

## R-matrix analysis of neutron scattering from ${}^6\text{Li}$

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The total elastic cross section,  ${}^6\text{Li}(n,\alpha){}^3\text{H}$  cross section, differential cross section, and polarization are calculated for the  $n$ - ${}^6\text{Li}$  system in the energy range 0–5 MeV by using general  $R$ -matrix theory and taking into account various coupled channels in addition to the direct channels. The results are compared with the experimental results and earlier calculations. Our calculations are helpful in estimating the contribution of dominant coupled channels to the cross sections in 0–5 MeV energy range.

[ NUCLEAR REACTIONS  $R$ -matrix theory,  $n$ - ${}^6\text{Li}$  system, cross sections, polarization. ]

### I. INTRODUCTION

The analysis of the  $n$ - ${}^6\text{Li}$  resonance reaction in the MeV region has been described using the compound nucleus formalism. Wigner's  $R$ -matrix theory is quite feasible to study the neutron scattering from  ${}^6\text{Li}$  in the low energy range. For the  $n$ - ${}^6\text{Li}$  system relatively few experimental and theoretical studies have been performed in the MeV region. This is perhaps due to the difficulty of interpreting the resonance structure above 1 MeV. For example, the excitation function in this energy range is smooth and exhibits no definite structure. The interest in this system in the MeV region has been increasing due to recent developments in the field of fusion-powered reactors. Moreover, the polarized neutron studies have proved to be highly successful<sup>1,2</sup> in providing measurements which are amenable to an interpretation in terms of a general  $R$ -matrix reaction theory. The analysis of neutron scattering from  ${}^6\text{Li}$  is complicated by the fact that  ${}^6\text{Li}$  is a spin-one nucleus so that both  $S=\frac{1}{2}$  and  $S=\frac{3}{2}$  spin channels are open. Furthermore, if we confine ourselves in the energy range 0–5 MeV, the  $(n,\alpha)$  channel also dominates along with the elastic channel. Of course, the other reaction channels in this energy range can be neglected in the analysis because they have an extremely small contribution to the cross sections. Recently Holt *et al.*<sup>3</sup> have studied the  $n$ - ${}^6\text{Li}$  reaction using the reduced  $R$ -matrix theory by varying the energy level parameters of the compound nucleus  ${}^7\text{Li}$  and the parameters of the  $R$  matrix. In their calculations they have assumed that the angular momentum, spin, and channel spins do not change between the entrance and exit channels. However, in the analysis of the  $n$ - ${}^6\text{Li}$  reaction in the low energy range it is not necessary to do this approximation; in fact the general  $R$ -matrix theory of Wigner which incorporates dominant cou-

pled channels is quite feasible in this case. We have calculated the total neutron cross section, total  ${}^6\text{Li}(n,\alpha){}^3\text{H}$  cross section, differential cross section, and polarization for the energy range 0–5 MeV by general  $R$ -matrix theory and have compared the results with the experimental data. For varying  $R$ -matrix parameters we have made an automatic search program<sup>4</sup> on the IBM 1130 computer to get the minimum  $\chi^2$  to fit the experimental data. Finally a comparative study of our results is made with the results of Holt *et al.* In Sec. II we give a brief discussion of  $R$ -matrix theory. Section III contains the discussion and conclusions.

### II. THEORY

In the collision theory, one can get the general expressions for the reaction cross sections but it has little specific physical content. It is based entirely on the  $S$  matrix, which gives the asymptotic form of the wave function of a system. To determine the  $S$  matrix, it is necessary to know something about the wave function in the internal region where the two particles actually interact. Then, using the property that the wave functions and their derivatives are continuous across the boundary, one can derive the  $S$  matrix. A similar but more general boundary formulation<sup>5</sup> has proved to be very important in the study of nuclear reactions. In brief, the  $S$  matrix is expressed in terms of an  $R$  matrix which involves various quantities evaluated on or just outside the surface of the region (internal) within which all  $A$  particles of the system may interact and outside which (external region) there is no further interchange of nucleons between the products of the reaction. Thus, the  $R$  matrix itself is still as formal a construct as the  $S$  matrix, but it is defined in terms of a hypothetical set of states of the system inside the inter-

action region. Therefore, once some physical picture of nuclear processes and nuclear structure has been attained, it can be connected more readily with the  $R$  matrix than with the  $S$  matrix. Thus we require (i) the expression for an  $R$  matrix involving some of the physical properties of the compound nucleus and (ii) the relation between  $R$  and  $S$  matrices.

The expression of the  $R$  matrix is given by<sup>5</sup>

$$R = \sum_{\lambda} \frac{\gamma_{\lambda} \times \gamma_{\lambda}}{E_{\lambda} - E}, \quad (2.1)$$

where the vector  $\gamma_{\lambda}$  has component  $\gamma_{\lambda c}$ , the reduced width amplitude for channel  $c$  and state  $\lambda$ ;  $E_{\lambda}$  is the level energy for the state  $\lambda$ . The relation between  $R$  and  $S$  matrices is as follows<sup>5</sup>:

$$S^J = \Omega W^J \Omega \quad (2.2)$$

with

$$W^J = 1 + 2i\mathfrak{B}^{1/2}(1 - R^J L^0)^{-1} R^J \mathfrak{B}^{1/2}, \quad (2.3)$$

$$L^0 = L - B, \quad (2.4)$$

where  $L$ ,  $\Omega$ ,  $\mathfrak{B}^{1/2}$  are diagonal matrices, and  $B$  is the real, diagonal boundary condition matrix. The components of  $L$ ,  $\Omega$ ,  $\mathfrak{B}^{1/2}$  are given by

$$\begin{aligned} L_c &= S_c + iP_c, \\ S_c &= [\rho_c(F_c F'_c + G_c G'_c)/(F_c^2 + G_c^2)]_{r_c=a_c}, \\ P_c &= [\rho_c/(F_c^2 + G_c^2)]_{r_c=a_c}, \\ \Omega_c &= e^{i(w_c - \phi_c)}, \\ \phi_c &= \tan^{-1}(F_c/G_c), \\ \mathfrak{B}_c &= P_c, \\ w_c &= w_{\alpha l} = \sum_{n=1}^l \tan^{-1}(\eta_{\alpha}/n). \end{aligned} \quad (2.5)$$

