Proton transfer studies of ^{172}Lu

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The level structure of the odd-odd nuclide ^{172}Lu has been studied by means of the (^{3}He, d) and (α, t) proton transfer reactions on ^{171}Yb. The spectrum has been interpreted in terms of two-quasiparticle states formed by coupling the transferred proton with the 1/2$^-$[521] neutron in the target ground state. Assignments have been proposed for the bands arising by parallel and antiparallel coupling of the target neutron with protons in the 7/2$^+$[404], 1/2$^-$[541], and 5/2$^+$[402] Nilsson orbitals. Some evidence is also presented for the assignment of the $\{1/2^-[521]_n \pm 9/2^-[514]_p\}$ levels. The Q value of the ^{171}Yb(^{3}He, d)^{172}Lu reaction is measured to be -792 ± 34 keV, yielding a mass excess of -56700 ± 37 keV for ^{172}Lu.

[NUCLEAR REACTIONS ^{171}Yb(^{3}He, d), $E = 27$ MeV; measured $\sigma(E_d)$. ^{171}Yb(α, t), $E = 28.5$ MeV; measured $\sigma(E_t)$. Model comparison. ^{172}Lu deduced levels I, π, K. Enriched target. Deduced ^{172}Lu mass.]

I. INTRODUCTION

This study of ^{172}Lu is part of a project utilizing single particle transfer reactions to identify two-quasiparticle states in odd-odd deformed nuclei. The nuclide ^{172}Lu can be reached by proton transfer on ^{171}Yb, and this paper reports measurements of the ^{171}Yb(^{3}He, d)^{172}Lu and ^{171}Yb(α, t)^{172}Lu reactions.

Available information on the level structure of ^{172}Lu comes mainly from γ-ray studies. Several low-spin levels have been identified by Sen in a study of γ rays following electron capture decay of ^{172}Hf (Ref. 1). A number of other levels are reported from studies of the ^{174}Yb(p, $3n\gamma$)^{172}Lu reaction.[2] Due to the selectivity of these different reactions there is unfortunately little overlap between either of these studies and the present one.

Considerable information is available on the low-lying proton Nilsson states in the neighboring odd-A lutetium isotopes ^{171}Lu and ^{173}Lu. Using this information, along with cross sections and l values from the present measurements, it has been possible to propose assignments for a total of 16 out of 20 levels observed below 800 keV excitation in the present experiment.

II. EXPERIMENTAL PROCEDURE AND RESULTS

The ^{171}Yb target material was obtained in an isotopically enriched oxide form from Oak Ridge National Laboratory. The isotopic composition was as follows: ^{168}Yb, <0.01%; ^{170}Yb, 0.63%; ^{171}Yb, 88.2%; ^{172}Yb, 8.13%; ^{173}Yb, 1.26%; ^{174}Yb, 1.47%; ^{176}Yb, 0.34%. The targets were made by heating the oxide with lanthanum metal in a carbon boat. The oxide was reduced and the Yb metal evaporated onto carbon backings of 20 μg/cm^2 in thickness. The target used was about 50 μg/cm^2 in thickness.

Charged particle beams of about 1 μA were provided by the University of Rochester MP tandem Van de Graaff accelerator. The (^{3}He, d) measurements were carried out at 30° and 60° with a 27 MeV ^{3}He beam and the (α, t) measurements at 30° and 60° with a 28.5 MeV α-particle beam. The reaction products were analyzed with an Enge split-pole spectrograph and recorded on Kodak type NTB50 nuclear emulsions. Data from the two reactions at these angles served to identify particle groups leading to states in ^{172}Lu, and to determine the transfer l values. Measurements were limited to two angles for each reaction because of the long exposure times required (approximately 10 h per angle), and the limited additional information to be deduced from data at other angles.

