nucleons; and s is defined as $\frac{1}{4}|\vec{r}_2+\vec{r}_1-(\vec{r}_3+\vec{r}_4)|$. This prescription, therefore, introduces N-N correlations between particle pairs (1, 2) and (3, 4). We have produced reasonable fits to the charge form factor (Fig. 1) for three sets of parameters: $r_0=1.35$ fm, $\rho=0.3$ fm^{-1}, $\sigma=1.0$ fm^{-1}; $r_0=1.39$ fm, $\rho=0.4$ fm^{-1}, $\sigma=0.5$ fm^{-1}; and $r_0=1.47$ fm, $\rho=0.6$ fm^{-1}, $\sigma=0.3$ fm^{-1}. We note, however, that this wave function ansatz is not truly symmetric in the nucleon coordinates, so that our calculation which takes into account Pauli correlations is not strictly correct. Nonetheless, we believe that the reduction is a reasonable estimate for the Pauli suppression factor.

We have attempted to estimate the effect of the final-state interaction among the nucleons by treating the interaction between the spectator pair within a zero-range wave function model. This

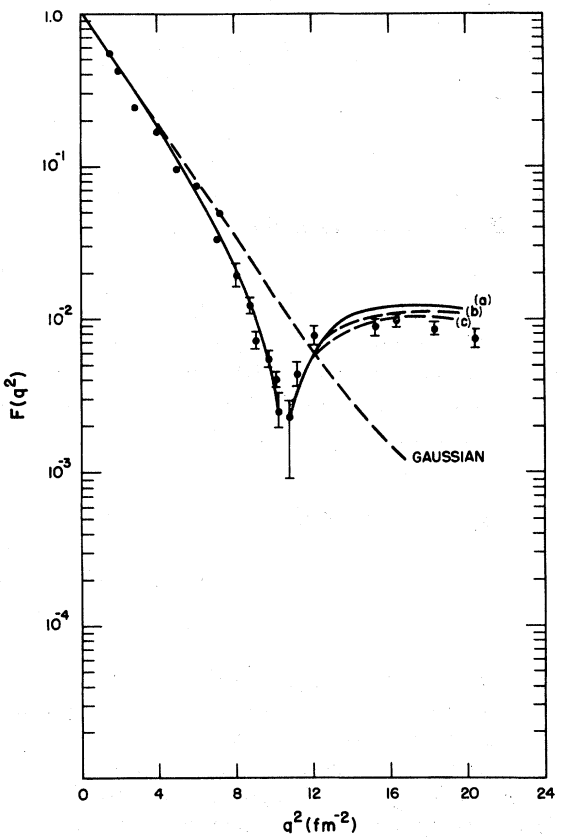

FIG. 1. Experimentally measured ^{4}He charge form factor (Ref. 10) plotted with that for the correlated density for three sets of parameters: (a) $\rho=0.3$ fm^{-1}, $\sigma=1.0$ fm^{-1}, $r_0=1.35$ fm; (b) $\rho=0.4$ fm^{-1}, $\sigma=0.5$ fm^{-1}, $r_0=1.39$ fm; (c) $\rho=0.6$ fm^{-1}, $\sigma=0.3$ fm^{-1}, $r_0=1.47$ fm. The charge form factor obtained for a Gaussian mass density having $\alpha=0.8715$ fm^{-1} is also shown.

results in a factor

$$e^{i\delta}e^{i\rho r_0}\sqrt{2\rho}\int_{r_0}^{\infty} r'^2 dr' \frac{\sin(qr'+\delta)}{qr'}\frac{e^{-\rho r'}}{r'}$$
$$= \frac{e^{i\delta}\sqrt{2\rho}}{\rho^2+q^2}[\cos(qr_0+\delta)+\frac{\rho}{q}\sin(qr_0+\delta)] \quad (15)$$

multiplying the amplitudes. For δ we have used the effective range formula

$$q\cot\delta = -1/a+\tfrac{1}{2}q^2 r_0 . \quad (16)$$

It is clear that this produces too large an effect, since the presence of the other strongly interacting particles modifies greatly the two-nucleon interaction. We emphasize that this is only a crude estimate intended to indicate just the size of the effect. There is, of course, interaction among other pairs of nucleons and it might seem strange, at first, to include only those interactions among the spectator pair. This, however, should be the largest effect, since the spectator nucleons are in a pure relative s state where the nucleon-nucleon interaction is strongest at low energy.

The formalism discussed so far has dealt with non-spin-flip scattering amplitudes as defined in Eqs. (2) and (3). However, we do include the possibility of a spin-flip occurring on one or both of the struck nucleons by adding spin-flip amplitudes of the form

$$f(\vec{p},\vec{q}) = \frac{2\pi}{ik}\lambda_s^1(\omega)\{\sqrt{2}\,\sigma_+[Y_{11}^*(\hat{p})Y_{10}(\hat{q})+Y_{10}^*(\hat{p})Y_{1-1}(\hat{q})]+\sqrt{2}\,\sigma_-[Y_{10}^*(\hat{p})Y_{11}(\hat{q})+Y_{1-1}(\hat{p})Y_{10}(\hat{q})]$$
$$+\sigma_z[Y_{11}^*(\hat{p})Y_{11}(\hat{q})-Y_{1-1}^*(\hat{p})Y_{1-1}(\hat{q})]\}v_1(p)v_1(q), \quad (17)$$

where $\lambda_s^1(\omega)$ is analogous to Eq. (4), but refers to the spin-flip π-N p-wave amplitude, $v_1(p)$ and $v_1(q)$ are the off-shell form factors as previously defined, and σ_+, σ_-, and σ_z are the Pauli spin matrices. This spin-flip amplitude is added to the non-spin-flip amplitude to give the total π-N charge-exchange amplitude. We may then write the π-N amplitudes which enter in Eq. (1) symbolically as

$$f_2(\vec{p},\vec{k}') = A + C\sigma_z^2 + D\sigma_+^2 + E\sigma_-^2 \quad (18)$$

and

$$f_1(\vec{k},\vec{p}) = A' + D'\sigma_+^1 + E'\sigma_-^1 , \quad (19)$$

where the coefficient of σ_z in f_1 is zero due to our choice of $\vec{k}$ lying along the z axis.