The quantities  $S_c$ ,  $P_c$ , and  $B_c$  (the components of  $B$ ) are called, respectively, the level shift factor, the penetration factor, and the boundary condition constant for channel  $C$ ; while  $-\phi_c$  is called the hard sphere scattering phase shift and  $\eta_{\alpha}$  is the Coulomb field parameter for the pair  $\alpha$ .  $F_c$  and  $G_c$  are regular and irregular wave functions for channel  $C$ , and  $F'_c$  and  $G'_c$  are, respectively, their derivatives with respect to  $\rho_c = k_c r_c$ . The channel radius is taken to be  $a_c = 1.4(1 + A^{1/3})$  where  $A$  is the atomic mass number of  ${}^6\text{Li}$ . Thus once the regular and irregular wave functions are known for all  $C$ , one can determine the  $S$  matrix and hence the total elastic cross section and reaction cross section<sup>5,6</sup> as follows

$$\begin{aligned} \sigma(n, n) &= \frac{\pi}{k_{\alpha}^2} \sum_{\substack{J \\ s s'}} g_J |\delta_{\alpha' \alpha} \delta_{s' s} \delta_{i' i} - S_{\alpha' s' i'; \alpha s i}^J|^2 \quad (\text{for } \alpha = \alpha') \\ &= \frac{\pi}{k_{\alpha}^2} \sum_J g_J \sum_{i s} \left( 1 - 2 \operatorname{Re} S_{\alpha' s' i'; \alpha s i}^J + \sum_{i' s'} |S_{\alpha' s' i'; \alpha s i}^J|^2 \right), \end{aligned} \quad (2.6)$$

$$\begin{aligned} \sigma(n, \alpha) &= \frac{\pi}{k_{\alpha}^2} \sum_{\substack{J \\ s s'}} g_J |S_{\alpha' s' i'; \alpha s i}^J|^2 \quad (\text{for } \alpha \neq \alpha') \\ &= \frac{\pi}{k_{\alpha}^2} \sum_J g_J \sum_{i s} \left( 1 - \sum_{i' s'} |S_{\alpha' s' i'; \alpha s i}^J|^2 \right), \end{aligned} \quad (2.7)$$

where the spin statistical factor is given by  $g_J = (2J+1)/[(2I+1)(2i+1)]$ ;  $i$  and  $I$  are the spins of the incident particle and the target nucleus.  $\operatorname{Re}$  in Eq. (2.6) stands for the real part of  $S_{\alpha' s' i'; \alpha s i}^J$ . The expressions for the differential cross section and polarization for  $\alpha' - \alpha$  collisions are given by

$$\sigma(\theta) = \left( \frac{1}{4k_{\alpha}^2 (2I+1)(2i+1)} \right) \sum_{s=|I-i|}^{I+i} \sum_{s'=|I'-i'|}^{I'+i'} \sum_{L=0}^2 B'_L(\alpha' s'; \alpha s) P_L(\cos\theta), \quad (2.8)$$

$$\sigma(\theta)P(\theta) = \left( \frac{[2(2i'+1)(i+1)/i]^{1/2}}{4k_{\alpha}^2 (2I+1)(2i+1)} \right) \sum_{s=|I-i|}^{I+i} \sum_{s'=|I'-i'|}^{I'+i'} \sum_{L=1}^2 C'_L(\alpha' s'; \alpha s) \left( \frac{(2L+1)}{2L(L+1)} \right)^{1/2} P_L^1(\cos\theta), \quad (2.9)$$

where  $P_L(\cos\theta)$  and  $P_L^1(\cos\theta)$  are Legendre polynomials and associated Legendre polynomials, respectively, and  $i'$  and  $I'$  are spins of outgoing particle and residual nucleus, respectively. The coefficients  $B'_L$  and  $C'_L$  are related to the  $S$  matrix by the expressions<sup>7,8</sup>

$$\begin{aligned} B'_L(\alpha' s'; \alpha s) &= (-1)^{s-s'} \sum_{\substack{J_1 J_2 J_1' J_2' \\ i_1' i_2'}} Z(l_1 J_1 l_2 J_2; sL) Z(l_1' J_1' l_2' J_2'; s'L) \\ &\quad \times \operatorname{Re} [(\delta_{\alpha' \alpha} \delta_{s' s} \delta_{i_1' i_1} - S_{\alpha' s' i_1'; \alpha s i_1}^J)^* (\delta_{\alpha' \alpha} \delta_{s' s} \delta_{i_2' i_2} - S_{\alpha' s' i_2'; \alpha s i_2}^J)], \end{aligned} \quad (2.10)$$