The incident beam was monitored by detecting elastically scattered particles from the target in an NaI scintillation detector fixed at 45° in the scattering chamber. Absolute reaction cross sections were determined with an estimated uncertainty of 25% using these elastic scattering data together with the calculated elastic cross section obtained from the code DWUCK.[3]

A spectrum for each reaction is shown in Fig. 1 with a common scale of excitation energy. The peak widths are about 13 keV full width at half maximum in each spectrum. The differences in relative peak heights reflect the different l dependence of the cross sections; the (α, t) reaction

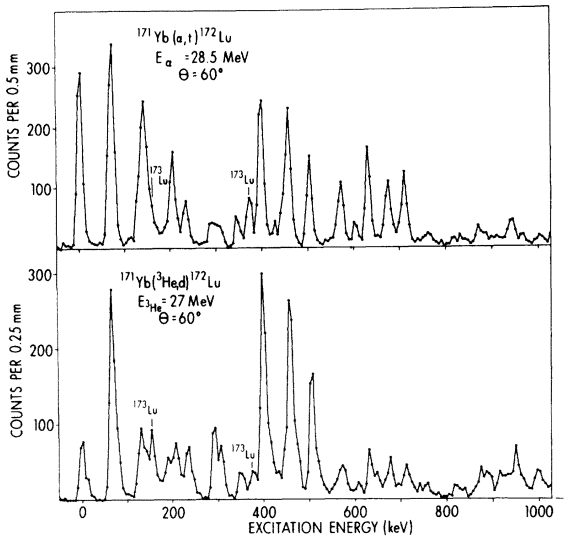

FIG. 1. Spectra obtained for proton stripping on ^{171}Yb.

favoring higher angular momentum transfer.

Groups arising from the ^{172}Yb impurity in the target were identified in the spectra and are labeled in Fig. 1. A comparison with earlier studies[4,5] of proton stripping on the Yb isotopes showed that other groups from the isotopic impurities would contribute a negligible amount.

The spectroscopic analysis of the target material indicated no other impurities at levels greater than 0.05%. The only additional heavy impurity which might have been introduced in target fabrication is the lanthanum used to reduce the oxide. Since the Q value for the ^{139}La(^{3}He, d)^{140}Ce reaction is larger than that for ^{171}Yb(^{3}He, d)^{172}Lu, tracks on the plates below the ^{172}Lu ground state could indicate the presence of lanthanum in the target. No evidence was found for groups near the expected positions for levels in ^{140}Ce, however. A comparison of the excitation energies measured at 30° and 60°, together with the observed lack of kinematic line broadening, indicated that none of the levels in Table I arise from impurities having $A < 150$.

Data reduction was carried out using a computer program[6] developed at Rochester. A standard line shape obtained from a strong isolated peak is fitted to others using a least-squares search routine. The centroid of the standard peak then serves to locate each line. Energies of excited states were calculated from the location of each group relative to the ground state group using the well-established spectrograph calibration. Levels in ^{172}Lu identified in these measurements are listed in Table I, along with the cross sections measured in both reactions. The uncertainty in excitation energies is estimated to be ±3 keV except for some of the

TABLE I. Measured differential cross sections for proton stripping on ^{171}Yb [μb/sr (c.m.)].

	(^{3}He, d)		(α, t)	
E_x	30°	60°	30°	60°
0	5.6± 0.6	5.7± 0.4	14.6± 0.6	18.3± 0.7
68	39.3± 1.6	21.0± 0.8	21.3± 0.8	27.2± 0.9
132	16.8± 1.0	9.9± 0.5	5.7± 0.4	9.1± 0.5
148	6.5± 0.6	2.9± 0.3	19.5± 0.7	19.3± 0.7
191	11.2± 0.9	5.5± 0.4	< 1.0	2.8± 0.3
210	6.6± 0.7	6.2± 0.4	13.7± 0.6	12.6± 0.6
237	6.7± 0.7	5.0± 0.4	3.0± 0.3	2.9± 0.3
296	17.5± 1.1	8.3± 0.5	3.1± 0.3	3.8± 0.3
314	5.2± 0.6	3.8± 0.3	2.4± 0.3	3.4± 0.3
355	4.4± 0.5	3.1± 0.3	4.0± 0.3	4.0± 0.3
406	46.3± 1.7	25.6± 0.9	7.6± 0.5	19.6± 0.7
437	5.2± 0.6	4.4± 0.4	1.1± 0.2	2.8± 0.3
466	49.3± 1.8	25.6± 0.9	12.8± 0.6	15.7± 0.7
513	32.1± 1.4	15.0± 0.7	5.1± 0.4	11.1± 0.6
581	8.0± 0.7	4.4± 0.4	7.5± 0.5	9.1± 0.5
614	2.7± 0.4	2.0± 0.2	3.5± 0.3	3.3± 0.3
640	4.1± 0.5	5.0± 0.4	11.8± 0.6	11.7± 0.6
663	3.9± 0.5	2.4± 0.3	2.7± 0.3	1.4± 0.2
685	7.1± 0.7	5.2± 0.4	10.4± 0.5	9.8± 0.5
720	23.7± 1.2	4.8± 0.4	7.7± 0.5	10.4± 0.5

weaker transitions. The quoted uncertainties in cross sections are those due to statistical fluctuations. As noted above, the overall uncertainty in absolute cross section is about 25%.

