Three-body unitary transformations, three-body forces, and trinucleon bound state properties

Michael I. Haftel
Naval Research Laboratory, Washington, D. C. 20375
(Received 26 April 1976)

A three-body unitary transformation method for the study of three-body forces is presented. Starting with a three-body Hamiltonian with two-body forces, unitary transformations are introduced to generate Hamiltonians that have both two- and three-body forces. For cases of physical interest, the two-body forces of the altered Hamiltonians are phase equivalent (for two-body scattering) to the original and the three-body force vanishes when any interparticle distance is large. Specific examples are presented. Applications for studying the possible role of three-body forces in accounting for trinucleon bound state properties are examined. Calculations of the ^{3}He and ^{3}H charge form factors and Coulomb energy difference with hyperspherical radial transformations and with conventional N-N potentials are performed. The form factor calculations demonstrate how the proposed method can help obtain improved agreement with experiment by the introduction of appropriate three-body forces. Calculations of the Coulomb energy difference confirm previous estimates concerning charge symmetry breaking in the N-N interaction.

[NUCLEAR STRUCTURE ^{3}H, ^{3}He; three-body forces introduced by unitary transformations; charge form factors and Coulomb energies calculated; five force models.]

I. INTRODUCTION

Faddeev calculations with realistic two-nucleon (N-N) potentials[1,2] do not yield predictions for the properties of the three-nucleon ($3N$) bound states in agreement with experiment. Realistic interactions predict neither the correct binding energy of ^{3}H ($E_T \approx 7$ MeV vs 8.5 MeV experiment) nor the correct ^{3}He charge form factor [$F_{ch}(q^2)$ (^{3}He)]. The position of the diffraction minimum (q_0^2) in $F_{ch}(q^2)$ (^{3}He) is not in agreement with experiment ($q_0^2 \gtrsim 14$ fm^{-2} vs 11.6 fm^{-2} experiment) and the height of the second maximum is at least a factor of 3 below experiment. If possible off-shell variations of realistic interactions are considered,[3,4] predictions of E_T may improve, but only at the cost of ruining the fit of $F_{ch}(q^2)$ (^{3}He) even more. Another discrepancy is that the Coulomb energy calculated from the $3N$ wave functions obtained from realistic interactions gives[5] 0.60–0.65 MeV compared with the experimental energy difference between ^{3}H and ^{3}He of ΔE (^{3}He-^{3}H) = 0.76 MeV.

The inability to explain the trinucleon bound state properties with pure two-body forces has suggested to some an important role for three-body forces in the $3N$ system.[3,6] From meson field theory such forces should exist and estimates of their contributions to nuclear binding energies range from 2.3 MeV for the triton[7] to anywhere from 0.1 to 5 MeV/A for nuclear matter.[8] Other possible reasons for the discrepancies mentioned above are the neglect of meson exchange currents [for $F_{ch}(q^2)$],[9] other relativistic effects,[10] and charge dependence and charge-symmetry-breaking in the N-N force.[11,12]

This paper employs a three-body unitary transformation method to generate three-body forces for use in investigating the properties of the $3N$ bound states. Section II describes the unitary transformation method and gives specific examples. The method is similar, in spirit, to the two-body unitary transformation method popular for studying off-shell effects.[13] The three-body forces generated are nonlocal and preserve the $3N$ binding energy (E_T) predicted by a starting "untransformed" two-body potential. While the transformations leave E_T invariant, the wave functions and properties that depend on them (electromagnetic form factors, Coulomb energy) change.

One of the main goals of this paper is to examine the influence of different nuclear force models, especially those containing three-body forces, on $3N$ bound state properties. In Sec. III we study the effects of certain transformations, and hence of certain three-body forces, on the $3N$ bound state properties. The "starting" N-N potential is the Graz preliminary (GRP) potential[14] since this potential gives both a satisfactory fit to the N-N data and approximately the correct value for E_T. We calculate the ^{3}He and ^{3}H charge form factors with the GRP potential and with two unitary

transformations as well as with two conventional N-N potentials [Reid (R)[15] and de Tourreil-Sprung A (S)[16]]. The principal purpose here is to determine how the three-body forces introduced might help explain the experimental values of *both* E_T *and* $F_{ch}(q^2)$. Previously, Brayshaw[6] deduced the necessity of three-body forces by studying the experimental ^{3}He and ^{3}H charge form factors and extracting the $3N$ wave function in a hyperspherical basis. He estimated that the required $3N$ force contributes about 2.5 MeV attraction to E_T. This work differs from Brayshaw's in that an explicit three-body force appears and the N-N potentials considered have tensor components. The three-body forces introduced are indeed helpful in fitting the form factor data in both ^{3}He and ^{3}H.

