PHYSICAL REVIEW C VOLUME 15, NUMBER 4 APRIL 1977

Low-momentum-transfer elastic electron scattering from ^3He†

Z. M. Szalata,* J. M. Finn,‡ J. Flanz, F. J. Kline,§ and G. A. Peterson
Department of Physics and Astronomy, University of Massachusetts, Amherst, Massachusetts 01003

J. W. Lightbody, Jr., X. K. Maruyama, and S. Penner
Center for Radiation Research, National Bureau of Standards, Washington, D. C. 20234
(Received 5 January 1977)

Elastic electron scattering cross sections for ^3He were measured relative to those of ^{12}C in the range of momentum transfer squared between 0.032 and 0.34 fm^{-2}. The ^3He rms charge radius was determined from the data to be 1.89 ± 0.05 fm.

[NUCLEAR REACTIONS $^3He(e,e)$, $E=28.8$–95.0 MeV; measured $\sigma(E)$ at $\theta=75°$; deduced rms charge radius.]

I. INTRODUCTION

We have measured low-momentum-transfer elastic electron scattering cross sections for ^3He. Our data yield a new determination of the root-mean-square charge radius R_{ch}, a quantity useful as a constraint in the calculations of three-body wave functions.[1] Our measurements were carried out at low-momentum transfers ($0.03<q^2<0.34$ fm^{-2}), whereas previous measurements of McCarthy *et al.*[2] covered the q^2 range from 0.25 to 20.0 fm^{-2}. In principle, low q^2 elastic (e,e) scattering experiments lend themselves to a model-independent determination of the rms charge radius,

$$R_{ch}{}^2 = -\lim_{q^2\to 0} 3\frac{dF_{ch}{}^2}{dq^2},$$

where F_{ch} is the Born approximation charge form factor. In the present work alone we were unable to obtain data at low enough momentum transfers, with sufficiently small uncertainties, to achieve a close estimate of the above limit. Curvature terms in the form factor, $\partial F^2/\partial q^4$, $\partial F^2/\partial q^6, \ldots,$ are significant for our data. These terms are related to higher moments of the radial charge distribution. The higher q data of Ref. 2 have been used to determine curvature terms so that their contributions to our data could be removed.

The previous data from McCarthy *et al.*[2] and from the 180° scattering experiment of Chertok *et al.*[3] included contributions from magnetic elastic scattering from the ^3He ground state magnetic moment distribution. The present experiment has been made quite insensitive to magnetic scattering by choosing a relatively forward scattering angle of 75°. Using results of the earlier work,[2] we have reduced the present data to include only longitudinal scattering effects.

II. EXPERIMENTAL PROCEDURE

The measurements were made at the linear accelerator facility[4] of the National Bureau of Standards. Data were taken for electron energies from 28.8 to 95.0 MeV, at a fixed scattering angle of 75° where longitudinal scattering predominates over magnetic scattering.

The ^3He cross sections were measured relative to those of ^{12}C by using a target cell[5] pressurized to about three atmospheres with an accurately known mixture of ^3He and CH_4 gases prepared in a special mixing chamber. An identical empty cell was used for background measurements. The mixing chamber was divided into two known volumes by a thin aluminized plastic foil. One volume was filled with ^3He while at the same time the other volume was filled with CH_4. A difference in pressure between the two volumes was detected by observing a deflection of a light beam reflected from the foil. When a null deflection was observed at the desired pressure, the valves were closed, the foil was ruptured, and the gases mixed. The mixture was then allowed to flow to the target cell. The ratio of molecular densities was then determined using known virial coefficients in the equations of state for the two gases.[6] The uncertainty in determining the relative density for the two gases in the target cell is ±0.9% and is common to the entire set of data. Other aspects of the experimental arrangements are described by Kan *et al.*[4]

III. ANALYSIS

The raw data were corrected for various dead-time and accidental counting effects, variation in spectrometer dispersion along the focal plane, and relative detector efficiencies. These corrected data were used to generate bin-sorted spectra.