$$\begin{aligned}
C'_L(\alpha's'; \alpha_s) = & \sum_{\substack{J_1 J_2 l_1 l_2 \\ l'_1 l'_2}} i^{l_2 - l_1 + l'_1 - l'_2} (-1)^{l_1 - l_2 - s + l'_1 + J_1 - s'} [(2l_1 + 1)(2l_2 + 1)(2l'_1 + 1)(2l'_2 + 1)]^{1/2} (2s' + 1) \\
& \times (2J_1 + 1)(2J_2 + 1) \langle l_1 l_2 00 | L0 \rangle \langle l'_1 l'_2 00 | L0 \rangle \\
& \times W(i's'i's'; I'1) W(l_1 J_1 l_2 J_2; sL) X(J_1 l'_1 s'; J_2 l'_2 s'; LL1) \\
& \times \text{Re} [i(\delta_{\alpha'\alpha} \delta_{s's} \delta_{l'_1 l_1} - S_{\alpha' s' l'_1 l_1}^J) * (\delta_{\alpha'\alpha} \delta_{s's} \delta_{l'_2 l_2} - S_{\alpha' s' l'_2 l_2}^J) + \alpha_{s l_2}], \quad (2.11)
\end{aligned}$$

where

$$Z(l_1 J_1 l_2 J_2; sL) = i^{L - l_1 + l_2} (2l_1 + 1)^{1/2} (2l_2 + 1)^{1/2} (2J_1 + 1)^{1/2} (2J_2 + 1)^{1/2} W(l_1 J_1 l_2 J_2; sL) \langle l_1 l_2 00 | L0 \rangle \quad (2.12)$$

and  $W(l_1 J_1 l_2 J_2; sL)$ ,  $\langle l_1 l_2 00 | L0 \rangle$ , and  $X(J_1 l'_1 s'; J_2 l'_2 s'; LL1)$  are Racah coefficients, Clebsch-Gordan coefficients, and  $X$  coefficients, respectively. In Eqs. (2.10) and (2.11),  $\text{Re}$  stands for the real part of the expressions in the brackets.

### III. NUMERICAL CALCULATIONS AND DISCUSSION

The analysis of  $n$ - ${}^6\text{Li}$  is made by an  $R$ -matrix fit to the total elastic cross section in the energy range 0–5 MeV. In this energy range, we have both elastic as well as reaction channels. However, the  $(n, \alpha)$  reaction channel dominates while the other reaction channels have an extremely small contribution to the cross section in the energy range 0–5 MeV as mentioned earlier so that we can neglect all other reaction channels except  $(n, \alpha)$  channel in the present analysis. Furthermore, we use  $S$  and  $P$  partial waves only in the elastic channel because other partial waves have a negligible contribution to the cross section in the above energy range. In the reaction channel we use  $S$  and  $P$  partial waves for  $J = \frac{1}{2}$ . For  $J = \frac{3}{2}$  and  $\frac{5}{2}$  we use  $D$  partial waves also in addition to  $S$  and  $P$  partial waves in order to get better agreement with the experimental data. In the analysis we vary the  $R$ -matrix parameters to get the minimum  $\chi^2$

$$\chi^2 = \sum_i \left( \frac{\sigma_{\text{exp}}^i - \sigma_{\text{cal}}^i}{\sigma_{\text{exp}}^i} \right)^2$$

to fit the experimental data. Here  $\sigma_{\text{exp}}^i$  and  $\sigma_{\text{cal}}^i$  are the experimental and calculated cross sections, respectively, at various energies. An automatic search program<sup>4</sup> was made on the IBM 1130 computer for varying the  $R$ -matrix parameters to get the least  $\chi^2$ .

The procedure of calculating the cross sections is as follows: we first calculate the  $R$  matrix from Eq. (2.1) using the energy levels  $E_\lambda$  as the states of the compound nucleus  ${}^7\text{Li}$  and the corresponding reduced width amplitudes  $\gamma_{\lambda c}$  as the variable parameters. In the energy range 0–5 MeV we have four states for  ${}^7\text{Li}$ : one for  $J$

$= \frac{1}{2}$  ( $\lambda = 1$ ), two for  $J = \frac{3}{2}$  ( $\lambda = 1, 2$ ), and one for  $J = \frac{5}{2}$  ( $\lambda = 1$ ). These states<sup>9</sup> are  $S_{1/2}$ ,  $S_{3/2}$ ,  $P_{3/2}$ , and  $P_{5/2}$  at energies –0.69, 2.45, 3.88, and 0.23 MeV, respectively. Corresponding to these energies, we take thirteen parameters for the reduced width amplitudes  $\gamma_{\lambda c}$ ; five for  $S_{1/2}$ , one for  $S_{3/2}$ , five for  $P_{3/2}$ , and two for  $P_{5/2}$ . The components of the diagonal matrices  $P$ ,  $S$ , and  $\Omega$  are evaluated by calculating the regular and irregular wave functions and their derivatives for elastic and reaction channels at  $r_c = a_c$  from Eq. (2.5) which are the Bessel functions and the Coulomb wave functions, respectively. The regular and irregular Coulomb wave functions and their derivatives are calculated by a power series expansion method.<sup>10</sup> The  $W$  and hence the  $S$  matrix are now calculated from Eqs. (2.3) and (2.2). The unitary condition of the  $S$  matrix  $\sum_{c''} S_{cc''} S_{c''c}^* = \delta_{cc'}$  is checked. Finally we calculate the total elastic cross section from Eq.

TABLE I.  $R$ -matrix parameters used in the analysis of neutron scattering from  ${}^6\text{Li}$ .