The last part of Sec. III investigates the Coulomb energy predictions obtained from wave functions of different force models. We relate the ΔE_C results with the form factor results and discuss what the experimental form factors may imply about ΔE_C. This type of investigation has been previously carried out by Friar[17] and by Fabre de la Ripell,[18] who estimate ΔE_C directly from the experimental charge form factors with the result $\Delta E_C \approx 0.64$ MeV. Our ΔE_C and form factor calculations yield a final estimate of ΔE_C in close agreement with Friar and Fabre de la Ripell, confirming previous estimates of charge symmetry breaking in the N-N force.

II. UNITARY TRANSFORMATION METHOD

The three-body unitary transformation method described here is an extension of the two-body unitary equivalence methods of Ekstein,[19] Baker,[20] Coester *et al.*,[13] and others. The three- and many-body extensions have been referred to by Villars[21] and by Ristig,[22] but they have never, to my knowledge, been employed for three- or many-nucleon calculations. Sáenz and Zachary[23] have considered the many-body method from the viewpoint of time-dependent scattering theory.

Analogous to the two-body unitary transformation method, we start with an untransformed Hamiltonian H for the three-body system

$$H = T + \sum_{i<j} V_{ij} \ , \tag{1}$$

where T is the total kinetic energy and V_{ij} are (for simplicity) two-body potentials. Since we assume translational and Galilean invariance throughout the discussion, we consider the Hilbert space of the three particles in the center-of-mass (c.m.) system. The Hamiltonian H yields a spectrum $\{E_n\}$ with eigenstates $\{\psi_n\}$ $(H\psi_n = E_n\psi_n)$ which may include bound states. Since we have the $3N$ system in mind, we assume one bound state with energy $-E_T$ and wave function ψ_T. (This restriction is not necessary in general.) From ψ_T one can calculate, for example, electromagnetic form factors [indicated collectively as $\{F(q^2)\}$].

We now consider a "transformed" Hamiltonian $\tilde{H} = UHU^\dagger$, where U is a translationally and Galilean invariant unitary operator consisting of two- and/or three-body parts. By unitarity $(UU^\dagger = U^\dagger U = 1)$ U preserves the spectrum $\{E_n\}$ but changes the wave functions $\{\tilde{\psi}_n = U\psi_n\}$. Properties that depend on the bound state wave functions may change (i.e., $\tilde{H} \to \tilde{\psi}_T \to \{\tilde{F}(q^2)\} \neq \{F(q^2)\}$). In general the transformed Hamiltonian can be written

$$\tilde{H} = UHU^\dagger = T + \sum_{i<j} \hat{V}_{ij} + H^{(3)} \ , \tag{2}$$

where $\hat{V}_{ij}$ is a new pair interaction, which may (or may not) differ from V_{ij}, and $H^{(3)}$ depends on the coordinates of all three particles, i.e., it is a three-body force. Of course, in transforming H to $\tilde{H}$, one usually would desire the same two-body scattering properties of the potentials V_{ij} and $\hat{V}_{ij}$. Sáenz and Zachary,[23] from time-dependent scattering theory, have derived general conditions on U such that H and $\tilde{H}$ are equivalent for three- (or many) body scattering and for which $\hat{V}_{ij}$ and V_{ij} are equivalent for two-body scattering. We present shortly specific examples of U for which V_{ij} and $\hat{V}_{ij}$ are phase-equivalent for two-body scattering.

As in the two-body unitary transformation method, we do not regard U as a complete unitary transformation of the quantum mechanical description of the three-body system. That would not be of any physical interest. Rather we exploit U as a means of manufacturing alternate Hamiltonians that preserve spectra and certain scattering properties. These alternate Hamiltonians are expressed in terms of unchanging operators representing physical observables. One could distinguish between these Hamiltonians with varying off-shell behavior and three-body forces, for example, by their predictions of bound state properties.

The main advantage of the proposed method over the *ad hoc* introduction of three-body forces is that one never has to solve the Faddeev equations with the generated three-body forces. One only has to transform wave functions obtained from a solution of the Faddeev equations from the starting potential. Of course, if one desires to assess the influence of $H^{(3)}$ in the Hamiltonian $\tilde{H}$, one may want to calculate $3N$ properties in the absence of $H^{(3)}$, i.e., with the Hamiltonian $\hat{H} = T + \sum \hat{V}_{ij}$. The

three-body forces produced in this method are, like the two-body forces generated in off-shell transformations, very nonlocal. In this respect they are quite different from the types of three-body forces derived from meson field theory.[7] We now present some examples.