This results in 12 terms which must be averaged over the spin wave functions:

$$f_2(\vec{p},\vec{k}')f_1(\vec{k},\vec{p}) = AA' + AD'\sigma_+^1 + AE'\sigma_-^1$$
$$+ CA'\sigma_z^2 + CD'\sigma_z^2\sigma_+^1 + CE'\sigma_z^2\sigma_-^1$$
$$+ DA'\sigma_+^2 + DD'\sigma_+^2\sigma_+^1 + DE'\sigma_+^2\sigma_-^1$$
$$+ EA'\sigma_-^2 + ED'\sigma_-^2\sigma_+^1 + EE'\sigma_-^2\sigma_-^1 . \quad (20)$$

We then evaluate $f_2(\vec{p},\vec{k}')f_1(\vec{k},\vec{p})$ between the initial- and final-spin states of Eqs. (9) and (10). This procedure is discussed in Appendix C. The incoherent sum of cross sections to the allowed final spin states give the double-charge-exchange cross section. In our results, as shall be discussed in Sec. III, we found that single spin-flip accounts for anywhere from 20–50% of the calculated cross sections, whereas the double spin-flip gives a negligible contribution.

III. RESULTS

We have performed calculations to compare with the single angle data at $E_{\text{init}} = 140$ MeV and at $E_{\text{final}} = 176$ MeV, as well as the angle integrated momentum distribution at 485 MeV. We have also calculated a total cross section curve.

A. $E_{\text{init}} = 140$ MeV ($\theta = 20°$)

Figure 2 shows the result of a calculation using a Gaussian density. In this case the Pauli correction is believable and is seen to be very large. Since the nucleons are bound and therefore moving (Fermi motion), the precise energy at which to evaluate the initial phase shifts is not well determined. Several prescriptions were tried and led to variations of the order of 20%. The agreement between theory and experiment is very good considering the obvious defects in the calculations. The discrepancy for large final pion energy could well be ascribed to the final state interactions among the nucleons.

Figures 3 and 4 show variations due to the three model correlated wave functions described earlier. The "Pauli" corrected curves can now only be regarded as estimates of this effect, since the initial-state spatial wave function is not symmetric in all of the nucleons. One may observe that a very strong dependence upon the correlation structure is exhibited by the double-charge-exchange two-body operator that is essentially unobservable with the one-body operator for elastic electron

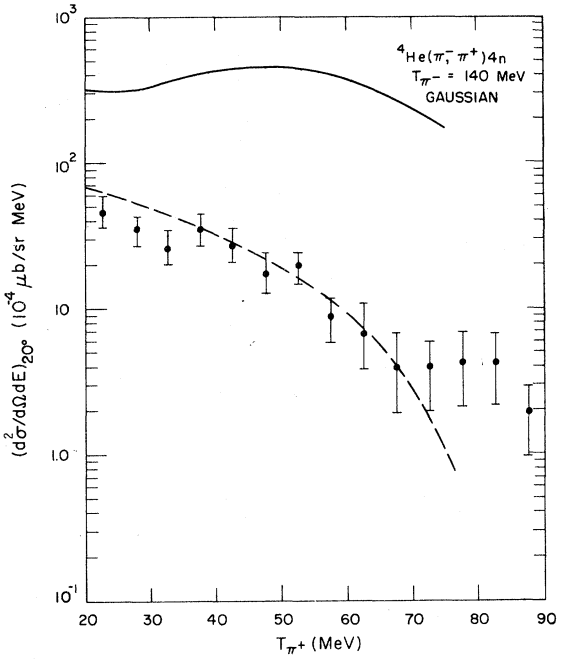

FIG. 2. Results using Gaussian ^{4}He density for both non-Pauli-suppression (solid curve) and Pauli-suppression (dashed curve) cases plotted with the data at 140 MeV of Kaufman *et al.* (Ref. 2).

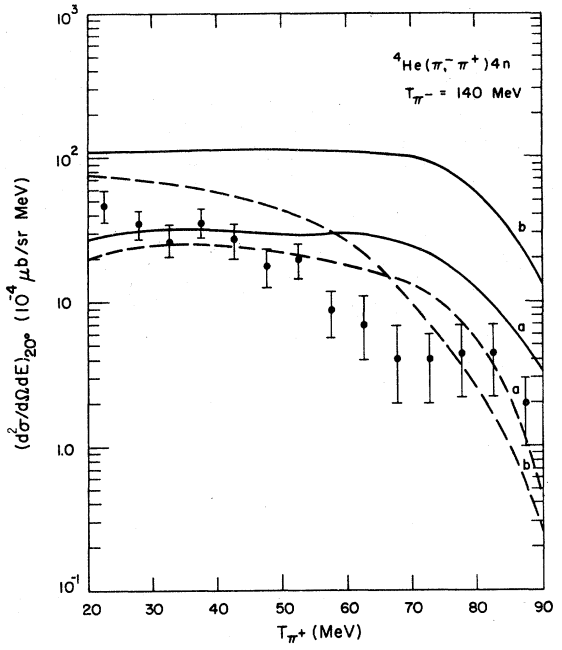

FIG. 3. Curves for two sets of parameters (a) and (b) for the correlated density as defined in Fig. 1, plotted with the 140 MeV data (Ref. 2). Solid and dashed curves as indicated in Fig. 2.

