Distorted wave analysis of (d,p) reactions to decaying states

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The distorted-wave Born approximation is used in the case of the stripping reaction $^{32}S(d,p)^{33}S^*$ leading to decaying states in order to explain the strong dependence of the ratio of the energy differential (d,p) stripping cross section to the total neutron elastic scattering cross section on the same target, on transferred angular momentum (l). The necessity of using a distorted-waves approximation instead of a plane-waves approximation, which is much simpler to calculate and which also shows "l dependence," is discussed in the context of identification of levels. The slowly converging overlap integral appearing in the distorted-wave Born approximation is treated by employing a Gaussian convergence factor. It is also found that in the plane-wave calculation, the inclusion of the unscattered part in the neutron form factor affects only the $l = 0$ resonances. The desirability of including the effect of the nuclear interior is discussed. The angular distribution for the reaction $^{16}O(d,p)^{17}O^*$ has been obtained employing the distorted-wave Born approximation method. The spectroscopic factor for this reaction is obtained and compared with that reported previously.

[NUCLEAR REACTIONS $^{32}S(d,p)$, $^{16}O(d,p)$ reactions, unbound states, calculated $[d^2\sigma(d,p)]/d\Omega dE$, $[d\sigma(d,p)]/d\Omega$, $E_D = 12$–13.3 MeV, DWBA analysis.]

I. INTRODUCTION

In the last few years, the experimental data[1-4] which have been accumulated in the field of stripping reactions to decaying states strongly suggest a close parallelism between the deuteron stripping to decaying states and the total neutron scattering cross section. Resonances observed in the outgoing proton spectrum are usually accompanied by resonances in the total neutron scattering cross section at the corresponding neutron energy. It is also found that the resonance stripping cross section depends very strongly on the angular momentum of the transferred particle compared to the dependence of the total neutron scattering cross section on the resonance l value. The reaction mechanism of most of the stripping reactions to the decaying states is believed to be direct, and Born approximations have been successfully employed in several cases.[5] Calculations have been reported by Shyam and Mukherjee,[6] using the plane-wave Born approximation (PWBA), which reproduce the essential features of experimental data for $l=1$ and $l=2$ transitions. However, the results for stripping to $l=0$ resonances are rather poor. Further, the PWBA results are critically dependent on the lower cutoff radius which has to be adjusted for each l value and a consistent choice is difficult to make. Hence, to test the effectiveness of the direct reaction technique and the other associated assumptions in describing the stripping reaction to decaying states, detailed calculations using proper distorted waves are necessary. The distorted-wave Born approximation (DWBA) method, being more exact, gives less ambiguous absolute cross sections which are essential for level identification and extraction of spectroscopic information for unbound states. In a different approach, Lipperheide and Möhring[7] have tried to explain the stripping process as a virtual deuteron breakup followed by an off-shell neutron scattering from the target nucleus. They have interpreted the strong dependence of the stripping cross section upon the resonating orbital angular momentum l as the off-shell continuation of the total neutron elastic scattering.

Unlike the bound state stripping, calculation of the T matrix in DWBA for unbound stripping is not straightforward. This is due to the slow convergence of the radial integrals in the T matrix of DWBA. Huby and Mines[8] have used a convergence factor $e^{-\alpha r}$ in the integrand and extrapolated the results to the limit in which $\alpha \to 0$. However, in order to get good accuracy, one has to perform the radial integrals up to very large distances.[9,10] It is seen here that faster convergence factors of the type $e^{-\alpha r^2}$ or $e^{-\alpha r^3}$ serve to overcome the above difficulty without sacrificing any numerical accuracy.

In the present investigation distorted wave analyses of deuteron stripping reaction to decaying states for ^{32}S and ^{16}O targets are presented. The ratio of the energy differential stripping cross section and total neutron elastic scattering cross section has been evaluated using the DWBA method for the ^{32}S nucleus. The results are compared with experiment and with those obtained by Shyam and Mukherjee[6] by employing the plane-wave Born approximation (PWBA) method. The stripping angular distribution has been calculated for the $^{16}O(d,p)^{17}O^*$ reaction to check the consistency of the DWBA method and also the procedure for handling the slowly convergent overlap integral.