One of the simplest types of three-body transformations are those that operate on the dependence of the wave function on the hyperspherical radius (R). Here

$$\langle Ry|U|R'y'\rangle = U(R,R')\,\delta(y-y')\ , \tag{3}$$

where $R^2 = r_i^{\,2} + \rho_i^{\,2}$, $\vec{r}_i = \vec{x}_j - \vec{x}_k$, $\vec{\rho}_i = (\vec{x}_j + \vec{x}_k - 2\vec{x}_i)/\sqrt{3}$ (i,j,k cyclic—assume equal masses) $\vec{x}_i$ is the position vector of particle i, and y represents all additional coordinates needed to describe the three-body system in the c.m. frame. The hyperspherical radius is exchange symmetric, i.e., it is independent of i. Therefore, U is exchange symmetric, and $\tilde{\psi} = U\psi$ satisfies the Pauli principle if ψ does. Furthermore, since R depends on the coordinates of all three particles, U is a pure three-body operator and $\hat{V}_{ij} = V_{ij}$. The hyperspherical radius R becomes infinite if any interparticle distance is infinite; thus, if U is short-ranged in R and R'

$$\left[\text{i.e., } U(R,R') \xrightarrow[R \text{ or } R'\to\infty]{} \delta(R-R')/R^5\right]$$

the three-body term ($H^{(3)} = UHU^\dagger - T - \sum \hat{V}_{ij}$) is a three-body force in the usual sense: It vanishes when any pair of particles is infinitely separated. Two examples of such operators are the rank-two separable hyperspherical transformation

$$U = 1 - (1-\cos\theta)\left(|g_1\rangle\langle g_1| + |g_2\rangle\langle g_2|\right) + \sin\theta\left(|g_1\rangle\langle g_2| - |g_2\rangle\langle g_1|\right) \tag{4}$$

$$[\langle g_i|g_j\rangle = \delta_{ij},\ \langle \vec{r}_k\,\vec{\rho}_k|g_i\rangle = g_i(R)]$$

and the radial scale distortion (Baker) transformation

$$\tilde{\psi}(R,y) = U\psi(R,y) = \left[\frac{f(R)}{R}\right]^{5/2}[f'(R)]^{1/2}\,\psi(f(R),y) \tag{5}$$

$$[f'(R) \geq 0,\quad f(R) \xrightarrow[R\to\infty]{} R]\ .$$

These are hyperspherical analogs to the two-body transformations described by Coester *et al.*[13]

An example of U (for nonidentical particles) with $\hat{V}_{ij} \neq V_{ij}$ is

$$\langle \vec{r}_1\,\vec{\rho}_1|U|\vec{r}_1'\,\vec{\rho}_1'\rangle = u(\vec{r}_1,\vec{r}_1')\,\delta(\vec{\rho}_1 - \vec{\rho}_1')\ , \tag{6}$$

where u is a unitary operator in the space of the relative motion of particles 2 and 3. Here $\hat{V}_{12} = V_{12}$, $\hat{V}_{13} = V_{13}$ and $\hat{V}_{23} = U(t_{23} + V_{23})\,U^\dagger - t_{23}$, where t_{23} is the relative kinetic energy of particles 2 and 3. The three-body force is $H^{(3)} = U(V_{12} + V_{13})\,U^\dagger - V_{12} - V_{13}$. If u is short ranged, $\hat{V}_{23}$ and V_{23} give equivalent two-body scattering results.

A three-body transformation with $\hat{V}_{ij} \neq V_{ij}$ suitable for identical particles is a three-body version of the Bohm-Baker-Gross transformation,[20,24] which is discussed by Sáenz and Zachary.[23] Here

$$\tilde{\psi}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = J^{1/2}(\vec{x}_1,\vec{x}_2,\vec{x}_3)\,\psi(\vec{y}_1,\vec{y}_2,\vec{y}_3)\ , \tag{7}$$

where

$$\vec{y}_i = \vec{x}_i + \sum_{j\neq i} F(|\vec{x}_i - \vec{x}_j|)\,(\vec{x}_i - \vec{x}_j)/|\vec{x}_i - \vec{x}_j|$$

and J is the Jacobian of the transformation. For this transformation $\hat{V}_{ij}$ is just the potential, phase shift equivalent to V_{ij}, obtained from the two-body radial scale distortion transformation[20,23] (of the relative displacement) $\vec{r}_k \to \vec{r}_k + 2F(r_k)\,\hat{r}_k$. All of the transformations proposed above satisfy the required symmetries (translational, rotational, etc.) and the scattering equivalence conditions of Sáenz and Zachary.[23] All three-body forces vanish when any interparticle distance is large.