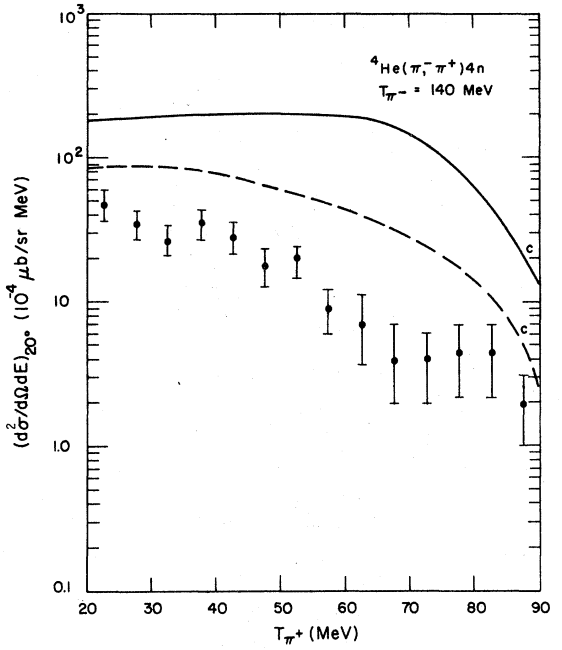

FIG. 4. Curves for the parameter set (c) (Fig. 1) for the correlated density. Data are from Ref. 2. Solid and dashed curves as noted in Fig. 2.

scattering (Fig. 1).

B. E_{final} = 176 MeV (θ = 0°)

Figure 5 shows the data of Gilly *et al.*[1] compared with our Gaussian calculation. In this experiment the incident beam had a rather poorly defined energy and included in this figure is a curve resulting from folding in a Gaussian resolution function with the quoted resolution (8%). Figures 6 and 7 show the corresponding calculations with the model correlated wave functions as before. As may be seen, the "Pauli" results are quite low.

The effect of our estimate of the final-state interaction is shown in Fig. 8 for one of the model densities. There are two effects of the final-state interactions. One is the actual increase of the matrix elements at low nucleon energies, leading to an increase of the non-Pauli cross section in this region. The second effect comes from the phase factor multiplying each of the matrix elements in the sums in Eqs. (11) and (12). This additional varying phase tends to reduce the Pauli cancellation.

C. E_{init} = 485 MeV

A comparison is made with the momentum distribution measured by Carayannopoulos *et al.*[3] and

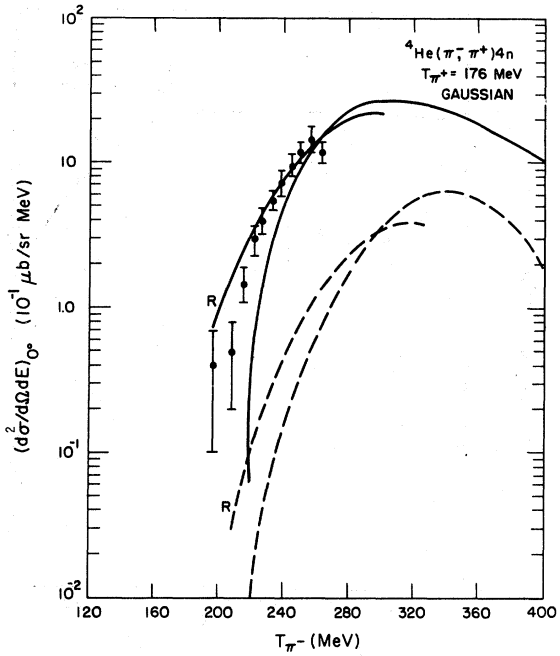

FIG. 5. Results for Gaussian ^{4}He density with the data of Gilly *et al*. (Ref. 1). Solid and dashed curves are as in Fig. 2. Curves marked "*R*" have the resolution function discussed in Sec. III folded in.

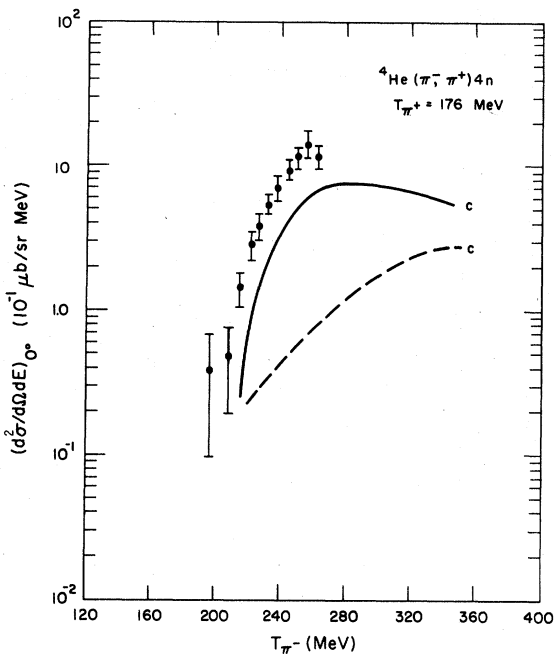

FIG. 7. Results using correlated density with parameter set (c). Dashed and solid curves are as defined in Fig. 2. Data are from Ref. 1.