II. THEORY

To evaluate the T-matrix element using the Born approximation near a resonance, we shall use the proper neutron scattering wave function that resonates in a single particle well as the form factor. If one assumes that the contribution from the nuclear interior is negligible and the unscattered part of the neutron wave function is also relatively weak near an isolated resonance, which is true for high l and at low neutron energy, one gets, following Cole and Huby,[11] a "model independent" expression for the cross section for the $A(d,p)B^*$ reaction for a particular (lj) value as

$$\frac{d^2\sigma_{lj}}{dE_p\,d\Omega_p} = \sin^2\delta_{lj}\,\frac{d^2\sigma_{lj}^{\mathrm{sp}}}{dE_p\,d\Omega_p}\,. \tag{1}$$

In this expression E_p is the center of mass (c.m.) kinetic energy of the proton leaving B^* at some excitation energy E_B in the continuum. It is also assumed that B^* can undergo only neutron decay to the ground state of A, which is true at low excitation energy. δ_{lj} is the phase shift for n-A scattering in the lth partial wave at energy corresponding to the formation of B^* with the excitation energy E_B and spin j. The quantity $d^2\sigma_{lj}^{\mathrm{sp}}/dE_p\,d\Omega_p$ is the stripping cross section which is calculated by the usual DWBA formula for the energy differential cross section using a single particle resonance wave function as follows: A real Woods-Saxon potential is adjusted in depth so that it produces an (lj) orbit resonating (i.e., having phase shift $\frac{1}{2}\pi$) at the neutron energy which corresponds to the formation of B^* with excitation energy E_B. The resonant wave function is normalized so that its radial part $f_l(r)$ behaves asymptotically as

$$f_l(r) \underset{r\to\infty}{\simeq} j_l(k_n r) + i e^{i\pi/2}\sin\tfrac{1}{2}\pi\, h_l^{(+)}(k_n r), \tag{2}$$

where the spherical Bessel and Hankel functions are denoted by j_l and $h_l^{(+)}$, and k_n is the wave number corresponding to the neutron energy into the continuum.

To get the angular distribution, we have to integrate Eq. (1) over the energy of the outgoing proton. Following the method given in Ref. 6, we get

$$\frac{d\sigma_{lj}}{d\Omega} = \frac{\pi}{2}\Gamma\,\frac{d^2\sigma_{lj}^{\mathrm{sp}}}{dE_p\,d\Omega_p}\,, \tag{3}$$

where it has been assumed that $d^2\sigma_{lj}^{\mathrm{sp}}/dE_p\,d\Omega_p$ varies very slowly with proton energy.

The total neutron scattering cross section near the resonance may be written as

$$\sigma_{lj}^{\mathrm{tot}}(n,n) = \frac{2\pi}{k_n^2}(2j+1)\sin^2\delta_{lj} \tag{4}$$

for a spin zero target, where j is the total spin of the resonance.

From Eqs. (1) and (4) the parallelism between the (d,p) stripping and the total neutron scattering cross section becomes immediately transparent. We can eliminate $\sin^2\delta_{lj}$ from the two equations to obtain

$$\frac{d^2\sigma_{lj}/dE_p\,d\Omega_p}{\sigma_{lj}^{\mathrm{tot}}(n,n)} = \frac{k_n^2}{2\pi(2j+1)}\,\frac{d^2\sigma_{lj}^{\mathrm{sp}}}{dE_p\,d\Omega_p}\,. \tag{5}$$

This ratio is called the "stripping enhancement factor (F_l)." Using PWBA one can get an analytic expression for F_l as[6]

$$F_l = \frac{1}{(2S+1)}\,\frac{m_i m_f}{(\pi\hbar^2)^3}\,\frac{k_f}{k_i}\,k_n^3 m_n D_0^2\,|I_l|^2\,, \tag{6}$$

where I_l is defined as

$$I_l = \frac{R_0^2}{k^2-k_n^2}\left[k h_l^{(+)}(k_n R_0)\, j_{l-1}(kR_0) - k_n h_{l-1}^{(+)}(k_n R_0)\, j_l(kR_0)\right] \tag{7}$$

and $k = |\vec{k}_i - \vec{k}_f|$. Here the symbols carry the same meaning as described in Ref. 6.

The numerical evaluation of $d^2\sigma_{lj}^{\mathrm{sp}}/dE_p\,d\Omega_p$ using the DWBA method presents some difficulties. This is due to the fact that both the proton and the neutron are free in the final state and are described by scattering wave functions in the T matrix. This makes the integrands of the overlap integral in the T matrix highly oscillatory, with slow convergence. This renders usual integration methods inadequate to calculate the T matrix for the unbound stripping.

