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Kaon-nucleus inelastic scattering*

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A theoretical investigation of kaon inelastic scattering is conducted for the specific reaction $^{12}C(K^+,K^{+\prime})^{12}C^*$ ($J^\pi = 2^+, E_x = 4.43$ MeV). The analysis utilizes the Kerman-McManus-Thaler multiple scattering formalism and the distorted wave impulse approximation. The distorting parameters are uniquely determined through the empirical K^+-nucleon scattering amplitudes. Numerical results demonstrate a clear sensitivity to choice of amplitudes. The cross sections, calculated over a wide range of energies, display a slow variation. Fermi motion effects are found to be quite important at kaon lab kinetic energies below 300 MeV. The magnitude of the K^+-nucleus elastic S matrix shows that the kaon is not strongly absorbed; however, cut-off calculations, which eliminate all internal form factor contributions, firmly establish the inelastic process is surface localized.

[NUCLEAR REACTIONS Kaon inelastic scattering. KMT and DWIA. Calculated $\sigma(E,\theta)$ for ^{12}C. Fermi motion effects. Comparison of different models. Strong absorption studies.]

For some time active theoretical and experimental interest has focused on pion-nucleus phenomena and the usefulness of conducting nuclear studies with this particle. Related to these investigations has been the recent theoretical attention[1,2] involving the K^+ meson as a probe of nuclear structure. As discussed in Ref. 1 and 2, the absence of resonance in the low energy (0–900 MeV) K^+ nucleon spectrum and a strangeness of +1 pronouncedly distinguishes the kaon from the pion. It is expected that these differences, as well as others (such as mass and isospin), should be reflected in the K^+-nucleus interaction which governs elastic and inelastic processes. The K^- meson, which is not considered in this work, also differs from the K^+ in strangeness and is not a member of the same isospin multiplet. Consequently, the K^+- and K^--nucleus interactions are not related by crossing symmetry. In fact, the K^--nucleus interaction should be somewhat more complicated and absorptive due to the rich resonant K^--*nucleon* spectrum. In contrast, current evidence[1,2] suggests that the K^+ should be weakly absorbed by the nucleus and thus be a good probe of the nuclear interior. The purpose of the present communication is to report preliminary findings from an investigation of this claim. The results are theoretical in nature, being a numerical study of K^+-nucleus inelastic scattering. Hopefully, this will encourage future theoretical activity and, more importantly, experimental observations of K^+-nucleus events using already existing beams of kaons.

The calculations reported in this work are for the specific reaction $^{12}C(K^+,K^{+\prime})^{12}C^*$ exciting the $J^\pi = 2^+$ state at $E_x = 4.43$ MeV. The total and differential inelastic cross sections were computed using the computer code DWPI,[3] a pion inelastic code, which was appropriately modified for kaons. The analysis utilizes the distorted wave impulse approximation. The distorted waves, which describe kaon elastic scattering in the initial and final states, are obtained by solving the Klein-Gordon equation using the standard Kisslinger model[4] for the nuclear distortions,

$$2EV(r) = -Ak^2 b_0 \rho(r) + Ab_1 \overleftarrow{\nabla}\cdot\rho(r)\overrightarrow{\nabla}\ . \qquad (1)$$

Here, E is the kaon c.m. energy, A is the nucleon number, k is the K^+-nucleus c.m. wave number (in MeV/c), $\rho(r)$ is the target matter density, and b_0, b_1 are complex parameters (in fm^3) described in detail below. The inelastic nuclear form factor was obtained by describing the target nucleus ^{12}C in a macroscopic collective model in which the nucleus is deformed with deformation parameter $\beta_2 = 0.56$ (corresponding to a deformation length $\delta = \beta R$ of 1.75 fm). The undeformed ground state density was taken[3] as

$$\rho(r) = \frac{4}{A(\sqrt{\pi}w)^3}\left[1 + \frac{(A-4)r^2}{6w^2}\right]e^{-r^2/w^2}\ , \qquad (2)$$

with $w = 1.62$ fm.