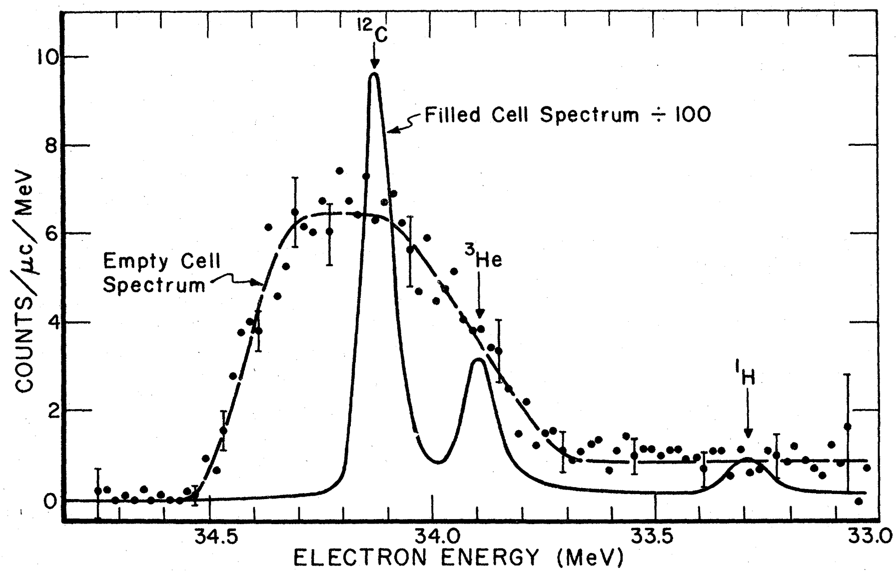

FIG. 1. Empty cell background spectrum compared to the mixed gas target spectrum. The filled cell spectrum shows three peaks due to elastic scattering from ^{12}C, ^{3}He, and ^{1}H. The incident electron energy is 34.27 MeV and the scattering angle is 75.14°.

TABLE I. Kinematic parameters, calculated carbon cross sections, measured ratios of ^{3}He to ^{12}C cross sections, Coulomb correction factors, ϵ_g, and resultant Born approximation ^{3}He charge form factors. The ^{12}C cross sections have been calculated by the HEINEL code (Ref. 8) using a harmonic oscillator charge distribution with parameters $a = 1.687$ and $\alpha = 1.067$. ϵ_g are the cross section ratios σ(Born approximation)/σ(phase shift) calculated using a Gaussian charge distribution with an rms radius of 1.89 fm.

E_0 (MeV)	θ (deg)	q^2 (fm^{-2})	$\left(\frac{d\sigma}{d\Omega}\right)_{^{12}C}$ (mb/sr)	$\left(\frac{d\sigma}{d\Omega}\right)^{exp}_{^{3}He} \Big/ \left(\frac{d\sigma}{d\Omega}\right)^{exp}_{^{12}C}$	ϵ_g	F_{ch}^2
28.83	75.31	0.0316	0.9930	0.1129(18)[a]	0.9858	0.991(16)
29.15	75.32	0.0323	0.9689	0.1080(17)	0.9851	0.945(15)
34.05	75.31	0.0441	0.6923	0.1115(18)	0.9849	0.952(15)
34.27	75.14	0.0445	0.6897	0.1092(12)	0.9850	0.932(10)
44.32	75.14	0.0741	0.3859	0.1105(14)	0.9862	0.884(11)
47.51	75.32	0.0855	0.3239	0.1150(19)	0.9861	0.898(15)
47.56	75.32	0.0859	0.3231	0.1161(14)	0.9860	0.906(12)
47.63	75.32	0.0867	0.3220	0.1156(15)	0.9854	0.902(12)
54.48	75.14	0.1117	0.2349	0.1198(18)	0.9874	0.885(13)
55.47	75.31	0.1162	0.2221	0.1158(15)	0.9877	0.847(11)
60.91	75.32	0.1400	0.1745	0.1186(15)	0.9887	0.824(11)
64.62	75.14	0.1568	0.1510	0.1234(20)	0.9892	0.826(14)
67.28	75.31	0.1705	0.1338	0.1199(19)	0.9891	0.779(12)
67.38	75.31	0.1710	0.1332	0.1220(17)	0.9891	0.792(11)
72.99	75.32	0.2004	0.1062	0.1353(20)	0.9902	0.824(12)
73.90	75.31	0.2053	0.1026	0.1303(18)	0.9906	0.786(11)
74.76	75.14	0.2093	0.1003	0.1284(20)	0.9909	0.767(12)
79.60	75.31	0.2379	0.0822	0.1314(18)	0.9918	0.737(10)
81.35	75.32	0.2484	0.0768	0.1408(20)	0.9916	0.771(11)
84.92	75.14	0.2693	0.0680	0.1330(21)	0.9930	0.697(11)
90.37	75.35	0.3060	0.0546	0.1435(21)	0.9939	0.693(10)
95.00	75.14	0.3362	0.0467	0.1456(27)	0.9948	0.658(12)

[a] Notation used here is such that 0.1129(18) stands for 0.1129 ± 0.0018.