This work applies two examples of hyperspherical transformations, one of the rank-two type and one of the Baker type, to study the possible effects of three-body forces on the $3N$ bound state observables. The forms of the transformation functions we choose are

$$g_i(R) \sim e^{-\alpha_i R}(\beta_{0i} + \beta_{1i}R + \beta_{2i}R^2)\ , \tag{8a}$$

$$f(R) = R + s(e^{-R/a} - e^{-R/b}) \tag{8b}$$

for the rank-two and Baker transformations, respectively [see Eqs. (4) and (5)]. We choose the Graz preliminary potential (GRP),[14] whose two-body properties are described in Ref. 14, as the "starting" potential, since this potential gives approximately the experimental E_T (see Sec. III). It also fits the two-body phase shifts up to 350 MeV and yields a qualitatively satisfactory deuteron quadrupole moment and electric form factor.[25] As with most two-body potentials that give more binding in the $3N$ bound state, this potential has a "softer" short-range (or off-shell) behavior[25] than "realistic" local potentials, and a lower deuteron D state probability[14] ($P_D = 3.7\%$ vs 6.5% of Reid[15]). We investigate the two unitary transforms, labeled GRPA and GRPB, whose parameters appear in Table I. We employ these transformations to consider the effects on $3N$ bound state properties of $3N$ wave functions that have the correct E_T yet not of the type that are normally obtainable from two-body forces that fit E_T. Predictions for the ^{3}He and ^{3}H charge form factors and the Coulomb energy (ΔE_C) under these transformations appear in the next section.

TABLE I. Three-body unitary transformation parameters.

Potential	
GRPA [a]	$\alpha_1=\alpha_2=2.1\ \text{fm}^{-1}$, $\beta_{01}=\beta_{02}=0$
	$\beta_{11}=-1.0\ \text{fm}^{-1}$, $\beta_{12}=1.5873\ \text{fm}^{-1}$,
	$\beta_{21}=0.3\ \text{fm}^{-2}$, $\beta_{22}=-1.0\ \text{fm}^{-2}$
GRPB [b]	$s=-3.2$ fm, $a=1.4$ fm, $b=1.0$ fm

[a] See Eq. (8a).
[b] See Eq. (8b).

The main weakness of our starting potential (GRP) is that it does not have a one-pion-exchange (OPE) tail (GRP is separable). To date, there does not exist a potential in the literature that has an OPE tail yet predicts the precise value of E_T. However, the one-boson-exchange model of Bryan and Gersten,[26] which has an OPE tail, gives $E_T = 7.8$ MeV, which is not in too poor agreement with experiment. Reasonably one could expect a moderate off-shell transformation of this potential to achieve the needed 0.6 MeV binding yet retain the OPE tail. We do not here consider the Bryan-Gersten potential, but merely point out its existence to add credence to the starting point of our method—that is, the necessity of starting with a two-body potential that fits E_T. We now consider the possible influences of unitary transformations on the $3N$ bound state.

III. RESULTS AND DISCUSSION

To calculate the $3N$ bound state properties, we first solve the three-channel Faddeev equations for E_T and the related off-shell Faddeev amplitudes, which essentially give the wave function in momentum space before antisymmetrization. We retain only the $^1S_0$ and $^3S_1+{}^3D_1$ interactions and the three trinucleon channels: $L=l=\mathcal{L}=0$, $S=0$, $\mathcal{S}=\frac{1}{2}$; $L=l=0$, $\mathcal{L}=0$, $S=1$, $\mathcal{S}=\frac{1}{2}$; $L=2$, $l=0$, $\mathcal{L}=2$, $S=1$, $\mathcal{S}=\frac{3}{2}$, where the notation is that of Harper and co-workers.[1,27] Table II gives the E_T values obtained in our three-channel calculations for each potential we consider and those obtained by more complete calculations. Generally speaking, the three-channel calculations underestimate E_T by about 0.6 MeV. Therefore, the GRP potential, which gives $E_T = 7.94$ MeV in the three-channel calculation, would probably give close to the experimental value (8.48 MeV) in a full calculation.