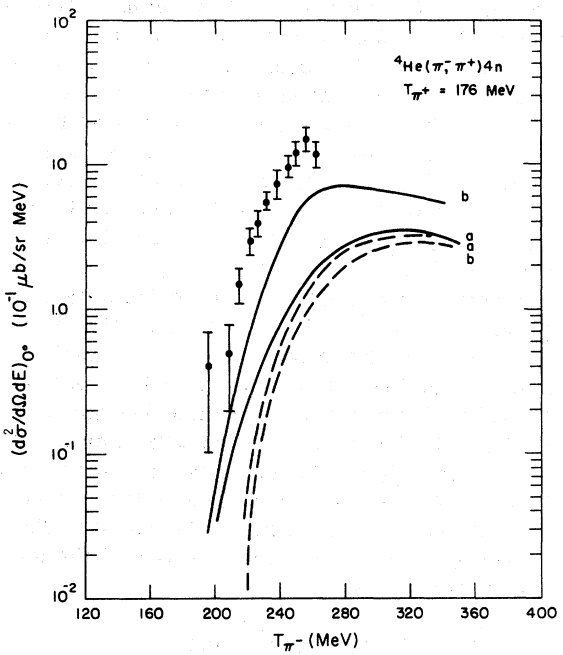

FIG. 6. Results using correlated density with parameter sets (a) and (b) (Fig. 1). Solid and dashed curves as in Fig. 2. Data are from Ref. 1.

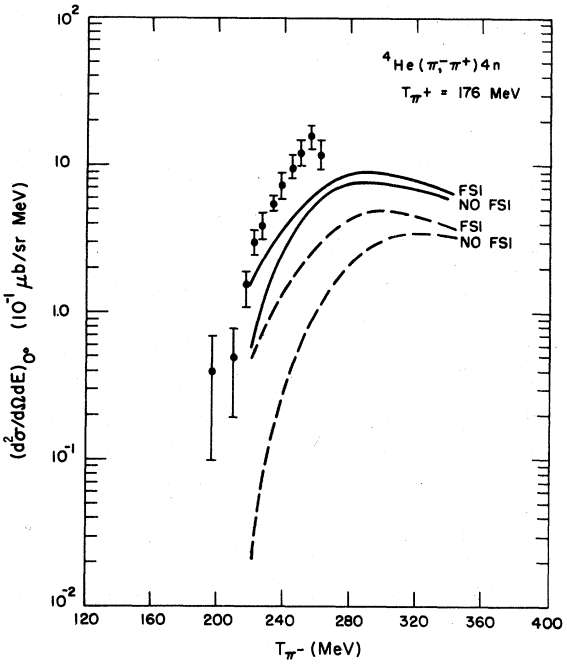

FIG. 8. Results using correlated density with parameter set (b) with and without the final-state interactions (FSI) discussed in Sec. II. Dashed and solid curves are as defined in Fig. 2. Data are from Ref. 1.

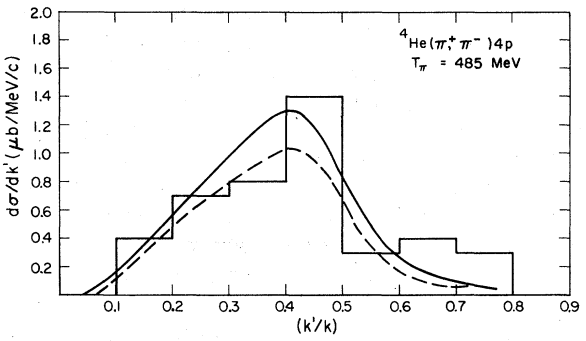

FIG. 9. Histogram of (k'/k) data of Carayannopoulos *et al.* (Ref. 3) plotted with our Pauli (dashed) and non-Pauli (solid) curves. The theoretical curves have been renormalized.

shown in Fig. 9. The shape is quite reasonable although the magnitude has been arbitrarily renormalized by a factor of 10.

D. Total cross section

A summary of the total cross section is given in Fig. 10, including the recent data of Falomkin *et al.*[4] Included also are the curves of Becker and Schmit[5] and Germond and Wilkin.[6] Our total cross section curves were calculated without any estimate of the final-state interaction.

IV. CONCLUSIONS

As may be seen from Fig. 10, questions of comparisons in this reaction involve orders of magnitude.

We expect our calculations to be best at low energies, since we include only s- and p-waves in the pion-nucleon interaction. The fact that we are far below the total cross section at 485 MeV is certainly to be expected, and this same restriction probably accounts for a large fraction of the discrepancy with the Gilly *et al.* data.[1] (Recall that the cross section measured at 250 MeV is strongly affected by the actual cross section at 300 MeV due to the resolution function.) These cross sections can be increased by the true d- (and higher) waves and relativistic effects which look like higher partial waves.

Thus, we believe that the (essential) agreement with the Kaufman *et al.* data,[2] the evident disagreement with the Gilly *et al.* data,[1] and the somewhat larger disagreement with the Carayannopoulos *et al.* data,[3] offer no surprises when account is taken of the deficiencies in our calculations. The disagreement with the Falomkin *et al.* data[4] is more serious since we expect our calculation to give reliable answers at those energies. In fact, it is very difficult to understand how any calculation can reconcile these data with the Kaufman *et al.* data.[2] If one makes a comparison of our calculation with the data of Gilly *et al.*, it is tempting to say that something additional is required. However, we do not believe our calculation is good enough at this energy to allow us to make this claim.

With regard to the calculation by Becker and Schmit,[5] we believe that the main difference lies in the off-shell t matrix. In their calculation, it would appear that the pion-nucleon amplitude for the second charge exchange is evaluated at an energy only slightly less than the incident pion energy, independent of the actual final energy, while we evaluate the phase shifts at the on-shell