	$J$	$\lambda$	$l s$	$E_{\lambda s J}$ (MeV)	$\gamma_{\lambda s J}$ (MeV) <sup>1/2</sup>
Elastic channels	$\frac{1}{2}$	1	$0 \frac{1}{2}$	–0.69	0.465
			$1 \frac{1}{2}$		0.180
			$1 \frac{3}{2}$		0.100
	$\frac{3}{2}$	1	$0 \frac{3}{2}$	2.45	1.280
			$1 \frac{1}{2}$		0.650
			$1 \frac{3}{2}$		0.200
$\frac{5}{2}$	1	$1 \frac{3}{2}$	0.245	1.030	
		$1 \frac{1}{2}$		0.150	
		$2 \frac{1}{2}$		0.100	
Reaction channels	$\frac{1}{2}$	1	$0 \frac{1}{2}$		0.600
			$1 \frac{1}{2}$		0.150
	$\frac{3}{2}$	2	$1 \frac{1}{2}$		0.150
			$2 \frac{1}{2}$		0.100
$\frac{5}{2}$	1	$2 \frac{1}{2}$		0.135	

(2.6) by varying the reduced width amplitude parameters. The fitted values of these parameters are listed in Table I. In our calculations we also take the boundary condition parameters  $B_{1,3/2,3/2}$ ,  $B_{1,3/2,5/2}$ , and  $B_{2,1/2,5/2}$  for  $B_{lsJ}$  as  $-0.90$ ,  $-0.918$ , and  $-0.35$ , respectively. The resonance energy level  $P_{5/2}$  is also shifted to  $0.245$  MeV as shown in Table I. The value of  $\chi^2$  for  $i=1$  to  $22$  for total

elastic cross section comes out to be  $0.497$ . The total  ${}^6\text{Li}(n, \alpha){}^3\text{H}$  cross section, differential cross section, and the neutron polarization are calculated by using the parameters which are fitted for the total elastic cross section.

For differential cross sections we use the following expression<sup>7</sup> for  $B'_L$  [Eq. (2.10)] to facilitate the computational calculations

$$\begin{aligned}
B'_L(\alpha's'; \alpha s) = & (-1)^{s-s'} \sum_{J=0}^{\infty} \sum_{l=|J-s|}^{J+s} \sum_{l'=|J-s'|}^{J+s'} Z(lJlJ; sL) Z(l'l'J'; s'L) |\delta_{\alpha's} \delta_{s's'} \delta_{l'l'} - S_{\alpha's'l'; \alpha sl}|^2 \\
& + 2(-1)^{s-s'} \sum_{J_1=0}^{\infty} \sum_{l_1=|J_1-s|}^{J_1+s} \sum_{l'_1=|J_1-s'|}^{J_1+s'} \left\{ \sum_{J_2=J_1+1}^{\infty} \sum_{l_2=|J_2-s|}^{J_2+s} \sum_{l'_2=|J_2-s'|}^{J_2+s'} Z(l_1J_1l_2J_2; sL) Z(l'_1J_1l'_2J_2; s'L) \text{Re}[\ ] \right. \\
& + \sum_{l_2=J_1+1}^{J_1+s} \sum_{l'_2=|J_1-s'|}^{J_1+s'} Z(l_1J_1l_2J_1; sL) Z(l'_1J_1l'_2J_1; s'L) \text{Re}[J_2=J_1] \\
& \left. + \sum_{l'_2=l'_1+1}^{J_1+s'} Z(l_1J_1l_1J_1; sL) Z(l'_1J_1l'_2J_1; s'L) \text{Re}[J_2=J_1, l_2=l_1] \right\}, \quad (3.1)
\end{aligned}$$

where  $\text{Re}[\ ]$  means the real part of the square bracket in Eq. (2.10);  $\text{Re}[J_2=J_1]$  and  $\text{Re}[J_2=J_1, l_2=l_1]$  are for the real part of the square bracket in Eq. (2.10) with  $J_2$  set equal to  $J_1$  and  $J_2=J_1, l_2=l_1$ , respectively. Similar expressions can be used for the neutron polarization [Eq. (2.11)]. In addition to the restrictions indicated on the various sums, the following restrictions reduce the number of actual terms

$$l_1 + l_2 - L = \text{even}; \quad l'_1 + l'_2 - L = \text{even}.$$

$(l_1 + l'_1)$  and  $(l_2 + l'_2) = [\text{even}, \text{odd}]$  if channels  $\alpha$  and  $\alpha'$  have [equal, opposite] parities.

Using the above restrictions and the properties of Racah coefficients the expressions for the  $n$ - ${}^6\text{Li}$  differential elastic cross section and polarization can be written as

$$\sigma(\theta) = \sum_{L=0}^2 B_L P_L(\cos\theta), \quad (3.2)$$

$$\sigma(\theta)P(\theta) = \sum_{L=1}^2 C_L P_L^1(\cos\theta), \quad (3.3)$$

where the Legendre coefficients  $B_L$  and  $C_L$  for neutron elastic scattering from  ${}^6\text{Li}$  are given below:

