Evaluation of the fluctuation enhancement factor*

P. A. Moldauer
Argonne National Laboratory, Argonne, Illinois 60439
(Received 15 March 1976)

The width fluctuation correction enhances, sometimes by large factors, small compound cross sections that compete with channels having much larger transmission coefficients. The effect is demonstrated by the results of statistical computer experiments. A simple numerical quadrature of the width fluctuation integral describes the effect much more accurately than the formula proposed by Tepel *et al.*

[NUCLEAR REACTIONS Average compound cross sections.]

The basic expression for the average fluctuation cross section $\sigma_{cd}^{\rm fl}$ is a product of the Hauser-Feshbach formula and the width fluctuation correction factor[1]

$$\sigma_{cd}^{\rm fl} = \sigma_{cd}^{\rm HF} W_{cd}, \tag{1}$$

where

$$W_{cd} = \left\langle \frac{\Gamma_{\mu c}\Gamma_{\mu d}}{\sum_e \Gamma_{\mu e}} \right\rangle \frac{\langle \sum_e \Gamma_{\mu e}\rangle_\mu}{\langle \Gamma_{\mu c}\rangle_\mu \langle \Gamma_{\mu d}\rangle_\mu}. \tag{2}$$

The "partial widths" $\Gamma_{\mu c}$ are randomly distributed in μ in a way that can be represented by a χ^2 distribution with ν_c degrees of freedom where ν_c lies between 1 and 2. The factor W_{cd} can be greater than unity, thus enhancing the fluctuation cross section, for one of two reasons.

The first type of enhancement arises from a correlation of the $\Gamma_{\mu c}$ and the $\Gamma_{\mu d}$ in Eq. (2). This leads to the well known enhancement of the compound elastic fluctuation cross section[2] by a maximum factor of

$$\langle \Gamma_{\mu c}^2\rangle / \langle \Gamma_{\mu c}\rangle^2 = 1 + 2/\nu_c. \tag{3}$$

Nonelastic fluctuation cross sections between directly coupled channels can be enhanced by the same mechanism.[3,4]

The second and less widely discussed enhancement arises from the fluctuations of the total "widths" $\Gamma_\mu = \sum_e \Gamma_{\mu e}$. When both channels c and d in Eq. (2) are very weakly absorbed (have small $\langle\Gamma_{\mu c}\rangle$ and $\langle\Gamma_{\mu d}\rangle$) but compete with strongly absorbed channels, then the fluctuations in Γ_μ will be essentially independent from those of the weak $\Gamma_{\mu c}$ and $\Gamma_{\mu d}$ in the numerator of Eq. (2), leading to a maximum enhancement factor of $\langle \Gamma_\mu^{-1}\rangle\langle\Gamma_\mu\rangle$. If there are N competing strongly absorbed channels, each fluctuating independently with ν degrees of freedom, then Γ_μ is distributed according to χ^2 distribution with $\nu_T = N\nu$ degrees of freedom and the maximum fluctuation enhancement factor is

$$\langle \Gamma_\mu^{-1}\rangle \langle \Gamma_\mu\rangle = \begin{cases} \infty, & N\nu \leq 2 \\ \left(1 - \dfrac{2}{N\nu}\right)^{-1}, & N\nu > 2. \end{cases} \tag{4}$$

Of course, the infinite enhancement is approached only as both $\langle\Gamma_{\mu c}\rangle$ and $\langle\Gamma_{\mu d}\rangle$ approach zero. But as we shall see, order of magnitude effects are entirely possible even in realistic situations, and 5% enhancements can still occur even when $N\nu$ is as large as 40.

The numerical evaluation of the fluctuation correction factor involves the following integration[1]

$$W_{cd} = (1 + 2\delta_{cd}/\nu_d) \times \int_0^\infty dt \prod_f (1 + 2t\nu_f^{-1}\langle\Gamma_{\mu f}\rangle\langle\Gamma_\mu\rangle^{-1})^{-(\nu_f/2 + \delta_{fc} + \delta_{fd})}, \tag{5}$$

which in general must be performed by numerical quadrature, though in many cases (particularly when all $\nu_f = 1$ or 2) the integral can be evaluated in terms of elementary functions.

Recently an algebraic evaluation of W_{cd} was proposed by Tepel *et al.*[4-6] which involves the solution of the following set of simultaneous quadratic equations for the X_c:

$$X_c \Sigma_d X_d + 2X_c^2/\nu_c = T_c = 2\pi\langle\Gamma_{\mu c}\rangle/D. \tag{6}$$

One then evaluates the average fluctuation cross section by

$$\sigma_{cd}^{\rm fl\,(T)} = X_c X_d + 2\delta_{cd} X_d^2/\nu_d. \tag{7}$$

This formula includes the correlation enhancement factor of Eq. (3) and also satisfies the unitarity condition $\Sigma_d \delta_{cd}^{\rm fl} = T_c$. But on evaluating the limiting fluctuation enhancement by Eqs. (6) and (7) we find the factor to be

$$\left(1 + \frac{2}{N\nu}\right), \quad \text{all } N\nu. \tag{8}$$

This factor is much smaller than the prediction of Eq. (4) when $N\nu$ is a small number. It is still small by 4% when $N\nu$ is as large as 10.

The numerical solution of Eq. (6) by either matrix methods of by successive approximations appears to present no substantial advantage over the numerical integration of Eq. (5). However an advantage would be obtained if Eq. (6) could be adequately solved by means of the approximation[5]

$$X_c \approx Y_c(\Sigma_d Y_d)^{-1/2}\,, \qquad Y_c = \frac{T_c}{1+(2/\nu_c)(T_c/\Sigma T)}\,. \tag{9}$$

Substitution of Eq. (9) in Eq. (7) leads to the same value [Eq. (8)] for the limiting fluctuation enhancement factor.