The parameters b_0 and b_1 were specified using a semiphenomenological approach. This scheme begins with the first-order expression provided by KMT multiple scattering formalism[5] for the meson-nucleus optical potential. The interaction is then further simplified through the "factorization" approximation.[6,7] In this approximation, the K^+-nucleon scattering matrix is factored out of the KMT integral expression for the K^+-nucleus optical potential, which thereby takes the product form of the nuclear density with the elementary amplitudes. Next, the exact kaon-nucleon scattering matrices, t, are replaced by the free kaon-nucleon amplitudes. The empirical amplitudes are constructed from phase shifts which have been adjusted to reproduce the experimentally observed K^+-nucleon cross section at different energies. In the present calculation only s and p wave phase shifts are retained in the partial wave expansion for t. Finally, expressions for b_0 and b_1 are obtained by comparing, in momentum space representation, this approximate, semi-phenomenological potential with the Kisslinger model, e.g., Eq. (1). In this sense the Kisslinger model is used predominately as a convenient vehicle which translates K-nucleon information into K-nucleus distortions. It is of interest to note that this prescription entails no adjustable parameters and hence presents an opportunity for further constraining the kaon-nucleon phase shifts which are not precisely known.

A major aim of the present analysis is to report some general features of kaon-nucleus inelastic scattering which, although of quantitative uncertainty, should be qualitatively valid. Equally important, however, is the determination of sensitivity (if any) of the theoretical cross section to variations in phase shifts. Accordingly, the calculations were performed using two different sets of phase shifts (or equivalently, different b_0 and b_1 parameters). The different sets correspond to the different phase shift analysis of Martin[8] (Set I and BGRT[9,10] (Set II). For comparison the effective b_0 and b_1 for these sets are displayed in Figs. 1 and 2 as a function of kaon kinetic lab energy T_L. The difference in strengths, b_0 and b_1, arising from the different choice of amplitudes is appreciable. Notice, however, that both sets of strengths vary slowly with T_L. Results are reported for $T_L \leq 700$ MeV. For lab energies below 400 MeV, the K^+-nucleus interaction is entirely determined from K^+-nucleon s and p wave amplitudes. Above 400 MeV d and f waves contribute but not significantly until around 700 MeV.[1,8] It is important not to confuse the few K^+-*nucleon* partial waves needed to construct the optical potential with the considerable number (typically 30–40) of K^+-*nucleus* partial waves needed to compute the inelastic cross section.

In contrast to the pion, the strength of b_0 is in general greater than the strength of b_1. Furthermore, no appreciable energy fluctuations exist, reflecting the absence of a resonance in the K^+-

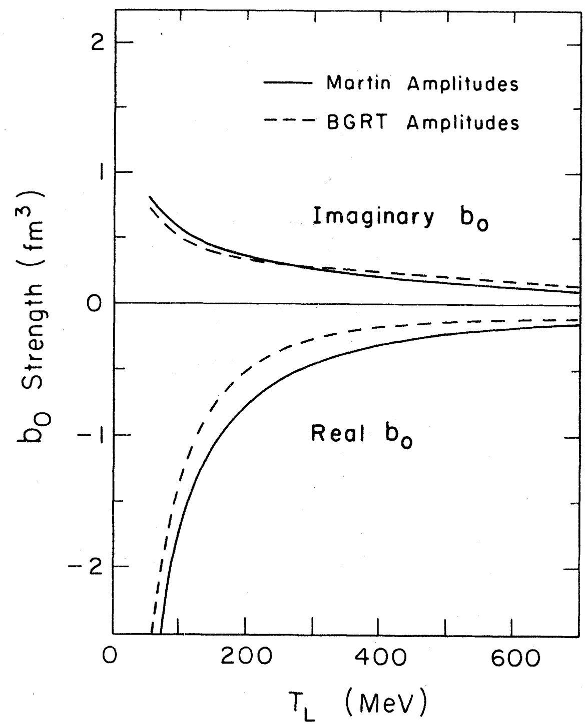

FIG. 1. Real and imaginary part of b_0 as a function of T_L, the kaon lab kinetic energy. The solid (dashed) curve was obtained using the Martin (BGRT) amplitudes. Note, the potential strength is given by the product $k^2 b_0$ which approaches zero for $T_L \to 0$.