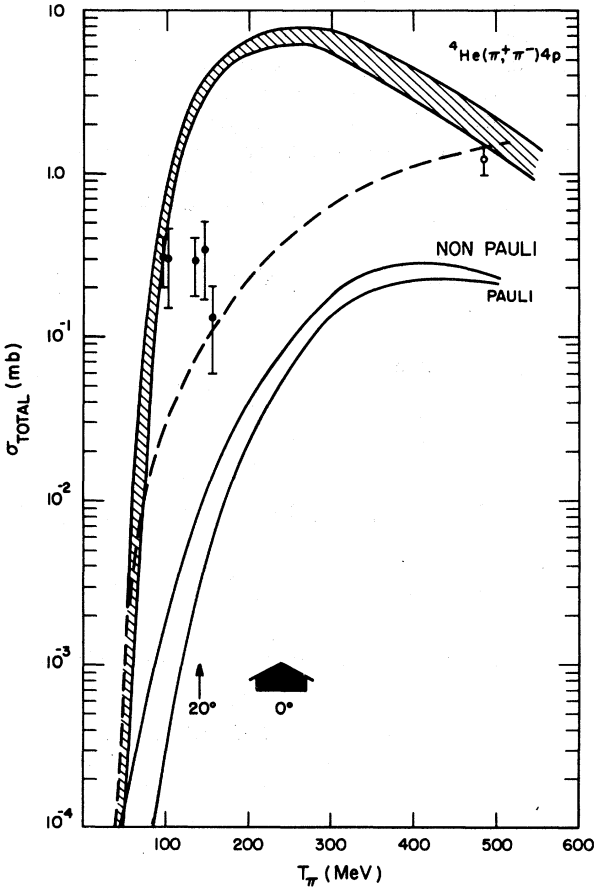

FIG. 10. Total cross section results of Becker and Schmit (Ref. 5) (dark band), Germond and Wilkin (Ref. 6) (dashed curve), and our results for Pauli and non-Pauli cases. Solid data points are from Ref. 4; open circle point is from Ref. 3. Arrow at 140 MeV indicates position of 20° energy spectrum of Ref. 2. Wide arrow indicates range of incident energies for energy spectrum of Ref. 1.

energy for each scattering. This difference alone is enough to account for about one order of magnitude of the discrepancy. The remainder of the difference is presumably due to their treatment of the momentum dependence of the t matrix, the final space, and ^{4}He wave functions. At 140 MeV they are too high by a factor of 2300 while we are in essential agreement with this data.

The calculation of Germond and Wilkin[6] did not include the effects of Pauli suppression in the final state, the five-body phase space, or form factors at the vertices. In addition, the estimate of π-π scattering is somewhat uncertain. Even with the reductions (and possible reductions) implied, their calculation will give a reasonably large cross section. It would appear (assuming only that our calculation gives a reasonable extrapolation from single angle data to total cross section) that the 140 MeV data of Kaufman *et al.*[2] rule out a contribution even as large as 10% of the one given by Germond and Wilkin at this energy. Of course, this also implies a disagreement with the data of Falomkin *et al.*,[4] which would tend to confirm a π-π contribution of the order predicted. Clearly, additional experimental information is needed around 140 MeV. At the higher energies measured to date, we cannot rule out contributions from π-π scattering of the predicted order.

We have seen that this reaction is very sensitive to correlations in the nuclear wave function but our present inability to treat the remainder of the problem in a totally believable fashion (predominantly the five-body final state) render this method impractical for their study. It does appear, however, that such calculations and experiments might make statements about the importance of "exchange current" contributions to the double-charge-exchange reaction.

APPENDIX A: MONTE CARLO INTEGRATION

In order to calculate the cross section per unit volume of phase space of the pion alone it was necessary to do the 12 dimensional integral over the coordinates of the four nucleons in the final state. With the definitions

$$E_0 \equiv \omega - \omega' - E_B, \quad \vec{q} \equiv \vec{k}' - \vec{k}, \tag{A1}$$

the integral can be written as

$$I = \int \delta\left(\frac{\sum k_i^2}{2M} - E_0\right) \delta^3\left(\sum \vec{k}_i + \vec{q}\right) \sigma(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) d\vec{k}_1 d\vec{k}_2 d\vec{k}_3 d\vec{k}_4 . \tag{A2}$$

Making the transformation

$$\vec{t}_i \equiv \frac{\vec{k}_i + \frac{1}{4}\vec{q}}{E'} , \tag{A3}$$

where $E' \equiv [2M(E_0 - q^2/8M)]^{1/2}$, we see that

$$I = 2ME'^7 \int \delta\left(\sum t_i^2 - 1\right) \delta^3\left(\sum \vec{t}_i\right) \sigma(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) d\vec{t}_1 d\vec{t}_2 d\vec{t}_3 d\vec{t}_4 . \tag{A4}$$

This can be converted to an average over the volume in t space by recognizing that

$$I_n \equiv \int \delta\left(\sum t_i^2 - 1\right) \delta^3\left(\sum \vec{t}_i\right) |\vec{t}_1 + \vec{t}_2|^n d\vec{t}_1 d\vec{t}_2 d\vec{t}_3 d\vec{t}_4 = \frac{2\pi^4}{(n+3)(n+5)(n+7)} \tag{A5}$$

is

$$2\pi^4/105 \text{ for } n = 0 .$$

Thus

$$I = 2ME'^7 (2\pi^4/105) \langle \sigma(\vec{k}_1, \vec{k}_2, \vec{k}_3, \vec{k}_4) \rangle_{\text{av}} , \tag{A6}$$

where the points in the averaging procedure are chosen uniformly dense under the condition that they satisfy the δ functions. This was accomplished as follows. First a set of 12 random components, in the range 0–1, was chosen. Next a constant vector (equal to the average of the original vectors selected) was subtracted from each $\vec{t}_i$ so that resultant $\vec{t}_i$'s have sum zero. Finally, the vectors were normalized by a constant such that $\sum t_i^2 = 1$.

The $\vec{k}_i$'s were then computed so that the appropriate cross section could be calculated.

In order to check this procedure and insure that no biases had been introduced, values of I_n/I_0 were calculated for $n=1$ and 2. Agreement with the analytic result to better than 1% was obtained.

APPENDIX B: PAULI PRINCIPLE EFFECTS

While the results of this section can be deduced from the permutation group (in fact, the final result is nothing more than the enumeration of the basis states of various representations), we obtain the expressions here by using spin-space-isospin expressions, letting the Clebsch-Gordan coefficients provide the relative weighting.

