Spreading width and branching ratios of isolated doorway resonances

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A lower limit for the spreading width of doorway resonances is derived as a function of absorption coefficients and of the branching ratio of the doorway to channels that are not directly coupled to it. Equations are obtained that provide a means of experimental determination of the fluctuating part of the cross sections without measurement of the fine structure.

[NUCLEAR REACTIONS Doorway states, fluctuation cross section, spreading width, and branching ratios discussed.]

I. INTRODUCTION

Isobaric analog states (IAS) have become a standard tool for the study of nuclear structure. Due to the fact that IAS are doorway states that couple to the more complicated $T_<$ states of the compound nucleus, cross sections may contain an appreciable amount of fluctuation cross section. This fluctuating part is neglected in nearly all analyses of experimental data on heavy nuclei. One reason is that it is difficult to calculate except in simple cases; the other is that from theoretical arguments using the Hauser-Feshbach theory one expects it to be small if the number of open neutron channels is big. [See Eqs. (13.9.12) and (13.9.14) of Ref. 1.] However the use of the conventional Hauser-Feshbach theory is not exactly justified and one would like a more direct experimental criterion. The fact that no fine structure of isobaric analogs (IAR) has been observed for nuclei above the Mo region[2] can be due to the absence of fine structure or insufficient experimental energy resolution. We want to derive here some relations that can be used to get an experimental determination of the fluctuating part of the cross section without measurement of the fine structure. Of course, the formulas derived here do not apply only to IAR, but more generally to doorway resonances. In the following we shall omit all geometrical factors associated with the cross section.

II. S MATRIX AND BRANCHING RATIOS

Following Ref. 1 we can write the energy averaged part of the S matrix for a single doorway resonance

$$\langle S_{cc'}\rangle = e^{i(\delta_c+\delta_{c'})}\left(\tau_c\,\delta_{cc'} - i\,\frac{\tilde{\Gamma}_c^{1/2}\tilde{\Gamma}_{c'}^{1/2}}{E-E_0+\frac{1}{2}i\Gamma_T}\right) \tag{1}$$

with

$$\tilde{\Gamma}_c = \Gamma_c e^{2i\theta_c}\,, \tag{2}$$

$$\Gamma_T = \Gamma^{\downarrow} + \sum_{c_p} \frac{2}{1+\tau_{c_p}}\cos 2\theta_{c_p}\Gamma_{c_p}\,, \tag{3}$$

where Γ_T is the total width, $\Gamma^{\downarrow}$ is the spreading width, E_0 is the resonance energy, θ_c is the resonance mixing phase, and δ_c and τ_c are the optical model phase shift and absorption coefficient, respectively. One can define $\Gamma^{\uparrow} = \sum_{c_p}\Gamma_{c_p}$. The sum in Eq. (3) is over all channels c_p that are directly coupled to the doorway (for IAR all proton channels).

From Eq. (1) it follows

$$\mathrm{Re}\langle S_{cc}\rangle = \tau_c\cos 2\delta_c + \frac{\Gamma_c}{(E-E_0)^2+\frac{1}{4}\Gamma_T^2}\left[\sin 2(\delta_c+\theta_c)(E-E_R) - \tfrac{1}{2}\Gamma_T\cos 2(\delta_c+\theta_c)\right]. \tag{4}$$

From unitarity it follows for the total reaction cross section in a channel c, omitting an eventual infinite contribution from elastic Coulomb scattering,

$$\sigma_{c,\,\mathrm{total}} = 2(1-\mathrm{Re}S_{cc}) \tag{5a}$$

and therefore

$$\langle\sigma_{c,\,\mathrm{total}}\rangle = 2(1-\mathrm{Re}\langle S_{cc}\rangle)\,. \tag{5b}$$

In the following we will suppose that the energy dependence of δ_c, θ_c, and τ_c is slow as compared with the resonant part of S. Then we can split $\langle\sigma_{c,\,\mathrm{total}}\rangle$ into an energy independent part and a resonating part:

$$\langle\sigma_{c,\text{total}}\rangle = \langle\sigma_{c,\text{total}}\rangle_{\text{res}} + \langle\sigma_{c,\text{total}}\rangle_{\text{off}} \ , \tag{6}$$

For the resonating part at $E = E_0$ we have

$$\langle\sigma_{c,\text{total}}\rangle_{\text{res}}\big|_{E=E_0} = 4\Gamma_c \cos 2(\delta_c + \theta_c)/\Gamma_T \ . \tag{7}$$

