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Nucleus-nucleus scattering at high energies

Victor Franco*
Physics Department, Brooklyn College of the City University of New York, Brooklyn, New York 11210

Girish K. Varma†
Physics Department, Brooklyn College of the City University of New York, Brooklyn, New York 11210 and Serin Physics Laboratory, Rutgers—The State University, Frelinghuysen Road, Piscataway, New Jersey 08854‡

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Nucleus-nucleus scattering is treated in the Glauber approximation. The usual optical limit result, generally thought to improve as the number of nucleons in the colliding nuclei increases, is found to be the first term of a series which diverges for large nuclei. Corrections to the optical limit are obtained which provide a means of performing realistic calculations for collisions involving light nuclei. Total cross section predictions agree well with recent measurements.

NUCLEAR REACTIONS Validity of the optical limit for heavy-ion scattering in the Glauber approximation. Effects of higher order corrections.

Interest in heavy-ion collisions at medium and high energies has increased rapidly in the past few years. In addition to being relevant to speculation regarding superheavy nuclei, these collisions are expected to provide a new tool for probing nuclear structure and for studying scattering mechanisms. The theoretical studies of such collisions have generally involved the Glauber approximation.[1-14] The exact Glauber scattering amplitude, however, is too difficult to evaluate for realistic forms of nuclear densities. An "optical limit" result,[4] therefore, has frequently been employed to study elastic scattering[8-12] and fragmentation processes[13]; it has also been used to test general concepts such as factorization of cross sections at high energies.[9-11]

A serious problem associated with the optical limit is that the elastic scattering cross sections diverge[4,7] at large momentum transfers when the center-of-mass correlation function is retained (as it should be, to get reasonable results at small momentum transfers). Furthermore, there appears to be a serious disagreement[15] between the optical limit predictions and the recent high energy ^{12}C-^{12}C total cross section measurements. In this paper, we examine the validity of the optical limit in the Glauber approximation. We obtain a series for the optical phase shift function (or the optical potential), where the first term is the usual optical limit result. As the number of nucleons in the colliding nuclei increases we find that, contrary to the prevalent belief, higher order terms begin to dominate and the optical limit result becomes inaccurate.

The elastic scattering amplitude for collisions between nuclei with mass numbers A_1 and A_2 can be written as[4-6]

$$F(q)=\frac{ik}{2\pi}K(q)\int d^2b\, e^{i\vec{q}\cdot\vec{b}}[1-e^{i\chi(b)}], \tag{1}$$

where $\hbar k$ is the incident momentum, b is the impact parameter, and $K(q)$ is a center-of-mass correlation function.[4] The total nucleus-nucleus phase shift function $\chi(b)$ is given by

$$i\chi(b)=\ln\langle\psi_{A_1}\psi_{A_2}|\left\{\prod_{i=1}^{A_1}\prod_{j=1}^{A_2}[1-\Gamma_{ij}(\vec{b}-\vec{s}_i+\vec{s}'_j)]\right\}|\psi_{A_1}\psi_{A_2}\rangle, \tag{2}$$

where ψ_{A_i} are the nuclear ground state wave functions, $\vec{s}_i$ and $\vec{s}'_j$ are the projections of the nucleon coordinates on the impact parameter plane, and Γ_{ij} are the nucleon-nucleon (NN) profile functions. By expanding Eq. (2) in powers of Γ_{ij}, we obtain the series[14]

$$i\chi(b)=i(\chi_1+\chi_2+\chi_3+\cdots), \tag{3}$$

with

$$i\chi_1=-\sum_{i=1}^{A_1}\sum_{j=1}^{A_2}\langle\psi_{A_1}\psi_{A_2}|\Gamma_{ij}(\vec{b}-\vec{s}_i+\vec{s}'_j)|\psi_{A_1}\psi_{A_2}\rangle, \tag{4}$$

etc. For simplicity, we shall assume that all NN amplitudes are equal [i.e., $f_{NN}(q)=f(q)$], and that

$$|\psi_{A_i}|^2=\prod_{j=1}^{A_i}|\phi_j(\vec{r}_j)|^2 . \tag{5}$$

We then obtain

$$i\chi_1=-A_1A_2C_0(b)\,,\qquad i\chi_2=\tfrac{1}{2}A_1A_2[(1-A_1-A_2)C_0{}^2(b)+(A_2-1)C_1(b)+(A_1-1)C_2(b)]\,, \tag{6}$$