The ^{172}Lu mass has not been measured previously. The ^{171}Yb(^{3}He, d)^{172}Lu Q value was obtained by locating the ground state groups relative to groups of known Q values from the ^{172}Yb(^{3}He, d)^{173}Lu reaction arising from the ^{172}Yb isotopic impurity in the target. The measured difference in ground state Q values is $Q(172 \rightarrow 173) - Q(171 \rightarrow 172) = 193 \pm 4$ keV. Using the published[7] Q value for ^{172}Yb(^{3}He, d) of -599 ± 34 keV yields a ground state Q value for the ^{171}Yb(^{3}He, d) reaction of -792 ± 34 keV. A mass excess of -56700 ± 37 keV is obtained for ^{172}Lu based on the known[7] mass of ^{171}Yb.

III. DATA ANALYSIS

The proposed identification of the states observed in these measurements is based largely on a comparison between the data and predictions of the Nilsson model for the two-quasiparticle states expected in this mass region. The following section presents specific details of the model calculations used in analyzing the present results. A more extensive discussion of the general theoretical formalism has been presented in an earlier paper.[8]

The distorted wave Born approximation (DWBA) calculations for transfer of a proton into a single particle state of given (l, j) were carried out using the computer code DWUCK.[3] The zero range ap-

proximation was used, and a spin-orbit term was included for the bound state. The parameters shown in Table II are those used by Lu and Alford[9] in the study of proton transfer on even tungsten targets, and originally came from work in the lead region.[10,11] A radial cutoff of 9.6 fm was used for the (^{3}He, d) calculations since it was found necessary by Lu and Alford in order to fit angular distributions. The usual renormalization factor $N = 4.42$ was used for the (^{3}He, d) calculations, and the value $N = 82$ was used for (α, t). The latter was taken from the previous study of the ^{183}W(α, t) reaction[8] where it was chosen to reproduce the $l = 2$ ground state cross section.

Recently it has been demonstrated[12,13] that indirect reaction processes are important for some states excited by single neutron transfer reactions in the deformed rare earth region. The effects on the cross sections are usually only seen for the weakest states, however, and are typically outside the angular range 30° to 60°. Coupled channels calculations, such as those made by Ascuitto *et al.*[12] (but much more difficult for odd targets) possibly could be used to resolve some of the discrepancies seen in this study, but it is expected that the band assignments would not be changed since they are based on the stronger band members.

The Nilsson expansion coefficients C_j^{Ω} were calculated with a program supplied by the Lund group.[14] The parameters used were $\epsilon_2 = 0.28$, $\epsilon_4 = -0.02$, $\kappa = 0.0632$, $\mu = 0.605$. The quadrupole and hexadecapole parameters were chosen so that the ordering and approximate spacing of levels in the neighboring nuclei were reproduced. The values of κ and μ used were those suggested by Nilsson[15] for $A = 172$.

Coriolis mixing calculations were made for all identified bands with a computer program[6] written at Rochester for odd-odd nuclei. The code makes a least-squares fit to the observed energies by searching over a specified combination of the following parameters: the unperturbed bandhead energies, the moment of inertia parameters, and, for $K = 0$ bands, the odd-even shifts.