$$\begin{aligned}
\bar{H}_{l's'J}^{l's} &= \frac{1}{k_\alpha^2} |\delta_{s's} \delta_{l'l'} - S_{\alpha's'l'; \alpha sl}^J|^2, \\
\bar{H}_{l_1'l_1'J_1s's'}^{l_2'l_2'J_2} &= \frac{1}{k_\alpha^2} \text{Re}[(\delta_{s's} \delta_{l_1'l_1'} - S_{\alpha's'l_1'; \alpha sl_1}^{J_1})^* (\delta_{s's} \delta_{l_2'l_2'} - S_{\alpha's'l_2'; \alpha sl_2}^{J_2})], \\
B_0 &= \frac{1}{12} \bar{H}_{ss}^{1/2} 1/2 + \frac{1}{12} \bar{H}_{pp}^{1/2} 1/2 + \frac{1}{12} \bar{H}_{pp}^{3/2} 1/2 + \frac{1}{6} \bar{H}_{ss}^{3/2} 3/2 + \frac{1}{6} \bar{H}_{pp}^{1/2} 3/2 + \frac{1}{6} \bar{H}_{pp}^{3/2} 3/2 + \frac{1}{4} \bar{H}_{pp}^{3/2} 5/2 + \frac{1}{12} \bar{H}_{pp}^{3/2} 1/2 \\
&+ \frac{1}{12} \bar{H}_{pp}^{1/2} 1/2 + \frac{1}{6} \bar{H}_{pp}^{3/2} 3/2 + \frac{1}{6} \bar{H}_{pp}^{1/2} 3/2, \\
B_1 &= \frac{1}{3} \bar{H}_{ss}^{3/2} 1/2 + \frac{1}{6} \bar{H}_{pp}^{3/2} 1/2 + \frac{1}{2} \bar{H}_{ss}^{5/2} 3/2 + \frac{1}{6} \bar{H}_{pp}^{1/2} 1/2 + \frac{1}{3} \bar{H}_{ss}^{3/2} 3/2, \\
B_2 &= \frac{1}{6} \bar{H}_{pp}^{3/2} 1/2 + \frac{8}{75} \bar{H}_{pp}^{3/2} 3/2 + \frac{7}{50} \bar{H}_{pp}^{5/2} 3/2 + \frac{1}{3} \bar{H}_{pp}^{3/2} 1/2 + \frac{1}{30} \bar{H}_{pp}^{3/2} 3/2 + \frac{3}{10} \bar{H}_{pp}^{5/2} 3/2 \\
&+ \frac{21}{50} \bar{H}_{pp}^{5/2} 3/2 - \frac{2}{15} \bar{H}_{pp}^{3/2} 1/2 - \frac{2}{15} \bar{H}_{pp}^{3/2} 3/2 - \frac{1}{3\sqrt{10}} \bar{H}_{pp}^{3/2} 1/2 - \frac{1}{3\sqrt{10}} \bar{H}_{pp}^{3/2} 3/2.
\end{aligned}$$

The Legendre coefficients for the differential polarization are given below

$$\begin{aligned}
\bar{H}_{l_1'l_1'J_1s's'}^{l_2'l_2'J_2} &= \frac{1}{k_\alpha^2} \text{Re}[i(\delta_{s's} \delta_{l_1'l_1'} - S_{\alpha's'l_1'; \alpha sl_1}^{J_1})^* (\delta_{s's} \delta_{l_2'l_2'} - S_{\alpha's'l_2'; \alpha sl_2}^{J_2})], \\
C_1 &= -\frac{1}{18} \bar{H}_{ss}^{3/2} 1/2 + \frac{5}{36} \bar{H}_{pp}^{3/2} 1/2 + \frac{1}{4} \bar{H}_{ss}^{5/2} 3/2 + \frac{1}{18} \bar{H}_{pp}^{1/2} 1/2 - \frac{1}{9} \bar{H}_{ss}^{3/2} 3/2,
\end{aligned}$$

$$C_2 = -\frac{1}{18} \frac{\Xi_{pp}^{3/2}}{\Xi_{pp}^{1/2} \Xi_{pp}^{1/2} \Xi_{pp}^{1/2}} + \frac{1}{180} \frac{\Xi_{pp}^{3/2}}{\Xi_{pp}^{1/2} \Xi_{pp}^{3/2} \Xi_{pp}^{3/2}} + \frac{2}{15} \frac{\Xi_{pp}^{5/2}}{\Xi_{pp}^{1/2} \Xi_{pp}^{3/2} \Xi_{pp}^{3/2}} + 0.044 \sqrt{7} \frac{\Xi_{pp}^{5/2}}{\Xi_{pp}^{3/2} \Xi_{pp}^{3/2} \Xi_{pp}^{3/2}}$$

$$- \frac{1}{18\sqrt{10}} \frac{\Xi_{pp}^{3/2}}{\Xi_{pp}^{1/2} \Xi_{pp}^{1/2} \Xi_{pp}^{3/2}} + \frac{1}{18\sqrt{10}} \frac{\Xi_{pp}^{3/2}}{\Xi_{pp}^{1/2} \Xi_{pp}^{3/2} \Xi_{pp}^{1/2}}.$$

Figures 1–5 illustrate the results of our calculations. Figure 1 shows the total elastic cross section in the energy range 0–5 MeV obtained by varying the  $R$ -matrix parameters as shown in Table I with the experimental points.<sup>3</sup> In Fig. 2 the total  ${}^6\text{Li}(n, \alpha){}^3\text{H}$  reaction cross sections are calculated in the same energy range with the experimental points.<sup>3</sup> Figures 3 and 4 illustrate the  $R$ -matrix predictions for differential cross sections at various energies with the experimental points.<sup>3,11</sup> In Fig. 4 the experimental data are not available for neutron energy greater than 2.0 MeV. In Fig. 5 we plot the neutron polarization at various energies along with the experimental points.<sup>3,11</sup> It is clear from the figures that a reasonably good agreement for the cross sections and polarization is obtained between the experimental data and the  $R$ -matrix prediction throughout the energy range 0–5 MeV.