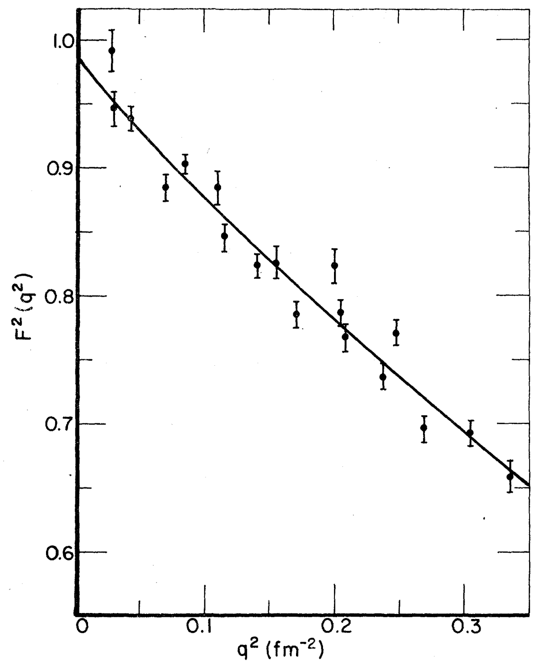

FIG. 2. ^{3}He charge form factor squared. The experimental points are the charged form factor squared from the data of Table I after corrections for Coulomb distortion effects have been applied. The solid line represents the best fit to the data using the polynomial and coefficients for q^4 and q^6 terms described in the text. The points at $q^2 = 0.044$, 0.086, and 0.171 fm^{-2} are averaged values of multiple data points.

The empty cell background, which was found to be significant for the lower incident electron energies, was removed from each bin-sorted spectrum. The empty cell background relative to the mixed gas target spectrum is shown in Fig. 1 for $E_0 = 34.3$ MeV and $\theta = 75°$.

The ratios R of the ^{3}He to the ^{12}C elastic peak areas were extracted from each spectrum by using a peak shape fitting method. The experimental ^{3}He elastic scattering cross sections were determined from these ratios by using the relation

$$\left(\frac{d\sigma}{d\Omega}\right)_{He} = \left(\frac{d\sigma}{d\Omega}\right)_C Rr,$$

where $(d\sigma/d\Omega)_C$ is the calculated ^{12}C elastic scattering cross section, and r is the ratio of the CH_4 to the ^{3}He gas molecular densities. The ratios r were corrected for a CH_4 dissociation effect in which the ratio of carbon to hydrogen is observed to decrease with accumulated beam charge. The latter correction was in the largest case 1.8%. The uncertainty in determining this correction contributes less than 0.4% to the uncertainty in the measurement of the ^{3}He form factor. The ^{12}C elastic scattering cross sections were calculated using the charge distribution parameters of Jansen, Peerdeman, and DeVries[7] in a phase shift calculation developed by Heisenberg.[8] The measured elastic scattering peak areas were corrected for radiative effects in the standard fashion. Magnetic scattering contributions to the ^{3}He elastic scattering cross sections were removed using the results of McCarthy *et al.*,[2] which showed that in our region of q^2 the charge and magnetic form factors are nearly equal. The Born approximation differential scattering cross section[2,9,10] for the spin-$\frac{1}{2}$ ^{3}He nucleus may then be written (where $\hbar = c = 1$)

$$\left(\frac{d\sigma}{d\Omega}\right)^{ch}_{He} = \left(\frac{d\sigma}{d\Omega}\right)_{He}\left[1+\frac{q^2}{2M^2}(1+K)^2(\tfrac{1}{2}+\tan^2\tfrac{1}{2}\theta)\right]^{-1}.$$

$K = -4.2$ is the anomalous magnetic moment of ^{3}He and M is the ^{3}He mass. Values of the charge form factor squared, $F_{ch}{}^2(q)$, are given in Table I:

$$F_{ch}{}^2(q) = \frac{(d\sigma/d\Omega)^{ch}}{(d\sigma/d\Omega)_{point}},$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{point} = \left(\frac{Z\alpha}{2E_0}\right)^2 \frac{\cos^2\frac{1}{2}\theta}{\sin^4\frac{1}{2}\theta}\left(1+\frac{2E_0}{M^2}\sin^2\tfrac{1}{2}\theta\right)^{-1}.$$

We have made Coulomb distortion corrections to our data in order to extract the rms radius from the standard expansion in powers of q^2 of the Born approximation (BA) form factor. These correction factors σ_{BA}/σ (phase shift) were computed using a Gaussian charge distribution model (see Table I). Other models were investigated yielding similar values for the correction within 0.1%.