To investigate the accuracies and ranges of validity of these various methods for calculating the fluctuation enhancement factor, we have evaluated them for a variety of two, three, and four channel models, each involving two groups of channels with transmission coefficients differing by a factor which ranges from 1 to 1000. Direct reactions were assumed to be absent. The calculated inelastic fluctuation cross sections between weak channels (weak channel elastic cross sections for the two channel case) are plotted in Fig. 1. They are compared there with the results of computer experiments employing the program STASIG, as described in Ref. 1. The statistical assumptions of these computer experiments correspond also to those reported in Refs. 4 and 5.

The following conclusions can be drawn from these calculations.

(1) Systematic differences between the predictions of the width fluctuation correction formula (5) and the Tepel formula Eqs. (6) and (7) can become very large when the ratios of competing transmission coefficients become large. These differences affect the small cross sections between weakly absorbed channels.

(2) The results of computer experiments support the predictions of Eq. (5) and are not consistent with Eqs. (6) and (7).

(3) When all competing transmission coefficients have values within a factor of 5 of each other, Eqs. (6) and (7) can be expected to yield cross sections within 5% of Eq. (5). For larger transmission ratios, Eqs. (6) and (7) can yield larger discrepancies. [Eqs. (6) and (9) cannot be relied on to better than 10%.]

(4) When ratios of competing transmission coefficients are within a factor of 100, the 20-point Gauss-Laguerre quadrature evaluation of the integral (5) yields results within 5% of the exact values. For larger transmission coefficient ratios, a precise calculation of small cross section values may require more exact methods for evaluating the integral (5).

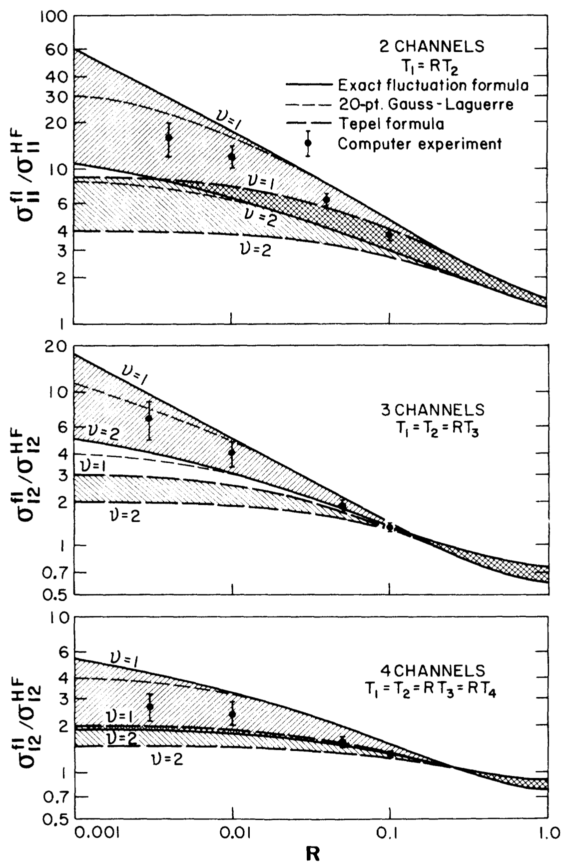

FIG. 1. Enhancements, compared to Hauser-Feshbach, of small compound cross sections for a variety of two, three and four channel cases. The predictions of each of the two theories lie within the shaded regions bounded by the curves for $\nu=1$ (all channels) above and $\nu=2$ (all channels) below. In each case the upper shaded region corresponds to the width fluctuation integral Eq. (5); the lower region corresponds to the Tepel formula Eqs. (6) and (7). Curves corresponding to the 20-point Gauss-Laguerre quadrature evaluation of the width fluctuation integral are shown for $\nu=1, 2$. The points show the average results and variances of computer experiments with T(large) = 0.91 (two channel cases) and T(large) = 0.84 (three and four channel cases).

(5) Under certain conditions the differences between Eq. (5) and Eqs. (6) and (7) can be compensated for by adjustment of the channel fluctuation indices ν_c. This probably accounts for some of the differences in the dependences of ν_c on transmission coefficients, as reported in Ref. 1 on the one hand and Refs. 4 and 5 on the other hand.

Because of the large relative effects involved, the total width fluctuation enhancement factor should have an appreciable influence on small compound nucleus cross sections, such as charged particle scattering or capture reactions below the Coulomb barrier, when they compete with several strong neutron channels.

*Work performed under the auspices of the Energy Research and Development Administration.

[1]P. A. Moldauer, Phys. Rev. C 11, 426 (1975).

[2]G. R. Satchler, Phys. Lett. 7, 55 (1963).

[3]P. A. Moldauer, MIT Report No. MIT-2098-149, 1965 (unpublished); Phys. Rev. C 12, 744 (1975).

[4]H. M. Hofmann, J. Richert, J. Tepel, and H. A. Weidenmüller, Ann. Phys. (N.Y.) 90, 403 (1975).

[5]J. W. Tepel, H. M. Hofmann, and H. A. Weidenmüller, Phys. Lett. 49B, 1 (1974).

[6]For the case $\nu = 2$, Eqs. (6) and (7) are equivalent to the formula of M. Kawai, A. K. Kerman, and K. W. McVoy, Ann. Phys. (N.Y.) 75, 156 (1973).