The form of the Coriolis matrix elements used in the program is

$$\langle I, K | H_{RPC} | I, K+1 \rangle = -(\hbar^2/2\mathcal{I})[(I-K)(I+K+1)]^{1/2} \langle K | J_- | K+1 \rangle \times (U_K U_{K+1} + V_K V_{K+1}) \eta_K \eta_{K+1} ,$$

where the mixing coefficient $\langle K | J_- | K+1 \rangle$ is calculated from the Nilsson wave functions.[6] The value of $\hbar^2/2\mathcal{I}$ used for each matrix element is taken as the average of the unperturbed moment of inertia parameters for the two bands. The pairing factor $(U_K U_{K+1} + V_K V_{K+1})$ tends to reduce the interaction for states which lie on opposite sides of the Fermi level. Standard results of the pairing theory[16] were used to calculate the U^2 using a Fermi energy 300 keV below the $\frac{7}{2}^+[404]$ orbital and a value of 800 keV for Δ. These parameters were found to give fairly good agreement for the cross sections in ^{171}Lu (Ref. 4) and ^{173}Lu (Ref. 5). The η_K and η_{K+1} are additional attenuation factors which are unity in the model, but are included here to allow a variation of the matrix elements that is sometimes required in order to fit the experimental data.[17,18]

Once the best fit to the energies has been obtained, the resultant wave functions are used to calculate[8] the perturbed cross sections for comparison with experiment. This comparison is shown in Table IV and Figs. 2 and 3.

IV. INTERPRETATION

A. Predicted spectra—general

The identification of the two-quasiparticle states excited in these measurements relies heavily on a knowledge of the proton Nilsson states expected at low excitation. The information is provided by studies of these states in neighboring odd-proton

TABLE II. Optical model parameters used in the DWBA calculations. $V(r) = V_c(r, r_{oc}) - V[(e^x+1)^{-1} + \lambda/45.2(\vec{l}\cdot\vec{s})1/r\, d/dr(e^x+1)^{-1}] - i(W - 4W_D\, d/dx')(e^{x'}+1)^{-1}$, $x = (r - r_0 A^{1/3})/a$, $x' = (r - r'_0 A^{1/3})/a'$, V_b is adjusted to reproduce the experimental binding energy.

Reaction	Particle	V (MeV)	r_0 (fm)	a (fm)	λ	W (MeV)	$4W_D$ (MeV)	r'_0 (fm)	a' (fm)	r_{oc} (fm)
(^{3}He,d)	^{3}He	175	1.14	0.732		17.5	0	1.60	0.81	1.4
	d	111	1.05	0.859		0	70.8	1.24	0.794	1.25
	p	V_b	1.25	0.65	6.0					
(α,t)	α	200	1.40	0.6		20	0	1.40	0.6	1.3
	t	200	1.40	0.6		50	0	1.40	0.6	1.3
	p	V_b	1.25	0.65	10.0					

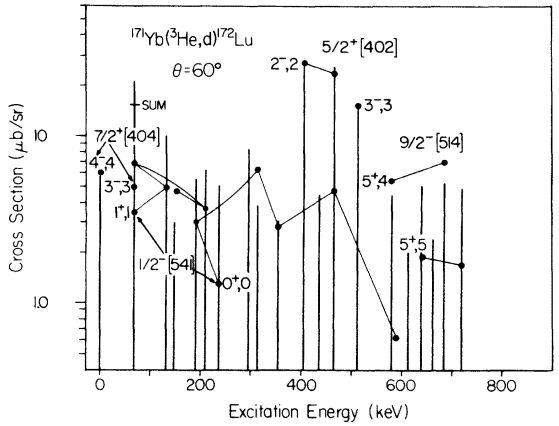

FIG. 2. Comparison of theoretical and experimental cross sections for the ^{171}Yb(^{3}He, d)^{172}Lu reaction. The vertical lines represent the experimental cross sections and the points represent predicted cross sections with the effects of mixing included. Lines connect the points which correspond to states within a rotational band. The lowest-spin level that is identified in a band is labeled with I^π,K. The proton orbital, which couples to the $\frac{1}{2}^-$[521] neutron, is specified for each pair of bands.

nuclei. The low-lying Nilsson states populated by proton transfer have been identified in ^{171}Lu by Gregory *et al.*[4] and in ^{173}Lu by O'Neil, Burke, and Alford.[5] Further information on the structure of these nuclides is provided by in-beam spectroscopy,[19] and numerous references to γ-ray studies are given by Bunker and Reich.[20] These studies have clearly established the Nilsson orbitals listed in Table III.

The two-quasiparticle states excited in ^{172}Lu are expected to be formed by the coupling of the proton orbitals given in Table III with the odd neutron in the ^{171}Yb target, which has the configuration $\frac{1}{2}^-$[521] (Ref. 20). The two bands arising from each pair of orbitals differ in K by one unit and will therefore be mixed by the Coriolis interaction. This mixing will tend to shift the stripping strength from the spin singlet band into the spin triplet band, since the latter is expected to lie lower in energy. In many cases the resulting perturbation is large.