With a symmetric operator, we need only antisymmetrize the initial state. The ground-state wave function of ^{4}He is taken as

$$\psi_i = \varphi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)\frac{1}{\sqrt{2}}\{|0000)|1100\rangle + |1100)|0000\rangle\}, \quad \text{(B1)}$$

where φ is a totally symmetric space state and

$$|T'T''TM) = \sum (\tfrac{1}{2}m_1\tfrac{1}{2}m_2|T'M')(\tfrac{1}{2}m_3\tfrac{1}{2}m_4|T''M'') \times (T'M'T''M''|TM) \times |\tfrac{1}{2}m_1)|\tfrac{1}{2}m_2)|\tfrac{1}{2}m_3)|\tfrac{1}{2}m_4), \quad \text{(B2)}$$

$$|S'S''SM\rangle = \sum (\tfrac{1}{2}m_1\tfrac{1}{2}m_2|S'M')(\tfrac{1}{2}m_3\tfrac{1}{2}m_4|S''M'') \times (S'M'S''M''|SM) \times |\tfrac{1}{2}m_1\rangle|\tfrac{1}{2}m_2\rangle|\tfrac{1}{2}m_3\rangle|\tfrac{1}{2}m_4\rangle. \quad \text{(B3)}$$

Note that $|)$ are isospin states and $|\rangle$ are spin states.

Since in the present calculation only two particles are affected by the operator, it can be expressed as a sum of two-body operators:

$$\theta = \sum_{i,j=1}^{4} \theta_{ij}(\vec{r}_i,\vec{r}_j)\tau_i^+\tau_j^+. \quad \text{(B4)}$$

The final state can be taken to be any product wave function which is complete in spin indices:

$$\psi_f = \prod_{i=1}^{4}|\tfrac{1}{2}\tfrac{1}{2})_i|S'S''SM\rangle\prod_{i=1}^{4}e^{i\vec{k}_i\cdot\vec{r}_i}. \quad \text{(B5)}$$

The appropriate linear combination of matrix elements is then determined by whether one starts from the first term in (B1) (called I) or the second term (called II), and by the final spin structure. The object of the present endeavor is to reduce the matrix element to a sum of matrix elements in which charge exchange takes place on particles 1 and 2 and spin-flip may occur on neither, on 1, or on 2. Note that the non-spin-flip can lead only to $S=0$ final states and the single spin-flip only to $S=1$ states, and thus they are incoherent (although the two spin-flips are coherent). The contributions from the two initial pieces can be written

TABLE I. Weighting of various matrix elements in their contribution to the whole. The numbers in parentheses to the left denote whether spin-flip takes place on the first (1) or second (2) nucleon. All matrix elements for I are to be multiplied by $\frac{1}{2}$; all elements for II, by $1/(2\sqrt{3})$.

$S'S''SM$	[1,2]	[2,1]	[1,3]	[3,1]	[1,4]	[4,1]	[2,3]	[3,2]	[2,4]	[4,2]	[3,4]	[4,3]
						I						
1100	0	0	+1	+1	−1	−1	−1	−1	+1	+1	0	0
(1) 111±1	0	0	+1	−1	−1	+1	−1	+1	+1	−1	0	0
(2) 111±1	0	0	−1	+1	+1	−1	+1	−1	−1	+1	0	0
(1) 011±1	0	0	+1	+1	−1	−1	+1	−1	−1	+1	0	0
(2) 011±1	0	0	+1	−1	−1	+1	−1	−1	+1	+1	0	0
(1) 101±1	0	0	+1	+1	−1	+1	−1	−1	+1	−1	0	0
(2) 101±1	0	0	−1	+1	−1	−1	+1	−1	+1	+1	0	0
						II						
0000	+2	+2	−1	−1	−1	−1	−1	−1	−1	−1	+2	+2
(1) 111±1	+2	−2	−1	+1	−1	−1	+1	+1	+1	−1	−2	+2
(2) 111±1	+2	−2	−1	+1	+1	+1	−1	−1	+1	−1	−2	+2
(1) 011±1	+2	+2	−1	−1	−1	+1	−1	−1	−1	+1	+2	−2
(2) 011±1	+2	+2	+1	−1	−1	−1	+1	−1	−1	−1	+2	−2
(1) 101±1	+2	−2	−1	−1	−1	−1	+1	−1	+1	−1	+2	+2
(2) 101±1	+2	−2	−1	+1	−1	+1	−1	−1	−1	−1	+2	+2

$$(\mathrm{I}) = \frac{1}{\sqrt{2}} \sum_{i,j} \prod_{k=1}^{4} {}_k(\tfrac{1}{2}\,\tfrac{1}{2}\,|\,\tau_i^+\tau_j^+\,|\,0000)\mu_{ij}\,,$$

$$(\mathrm{II}) = \frac{1}{\sqrt{2}} \sum_{i,j} \prod_{k=1}^{4} {}_k(\tfrac{1}{2}\,\tfrac{1}{2}\,|\,\tau_i^+\tau_j^+\,|\,1100)\eta_{ij}\,, \qquad \text{(B6)}$$

where

$$\begin{Bmatrix}\mu_{ij}\\ \eta_{ij}\end{Bmatrix} = \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 e^{i(\vec{k}_1\cdot\vec{r}_1+\vec{k}_2\cdot\vec{r}_2+\vec{k}_3\cdot\vec{r}_3+\vec{k}_4\cdot\vec{r}_4)} \times \varphi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)\langle S'S''SM| \times \theta_{ij}(\vec{r}_i,\vec{r}_j)\begin{Bmatrix}|1100\rangle\\ |0000\rangle\end{Bmatrix}. \qquad \text{(B7)}$$

The coefficients in Eqs. (B6) can be evaluated in a straightforward manner and are tabulated in Table (I) under I-1100 and II-0000. For non-spin-flip the operator in (B7) is diagonal and there are no changes in the coefficients. Since the spin operator is assumed to operate on particle 1 or particle 2 there are additional signs which come from permuting $i \to 2$ or $j \to 1$. These are reflected in the remainder of the table.