As first pointed out by Zel'dovich[12] and later by Berggren,[13] such integrals can be defined as the limit

$$\lim_{\alpha\to 0+}\int e^{-\alpha r^2}(\text{Integrand})\,dr\,.$$

Introduction of the convergence factor makes possible the integration by usual methods.

III. NUMERICAL RESULTS AND DISCUSSIONS

We shall briefly summarize the results obtained by using PWBA for ^{32}S described in a previous communication.[6] The stripping enhancement factor F_l has been calculated by using PWBA, in which the cutoff radius R_0, defined as

$$R_0 = (1.4A^{1/3} + \Delta)\ \mathrm{fm}, \tag{8}$$

is varied by varying Δ. Good fits to the experimental data of Bommer *et al.*[4] are obtained for the resonances corresponding to $l=1$ with $\Delta=2.0$ and for $l=2$ with $\Delta=3.8$. These results are given in Fig. 1.

In the present investigation the PWBA method is applied to calculate the stripping enhancement factor for $l=0$ resonances. It is observed from Fig. 1 that the theoretical prediction is rather poor for $l=0$ resonances, in comparison to those for $l=1$ and $l=2$ cases. The theoretical curve can be brought closest to the experiment for the $l=0$ case by putting $\Delta=0$. However, the choice of smaller Δ is against the wisdom of the PWBA.

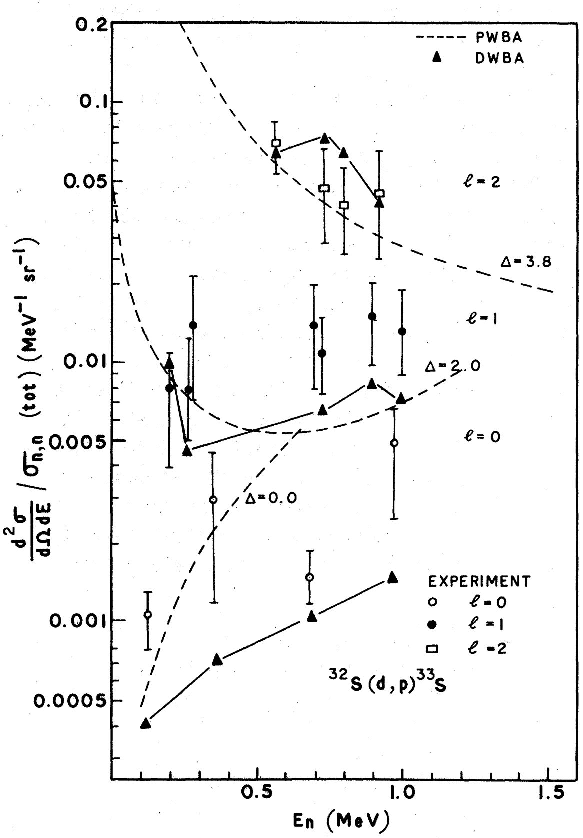

FIG. 1. Variation of the stripping enhancement factor $(d^2\sigma/d\Omega dE)/\sigma_{\mathrm{tot}}(n,n)$ for a ^{32}S target with the neutron resonance energy for $l=0$, 1, and 2. The energy differential cross section is measured at 10° for 12 MeV incident deuteron energy. The dotted line shows the results obtained by the plane-wave Butler cutoff method. Solid triangles show the results for DWBA calculations. The DWBA points are connected by a free-hand curve just to guide the eye. Experimental points are taken from Ref. 4.

The reason for this discrepancy can be understood if one includes the unscattered part $j_l(k_n r)$ in the neutron form factor. The unscattered part for $l=0$ neutrons is much larger than that for higher l values at the energy interval concerned. This is apparent from the behavior of the Bessel and Hankel functions at small arguments and shown explicitly by calculating the overlap integral I_l with and without $j_l(k_n r)$ and plotting $|I_l|^2$ against neutron energy. This is shown in Fig. 2. It is observed that the effect of including j_l is quite drastic for $l=0$ as compared to $l=1$ and $l=2$. This gives rise to two problems in using the Butler approximation for $l=0$ stripping. First, one is not justified in using plane waves when a significant contribution comes from the region much closer to the nuclear surface compared to the higher l transitions, and secondly, the unscattered part should not be neglected. These considerations lead one to the DWBA method of calculation using the full neutron wave function.

