Macroscopic description of exchange effects in light-ion interactions with ^{16}O†

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(Received 8 December 1976)

Exchange effects in the $\alpha + {}^{16}O$, $^3He + {}^{16}O$, and $p + {}^{16}O$ systems are examined by studying the behavior of phase shifts and cross sections calculated with the resonating-group method. The result shows that these effects can be approximately represented by an exchange potential $V_{ex} = V_a + (-1)^l V_b$. The term V_a, representing mainly the knockout process, is found to have an appreciable magnitude and to have a range shorter than that of the direct nuclear potential. The term $(-1)^l V_b$, representing mainly the heavy-particle pickup process, is, on the other hand, rather small in all these three systems. Based on the findings obtained from this and previous investigations, an empirical rule is given which indicates the situation under which the heavy-particle pickup process is expected to be important.

[NUCLEAR REACTIONS $^{16}O(\alpha, \alpha)$, $^{16}O(^3He, ^3He)$, $^{16}O(p, p)$. Potential-model analysis of resonating-group results.]

I. INTRODUCTION

The folding model[1] has frequently been employed to analyze elastic-scattering data. In this model, the real central part of the effective potential between two nuclei is obtained by using a simple folding procedure involving essentially the direct part of the nucleon-nucleon potential and the nuclear matter distributions. As has been pointed out,[1] one of the principal defects of this model is that exchange effects, arising from the antisymmetrization of the wave function with respect to the interchange of nucleons in different nuclei, are not accounted for. In the present investigation, we examine in detail the properties of the interactions of the light ions p, ^3He, and α with the nucleus ^{16}O in order to see if these effects can be simply incorporated into the folding model.

We choose to study the interactions of these particular light ions with ^{16}O because, with the development of generator-coordinate techniques,[2] resonating-group calculations[3-5] for these systems have been made recently. As is well known, the chief advantage of the resonating-group method is that the antisymmetrization of the wave function is properly carried out. Thus, by studying the features of the calculated resonating-group phase shifts and cross sections, one can hope to gain valuable information concerning exchange effects and thereby to construct simple local potentials which can approximately represent these effects.

The procedure for analyzing the resonating-group results is described in Sec. II. There also is given a discussion of the features of the effective local potential which can best represent the resonating-group, intercluster, nonlocal interaction. In Sec. III, possible extrapolations of the results of the present analysis are presented and concluding remarks are made.

II. ANALYSIS OF RESONATING-GROUP RESULTS AND FEATURES OF EFFECTIVE LOCAL POTENTIALS

The descriptions of resonating-group calculations for the $p + {}^{16}O$, $^3He + {}^{16}O$, and $\alpha + {}^{16}O$ systems are given in Refs. 3–5 and hence will not be repeated here.[6] It suffices to say that the radial function $f_l(R)$ for the lth partial wave of the relative motion between the clusters satisfies an integrodifferential equation of the form

$$\left\{\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dR^2} - \frac{l(l+1)}{R^2}\right] + E - V_N(R) - V_C(R)\right\} f_l(R) = \int_0^\infty [k_l^N(R,R') + k_l^C(R,R')] f_l(R')\,dR', \quad (1)$$

where E is the relative energy of the clusters in the c.m. system, and V_N and V_C represent, respectively, the direct nuclear and direct Coulomb potentials, obtained by a folding procedure as mentioned in Sec. I. The kernel functions k_l^N and k_l^C represent the nonlocal interactions between the clusters arising from the antisymmetrization procedure. The expressions for these kernel functions are quite lengthy and are rather tedious to derive even with the help of generator-coordinate techniques. Therefore, from a practical viewpoint, it would be desirable if one could analyze scattering data by employing instead a simpler equation of the form

$$\left\{\frac{\hbar^2}{2\mu}\left[\frac{d^2}{dR^2}-\frac{l(l+1)}{R^2}\right]\right.$$

$$\left.+E-V_N(R)-V_{ex}(R)-V_C(R)\right\} f_l(R)=0\,, \qquad (2)$$

where V_{ex} is an exchange potential introduced to represent as well as possible the effects of the nonlocal interactions contained in Eq. (1).