Once the Faddeev equations are solved, wave functions are obtained by antisymmetrization of the Faddeev amplitudes. From the wave functions we can calculate electromagnetic form factors, Coulomb energies, and other properties. A complete discussion of the $3N$ bound state equations, from the Faddeev equations to obtaining the form factors, appears in a series of articles by the Purdue group.[1,27] We adhere to their notation throughout this section. Of course, for the transformed potentials GRPA and GRPB we do not have to solve the Faddeev equations again, but merely transform the $3N$ wave functions obtained from the GRP potential.

In calculating $3N$ bound state properties, we retain the wave function components listed in Table III. The notation is of Harper, Kim and Tubis.[1,27] with the spin-isospin states $W_{\mathcal{S}}^{\tau}(S, I)$ listed in the Blatt-Derrick basis.[28] We operate with the unitary transformations only on the spatially symmetric S-state component of the wave function. One can do this while satisfying unitarity and the Pauli principle thanks to the exchange symmetries of the hyperspherical transformations and of the Blatt-Derrick state $W_{\mathcal{S}}^{\tau}=A$ (spatially symmetric, spin isospin and antisymmetric). Of the states listed in Table III, only state 1 (spatially symmetric S-state with $l=L=\mathcal{L}=0$) is changed by the unitary transformation GRPA and GRPB. This state accounts for over 95% of the wave function.[25]

Figure 1 illustrates the charge form factors for

TABLE II. Triton binding energies.

Potential	E_T (three channel) (MeV) [a]	E_T (MeV)—type of calculation
Reid (R)	6.34	6.7—Five channel Faddeev [a] (Ref. 1) 7.0—Faddeev r space (Ref. 2)
de Tourreil-Sprung (S)	7.02	7.64—Faddeev r space (Ref. 2)
Graz preliminary (GRP)	7.94	8.5—Estimate
GRPA [b]	7.94	8.5—Estimate
GRPB [b]	7.94	8.5—Estimate

[a] Only $^1S_0$ and $^3S_1+{}^3D_1$ interaction retained.
[b] GRPA and GRPB must give exactly the same E_T as GRP.

TABLE III. Three-nucleon bound state wave function components.

Component	L	l	$\mathcal{L}$	$\mathcal{S}$	r
1	0	0	0	$\frac{1}{2}$	A
2	0	0	0	$\frac{1}{2}$	−
3	1	1	0	$\frac{1}{2}$	+
4	2	0	2	$\frac{3}{2}$	−
5	1	1	2	$\frac{3}{2}$	+

^{3}He and ^{3}H for the GRP potential and the unitary transformations GRPA and GRPB. Also included for comparison are results for the Reid soft-core[15] (R) and de Tourreil-Sprung A[16] (S) potentials—two potentials that fit the N-N data but underbind the triton. All form factor calculations for Fig. 1 employ the impulse approximation, the expression for which appears in Eq. (1) of Harper *et al.*[1] For the two-body potentials (R, S, GRP), S gives a good fit to the ^{3}He, ^{3}H data for $q^2 \lesssim 10$ fm^{-2}, while R slightly underestimates the experimental form factors and GRP seriously overestimates them, especially for $q^2 > 6$ fm^{-2}. Potential GRP exhibits a typical trend for two-body forces that give the correct E_T: The trinucleon charge form factors are poorly predicted. This trend has been cited as evidence for three-body forces.[3,6] Indeed, models GRPA and GRPB, which contain $3N$ forces, reproduce closely the good agreement with experiment obtained by potential S, yet are more consistent with the experimental E_T. All models we consider predict the position of the ^{3}He diffraction minimum (q_0^2) too far out and the height of the second maximum an order of magnitude too low. All unitary transformations we have tried that move in q_0^2 substantially further can do so only at the cost of ruining the low q^2 fit and none appreciably raises the height of the second maximum. Nevertheless, the q_0^2 predicted by GRPA and GRPB are much more satisfactory than that given by the two-body potential GRP.

Of course, one could fault the above calculations for their neglect of meson exchange currents.