The DWBA calculations have been performed using a convergence factor $e^{-\alpha r^2}$ in the radial integral as described earlier. The cross sections

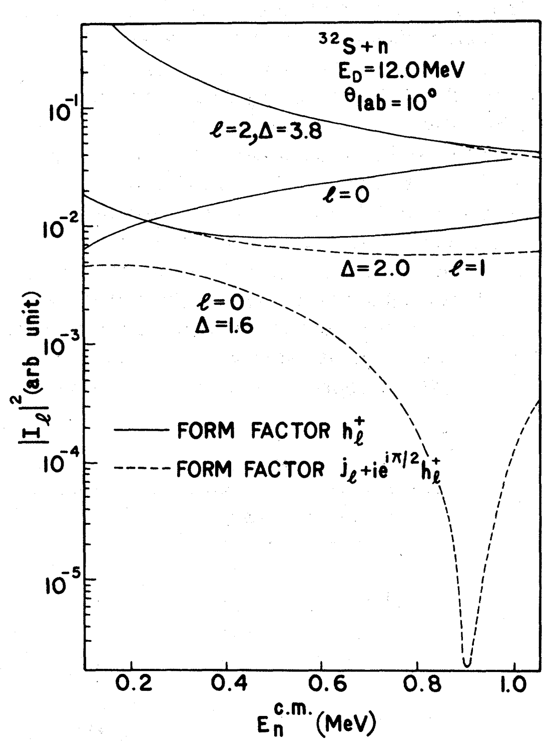

FIG. 2. Variation of stripping overlap integral $|I_l|^2$ for the plane waves with the neutron resonant energy for $l=0$, 1, and 2. The solid curve represents the results when the form factor does not include the unscattered part, whereas the dotted lines are the results when this is included in the form factor.

have been computed for various values of α and the extrapolated value for $\alpha = 0$ is used for comparison with the experiment. The stability of the computed values against the changes of upper limit of the radial integral and number of the partial waves used must be ensured. In fact, this consideration limits one in taking α arbitrarily small as ideally should be the case. Typical values of α used in our calculations are between 0.001 and 0.005. The small variation of the computed cross section with α (in the range mentioned above) indicates that the extrapolated value must be very close to the mathematically defined limiting value.

The well depth for neutron potential for each resonance energy has been obtained with the help of an optical model computer code MAIN[14] which automatically searches over the depth of a Woods-Saxon potential to produce a $\frac{1}{2}\pi$ phase shift for the resonating partial waves. The single particle level width is also simultaneously calculated by this code. This neutron potential is then used in the DWBA code DWUCK, which has been modified for this purpose. The deuteron optical potential is taken from Ref. 9 and the proton optical potential from Becchetti and Greenlees.[15] The values of the parameters for the various potentials are shown in Table I. The spin-orbit term has not been included in the optical potentials describing the interaction both in deuteron and proton channels, and j has been taken equal to l, except for $l = 0$ stripping where $j = \frac{1}{2}$ has been used.

The results of DWBA calculations for the stripping enhancement factor for the ^{32}S nucleus are shown in Fig. 1 for various values of l and E_n. We see that l enhancement is clearly reproduced. It is also observed from the experimental data that the l enhancement decreases at higher neutron energies. DWBA reproduces this trend as well.

Close agreement between theoretical and experimental points is obtained for $l = 2$ transition. However, for $l = 1$ transition the agreement is not so good and for $l = 0$ transition the calculated values are smaller than the experimental values by nearly an order of magnitude. This clearly shows that the model independent approximation works better for higher l values.

Like stripping to a bound state, the spectroscopic factor for stripping to an unbound state may be defined as the ratio of the experimental and theoretical cross sections at forward angles. However, if the left-hand side of Eq. (3) is taken as the experimental value of the cross section and it is fitted with the theoretical value on the right-hand side, one can obtain a value for the level width Γ. The comparison of this value of the level width with the value obtained by neutron scattering from the same target may provide a means to check the accuracy of the DWBA method of calculation. No such check is possible in case of bound state stripping.

We have calculated in the present study the angular distribution for the $^{16}O(d,p)^{17}O^*$ reaction leading to the unbound state at excitation energy $E_X = 5.083$ MeV for incident deuteron energy $E_D = 13.3$ MeV. The level width $\Gamma_{d,p}$ required to fit the experimental value[16] of the cross sections at the forward angles with the calculated values is 122 keV. This value is slightly greater than that obtained from neutron scattering measurements (90 ± 5 keV). The theoretical analysis including the spin-orbit force, recently made by Darden *et al.*[16] of the above (d,p) data, yields $\Gamma_{d,p} = 96$ keV. It is likely that our results will also improve with the inclusion of the spin-orbit interaction.