and so on, where

$$\begin{aligned} C_0(b)&=(2\pi ik_N)^{-1}\int d^2q\, e^{-i\vec{q}\cdot\vec{b}}f(\vec{q})S_{A_1}(\vec{q})S_{A_2}(-\vec{q})\,,\\ C_1(b)&=(2\pi ik_N)^{-2}\int d^2q\,d^2q'e^{-i(\vec{q}+\vec{q}')\cdot\vec{b}}f(\vec{q})f(\vec{q}')S_{A_1}(\vec{q}+\vec{q}')S_{A_2}(-\vec{q})S_{A_2}(-\vec{q}')\,,\end{aligned} \tag{7}$$

and $C_2(b)$ is obtained from $C_1(b)$ by letting $S_{A_1}\leftrightarrow S_{A_2}$, where $S_{A_i}(q)$ are the nuclear form factors. We have also calculated the terms χ_3 and χ_4 but the results are too lengthy to write down here.

The usual optical limit result corresponds to truncating the series (3) at $i\chi=i\chi_1$. To illustrate the effects of $\chi_2,\ldots,\chi_4$, we shall take the simple case

$$S_{A_i}(q)=e^{-q^2R_i^2/4}\,,\quad f(q)=\frac{k_N\sigma(i+\rho)}{4\pi}e^{-aq^2/2}\,, \tag{8}$$

which leads to the result

$$C_0(b)=ye^{-b^2/R^2}\,;\quad R^2=R_1{}^2+R_2{}^2+2a\,,\quad y=\frac{\sigma(1-i\rho)}{2\pi R^2}\,,\quad C_j(b)=y^2[1-(R_j/R)^4]^{-1}e^{-2b^2/(R^2+R_j{}^2)}\,;\quad j=1,2\,. \tag{9}$$

Again, the general expressions for $\chi_2,\ldots,\chi_4$ are lengthy and will not be written down here. The optical limit ($\chi=\chi_1$) is usually considered to be a good approximation for collisions between large nuclei. Let us, therefore, estimate the size of the correction terms for the case $A_1=A_2\gg1$, $R_1{}^2=R_2{}^2\gg a$. We obtain

$$\begin{aligned} &i\chi_1(b)\to-A^2ye^{-b^2/R^2}\,,\qquad i\chi_2(b)\to i\chi_1(b)(Ay)[e^{-b^2/R^2}-\tfrac{4}{3}e^{-b^2/3R^2}]\,,\\ &i\chi_3(b)\to i\chi_1(b)(Ay)^2[\tfrac{5}{3}e^{-2b^2/R^2}-4e^{-4b^2/3R^2}+2e^{-b^2/R^2}+\tfrac{2}{3}e^{-b^2/2R^2}]\,,\end{aligned} \tag{10}$$

TABLE I. Nucleus-nucleus total cross sections at 0.87 and 2.1 GeV/n. The values in parentheses are the total cross sections at 0.87 GeV/n. The quoted experimental errors are statistical only. The two experimental values in the second row correspond to ^{4}He-^{12}C and ^{12}C-^{4}He scattering, respectively. The nucleon-nucleon parameters used in our calculations are $\sigma=42.7$ mb, $a=6.2$ (GeV/c)$^{-2}$, $\rho=-0.28$ at 2.1 GeV/n; and $\sigma=42.4$ mb, $a=5$ (GeV/c)$^{-2}$, $\rho=-0.2$ at 0.87 GeV/n (Ref. 17). The nuclear rms radii were taken from Ref. 18 and corrected for the finite proton size and c.m. recoil.

Nuclei	σ_{tot} (mb) with χ equal to				σ_{tot} (mb)
A_1–A_2	χ_1	$\chi_1+\chi_2$	$\chi_1+\chi_2+\chi_3$	$\chi_1+\chi_2+\chi_3+\chi_4$	Experiment
4–4	429	384	387	386	408 ±2.5
	(420)	(373)	(377)	(375)	(390 ±4.2)
4–12	909	788	810	802	835 ±5, 826 ±5.9
	(885)	(767)	(792)	(781)	(820 ±13, 790 ± 7.0)
4–24	1387	1217	1260	1244	
4–40	1939	1720	1778	1757	
12–12	1605	1365	1453	1420	1347 ± 25
	(1580)	(1329)	(1430)	(1384)	(1256 ±31)
12–24	2272	1931	2086	2026	
12–40	3010	2584	2786	2709	
24–24	3077	2607	2861	2765	
24–40	3949	3385	3698	3577	
40–40	4941	4307	4662	4512	