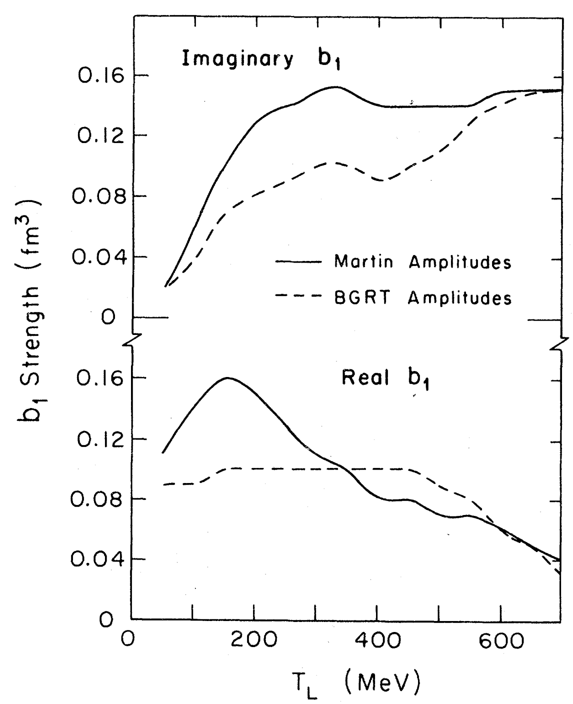

FIG. 2. Same plot as Fig. 1 for b_1.

nucleon low-energy spectrum. A detailed examination of the four contributing amplitudes $t_{L,T}$ ($L=0$, 1 and $T=0,1$) reveals that a cancellation "conspiracy" is present since the energy variation of the individual $t_{L,T}$ is more appreciable than the combined behavior displayed in Figs. 1 and 2. The significant difference between Set I and Set II parameters is also reflected in the total inelastic cross section σ_T, which is given in Fig. 3. This result demonstrates a definite sensitivity to choice in K^+-nucleon amplitudes, especially at low energies where the Martin amplitudes, Set I, predict a σ_T which is up to 50% larger than that calculated with Set II. A disparity of this size could be resolved by experiment.

Both sets of K^+-nucleon amplitudes predict an energy dependence of σ_T which scales accurately as $1/T_L$ up to $T_L=400$ MeV. For higher energies, the two models predict a constant or slightly increasing σ_T. Including d and f wave amplitudes will yield an even larger σ_T at energies beyond 600–700 MeV. Also of interest are the differential inelastic cross sections which are given in Fig. 4 for the Martin amplitudes at $T_L=100$, 300, and 600 MeV and in Fig. 5 for both sets of amplitudes

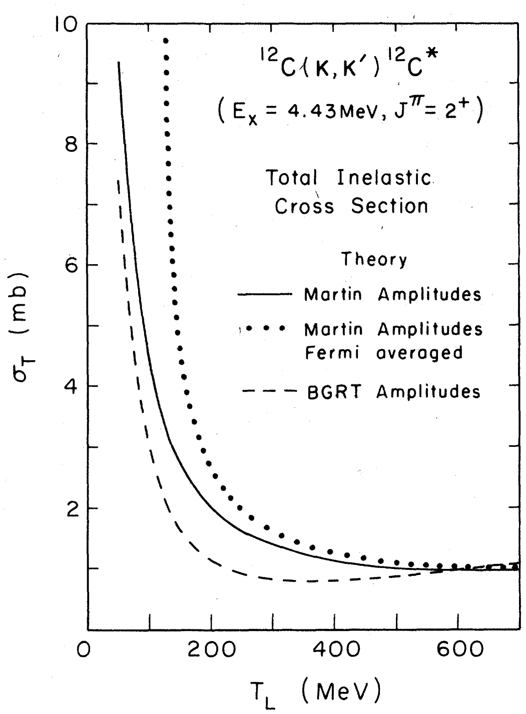

FIG. 3. Total inelastic cross section for kaon inelastic scattering from ^{12}C plotted as a function of T_L. The solid (dashed) curve was obtained using the Martin (BGRT) amplitudes. The dotted curve was generated using Martin amplitudes which were Fermi averaged. The inelastic cross sections are proportional to $(k^2b_0)^2$ and approach zero for $T_L \to 0$.