In order to have a convenient measure of the single particle strength of an unbound state, the spectroscopic factor may also be defined as[17]

TABLE I. Potential parameters. All quantities are either in MeV or in fm.

Nucleus	Energy	V_0	r_0	a_0	W^a	r_w	a_w
		Deuteron optical potential					
^{32}S	12.0	110.9	1.005	0.875	20.5	1.417	0.584
^{16}O	13.3	85.25	1.25	0.606	12.75	0.958	1.578
		Proton optical potential					
^{33}S		53.4	1.17	0.75	9.8	1.32	0.510
^{17}O		59.99	1.25	0.501	5.729	1.517	0.474
		Neutron potential					
^{32}S		b	1.22	0.7	···	···	···
^{16}O		b	1.30	0.60	···	···	···

[a] Volume type

[b] Found by searching as described in the text.

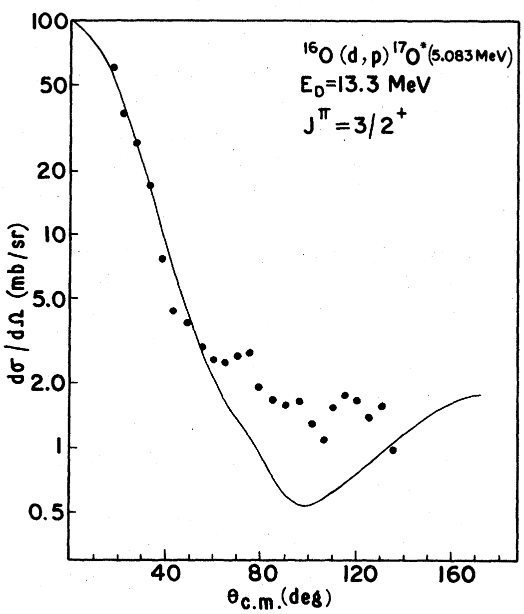

FIG. 3. Angular distribution for $^{16}O(d,p)^{17}O^*$ (5.083 MeV) reaction obtained by the DWBA method. The experimental points are taken from Ref. 16.

$$S=\Gamma_{d,p}/\Gamma_{sp}\,, \qquad (9)$$

where Γ_{sp} is the single particle level width.

The single particle level width, calculated by the code MAIN for the level ($E_X=5.083$ MeV), is 110 keV. Hence the corresponding value of the spectroscopic factor is 1.11. The angular distribution for the above reaction as obtained in the present calculation is shown in Fig. 3 along with that obtained experimentally. The theoretical curve has been normalized to the experimental cross sections at forward angles.

Since the DWBA calculations have been performed including the entire space, one would like to investigate whether the contribution from the inside region is really small, because this is one of the approximations made in writing Eq. (1). It has been done in the following way: We obtain two depths ($V_0=73.385$ and 28.943 MeV) for the neutron well, both producing resonance for $l=2$ at $E_n^{c.m.}=0.792$ MeV, the deeper potential accommodating one-half wavelength more inside the nucleus than the shallower one. The DWBA calculations have been performed using both the potential wells and the results are shown in Fig. 4. The calculated cross sections are remarkably similar. Similar observation has been made for $l=0$ resonances as well. This implies that the DWBA matrix elements are not sensitive to the interior form factor. It must be noted that Baur, Rösel, and Trautmann[18] reached a different conclusion regarding the inside contribution for the reaction $^{15}N(d,p)^{16}N^*$. They found that for this reaction the nuclear interior does contribute significantly.

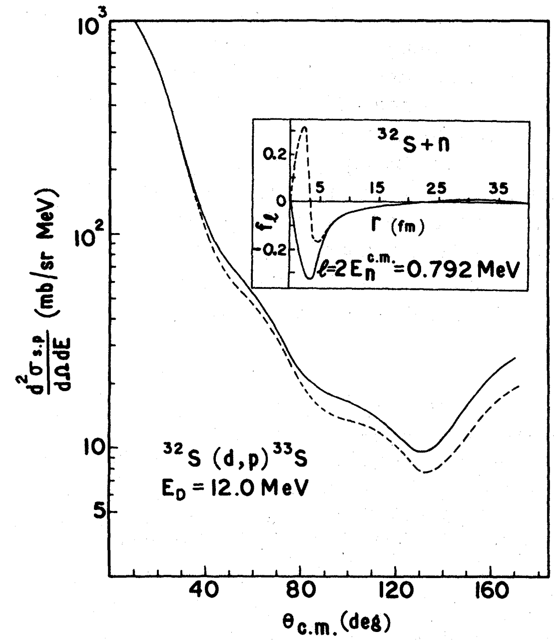

FIG. 4. Variation of the energy differential cross section $d^2\sigma_{sp}/d\Omega dE$ with angle, obtained with two neutron depths: $V_0=73.385$ MeV (deep well) shown by a dotted line and $V_0=28.943$ MeV (shallow well) shown by a solid line. The other potential parameters are $r_0=1.22$ fm and $a_0=0.7$ fm. The neutron wave functions obtained by the deep (dotted line) and the shallow (solid) potentials are also shown in the figure.