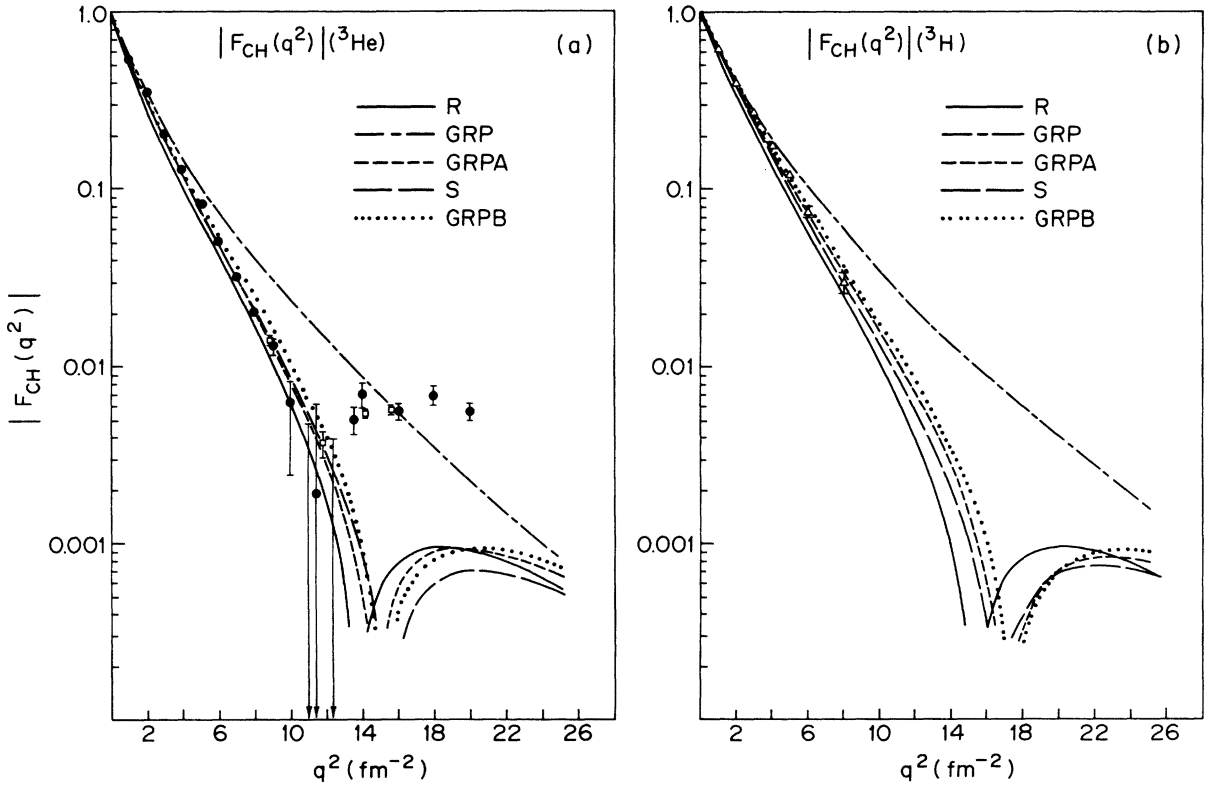

FIG. 1. The (a) ^{3}He and (b) ^{3}H charge form factors for various potentials and unitary transformations. The abbreviations are as follows: R—Reid soft core; S—de Tourreil-Sprung A, GRP—Graz preliminary, GRPA, GRPB—unitary transformations of Table I applied to the GRP wave function. The proton and neutron charge form factors used in the calculations are taken from Ref. 30. The experimental points are taken from Ref. 32: heavy dots—McCarthy *et al.*, squares—Bernheim *et al.*, triangles—Collard *et al.*

Future publications[25,29] consider the effect of these unitary transformations on $F_{ch}(q^2)$ when exchange currents are taken into account. The authors demonstrate that the main effect of the exchange currents in ^{3}He is to move in the predicted q_0^2 close to the experimental point and to improve the behavior of the second maximum for all potentials. For $q^2 \lesssim 10$ fm^{-2} they show that GRPB gives a good fit in ^{3}He, GRPA is slightly below the ^{3}He data, and GRP is considerably above the ^{3}He data (but not as far above as in the impulse approximation). The exchange effects are small in ^{3}H and do not change the previous remarks concerning impulse approximation results in ^{3}H. The $3N$ forces are evidently critical in simultaneously fittings E_T and $F_{ch}(q^2)(^3\mathrm{H})$.