Sometimes it is possible to make an unambiguous identification of a configuration with the transfer data alone. More usually, however, only one or two levels will be strongly excited, or Coriolis mixing effects will dominate, and the identification is less direct. In such cases, it will be possible to state whether a proposed assignment is consistent with the data, even though the particle transfer data cannot independently provide a final assignment.

In the following sections, the available evidence for the assignment of the expected two-quasiparticle states is discussed. The proposed assignments are summarized in Table IV and Figs. 2 and 3.

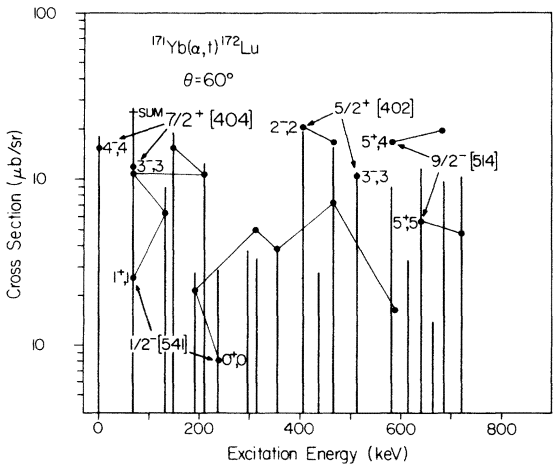

FIG. 3. Comparison of theoretical and experimental cross sections for the ^{171}Yb(α,t)^{172}Lu reaction. The proton orbital given is coupled to the $\frac{1}{2}^-$[521] neutron orbital. The notation is explained in the caption to Fig. 2.

B. States $\{\frac{1}{2}^-[521]_n \pm \frac{7}{2}^+[404]_p\}$ $K^\pi = 4^-,3^-$

The measured cross sections to the ground state are consistent with its assignment as the bandhead of the $K=4$ spin triplet band. The 5^- and 6^- members of the band are located at 118 and 259 keV by the observation of γ transitions from the ^{174}Yb-(p, $3n\gamma$)^{172}Lu reaction. These states should be only weakly populated in proton stripping however, and were not observed in the present measurements.

In ^{174}Lu these same configurations were identified, and the bandhead of the $K=3$ band was observed at an excitation energy 68 keV above the $K=4$ bandhead. Since the $\frac{7}{2}^+$[404] orbital shows

TABLE III. Bandhead energies (in keV) of proton states in two odd mass lutetium isotopes.

Proton orbital	^{171}Lu	^{173}Lu	Character
$\frac{7}{2}^+$[404]	0	0	Particle
$\frac{1}{2}^-$[541]	71	128	Particle
$\frac{1}{2}^+$[411]	208	425	Hole
$\frac{5}{2}^+$[402]	296	357	Particle
$\frac{9}{2}^-$[514]	469	≈446	Particle
$\frac{7}{2}^-$[523]	622		Hole

TABLE IV. Spectroscopic information for ^{172}Lu.