The use of a single particle model to describe an $l=0$ resonance for the neutron may be questioned, since it is well known[19] that a simple Woods-Saxon type of potential cannot produce an $l=0$ resonance for a neutron. This can be answered from the previous observation that the neutron wave function is only really important in the outside region and this is described uniquely by phase shift alone, independent of the model used to generate the entire wave function.

CONCLUSION

In summary, we can say that the DWBA method using a resonating wave function is quite successful in reproducing the observed l enhancement in stripping to unbound levels. This l enhancement can be used in conjunction with DWBA calculation to supplement the existing methods of spin and parity assignment to the resonant states. The present method of DWBA calculation is comparatively easy to adapt and its reliability has been established. The effect of including spin in different channels is expected to improve the agreement with the experiment.

It must be remarked that though in the present calculation the effect of the interior region is found negligible, this may not be the case with other nuclei as already observed by Baur and Trautmann.[20] In such cases one can test various models to describe the resonant states. In this way the theory of unbound state stripping, which is itself a direct reaction, may serve as a probe of resonant structure.

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[1]H. Fuchs, H. Homeyer, T. Lorenz, and H. Oeschler, Phys. Lett. 37B, 285 (1971); H. Fuchs, H. Homeyer, H. Oeschler, R. Lipperheide, and K. Möhring, Nucl. Phys. A196, 286 (1972).

[2]J. Bommer, M. Ekpo, H. Fuchs, K. Grabisch, and H. Oeschler, Phys. Lett. 52B, 421 (1974).

[3]J. Bommer, M. Ekpo, H. Fuchs, K. Grabisch, and H. Kluge, Nucl. Phys. A251, 246, 257 (1975).

[4]J. Bommer, M. Ekpo, K. Grabisch, and H. Kluge, Nucl. Phys. (to be published).

[5]G. Baur and D. Trautmann, Z. Phys. 267, 103 (1974); Nucl. Phys. A199, 218 (1973); A211, 333 (1973); J. Bang and J. Zimanyi, Nucl. Phys. A139, 534 (1969); F. S. Levin, Ann. Phys. (N.Y.) 46, 41 (1968).

[6]R. Shyam and S. Mukherjee, Phys. Rev. C 13, 2099 (1976).

[7]R. Lipperheide and K. Möhring, Phys. Lett. 39B, 323 (1972).

[8]R. Huby and J. R. Mines, Rev. Mod. Phys. 37, 406 (1965).

[9]C. M. Vincent and H. T. Fortune, Phys. Rev. C 2, 782 (1970).

[10]L. G. Keller and W. Haeberli, Nucl. Phys. A172, 625 (1971).

[11]B. J. Cole and R. Huby, Phys. Lett. 46B, 55 (1973).

[12]Ya. B. Zel'dovich, Zh. Eksp. Teor. Fiz. 39, 776 (1970) [Sov. Phys.-JETP 12, 542 (1971)].

[13]T. Berggren, Nucl. Phys. A109, 265 (1968); A169, 353 (1971).

[14]Y. P. Viyogi and N. K. Ganguly, Report, Bhabha Atomic Research Centre (unpublished).

[15]F. D. Becchetti and G. W. Greenlees, Phys. Rev. 182, 1190 (1969).

[16]S. E. Darden, S. Sen, H. R. Hiddleston, J. A. Aymar, and W. A. Yoh, Nucl. Phys. A208, 77 (1973).

[17]R. Huby and D. Kelvin, J. Phys. 1, 203 (1975); J. P. Schiffer, Nucl. Phys. 46, 246 (1963).

[18]G. Baur, F. Rösel, and D. Trautmann, Nucl. Phys. A252, 77 (1975).

[19]K. W. McVoy, Ann. Phys. (N.Y.) 43, 191 (1967).

[20]G. Baur and D. Trautmann, Phys. Rep. 25L, 294 (1976).