The table may be generated by using the following rules:

$$\begin{aligned}
&\langle S'S''SM|\vec{\sigma}_2|1100\rangle = (-1)^{1-S'}\langle S'S''SM|\vec{\sigma}_1|1100\rangle\,,\\
&\langle S'S''SM|\vec{\sigma}_3|1100\rangle = (-1)^{S'+S''-S}\langle S'S''SM|\vec{\sigma}_1|1100\rangle\,,\\
&\langle S'S''SM|\vec{\sigma}_4|1100\rangle = (-1)^{S'+S+1}\langle S'S''SM|\vec{\sigma}_1|1100\rangle\,,\\
&\langle S'S''SM|\vec{\sigma}_2|0000\rangle = (-1)^{S'}\langle S'S''SM|\vec{\sigma}_1|0000\rangle\,,\\
&\langle S'S''SM|\vec{\sigma}_3|0000\rangle = (-1)^{S'+S''-S}\langle S'S''SM|\vec{\sigma}_1|0000\rangle\,,\\
&\langle S'S''SM|\vec{\sigma}_4|0000\rangle = (-1)^{S'-S}\langle S'S''SM|\vec{\sigma}_1|0000\rangle\,.
\end{aligned} \qquad \text{(B8)}$$

Having removed the specific spin and isospin dependence, one is left with an operator which is a function of space coordinates. Let Q denote one of these operators (i.e., non-spin-flip, spin-flip on first particle, or spin-flip on second particle). Then

$$\bar{Q}_{ij} = \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 e^{i(\vec{k}_1\cdot\vec{r}_1+\vec{k}_2\cdot\vec{r}_2+\vec{k}_3\cdot\vec{r}_3+\vec{k}_4\cdot\vec{r}_4)} Q(\vec{r}_i,\vec{r}_j)\varphi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4)\,. \qquad \text{(B9)}$$

Change variables: $\vec{r}_1 = \vec{r}_j$, $\vec{r}_2 = \vec{r}_i$, $\vec{r}_3 = \vec{r}_l$, and $\vec{r}_4 = \vec{r}_m$, where l and m are chosen from the set 1, 2, 3, 4 but not equal to i or j or each other. Then

$$\bar{Q}_{ij} = \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 e^{i(\vec{k}_j\cdot\vec{r}_1+\vec{k}_i\cdot\vec{r}_2+\vec{k}_l\cdot\vec{r}_3+\vec{k}_m\cdot\vec{r}_4)} Q(\vec{r}_2,\vec{r}_1)\varphi(\vec{r}_1,\vec{r}_2,\vec{r}_3,\vec{r}_4) = \bar{Q}(\vec{k}_i,\vec{k}_j) \equiv [i,j]\,, \qquad \text{(B10)}$$

TABLE II. Contribution of the various types of spin-flip amplitudes to the different final states.

$S'S''SM$	
0000	$\left[\frac{1}{\sqrt{2}}AA' - \frac{1}{2\sqrt{2}}DE' - \frac{1}{2\sqrt{2}}ED'\right]$
1011	$\left[\frac{1}{2}AD' - \frac{1}{2}DA' + \frac{1}{2}CD'\right]$
101-1	$\left[-\frac{1}{2}AE' + \frac{1}{2}EA' + \frac{1}{2}CE'\right]$
1010	$\left[\frac{1}{2\sqrt{2}}CA' - \frac{1}{2\sqrt{2}}DE' + \frac{1}{2\sqrt{2}}ED'\right]$
1100	$\left[\frac{1}{\sqrt{2}}AA' + \frac{1}{6\sqrt{2}}DE' + \frac{1}{6\sqrt{2}}ED'\right]$
0111	$\left[\frac{1}{2\sqrt{3}}AD' + \frac{1}{2\sqrt{3}}DA' + \frac{1}{3\sqrt{2}}CD'\right]$
1111	$\left[\frac{1}{\sqrt{6}}AD' - \frac{1}{\sqrt{6}}DA'\right]$
011-1	$\left[\frac{1}{2\sqrt{3}}AE' - \frac{1}{2\sqrt{3}}EA' + \frac{1}{3\sqrt{2}}CE'\right]$
111-1	$\left[\frac{1}{\sqrt{6}}AE' + \frac{1}{\sqrt{6}}EA'\right]$
1110	$\left[\frac{1}{2\sqrt{3}}CA'\right]$
0110	$\left[-\frac{1}{2\sqrt{6}}CA' - \frac{1}{2\sqrt{6}}DE' + \frac{1}{2\sqrt{6}}ED'\right]$
1122	$\left[\frac{1}{\sqrt{6}}DD'\right]$
1121	$\left[-\frac{1}{\sqrt{6}}CD'\right]$
1120	$-\frac{1}{6}[DE' + ED']$
112-1	$\left[\frac{1}{\sqrt{6}}CE'\right]$
112-2	$\left[\frac{1}{\sqrt{6}}EE'\right]$

since i and j are sufficient to specify the entire matrix element.

It is interesting to see the interplay of the available phase space (as supplied by the Monte Carlo technique described in Appendix A) and the strength of the Pauli effect. If there is little phase space available then all of the nucleon momenta are approximately equal (≈ 0) and the terms all cancel. If the phase space expands so that all momenta are (usually) very different, then only one term in each Pauli sum will survive and all will enter incoherently. Thus one may observe that for large amounts of energy given to the nucleon system the effect of the Pauli principle automatically vanishes.