The energy averaged cross section $\langle\sigma_{c,c_p}\rangle$ can be written, using $S_{cc_p} = \langle S_{cc_p}\rangle + S^{\text{fl}}_{cc_p}$,

$$\langle\sigma_{cc_p}\rangle = |\delta_{cc_p} - \langle S_{cc_p}\rangle|^2 + \langle |S^{\text{fl}}_{cc_p}|^2\rangle \tag{8}$$

$$= \delta_{cc_p}(1 - 2\tau_c \cos 2\delta_c + \tau_c^2) + \frac{\Gamma_c \Gamma_{c_p}}{(E-E_0)^2 + (\frac{1}{2}\Gamma_T)^2} + \delta_{cc_p} \frac{2\Gamma_c}{(E-E_0)^2 + (\frac{1}{2}\Gamma_T)^2}$$

$$\times \{\tau_c[\sin 2\theta_c (E-E_0) - \tfrac{1}{2}\Gamma_T \cos 2\theta_c] - (E-E_0)\sin 2(\theta_c + \delta_c) + \tfrac{1}{2}\Gamma_T \cos 2(\theta_c + \delta_c)\} + \langle |S^{\text{fl}}_{cc_p}|^2\rangle \ . \tag{9}$$

Splitting as before $\langle\sigma_{cc'}\rangle$ into a resonant part $\langle\sigma_{cc'}\rangle_{\text{res}}$ and a background term $\langle\sigma_{cc'}\rangle_{\text{off}}$ one has for $E = E_0$

$$\langle\sigma_{cc_p}\rangle_{\text{res}}\big|_{E=E_0} = \frac{4\Gamma_c \Gamma_{c_p}}{\Gamma_T^2} + \delta_{cc_p} \frac{4\Gamma_c}{\Gamma_T}[\cos 2(\delta_c + \theta_c) - \tau_c \cos 2\theta_c] + \langle |S^{\text{fl}}_{cc_p}|^2\rangle_{\text{res}}\big|_{E=E_0} \ .$$

We can now define a branching ratio to the channel c_p

$$B_{c_p} = \frac{\langle\sigma_{c,c_p}\rangle_{\text{res}}|_{E=E_0}}{\langle\sigma_{c,\text{total}}\rangle_{\text{res}}|_{E=E_0}}$$

$$= \frac{\Gamma_{c_p} + \delta_{c,c_p}\Gamma_T[\cos 2(\delta_c + \theta_c) - \tau_c \cos 2\theta_c] + (\Gamma_T^2/4\Gamma_c)\langle |S^{\text{fl}}_{cc_p}|^2\rangle_{\text{res}}|_{E=E_0}}{\Gamma_T \cos 2(\delta_c + \theta_c)} \ . \tag{10}$$

The branching ratio B_{c_p} can be directly measured by $(p,n\bar{p})$ or $(^3\text{He},d\bar{p})$ reactions. The values of Γ_{c_p}, Γ_T, δ_{c_p}, τ_{c_p}, and θ_{c_p} can be obtained from analysis of (p,p) and (p,p') data. Most convenient are elastic scattering data, since this channel is dominated by the interference between the Coulomb amplitude and $\langle S_{cc}\rangle$, and where thus $|S^{\text{fl}}_{cc}|^2$ can be neglected in nearly all practical cases. Then Eq. (10) can be used to determine $\langle |S^{\text{fl}}_{cc_p}|^2\rangle_{\text{res}}$. If one uses the resonance integral $\int\langle\sigma_{cc_p}\rangle_{\text{res}} dE$ instead of the on-resonance cross section it is easy to see (see also Ref. 3) that the same relation (10) for the branching ratio is obtained except that the term $\Gamma_T^2\langle |S^{\text{fl}}_{c,c_p}|^2\rangle_{\text{res}}|_{E=E_0}/(4\Gamma_c)$ must be substituted by $\int\langle |S^{\text{fl}}_{c,c_p}|^2\rangle_{\text{res}} dE\, \Gamma_T/(2\pi\Gamma_c)$. The branching ratio B_p to channels that are directly coupled to the doorway (proton channels) is given by $B_p = \sum_{c_p} B_{c_p}$. This implies for the branching ratio to neutron channels that $B_n = 1 - B_p$. If now, $\langle |S^{\text{fl}}_{c,c_p}|^2\rangle_{\text{res}}$ is positive, Eq. (10) can be used to obtain some inequalities. In Ref. 4 the following approximate relation is given for the resonating part of the fluctuating cross sections

$$\langle\sigma^{\text{fl}}_{c,c_p}\rangle_{\text{res}} = \frac{\Gamma_c \Gamma_{c_p}}{(E-E_0)^2 + \frac{1}{4}\Gamma_T^2} \frac{\Gamma_T \bar{\tau} - \Gamma^{\dagger}}{\Gamma^{\dagger}} , \tag{11}$$