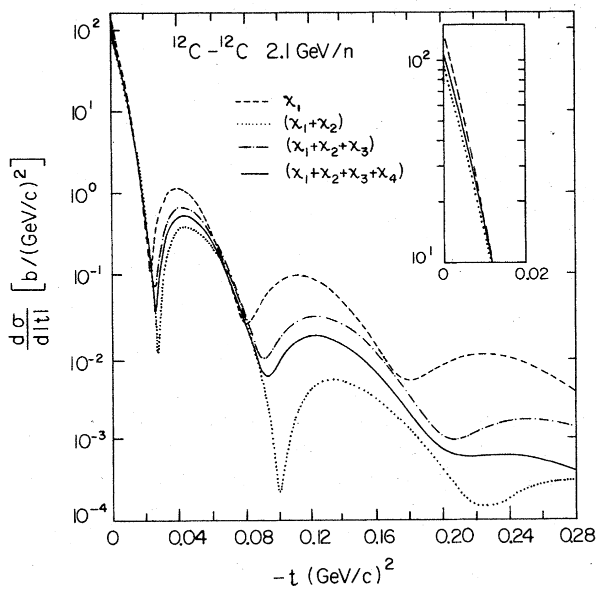

FIG. 1. $^{12}C-^{12}C$ elastic scattering at 2.1 GeV/n. The curves illustrate the effects of the higher order corrections χ_2, χ_3, and χ_4, to the usual optical limit result.

etc. For example,

$$\chi_2(0) \sim \chi_1(0)[-\tfrac{1}{3}(Ay)], \quad \chi_2(R) \sim \chi_1(R)[-0.59(Ay)],$$

$$\chi_3(0) \sim \chi_1(0)[\tfrac{1}{3}(Ay)^2], \quad \chi_3(R) \sim \chi_1(R)[0.31(Ay)^2],$$

$$\chi_4(0) \sim \chi_1(0)[-\tfrac{1}{2}(Ay)^3], \quad \chi_4(R) \sim \chi_1(R)[-0.28(Ay)^3].$$

Since $R \sim A^{1/3}$, we have $Ay \sim A^{1/3}$. (If we relate R_i^2 to the mean square radii $\langle r_i^2\rangle$ by $R_i^2 = \frac{2}{3}\langle r_i^2\rangle$, we obtain for Pb-Pb collisions $Ay \sim 3.3$.) Thus for large A $|\chi_4| > |\chi_3| > |\chi_2| > |\chi_1|$, and the series (3) appears to diverge. Furthermore, the series is oscillatory and a truncation of the series at χ_n (n even) for large A may lead to unphysical results (for example, $|e^{i\chi}|>1$). However, the series (3) is still useful for light- and medium-A nuclei (where $|\chi_1|>|\chi_2|>|\chi_3|\cdots$).

We emphasize that these simple estimates are valid *only* for the case $A_1, A_2 \to \infty$, and have been obtained only to illustrate that χ does not approach χ_1 in that limit (in contradiction to what has been argued in the past). For finite A_1, A_2, $\chi_1, \ldots, \chi_4$ should be evaluated without any approximation. We find that the higher order corrections are most important at small b and tend to become smaller at larger b. Therefore, one may expect the corrections to be less significant for total cross sections, for example, which depend mostly upon peripheral collisions. They should, however, become extremely important for differential cross sections away from the forward direction, which probe collisions at smaller impact parameters.

In Table I we show the results for total cross sections at 2.1 and 0.87 GeV/n, together with the available experimental measurements.[15] The four columns of calculated results illustrate the effects of χ_2, χ_3, and χ_4. The disagreement between optical limit prediction and the data for $^{12}C-^{12}C$ scattering is very significantly reduced by including the correction terms. Figure 1 illustrates the inadequacy of the optical limit for the $^{12}C-^{12}C$ elastic scattering angular distribution. It is also evident, however, that accurate results can be obtained by retaining sufficient terms in series (3). Convergence is more rapid for α-α collisions where the curves corresponding to terms up to χ_3 and χ_4, respectively, cannot be distinguished out to the second minimum.

We thus see that the usual optical limit is quite inaccurate for the description of heavy-ion scattering. For large A_1 and A_2 the higher order corrections to the optical limit begin to dominate (even when nuclear correlations are neglected) and render the series (3) useless. However, for light- and medium-A nuclei the series given by Eq. (3) provides a basis for accurate calculations with realistic forms of nuclear densities.

We also point out that for hadron-hadron scattering (in a model where hadrons are considered to be made up of an infinite number of constituents), the appropriate limit is $\sigma A_1 A_2 \to$ const, as $\sigma \to 0$ and $A_1, A_2 \to \infty$. In this case $Ay \sim 1/A$ and $i\chi$ goes over to $i\chi_1$ (the optical limit), a result which is also equivalent to the Chou-Yang model.[12,16]

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‡Present address.

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