Proton orbital [a]	I^π,K	Energy (keV) Unpert	Pert	Exp	$d\sigma/d\Omega(^3\text{He},d)$ 60° (μb/sr) Unpert	Pert	Exp	$(d\sigma/d\Omega(\alpha,t)$ 60° (μb/sr) Unpert	Pert	Exp	U^2	$\hbar^2/2\mathcal{I}$ (keV) Unpert	Exp
$\frac{7}{2}^+[404]$	$4^-,4$	7	0	0	5.4	6.0	5.7	13.7	15.3	18.3	0.67	12.7	11.8
	$5^-,4$	134	118	118	<0.1	<0.1		0.2	0.2				
	$6^-,4$	286	259	259	<0.1	<0.1		<0.1	<0.1				
$\frac{7}{2}^+[404]$	$3^-,3$	69	68	68	4.8	4.9	6.7 [b]	11.8	11.9	12.7 [b]	0.67	11.4	12.5
	$4^-,3$	161	167	168	0.8	0.2		1.8	0.4				
	$5^-,3$	277	291	290	<0.1	<0.1		<0.1	<0.1				
$\frac{1}{2}^-[541]$	$1^+,1$	121	70	66 [c]	3.2	3.4	4.7 [b]	2.4	2.6	2.8 [b]	0.75	9.6	d
	$2^+,1$	159	131	132	4.1	4.9	9.9	4.3	6.2	9.1			
	$3^+,1$	217	70	68	3.7	6.9	9.5 [b]	6.0	11.0	11.7 [b]			
	$4^+,1$	294	212	210	1.8	3.7	6.2	4.5	10.7	12.6			
	$5^+,1$	390	145	148	1.6	4.7	2.9	5.4	15.5	19			
$\frac{1}{2}^-[541]$ [e]	$0^+,0$	240	240	237	1.3	1.3	5.0	0.8	0.8	2.9	0.75	11.8	d
	$1^+,0$	147	191	191	2.9	3.1	5.5	2.0	2.1	2.8			
	$2^+,0$	311	311	314	4.3	6.3	3.8	4.5	5.0	3.4			
	$3^+,0$	264	357	355	3.4	2.8	3.1	4.4	3.9	4.0			
	$4^+,0$	476	462		2.1	4.7		4.7	7.1				
	$5^+,0$	476	589	(614)	1.5	0.6	(2.0)	4.1	1.6	(3.3)			
$\frac{5}{2}^+[402]$	$2^-,2$	406	406	406	27.1	27.1	25.6	20.1	20.1	19.6	0.88	11.8	10.0
	$3^-,2$	477	466	466	5.8	23.8	25.6	4.3	16.8	15.7			
$\frac{5}{2}^+[402]$	$3^-,3$	502	513	513	33.7	15.3	15.0	22.6	10.4	11.1	0.88	11.8	
$\frac{9}{2}^-[514]$ [f]	$4^+,4$	497	493		<0.1	<0.1		<0.1	<0.1		0.90	10.0	8.7
	$5^+,4$	597	583	581	5.9	5.4	4.4	17.7	16.2	9.1			
	$6^+,4$	716	682	685	1.2	7.0	5.2	3.4	19.8	9.8			
$\frac{9}{2}^-[514]$ [f]	$5^+,5$	638	640	640	0.6	1.9	5.0	1.8	5.6	11.7	0.90	5.9	6.7
	$6^+,5$	709	722	720	6.7	1.7	4.8	18.5	4.7	10.4			

[a] The bands are formed with this proton orbital coupled to the $\frac{1}{2}^-[521]$ neutron orbital.

[b] The 68 keV group is wider than nearby levels, but the cross sections to each component could not be obtained; instead the total cross section is divided in the theoretical ratio.

[c] Energy taken from Ref. 1.

[d] The experimental moment of inertia is meaningless for these bands because they are so strongly mixed (see text).

[e] Unperturbed energies calculated with $a_p a_n = 2.84$ and $B = 92$ keV.

[f] The assignments for these bands are tentative because of the poor cross section agreement (see text).

little Coriolis mixing with other orbitals, a similar splitting is expected in ^{172}Lu. A group is observed in the present measurements at 68 keV, but the cross section is larger than expected for the $K=3$ bandhead. This peak is broader than the ground state in all the spectra, however, so that it is reasonable to assume the existence of unresolved levels at this energy and to identify the $I^\pi,K=3^-,3$ state with one of them. γ rays were seen in the ^{174}Yb$(p,3n\gamma)$ reaction study that are thought to arise from the decay of the 4^- and 5^- members of this band. If this assignment is correct, these levels would be located at 168 and 290 keV. Both levels are predicted to be weakly populated in stripping, and would not be observable in the present measurements. The $3^-,3 \rightarrow 4^-,4$ γ transition was not observed, a finding which is consistent with the assignment of the $3^-,3$ state at 68 keV since γ rays below 100 keV were obscured by atomic x rays in that study.

C. States $\{\frac{1}{2}^-[521]_n \pm \frac{5}{2}^+[402]_p\}$ $K^\pi = 2^-,3^-$

Strong $l=2$ transitions are expected to these bands in the region 300–500 keV. The only strong levels below 1 MeV with (α,t) to $(^3\text{He},\alpha)$ cross section ratios consistent with $l=2$ are seen at 406, 466, and 513 keV. The $K^\pi=2^-$ spin triplet bandhead is assigned to the lowest of the three at 406 keV, and the $I=3$ member is assigned to the 466 keV level. The $K^\pi=3^-$ bandhead, assigned to the weakest level at 513 keV, has much of its strength shifted into the $I^\pi,K=3^-,2$ state by Coriolis mixing. The cross sections to all higher band members are predicted to be less than 1 μb/sr.