where $\bar{\tau}$ is the mean absorption in the channels c_p. Even if (11) is approximate, it should supply a good criterion under what conditions $\langle |S^{\text{fl}}_{c,c_p}|^2\rangle_{\text{res}}$ is positive. As one sees from Eq. (11) this is always the case for $\bar{\tau} \geq \Gamma^{\dagger}/\Gamma_T$. For analog states one has typically $\Gamma^{\dagger}/\Gamma_T \lesssim 0.5$. This means the condition for a positive value of $\langle\sigma^{\text{fl}}_{c,c_p}\rangle_{\text{res}}$ is $\tau > 0.5$. This condition of moderate absorption holds for most experimental cases. With the assumption $\langle |S^{\text{fl}}_{c,c_p}|^2\rangle_{\text{res}} \geq 0$ one obtains from Eq. (10)

$$B_p \geq \frac{\sum_{c_p}\Gamma_{c_p} + \Gamma_T[\cos 2(\delta_c + \theta_c) - \tau_c \cos 2\theta_c]}{\Gamma_T \cos 2(\delta_c + \theta_c)} . \tag{12}$$

Equation (12) is the same if resonance integrals are used instead of the on-resonance cross section. With $B_n = 1 - B_p$ one gets from Eq. (12)

$$B_n \leq \frac{\Gamma_T \tau_c \cos 2\theta_c - \Gamma^{\dagger}}{\Gamma_T \cos 2(\delta_c + \theta_c)} . \tag{13}$$

Using Eq. (3) one can write Eq. (13) in another form

$$\Gamma^{\dagger} \geq \frac{1}{\tau_c \cos 2\theta_c - B_n \cos 2(\theta_c + \delta_c)}$$

$$\times \sum_{c_p} \Gamma_{c_p}\left(1 + \frac{2B_n}{1 + \tau_{c_p}} \cos 2\theta_{c_p} \cos 2(\theta_c + \delta_c) + \frac{2}{1 + \tau_{c_p}} \tau_c \cos 2\theta_c \cos 2\theta_{c_p}\right) . \tag{14}$$

This establishes a lower limit for $\Gamma^{\dagger}$. We will discuss Eq. (14) with some simplifying assumptions. Let's suppose $\cos 2\theta_{c_p} \sim 1$, $\cos 2(\theta_c + \delta_c) \sim 1$, and $\tau_{c_p} \sim \bar{\tau}$, which is justified when the phase shifts are small and the absorption coefficients do not vary much in the different channels c_p. Then (14) simplifies to

TABLE I. The nucleus is the compound nucleus of the IAR. l is the multipolarity of the transition of the proton to the ground state of the final nucleus. B_p is the branching ratio to proton channels as determined by $(p,n\bar{p})$ or $(^3\mathrm{He},d\bar{p})$. B_{pp} is the ratio $\sum\Gamma_p/\Gamma_T$ as determined by analysis of (p,p') data. E_n is the energy of the IAR above neutron threshold.

Nucleus	l	B_p	Reaction	B_{pp}	E_n (MeV)
^{93}Tc	2	1.0 ± 0.03 [a]	$(^3\mathrm{He},d\bar{p})$	0.05 [b]	−3.42
^{95}Tc	2	1.0 [c]	$(p,n\bar{p})$	0.03 [d]	−0.1
^{97}Tc	2	0.09 ± 0.02 [c]	$(p,n\bar{p})$	0.08 [d]	1.7
^{112}In	0	0.32 ± 0.05 [c]	$(p,n\bar{p})$	0.40 [e]	3.45
^{115}Sb	0	1.0 [c]	$(p,n\bar{p})$	0.21 [f]	−0.73
^{117}Sb	0	0.31 ± 0.05 [c]	$(p,n\bar{p})$	0.38 [f]	1.50
^{119}Sb	0	0.38 ± 0.05 [c]	$(p,n\bar{p})$	0.34 [f]	2.74
^{125}I	0	0.30 ± 0.05 [c]	$(p,n\bar{p})$	0.27 [g]	3.48
^{208}Bi	1	1.04 ± 0.3 [i]	$(p,n\bar{p})$	0.61 ± 0.05 [h]	8.36
		1.9 ± 0.2 [j]	$(p,n\bar{p})$		
		$0.6-2$ [k]	$(p,n\bar{p})$		