We calculate the Coulomb energy difference (ΔE_C) by evaluating the difference in the Coulomb interaction matrix element between that of the ^{3}He wave function and that of the ^{3}H wave function. If the ^{3}He and ^{3}H wave functions are assumed equal (aside from total isospin projection), the expression for ΔE_C becomes, in momentum space

$$\Delta E_C = \frac{3e^2}{\pi} \sum_{\substack{Ll\mathfrak{L}S \\ rr'}} \Bigg[\int_0^\infty q^2 dq p^2 dp p'^2 dp' \psi_B^*(p, q, (Ll)\mathfrak{L} W_S^r; J) \psi_B(p', q(Ll)\mathfrak{L} W_S^{r'}; J) \times \int_{-1}^{1} dx P_l(x) \frac{[f_p^2(p^2+p'^2-2pp'x) - f_n^2(p^2+p'^2-2pp'x) - 1]}{p^2+p'^2-2pp'x}$$

$$+ \int_0^\infty q^2 dq p dp dp' \psi_B^*(p, q, (Ll)\mathfrak{L} W_S^r; J) \psi_B(p', q, (Ll)\mathfrak{L} W_S^{r'}; J) Q_l \left(\frac{(p^2+p'^2)}{2pp'} \right) \Bigg]$$

$$\times {}_1\langle W_S^r(S, I=\tfrac{1}{2}, I_z=\pm\tfrac{1}{2} | \tfrac{1}{4}(1\pm\tau_{2z})(1\pm\tau_{3z}) | W_S^{r'}(S, I=\tfrac{1}{2}, I_z=\pm\tfrac{1}{2}) \rangle_1 . \quad (9)$$

In this expression ψ_B is the bound state wave function, W_S^r are the spin-isospin functions in the Blatt-Derrick basis,[28] I and I_z are the total isospin quantum numbers, and τ_{iz} is twice the isospin projection of particle i. All wave functions for the $3N$ bound state are coupled to total angular momentum $J=\frac{1}{2}$, are independent of J_z, and are normalized by

$$\sum_{Ll\mathfrak{L}Sr} \int q^2 dq p^2 dp |\psi_B(p, q, (Ll)\mathfrak{L}, W_S^r; J)|^2 = 1.$$

The second integral term in Eq. (9) (the one involving Q_l) is the result for point charge protons while the first term takes into account the extended charge distributions of the proton and neutron through the proton and neutron charge form factors $[f_p(q^2), f_n(q^2)]$. The Q_l in the point charge term is a Legendre function of the second kind and has a logarithmic singularity at $p=p'$. We handle this singularity by standard subtraction techniques.

Table IV gives the Coulomb matrix elements and total Coulomb energy for each potential for both the point-charge case and the extended proton case. We take $f_p(q^2)$ from the analysis of Janssen *et al.*[30] We take $f_n(q^2)=0$; the inclusion of the $f_n(q^2)$ of Janssen *et al.* has a miniscule effect (less than 0.1%) on ΔE_C. The result of 0.580 MeV for the Reid potential and = 0.604 MeV for the de Tourreil-Sprung A potential are both very close to the calculations of Gignoux and Laverne,[2] (0.575 MeV for R, 0.611 MeV for S) which take into account more three-body states.

We see from Table IV that some sensitivity of ΔE_C to the wave function exists in the point-charge case. Percentage-wise, this sensitivity is somewhat less than that of E_T for the two-body potentials. Nevertheless, if GRP satisfactorily describes the $3N$ wave function, the Coulomb energy anomaly in ^{3}He-^{3}H could be explained—for a point Coulomb interaction. For the true extended proton case, however, the sensitivity of ΔE_C is about 60–70% that of the point-charge case for most potential comparisons, and in all cases ΔE_C is reduced. The strong (singular at $r=0$) short-range Coulomb interaction here is "smeared out" becoming less a probe of the short-range N-N behavior. Given the results for the wave functions that satisfactorily explain the ^{3}He and ^{3}H charge form factors (S, GRPA, GRPB), the data would seem to favor a value of $\Delta E_C \approx 0.62$ MeV—only 0.04 MeV more than the Reid value. Calculations of $F_{ch}(q^2)(^3\mathrm{He}, ^3\mathrm{H})$ in Refs. 25 and 29 indicate that exchange currents would change this estimate by at most to 0.63 MeV. Even the wave function of GRP, which is definitely excluded by the data, brings ΔE_C only up to 0.67 MeV for the extended proton case. With an estimated possible error of 0.02 MeV due to our various approximations, our estimate of ΔE_C is certainly consistent with Friar[17] and Fabre de la Ripell.[18]

Given the relative "stiffness" of ΔE_C to the wave function, especially to those that fit the electron scattering data, a certain amount of charge asymmetry must exist in the nuclear force. The re-

TABLE IV. Three-nucleon Coulomb energy difference and contributions from component pair matrix elements.