The results of a Coriolis mixing calculation that included these two bands together with the $\{\frac{1}{2}^-[521]_n \pm \frac{7}{2}^+[404]_p\}$ bands assigned above are given in Table IV. The energies of the three $\frac{5}{2}^+[402]$ levels were fitted exactly by adjusting the $K=2$

moment of inertia and the bandhead energies. The moment of inertia for the $K=3$ band was held equal to the $K=2$ value.

In order to reproduce the ratio of the strengths to the 3^- states in the $K=2$ and $K=3$ bands, it is necessary to assume that the Coriolis attenuation factors η_K [see Eq. (1)] are significantly less than unity. An upper limit is $\eta_2\eta_3=0.75$. This value could be as small as 0.65 if a level from the $\{\frac{1}{2}^-[521]_n - \frac{1}{2}^-[541]_p\}$ configuration (discussed in the next section) is not resolved from the 3^-, 2 level.

D. States $\{\frac{1}{2}^-[521]_n \pm \frac{1}{2}^-[541]_p\}$ $K^\pi = 1^+, 0^+$

There are a number of levels in the spectra below the $\frac{5}{2}^+[402]$ bands which probably arise from the $\frac{1}{2}^-[541]$ orbital. Bands arising from the $\frac{1}{2}^+[411]$ orbital may contribute to the strength in this region, although these bands are predicted to be weakly populated because of their hole character. Since the $\frac{1}{2}^-[541]$ proton orbital originates in the $h_{9/2}$ shell, much of the strength to these bands will be in the $l=5$ transitions. The wave function also contains appreciable $l=1$ and $l=3$ components however, so that all of the band members up to spin 5 should be observed. This spreading of the strength, together with strong Coriolis mixing between the two bands and an unknown odd-even shift in the $K=0$ band, leads to difficulties in the identification of these bands.

The electron capture decay of ^{172}Hf studied by Harmatz and Handley[21] and by Sen,[1] is expected to populate some of the low-spin states of the $\{\frac{1}{2}^-[521]_n \pm \frac{1}{2}^-[541]_p\}$ configurations. Sen assigns to these configurations levels at 66, 148, 192, and 232 keV, which are close to the energies of levels observed in the present measurements. The 66 keV level is identified as the 1^+, 1 bandhead. This is populated in the present measurements as part of the broad group at 68 keV.

Effects of Coriolis mixing must be considered in order to locate the other members of the 1^+ band. In addition to the $\frac{1}{2}^-[541]$ bands, the four bands involving the $\frac{1}{2}^-[530]$ and $\frac{3}{2}^-[532]$ proton orbitals were included in the mixing calculations. When placed at estimated energies near 1 MeV, the major effects of these unidentified bands are to increase the cross sections to the $\frac{1}{2}^-[541]$ bands and to decrease the moment of inertia parameters. A number of parameters were varied within reasonable limits in an attempt to reproduce the spectrum. The mixing attenuation factor $\eta_0\eta_1$ was held near 1 and the moment of inertia parameters were varied in the range 9 to 14 keV. The only restriction on the unperturbed bandhead energies was that the $K^\pi=0^+$ band lie above the 1^+ spin triplet band.

For these bands having $|\Omega_p| = |\Omega_n| = \frac{1}{2}$, the product of the proton and neutron decoupling parameters $a_p a_n$ enters into the expression for the unperturbed energies[8] with a $(-1)^I$ dependence given by $[(\hbar^2/2\mathcal{I})a_p a_n - B]$. A calculated value of 2.84 for $a_p a_n$ was used in this analysis. The odd-even shift (B) of the $K=0$ band, an effect of the residual interaction, was varied within the limits -100 to 100 keV. Values for B have been reported in the range -56 to 69 keV,[22] and it would not seem unreasonable to find a band having a value of B as large as 100 keV.