The statistical uncertainty in each of our cross section measurements is less than 0.5%. The uncertainties presented in Table I result from adding (i) the statistical uncertainty in quadrature with uncertainties from (ii) the dissociation effect corrections and (iii) the relative He-CH_4 density ratio. In principle, the latter two effects should not be included as random uncertainties. One can treat them by allowing the form factor normalization factor to be free in the fitting procedure. However, we think that there may be additional uncertainties in the effective density ratio coming from other effects such as time variations in the local density because of beam heating of the gas, or possibly from beam induced differential diffusion causing a separation of the mixed gases. In the present experiment we did not control these variables. We have retained item (iii) as an independent contribution to the total uncertainty on each measured cross section ratio in order to account for possible variations in the above mentioned effects. In addition, we allowed the overall cross section normalization to be a free parameter.

A polynomial of the following form was fitted to the Born form factors

$$F_{BA}{}^2(q^2)=A_0[1-A_1q^2+A_2q^4-A_3q^6],$$

where A_0 is a free overall normalization factor and A_1 is related to the rms charge radius ($R_{ch}{}^2=3A_1$). A_2 and A_3 were taken to be the coefficients of the q^4 and q^6 terms in the expansion of the analytical expression for the Born charge form factor squared reported by McCarthy *et al.*[2] Higher order terms were ignored in our analysis because their contributions to the form factors at our highest momentum transfers are less than 0.2%. The uncertainty associated with A_2 and A_3 contributes a 0.7% uncertainty to our determination of R_{ch}. The normalization factor A_0 was determined to 0.990 ± 0.005 and our best fit value of the ^{3}He rms charge radius is

$$R_{ch}=1.89\pm0.03 \text{ fm},$$

with a reduced χ^2 of

$$\chi^2/n=2.76, \quad n=20 \text{ (degrees of freedom)}.$$

This large value for χ^2 implies that an additional uncertainty must be included in our data. We chose to scale up the uncertainty on each of our data points by $(\chi^2/n)^{1/2}$. By making this assumption, we affect only the standard deviation of our rms radius, hence

$$R_{ch}=1.89\pm0.05 \text{ fm}.$$

The data and best fit polynomial are shown in Fig. 2. Our result is in statistical agreement with the previous high energy measurements of the rms radius by McCarthy *et al.*[2] and by Collard *et al.*,[9] who obtained $R_{ch}=1.88\pm0.05$ fm and 1.87 fm, respectively.

---

†Work supported in part by the National Science Foundation and the Energy Research and Development Administration.

*Present address: American University, Washington, D. C. 20016.

‡Present address: Louisiana State University, Baton Rouge, Louisiana 70803.

§Present address: National Bureau of Standards, Washington, D. C. 20234.

[1]J. A. Tjon, B. F. Gibson, and J. S. O'Connell, Phys. Rev. Lett. 25, 540 (1970).

[2]J. S. McCarthy, I. Sick, R. R. Whitney, and M. R. Yearian, Phys. Rev. Lett. 25, 884 (1970).

[3]B. T. Chertok, E. C. Jones, W. L. Bendel, and L. W. Fagg, Phys. Rev. Lett. 23, 34 (1969).

[4]P. T. Kan, G. A. Peterson, D. V. Webb, Z. M. Szalata, J. S. O'Connell, S. P. Fivozinsky, J. W. Lightbody, Jr., and S. Penner, Phys. Rev. C 12, 1118 (1975).

[5]D. V. Webb, G. A. Peterson, Z. M. Szalata, and P. T. Kan, Nucl. Instrum. Methods 120, 359 (1974).

[6]A. J. Vennix and R. Kobayashi, J. Am. Inst. Chem. Eng. 15, 926 (1969).

[7]J. A. Jansen, R. T. Peerdeman, and C. DeVries, Nucl. Phys. A188, 337 (1972).

[8]J. Heisenberg (private communication).

[9]H. Collard, H. Hofstadter, E. B. Hughes, A. Johansson, M. R. Yearian, R. B. Day, and R. T. Wagner, Phys. Rev. 138, B57 (1965).

[10]In the expression used here and in Refs. 2 and 9, the ^{3}He nucleus is treated as as elementary fermion. The magnetic moment of ^{3}He is $\mu_{3\text{He}}=(1+K)\mu_0=-2.13\mu_N$, where μ_0 is the Dirac theory magnetic moment for ^{3}He given by $\mu_0=Ze\hbar/2Mc$.