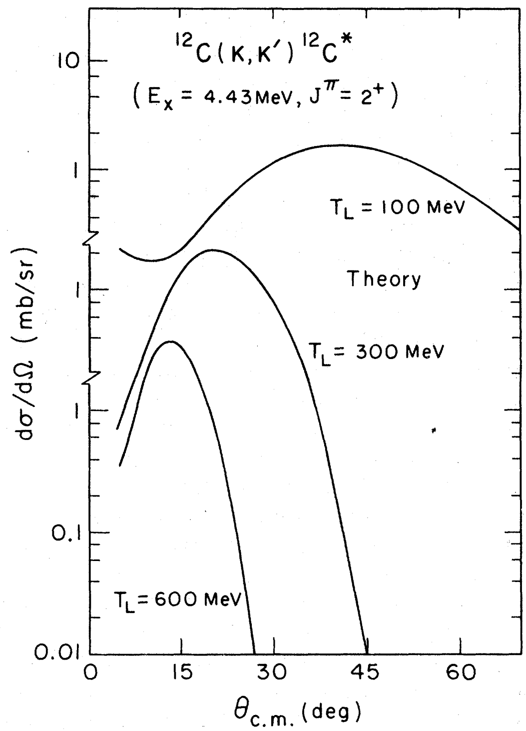

FIG. 4. Differential inelastic cross sections, using Martin amplitudes, for kaon inelastic scattering from ^{12}C at $T_L=100$, 300, and 600 MeV.

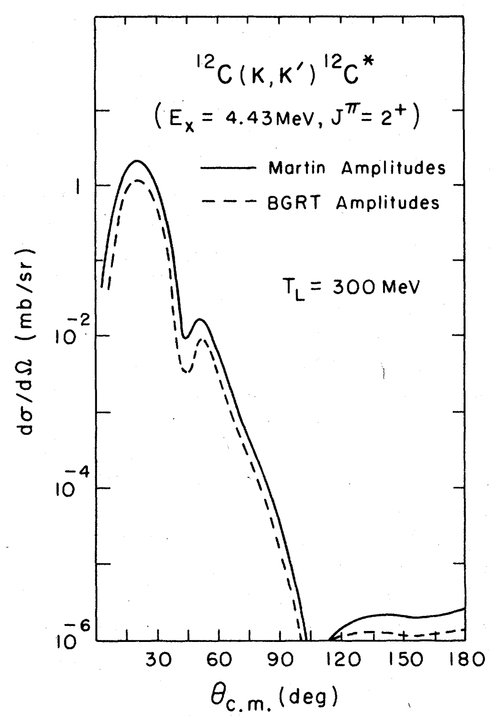

FIG. 5. Comparison of differential inelastic cross sections at $T_L=300$ MeV using Martin (solid curve) and BGRT (dashed curve) amplitudes.

at $T_L = 300$ MeV. The most noticeable features are the forward shift in the first maximum with increasing T_L and the change in angular pattern from the familiar bell shape at low energies to diffractive at higher energies. Such aspects are usually characteristic of processes that are dominated by strong absorption and large momentum transfer. Again, the difference predicted by the two sets of amplitudes is sizable and, in conjunction with experimental measurement, provides additional information for determining an optimum set of kaon-nucleon phase shifts.

As mentioned earlier, use has been made of the factorization approximation. In this approximation the nucleons in the nucleus are assumed frozen and are fixed points from which the kaon scatters. Such an approximation neglects the Fermi motion of the individual nucleons. To assess this effect, which may be important at low T_L, we have performed several calculations with K^+-nucleon amplitudes which have been crudely Fermi averaged. The averaged strengths $\langle b_0 \rangle$ and $\langle b_1 \rangle$ were obtained using K^+-nucleon amplitudes which have been averaged over the momentum distribution[11] of the target nucleons. For this purpose it is necessary to evaluate the amplitudes at the appropriate kaon-nucleon relative momentum which is obtained from the kaon-nucleus lab momentum through a Lorentz transformation. The effect of Fermi averaging is shown in Fig. 3 which compares σ_T using the Martin amplitudes with and without Fermi averaging. The calculations reveal that Fermi averaging produces a large enhancement in σ_T, especially at low energies. At higher energies, $T_L = 400$ to 700 MeV, the effects are small and appear not to be important. The reason for the above energy behavior stems from a non-negligible probability of a nucleon having a momentum between 200 and 300 MeV/c, a momentum comparable to the kaon's momentum at low energies. Consequently, the Fermi averaging includes amplitudes which are evaluated at relative momenta differing by several hundred MeV/c. At high kaon energies (corresponding to high kaon momenta) the amplitudes are relatively constant; however, at lower energies there is some variation with energy (see Fig. 1). Hence Fermi averaging is mainly important at low T_L, below 300 MeV, and for such energies caution is urged in applying the factorization approximation. At higher kaon energies the approximation appears reasonable. From a present theoretical viewpoint these would be the logical energies for performing measurements. Furthermore, because both the Martin and BGRT amplitudes predict cross sections which also agree at higher energies (500–700 MeV) this would constitute a stringent test of the overall theory.