APPENDIX C: SPIN AVERAGING OF DCX AMPLITUDE

From Eqs. (9) and (10) we can write the initial- and final-spin functions as

$$\chi_{\text{init}} = \begin{Bmatrix} |0000\rangle \\ |1100\rangle \end{Bmatrix}, \tag{C1}$$

$$\chi_{\text{final}} = |\bar{S}'\bar{S}''\bar{S}\bar{M}\rangle . \tag{C2}$$

In Eq. (C1) the $S' = S'' = 0$ piece corresponds to the initial (1100) isospin configuration; likewise, the $S' = S'' = 1$ piece goes with the (0000) isospin configuration. We must evaluate Eq. (18) between these spin functions. This procedure yields (using the fact that $S' = S'' = \bar{S}''$):

$$\begin{aligned}
&\langle f_1(\vec{p},\vec{k}')f_2(\vec{k},\vec{p})\rangle_{\text{spin}} \\
&= AA'\delta(\bar{S},0)\delta(\bar{M},0)[\delta(S',1)\delta(\bar{S}',1)+\delta(S',0)\delta(\bar{S}',0)] \\
&+ AD'\delta(\bar{S},1)\delta(\bar{M},1)\left[\frac{1}{\sqrt{2}}\delta(S',0)\delta(\bar{S}',1)-\frac{1}{\sqrt{6}}\delta(S',1)\delta(\bar{S}',0)-\frac{1}{\sqrt{3}}\delta(S',1)\delta(\bar{S}',1)\right] \\
&+ AE'\delta(\bar{S},1)\delta(\bar{M},-1)\left[-\frac{1}{\sqrt{2}}\delta(S',0)\delta(\bar{S}',1)+\frac{1}{\sqrt{6}}\delta(S',1)\delta(\bar{S}',0)+\frac{1}{\sqrt{3}}\delta(S',1)\delta(\bar{S}',1)\right] \\
&+ CA'\delta(\bar{S},1)\delta(\bar{M},0)\left[\frac{1}{2}\delta(S',0)\delta(\bar{S}',1)+\frac{1}{\sqrt{6}}\delta(S',1)(\bar{S}',1)-\frac{1}{2\sqrt{3}}\delta(S',1)\delta(S',0)\right] \\
&+ DA'\delta(\bar{S},1)\delta(\bar{M},1)\left[-\frac{1}{\sqrt{2}}\delta(S',0)\delta(\bar{S}',1)+\frac{1}{\sqrt{6}}\delta(S',1)\delta(\bar{S}',0)-\frac{1}{\sqrt{3}}\delta(\bar{S}',1)\delta(\bar{S}',1)\right] \\
&+ EA'\delta(\bar{S},1)\delta(\bar{M},-1)\left[\frac{1}{\sqrt{2}}\delta(S',0)\delta(\bar{S}',1)-\frac{1}{\sqrt{6}}\delta(S',1)\delta(\bar{S}',0)+\frac{1}{\sqrt{3}}\delta(S',1)\delta(\bar{S}',1)\right] \\
&+ CD'\delta(\bar{M},1)\left[\frac{1}{\sqrt{2}}\delta(S',0)\delta(\bar{S}',1)\delta(\bar{S},1)+\frac{1}{\sqrt{6}}\delta(S',1)\delta(\bar{S}',0)\delta(\bar{S},1)-\frac{1}{\sqrt{3}}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},2)\right] \\
&+ CE'\delta(\bar{M},-1)\left[\frac{1}{\sqrt{2}}\delta(S',0)\delta(\bar{S}',1)\delta(\bar{S},1)+\frac{1}{\sqrt{6}}\delta(S',1)\delta(\bar{S}',0)\delta(\bar{S},1)+\frac{1}{\sqrt{3}}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},2)\right] \\
&+ DE'\delta(\bar{M},0)\left[-\frac{1}{2}\delta(S',0)\delta(\bar{S}',0)\delta(\bar{S},0)-\frac{1}{2\sqrt{3}}\delta(S',1)\delta(\bar{S}',0)\delta(\bar{S},1)-\frac{1}{2}\delta(S',0)\delta(\bar{S}',1)\delta(\bar{S},1)\right. \\
&\qquad \left.+\frac{1}{6}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},0)-\frac{1}{3\sqrt{2}}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},2)\right] \\
&+ ED'\delta(\bar{M},0)\left[-\frac{1}{2}\delta(S',0)\delta(\bar{S}',0)\delta(\bar{S},0)+\frac{1}{2\sqrt{3}}\delta(S',1)\delta(\bar{S}',0)\delta(\bar{S},1)+\frac{1}{2}\delta(S',0)\delta(\bar{S}',1)\delta(\bar{S},1)\right. \\
&\qquad \left.+\frac{1}{6}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},0)-\frac{1}{3\sqrt{2}}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},2)\right] \\
&+ \frac{EE'}{\sqrt{3}}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},2)\delta(\bar{M},-2)+\frac{DD'}{3}\delta(S',1)\delta(\bar{S}',1)\delta(\bar{S},2)\delta(\bar{M},2),
\end{aligned} \tag{C3}$$

where the coefficients A, A', etc., are obtained from the expressions for the scattering amplitude, Eqs. (2), (3), and (15). These coefficients depend on $\vec{p}$ and must be used in Eq. (1) to obtain an expression analogous to Eq. (15) involving the Z functions, the radial coordinates, and the pion momenta for the full spin-dependent amplitudes.

We must calculate the cross sections to all of the allowed final states and add them incoherently to obtain the total result. In Table II we show the combinations of amplitudes along with the final states labeled by $\bar{S}'$, $\bar{S}''$, $\bar{S}$, and $\bar{M}$, where we have included the $1/\sqrt{2}$ factor from Eq. (9).

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*Present address, Department of Physics, University of Wisconsin, Madison, Wisconsin.

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