When B is positive and larger than $(\hbar^2/2\mathcal{I})a_p a_n \approx 30$ keV, the predicted energies of the odd $K=0$ band members are lowered with respect to the even ones. As a result of the strong Coriolis mixing with the $K=1$ band and the $(-1)^I$ dependence of the mixing coefficient,[6] the odd members of the $K=1$ band are also depressed. In particular, when B is the range 70 to 100 keV a reasonable adjustment of the other parameters can be used to place the $I^\pi, K=3^+, 1$ state at about 68 keV, near the 1^+, 1 bandhead. When this assignment is made, the (α, t) cross section to this strong wide peak is accounted for, and the predicted $(^3\mathrm{He}, d)$ cross section is only 25% low.

By adjusting the odd-even shift and the energy of the $K=0$ bandhead in the mixing calculation, the other members of the $K=1$ band can be fitted to peaks in the spectrum. The 2^+, 1 state is assigned to an unresolved level at 132 keV. This low-l level is obscured in the (α, t) spectrum by the $l=5$ level at 148 keV, however, it is better resolved in $(^3\mathrm{He}, d)$. The 4^+ and 5^+ states are assigned to the high-l levels at 210 and 148 keV, respectively.

In order for the mixing calculation to predict the locations of the $K=1$ band members assigned above, the energies of the $K=0$ states must be somewhat restricted. Levels with energies near those predicted are seen to account for five of the first six members of the $K=0$ band. The $0^+, 1^+, 2^+, 3^+$ members can be identified with levels at 237, 191, 314, 355 keV, respectively. The 4^+ state is predicted to be near 460 keV, where it would be obscured by the strong 3^- level there. A level seen at 614 keV could be the 5^+ member of this band. Sen assigns levels at 191.7 and 232.4 keV to the $I^\pi, K=0^+, 0$ and $1^+, 0$ states, respectively. If it is assumed that these are the same levels seen here at 191 and 237 keV, the assignments of Sen must be interchanged. The only inconsistency which results arises from the 232 – 109 keV transition which cannot go between two 0^+ states. However, the 0^+ spin assignment of the 109.35 keV level is based on the Nilsson model only; the β-decay data would also support a 1^+ assignment.

E. States $\{\frac{1}{2}^-[521]_n \pm \frac{9}{2}^-[514]_p\}$ $K^\pi = 4^+, 5^+$

Since the $\frac{9}{2}^-[514]$ orbital originates in the $h_{11/2}$ shell and is a particle state here, strong $l=5$ transitions are expected to members of these bands. Four strong levels are seen above the $\frac{5}{2}^+[402]$ bands where the $\frac{9}{2}^-[514]$ strength is expected, and the (α, t) to $(^3\mathrm{He}, d)$ cross section ratios indicate l values near 5.

Although these levels probably arise from the $\frac{9}{2}^-[514]$ proton states, it has not been possible to assign levels so that the observed cross sections are accounted for. The difficulty arises from the fact that this orbital shows strong Coriolis coupling with the $\frac{7}{2}^-[523]$ and $\frac{11}{2}^-[505]$ orbitals. The latter is expected to be at about 1500 keV excitation, and its precise location is not critical. The former is expected to appear as a hole state close to the $\frac{9}{2}^-[514]$ orbital however, and mixing effects are very sensitive to its location. Although assignments are proposed in Table IV, they are considered quite tentative.

V. SUMMARY OF BANDS IDENTIFIED IN ^{172}Lu

Figures 2 and 3 summarize the comparison between the experimental and theoretical cross sections for the two reactions leading to ^{172}Lu. States arising from the four known proton particle orbitals below 1 MeV have been identified and assignments have been made for most of the levels seen below 1 MeV. Although some of the assignments are considered tentative, it can be seen that most of the stripping strength has been well accounted for. The hole configurations arising from the $\frac{1}{2}^+[411]$ and $\frac{7}{2}^-[523]$ proton orbitals were predicted to be weakly excited, and were not identified. Some of the unassigned levels may come from these orbitals.

The only levels for which the assignments can be considered definite are the ground state and the three levels arising from the $\frac{5}{2}^+[402]$ proton orbital. The identification of the bands arising from the $\frac{1}{2}^-[541]$ orbital was difficult because of the strong Coriolis mixing and the energy shift of the even band members with respect to the odd ones. The $l=5$ strength expected from the $\frac{9}{2}^-[514]$ proton transfer seems to be identified in the region near 600 keV excitation. The distribution of strength among the proposed band members is not in very good agreement with that predicted, however. Consequently the proposed assignments for these bands must be considered tentative.

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