As a final note, Dover[1] has provided good arguments for utilizing the K^+ as an appealing probe of nuclear structure. Specifically, he mentions that because of the rather weak K^+-nucleon interaction and correspondingly long mean free path, the K^+ should be sensitive to the entire nuclear volume and not just the nuclear periphery as are the strongly absorbed particles. To verify this claim two further investigations have been performed: a study of the magnitude of the elastic kaon-nucleus S matrix, $|S_L|$, and radial cut-off calculations where the inelastic radial form factor is set to zero for $r < R_c$, the cut-off radius. Because the inelastic radial form factor peaks at the nuclear surface, appreciable sensitivity to the interior of the nucleus is, of course, not expected. However, the kaon may be sensitive to the inner tail of the form factor which does extend appreciably into the nuclear interior. The first study, an examination of $|S_L|$, supports Dover's contention in that $|S_L| > 0.6$ for all partial waves. Strongly absorbed particles, such as the pion, α particle, or heavy ions, typically have $|S_L| \simeq 0.1$ to 0.2 for the lower, absorbed partial waves. The results from the cut-off study, however, demonstrate that the K^+ is insensitive to the nuclear interior even when the form factor is appreciable. These results are given in Fig. 6 where both σ_T and $d\sigma/d\Omega$ (at the first maximum) are plotted as a function of the cut-off radius R_c. Clearly, no contributions to σ_T and $d\sigma/d\Omega$ come from the region

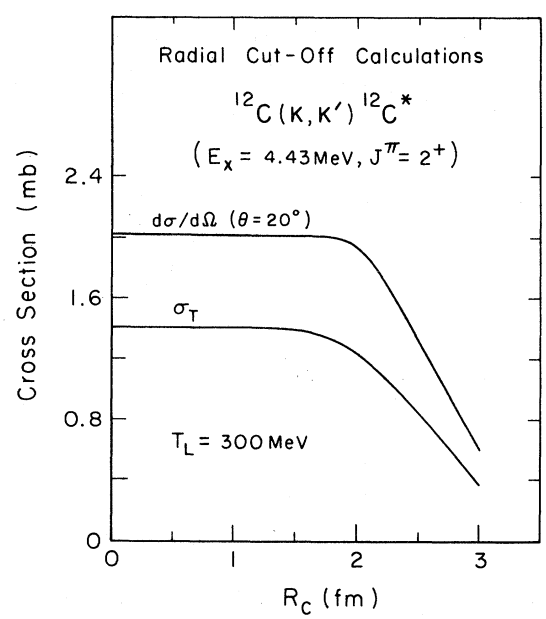

FIG. 6. Total and differential (at first maximum) cross section plotted as a function of the cut-off radius R_c.

$r=0$ to 2 fm. Since the nuclear radius for ^{12}C used in these calculations is about 2.8 fm it would appear that the K^+ is predominantly sampling the nuclear surface. Although no single explanation for this behavior has emerged from our analysis, several effects are present which could give surface localization: the repulsive K^+-nucleus interaction, phase averaging from large momentum transfer, surface peaking of the inelastic form factor, and the slow falloff of the inelastic transition amplitude with increasing partial wave. The last effect strongly favors surface localization since the higher, surface partial waves are weighted by $2L+1$ and thus contribute more to the cross section.

In summary, the major thrust of this work has been to report quantitative results which should represent some of the general qualitative aspects of kaon-nucleus inelastic scattering. The calculations have not only revealed a marked sensitivity of the cross section to the choice of K^+-nucleon phase shifts (20–50% effect) but have also cast doubt upon the validity of the factorization approximation at low energies where the effects from Fermi averaging are dramatic. Furthermore, the radial cut-off analysis has shown that the inelastic process is predominantly a surface phenomena and is a means for studying the nuclear surface. Apparently other reactions such as knock-out or particle production would be more appropriate for investigating the nuclear interior. Experimental information would prove invaluable in confirming all of the above predictions. For this reason it is strongly recommended that K^+-nucleus elastic and inelastic measurements be performed in the near future. Also for future theoretical and experimental consideration should be the charge exchange processes (K^+, K^0) and (K^+, K^-), as well as K^- induced nuclear reactions exciting strangeness analog states which are believed to exist.[12]

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