To find the features which V_{ex} should possess, we first write it in the form

$$V_{ex}(R)=C_l V_N(R)\,, \qquad (3)$$

with C_l being an l-dependent constant. This constant is then adjusted to yield exactly the phase shift δ_l obtained from the corresponding resonating-group calculation. The resultant behavior of C_l as a function of l at an energy of 25 MeV is shown in Fig. 1 for both the $\alpha+{}^{16}\mathrm{O}$ and ${}^3\mathrm{He}+{}^{16}\mathrm{O}$ systems. From this figure, one notes the following salient features: (i) The value of C_l is significantly larger than 0 for most l values, indicating that exchange effects result in an additional *attractive* interaction between the clusters. (ii) For $l\leq 7$ in the $\alpha+{}^{16}\mathrm{O}$ system and $l\leq 5$ in the ${}^3\mathrm{He}+{}^{16}\mathrm{O}$ system, the interactions in odd-l states are stronger than those in neighboring even-l states. (iii) For values of l larger than those mentioned in (ii), the odd-even effect seems to have a behavior opposite to that mentioned in (ii). (iv) There is a general tendency for C_l to decrease with increasing value of l. The odd-even feature mentioned in (ii) has in fact previously been noted by Buck, Dover, and Vary[7] in a phenomenological study of the rotational bands in ${}^{20}\mathrm{Ne}$, and the general magnitude of the odd-even difference found by these authors is similar to that obtained here.

Because of the presence of the odd-even effect and the general decreasing trend of C_l with l, it seems reasonable to assume for V_{ex} the form

$$V_{ex}=V_a(R)+(-1)^l V_b(R)\,, \qquad (4)$$

with

$$V_a(R)=C_a e^{-\beta_a R^2} V_N(R) \qquad (5)$$

and

$$V_b(R)=C_b e^{-\beta_b R^2} V_N(R)\,. \qquad (6)$$

The four parameters C_a, C_b, β_a, and β_b can then be adjusted to yield a best fit of the calculated differential cross section to the corresponding resonating-group result.

A search over these four parameters shows that, for all three systems, the best value of C_b is close to zero. This is of course a consequence of the fact that the odd-even effect has different behavior for smaller and larger values of l [see features (ii) and (iii) mentioned above]. In Fig. 2, we show a comparison between the potential-model results obtained with $C_b=0$ (solid curves) and the resonating-group results (solid dots) for $\alpha+{}^{16}\mathrm{O}$, ${}^3\mathrm{He}+{}^{16}\mathrm{O}$, and $p+{}^{16}\mathrm{O}$ scattering at 25 MeV. The values of the parameters C_a and β_a used in Fig. 2 are given in Table I, where we have also listed the allowable range of β_a, the rms radius R_e of the potential V_N+V_a, and the difference between R_N and R_e, with R_N being the rms radius of the direct nuclear potential V_N (the folding potential). From Fig. 2 one sees that the agreement between the model results and the resonating-group results is quite satisfactory, even when no odd-even potential term V_b is included. This reflects the fact that in these systems the odd-even effects are relatively weak, as can be seen from the behavior of C_l shown in Fig. 1.

From Table I one notes that the difference (R_N-R_e) is not at all negligible. This means that caution must be exercised in using a folding-type approach to extract information about the neutron distribution from scattering experiments involving strongly interacting fermion systems. For ex-