In the literature, relatively few experimental and theoretical studies have been made on the  $n$ - ${}^6\text{Li}$  system as mentioned earlier. Recently Holt *et al.*<sup>3</sup> have made experimental and theoretical studies on this system in the energy range 0–5 MeV. They have calculated the total elastic cross section, total  ${}^6\text{Li}(n, \alpha){}^3\text{H}$  cross section, differential elastic cross section, and the neutron polarization in the above energy range. In their

analysis, they have expressed  $\sigma(n, n)$  and  $\sigma(n, \alpha)$  using the optical model expression for the elastic and reaction channels. This could be done by them because of the fact that in the reaction channels only the  $\sigma(n, \alpha)$  process was dominant. Furthermore, in their calculations, they have assumed that the angular momentum, spin, and channel spins do not change between the entrance and exit channels. That is, they have considered only diagonal elements of the scattering matrix. The effect of the reaction channel  $(n, \alpha)$  was taken into account using the complex “reduced  $R$  matrix”<sup>5,12</sup>

$$R_{l_s J} = \sum_{\lambda} \frac{\gamma_{\lambda l_s J}^2}{E_{\lambda l_s J} - E - \frac{1}{2} i \Gamma_{\lambda l_s J}^{\alpha}} + R_{l_s J}^{\infty},$$

where  $\gamma_{\lambda l_s J}^2$ ,  $E_{\lambda l_s J}$ , and  $\Gamma_{\lambda l_s J}^{\alpha}$  are the reduced widths, level energies, and the  $\alpha$ -particle widths, respectively.  $R_{l_s J}^{\infty}$  is the complex contribution to the  $R$  matrix from the distant levels. Holt *et al.* have varied these parameters along with the boundary condition parameters to fit the experimentally observed values of the cross sections and obtained a good fit. Our calculations show that in the case of the  $n$ - ${}^6\text{Li}$  analysis at low energies, it is possible to incorporate the contribution of non-diagonal terms of the scattering matrix. In our analysis we calculate the cross sections and po-

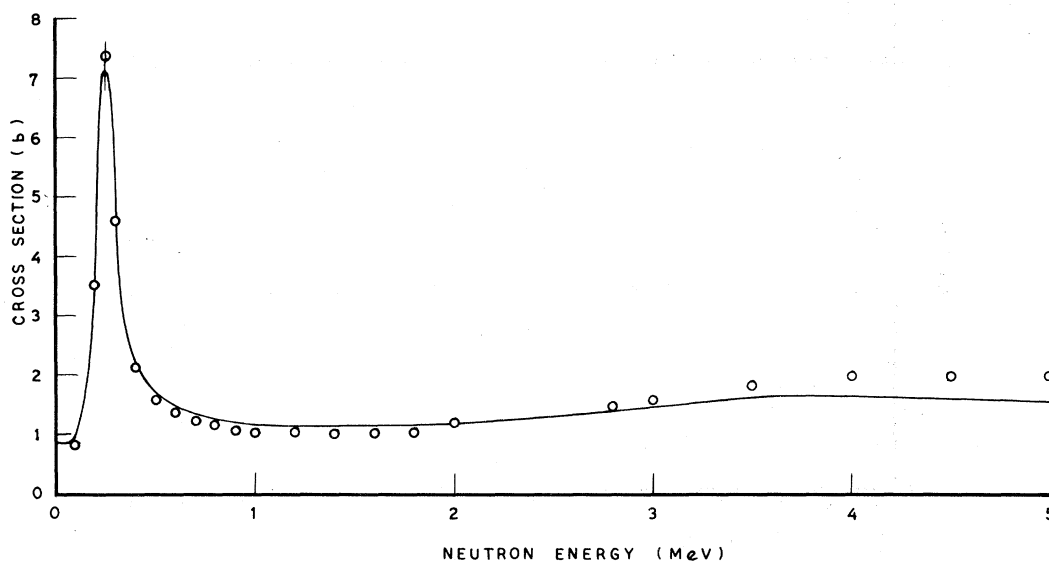


FIG. 1. The total neutron cross section for the  $n$ - ${}^6\text{Li}$  system in the energy range 0–5 MeV with the experimental points.

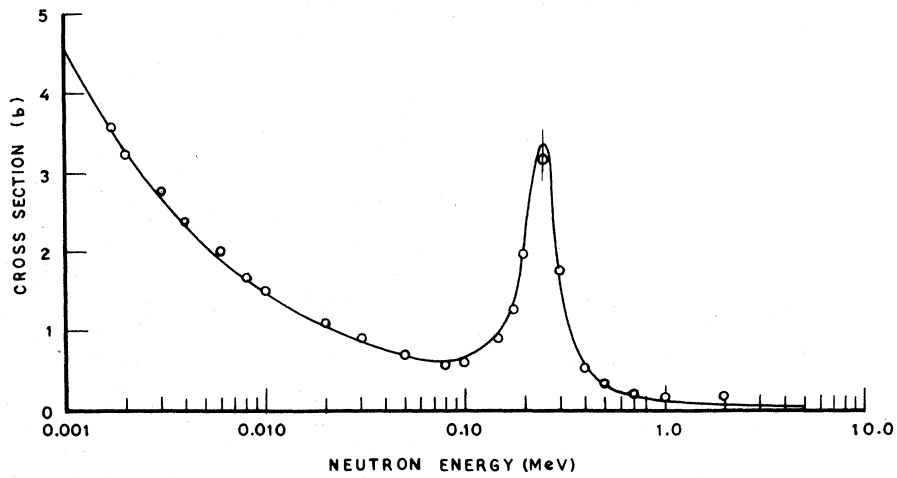


FIG. 2. The total  ${}^6\text{Li}(n, \alpha){}^3\text{H}$  cross section in the energy range 0-5 MeV with the experimental points.