Potential	Coulomb matrix element [a,b] (MeV) (1,1)	(1,2)+(2,1)	(2,2)	(3,3)	(5,5)	Total
Reid (R)						
Point protons	0.616	−0.042	0.007	0.003	0.024	0.608
Extended protons	0.584	−0.037	0.007	0.003	0.023	0.580
de Tourreil-Sprung (S)						
Point protons	0.651	−0.037	0.006	0.003	0.018	0.640
Extended protons	0.611	−0.032	0.005	0.003	0.018	0.604
Graz Preliminary (GRP)						
Point protons	0.780	−0.078	0.013	0.004	0.016	0.735
Extended protons	0.686	−0.045	0.008	0.004	0.015	0.668
GRPA						
Point protons	0.711	−0.074	0.013 [c]	0.004 [c]	0.016 [c]	0.670
Extended protons	0.643	−0.045	0.008 [c]	0.004 [c]	0.015 [c]	0.625
GRPB						
Point protons	0.716	−0.074	0.013 [c]	0.004 [c]	0.016 [c]	0.674
Extended protons	0.647	−0.045	0.008 [c]	0.004 [c]	0.015 [c]	0.629

[a] The wave function component numbers in the matrix elements correspond to the component numbers in Table III.

[b] Component pairs that do not contribute to ΔE_C, such as (1,3) and (4,4), etc., do not appear.

[c] Exactly the same as GRP.

sults above confirm the previously estimated amount of charge asymmetry[11]—enough to account for a 0.10–0.15 MeV discrepancy in the ^{3}He-^{3}H energy difference. Extrapolating from the work of Gibson and Stephenson,[12] this amount of charge-symmetry breaking (CSB) would be consistent with $a^s_{pp} - a^s_{nn} \approx 5$ fm or $r^s_{pp} - r^s_{nn} \approx 0.1$ fm. The former eventuality is very unlikely since estimates of CSB from the field-theoretic origins of the nuclear force[31] give $a^s_{pp} - a^s_{nn} \approx -1$ fm. The more likely case is that $r^s_{pp} - r^s_{nn} \approx 0.1$ fm.

IV. CONCLUSIONS

We have calculated some of the properties of the $3N$ bound state with both typical two- and three-body forces generated from the unitary transformation method described in Sec. II. The results in Sec. III demonstrate the usefulness of the unitary transformation method in studying the possible role of three-body forces in the $3N$ bound state. Sizable changes occur in the ^{3}He and ^{3}H charge form factors when one, through this method, introduces alternate $3N$ wave functions consistent with the experimental E_T to those typically obtained by two-body forces that fit E_T. Much improved fits to the ^{3}H and ^{3}He charge form factors occur over those yielded by two-body potentials that fit E_T. The form factors predicted by the unitary transformed cases (GRPA, GRPB) of the GRP potential are quite similar to those predicted by the de Tourreil-Sprung potential,[16] which also gives satisfactory charge form factors. The de Tourreil-Sprung potential, however, gives 0.9 MeV less binding than GRP (or GRPA, GRPB), and also underbinds the triton by about this amount.

We also investigated the $3N$ Coulomb energy difference (ΔE_C) for various $3N$ wave functions, including those generated in the unitary transformation method. The Coulomb energy is not very sensitive to the $3N$ wave functions considered if one uses an extended proton charge distribution. Even the extreme wave function of the GRP potential fails by almost 0.1 MeV to explain the Coulomb energy anomaly in ^{3}He-^{3}H. Wave functions favored by the experimental charge form factor favor values of $\Delta E_C \approx 0.62$ MeV—close to those estimated by Friar[17] and by Fabre de la Ripell[18] as well as those calculated from "realistic" potentials. Most previous estimates of the amount of charge symmetry breaking in the N-N force are thereby confirmed in this work. Results of calculations with exchange currents,[25,29] to be published shortly, do not appreciably change the conclusions regarding $F_{ch}(q^2)$ and ΔE_C in this paper.

ACKNOWLEDGMENTS

The author gratefully acknowledges many profitable discussions with A. W. Sáenz. He would also like to thank W. M. Kloet and M. Rosen for helpful discussions and comments. The statements made concerning form factor results with exchange currents are based on calculations carried out by W. M. Kloet.

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