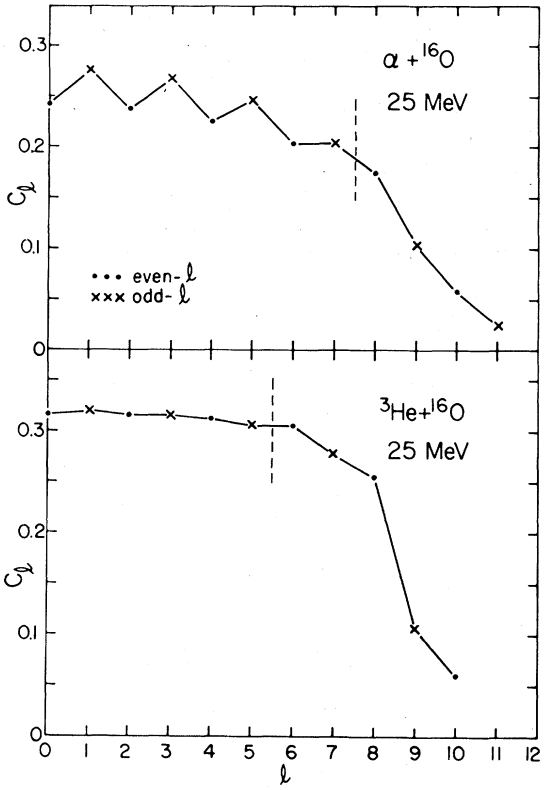

FIG. 1. C_l of Eq. (3) as a function of l for the $\alpha+{}^{16}\mathrm{O}$ and ${}^3\mathrm{He}+{}^{16}\mathrm{O}$ systems at 25 MeV. The dashed lines separate the l space into two regions where the odd-even effect has different characteristics.

ample, using such an approach to analyze $\alpha + {}^{208}\mathrm{Pb}$ scattering, Gils and Rebel[8] reached the conclusion that in $^{208}\mathrm{Pb}$ the rms radius of the neutron distribution is larger than that of the proton distribution by 0.30 ± 0.07 fm. In view of the fact that the importance of exchange effects in this scattering problem is entirely unknown at present, we must conclude that their result is undoubtedly subject to considerable uncertainty.

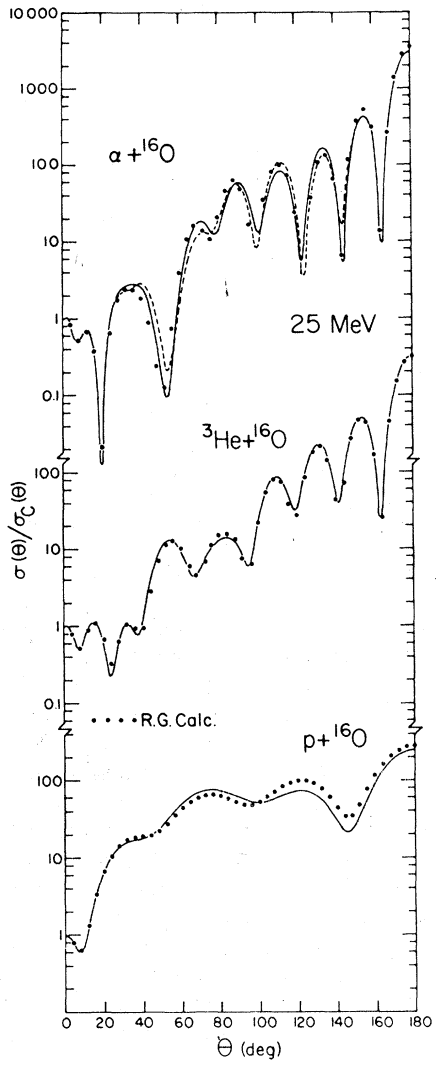

FIG. 2. Comparison of cross sections calculated using the resonating-group method with those of the potential model of Eqs. (4), (5), and (6) for the $\alpha + {}^{16}\mathrm{O}$, $^3\mathrm{He} + {}^{16}\mathrm{O}$, and $p + {}^{16}\mathrm{O}$ systems at 25 MeV. The solid curves represent the results obtained with C_a and β_a given by Table I and $C_b = 0$, while the dashed curve represents the results obtained with the parameter set of Eq. (7). Resonating-group (R.G.) results are represented by solid dots.

TABLE I. Values of C_a, β_a, R_e, and $R_N - R_e$, with $C_b = 0$.

System	C_a	β_a (fm^{-2})	Range of β_a (fm^{-2})	R_e (fm)	$R_N - R_e$ (fm)
$\alpha + {}^{16}\mathrm{O}$	0.40	0.10	0.07–0.10	3.55–3.56	0.11–0.12
$^3\mathrm{He} + {}^{16}\mathrm{O}$	0.40	0.04	0.02–0.05	3.62–3.66	0.07–0.11
$p + {}^{16}\mathrm{O}$	0.45	0.03	0–0.05	3.25–3.34	0–0.09

We should mention here that the finding of β_a being greater than zero is consistent with the result of a previous $n + \alpha$ investigation.[9] There it was found that, in the Born approximation, the nonlocal interaction can be represented exactly by a local exchange potential which is of *shorter* range than the direct nuclear potential.