[a] S. Galès (private communication).
[b] D. C. Kocher, Nucl. Data B8, 527 (1972).
[c] P. S. Miller and G. T. Garvey, Nucl. Phys. A163, 65 (1971); in this work only the $\bar{p}$ cross section has been measured. The branching ratio B_p has been deduced using a calculated $\sigma(p,n)$ cross section.
[d] C. F. Moore, P. Richard, C. E. Watson, D. Robson, and J. D. Fox, Phys. Rev. 141, 1166 (1966).
[e] E. Abramson, R. A. Eisenstein, I. Plesser, and Z. Vager, Nucl. Phys. A138, 609 (1969).
[f] P. Richard, C. F. Moore, J. A. Becker, and J. D. Fox, Phys. Rev. 145, 971 (1966).
[g] J. L. Foster, P. J. Riley, and C. F. Moore, Phys. Rev. 175, 1498 (1968).
[h] E. C. Booth and B. S. Madsen, Nucl. Phys. A206, 293 (1973).
[i] T. J. Woods, G. J. Igo, C. A. Whitten, W. Dunlop, and G. W. Hoffman, Phys. Lett. 34B, 594 (1971).
[j] T. J. Woods, C. A. Whitten, and G. J. Igo, Nucl. Phys. A198, 542 (1972).
[k] A. Galonsky, G. M. Crawley, P. S. Miller, R. R. Doering, and D. M. Patterson, Phys. Rev. C 12, 1072 (1975); G. M. Crawley and P. S. Miller, *ibid.* 6, 306 (1972); the authors point out that B_p depends critically on the underlying background in the $\bar{p}$ spectra.

$$\Gamma^{\downarrow} \geq \frac{1-\bar{\tau}+2B_n}{(\bar{\tau}-B_n)(1+\bar{\tau})}\,\Gamma^{\uparrow} = \frac{1-\bar{\tau}+2B_n}{1+\bar{\tau}}\,\Gamma_T. \qquad (15)$$

For weak absorption $\bar{\tau}\sim 1$, an even simpler expression is obtained

$$\Gamma^{\downarrow} \geq \frac{B_n}{1-B_n}\,\Gamma^{\uparrow} = B_n\Gamma_T. \qquad (16)$$

We note here, that in the Eqs. (12)–(16) the equality is valid when and only when the fluctuating part of the cross section is zero. Therefore all these equations can be used to determine whether or not the fluctuating part of the cross sections can be neglected. However, Eq. (10) seems best suited to determine the fluctuating cross section because the values that enter in (10) are more easily determined experimentally.

If the fluctuating part of the cross section can be neglected, experimental data can be analyzed using the S matrix of Eq. (1) only, and the analysis of (p,p') data will give $B_{pp}=\sum_{c_p}\Gamma_{c_p}/\Gamma_T=\Gamma^{\uparrow}/\Gamma_T$. In Eqs. (12)–(16) the equality holds in this case, and for weak absorption and small phase shifts Eq. (16) gives $\Gamma^{\downarrow}=B_n\Gamma_T$, or $\Gamma^{\uparrow}/\Gamma_T=B_p$. Therefore, for small absorption and small phase shifts, a *necessary* condition for neglecting of the fluctuating part of the cross section is $B_{pp}=B_p$. In Table I some experimental values for B_{pp} and B_p are given. Within experimental errors, the equality seems to hold, as soon as the IAR is above neutron threshold, with perhaps the exception of ^{208}Bi, where the difference between B_{pp} and B_p is somewhat out of error bars. It would be better to use Eq. (14) or Eq. (15) instead of Eq. (16), because they involve less assumptions [Eq. (15) gives $B_p-(1-\bar{\tau})=\Gamma^{\uparrow}/\Gamma_T=B_{pp}$] but the low experimental precision does not seem to justify the extra effort implied. Better precision measurements would be necessary.

III. CONCLUSION

For moderate absorption ($\tau \gtrsim \Gamma^{\uparrow}/\Gamma_T$) lower limits have been derived for the spreading width. Equations have been obtained that permit the experimental determination of the fluctuating part of the cross section without measurement of the fine structure. Available experimental information is not precise enough to provide a good test of the equations derived. Good quality $(^3\mathrm{He},d\bar{p})$ and $(p,n\bar{p})$ measurements of branching ratios would be welcome.

Note added in proof. Since completion of the manuscript, good $(^3\mathrm{He},d\bar{p})$ data have been published by S. Galès *et al.* Analysis of these data is in course.

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[1] C. Mahaux and H. A. Weidenmüller, *Shell Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, 1969), and references cited.
[2] G. M. Temmer, in *Proceedings of the International Symposium on Nuclear Structure, Dubna 1968* (IAEA, Vienna, 1969).
[3] W. Mittig, Phys. Rev. C 15, 325 (1977).
[4] M. S. Hussein, Phys. Rev. C 13, 1420 (1976).