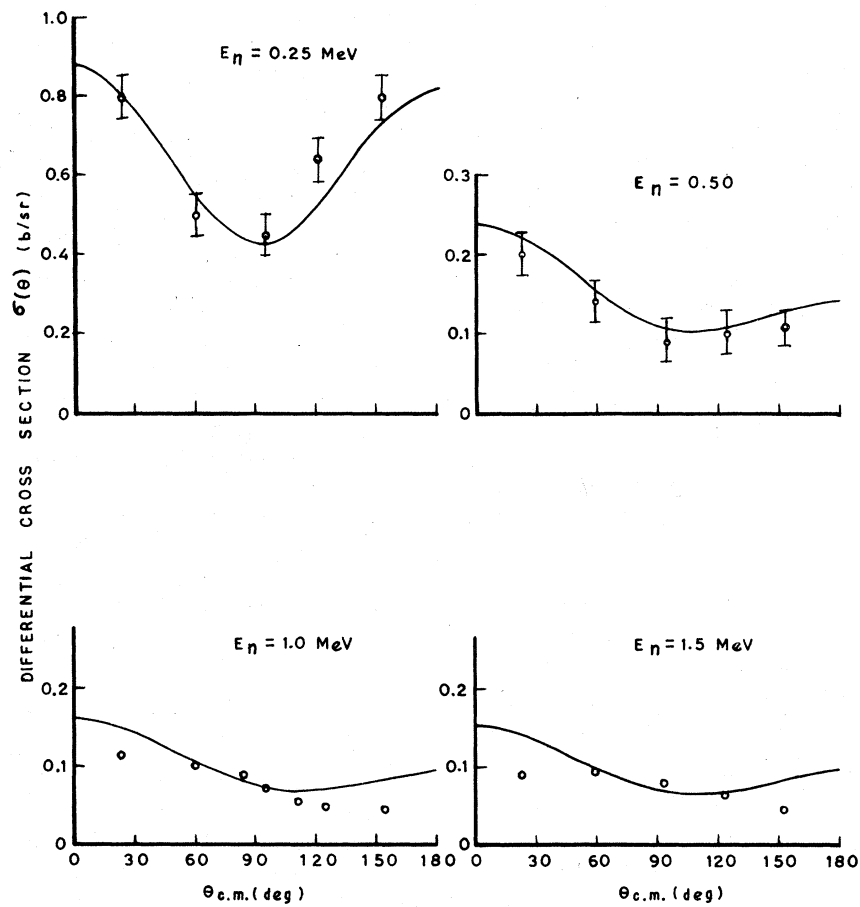


FIG. 3. The differential cross sections for the  $n-{}^6\text{Li}$  system below 2 MeV with the experimental points.

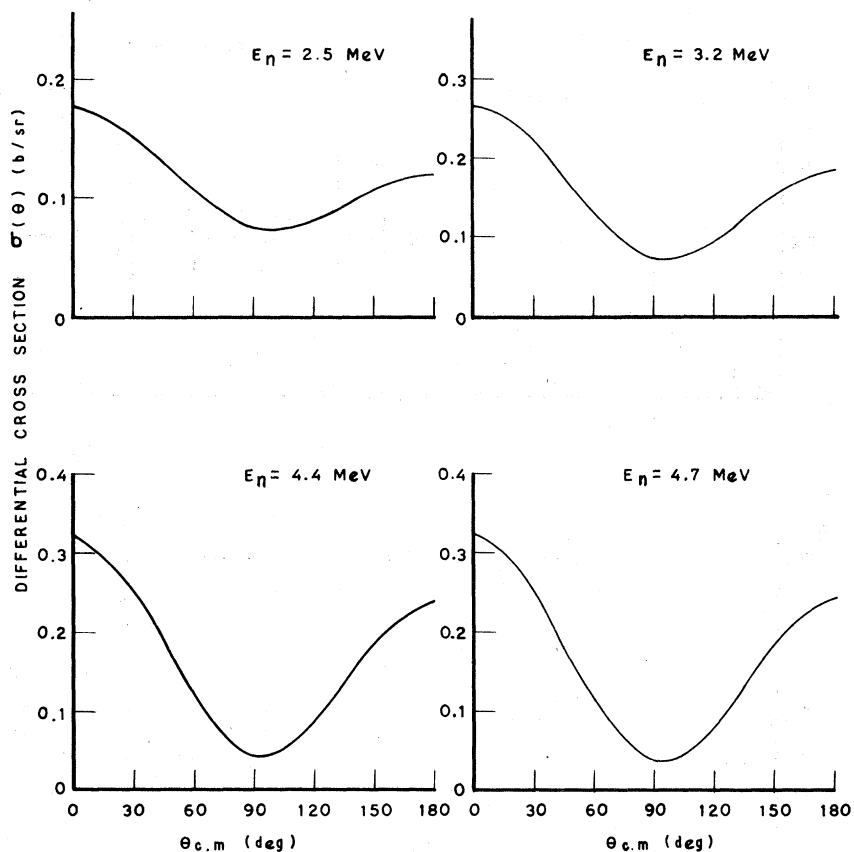


FIG. 4. The differential cross sections for the  $n$ - ${}^6\text{Li}$  system in the energy range 2-5 MeV.