In Fig. 2, we also show (dashed line) the result obtained for $\alpha + {}^{16}\mathrm{O}$ scattering with the following parameter set:

$$C_a = 0.40, \quad \beta_a = 0.1\ \mathrm{fm}^{-2}, \qquad C_b = -0.04, \quad \beta_b = 0.2\ \mathrm{fm}^{-2}. \tag{7}$$

With this set, the potential model yields phase shifts which agree very well with the corresponding resonating-group phase shifts for $l \leq 7$. On the other hand, it is noted from Fig. 2 that the fit to the resonating-group cross section is no better than that when C_b is chosen as zero. The reason for this is that in states with $l \geq 8$ a positive value of C_b is to be preferred, and therefore an l-independent choice for $V_b(R)$ cannot be expected to yield satisfactory results for all the relevant phase shifts.

If one wishes to obtain nearly perfect agreement with the resonating-group phase shifts in all orbital-angular-momentum states, then one must introduce more freedom into the potential model of Eqs. (2) and (4)–(6). One possible way to do this, as has been previously suggested,[10-12] is to introduce Pauli repulsive cores into the intercluster effective potential. These cores have strengths and shapes given by a well-defined prescription, but do contain adjustable cutoff radii. In the $\alpha + {}^{16}\mathrm{O}$ case,[12] for instance, one would add l-dependent repulsive potentials into states with $l \leq 7$. In this respect, it is interesting to note that $l = 7$ also happens to be the angular-momentum value which divides the l space into regions of different odd-even behavior. With the introduction of these repulsive cores, one could then adopt a small positive value of C_b and rely on the adjustable parameters in the repulsive potentials to yield satisfactory phase-shift values for $l \leq 7$. In addition, it should be mentioned that the presence of such cores also results in another important

advantage; i.e., it eliminates the presence of spurious bound states which exist in folding-type or in commonly employed phenomenological optical potentials.[13] On the other hand, when repulsive cores are included, there is the obvious disadvantage that more adjustable parameters are involved. Therefore, in most cases it might be preferable simply to adopt the potential model of Eqs. (2) and (4)–(6) with C_b properly chosen; for example, if the odd-even effects are relatively minor, then C_b could be chosen as zero. Indeed, our present investigation in the $\alpha+{}^{16}\mathrm{O}$, ${}^3\mathrm{He}+{}^{16}\mathrm{O}$, and $p+{}^{16}\mathrm{O}$ cases shows that, as far as scattering states are concerned, the simplification of omitting Pauli repulsive cores certainly does not lead to any great compromise in fit to the resonating-group cross-section results.

Finally, it can be easily inferred from Fig. 1 that the choice of $\beta_a=0$ will lead to unsatisfactory results. In Fig. 3, the dashed curve shows the result obtained with $C_l=0$ (double-folding model), while the solid curve shows the result obtained with $C_l=0.25$ (strength-adjusted double-folding model). Quite obviously, both these calculations yield rather poor agreement with the corresponding resonating-group result, indicating that the freedom introduced by allowing the range of V_{ex} to be different from that of V_N is quite important.