larization by general  $R$ -matrix theory taking the nondiagonal terms of  $S$  matrix and elastic and reaction channels into account. That is, we vary the reduced width amplitude parameters  $\gamma_{\lambda c}$  of both elastic and reaction channels to get the experimental fit. It is clear from Table I that the parameters  $\gamma_{\lambda c}$  for the reaction channels are comparable to the parameters  $\gamma_{\lambda c}$  of the elastic channels. These parameters are eliminated by Holt *et al.* in the reduced  $R$ -matrix theory by making the  $R$  matrix complex and introducing the  $\alpha$ -particle width parameters in the  $R$ -matrix expression. The contribution of the nondiagonal terms in their calculations may come implicitly from the parameters  $R_{isJ}^{\alpha}$ . In fact, it is found that the contribution of the nondiagonal terms is significant. However, in the parameter fitting type calculations, the experimental data can be reproduced by varying a reasonable number of parameters and neglecting the effect of the nondiagonal terms. Thus our procedure has the advantage in the sense that we are

getting quite reasonable results for the cross sections and polarization, varying roughly the same number of parameters as used by Holt *et al.*, taking nondiagonal terms into account, and showing the contribution of reaction channels more explicitly. Moreover, in our analysis we keep the energy level parameters  $E_{\lambda isJ}$  fixed at the values of compound nucleus  ${}^7\text{Li}$  levels, except a small shift in the resonance energy level  $P_{5/2}$ . Also, the parameters  $R_{isJ}^{\alpha}$  are not included in our analysis. Finally we can say that the general  $R$ -matrix theory is quite feasible for analyzing the  $n$ - ${}^6\text{Li}$  reaction in the energy range 0-5 MeV taking the real  $R$  matrix in contrast to the complex reduced  $R$  matrix, and for giving the contribution of nondiagonal terms of the scattering matrix and the reaction channels in a more explicit way.

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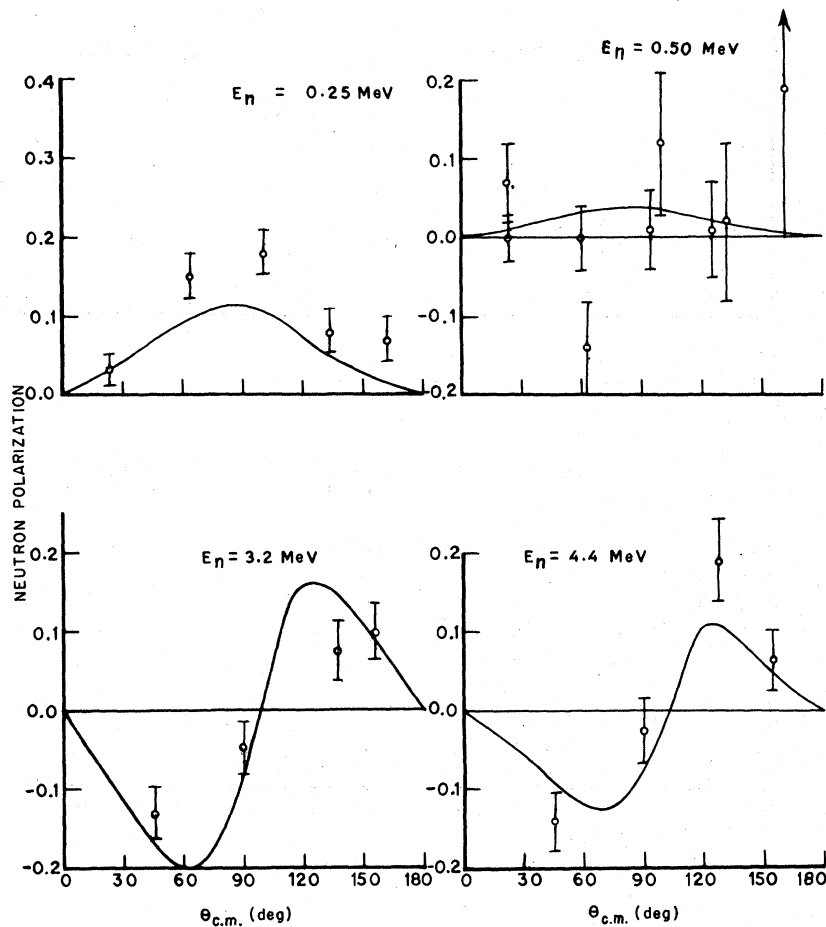


FIG. 5. Neutron polarization for  $n$ - ${}^6\text{Li}$  system in the energy range 0–5 MeV with the experimental points.

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<sup>1</sup>R. J. Holt, F. W. K. Firk, R. Nath, and H. L. Schultz, Nucl. Phys. **A213**, 147 (1973).

<sup>2</sup>G. T. Hickey, F. W. K. Firk, R. J. Holt, and R. Nath, Nucl. Phys. **A225**, 470 (1974).

<sup>3</sup>R. J. Holt, F. W. K. Firk, G. T. Hickey, and R. Nath, Nucl. Phys. **A237**, 111 (1975).

<sup>4</sup>M. A. Melkanoff, T. Sawada, and J. Raynal, *Methods in Computational Physics*, edited by B. Adler, S. Fernbach, and M. Rotenberg (Academic, New York, 1966), Vol. 6.

<sup>5</sup>A. M. Lane and R. G. Thomas, Rev. Mod. Phys. **30**, 257 (1958).

<sup>6</sup>M. A. Preston, *Physics of the Nucleus* (Addison-Wesley, Reading, Massachusetts, 1962).

<sup>7</sup>J. M. Blatt and L. C. Biedenharn, Rev. Mod. Phys. **24**, 258 (1952).

<sup>8</sup>A. Simon and T. A. Welton, Phys. Rev. **90**, 1036 (1953).

<sup>9</sup>T. Lauritsen and F. Ajzenberg-Selove, Nucl. Phys. **78**, 1 (1966).

<sup>10</sup>C.-E. Fröberg, Rev. Mod. Phys. **27**, 399 (1955).

<sup>11</sup>R. O. Lane, A. J. Elwyn, and A. Langsdorf, Jr., Phys. Rev. **136**, B1710 (1964).

<sup>12</sup>R. G. Thomas, Phys. Rev. **97**, 224 (1955).