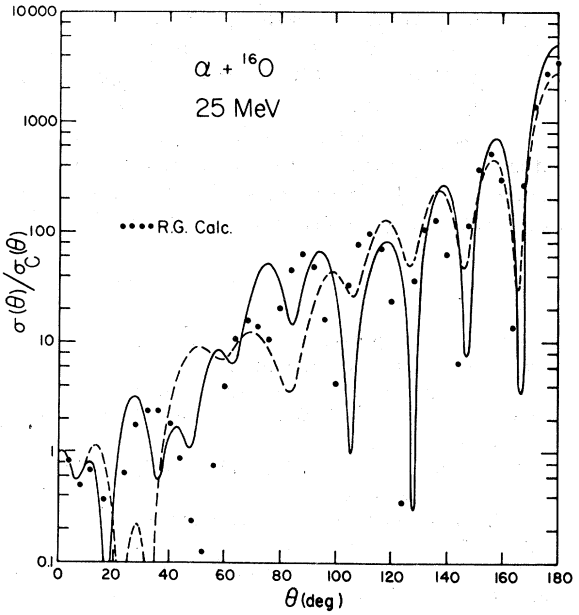

FIG. 3. Comparison of $\alpha+{}^{16}\mathrm{O}$ cross sections at 25 MeV calculated using the resonating-group method (solid dots) with those of the potential model of Eq. (3) with $C_l=0.25$ (solid curve) and $C_l=0$ (dashed curve).

III. DISCUSSION AND CONCLUSION

The purpose of this investigation was to see how exchange effects, which arise as a consequence of the indistinguishability of the nucleons, can be approximately incorporated into a simple folding-type model involving an effective local potential. To this end, we have carefully examined the behavior of the $\alpha+{}^{16}\mathrm{O}$, ${}^3\mathrm{He}+{}^{16}\mathrm{O}$, and $p+{}^{16}\mathrm{O}$ systems and have studied the necessary features which this effective local potential may need to have. One of the interesting findings of this investigation is that in all these systems the odd-even effect, represented by the potential term $(-1)^l V_b(R)$ in Eq. (4), has generally a relatively minor influence on the scattering cross section. Together with our previous finding that this effect is also rather minor in the $n+{}^{40}\mathrm{Ca}$ system[14] but is quite significant in the ${}^3\mathrm{He}+\alpha$ system,[15] we are led to make the following important assertion. Since the odd-even effect is mainly associated with a heavy-particle pickup process[9,16] we contend that it becomes important when the ratio M/m involved in this process is large, where M is the mass of the nucleus which does the pickup and m is the mass of the cluster being picked up. This assertion is quite reasonable from an intuitive viewpoint, and if it should turn out to be generally true, one would expect the odd-even effect to have a large influence not only for ${}^3\mathrm{He}+\alpha$ scattering ($M/m=3$), but also for heavy-ion scattering such as ${}^{12}\mathrm{C}+{}^{13}\mathrm{C}$ (Ref. 17) ($M/m=12$), and to have a much smaller influence not only in the $p+{}^{16}\mathrm{O}$ system ($M/m=\frac{1}{15}$) but also in light-ion scattering by medium- and heavy-weight nuclei.

We should mention that, even though the quantity M/m is likely to be a dominant factor in determining the importance of the odd-even effect, there are undoubtedly other factors involved. Thus, while M/m takes on the same value of $\frac{1}{3}$ for the $n+\alpha$ and $\alpha+{}^{16}\mathrm{O}$ systems, resonating-group calculations[9,14] seem to indicate that the odd-even effect is somewhat more important in the $n+\alpha$ system than in the $\alpha+{}^{16}\mathrm{O}$ system. In addition, it should be pointed out that there are certain situations in which the odd-even effect might be important, even when M/m is small. In $\alpha+{}^{16}\mathrm{O}$ scattering, for example, such situations will occur at energies higher than about 200 MeV where, at large angles, the heavy-particle pickup amplitude, though small, may turn out to be the dominant part of the total scattering amplitude.

The term $V_a(R)$ in the exchange potential $V_{\mathrm{ex}}(R)$ of Eq. (4) is introduced to represent mainly the knockout exchange process.[9] From our investigation we find that, in all the systems considered

here, this potential is quite appreciable in comparison with the direct nuclear potential $V_N(R)$, indicating that the knockout process is generally important.[18] In addition, our detailed analysis shows that $V_a(R)$ has a shorter range than $V_N(R)$. Although this latter feature is found by specifically examining the α+, ^{3}He +, and $p + {}^{16}$O systems, we believe that it is generally true and should be incorporated into any potential-model analysis of scattering data.

In conclusion, we feel that the present investigation has yielded useful information concerning exchange effects,[19] and this information should be helpful in constructing local-potential models for the description of scattering between complex nuclei.

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†Work supported in part by the U.S. Energy Research and Development Administration.

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