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Alpha-transfer reactions in light nuclei. I. An exact-finite-range coupled-channels analysis*

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The importance of including multistep inelastic processes in α-particle-transfer reactions in light nuclei is discussed, and the necessity of using an exact-finite-range analysis of such reactions is emphasized. A formalism for the treatment of such reactions in an exact-finite-range Born approximation which includes inelastic effects to all orders is presented. An example from the ^{24}Mg(^{3}He,^{7}Be)^{20}Ne reaction in which it is necessary to use this analysis in order to understand the experimental data is discussed.

[NUCLEAR REACTIONS α-transfer reactions, importance of multistep inelastic processes. Formalism for exact-finite-range coupled-channels analysis.]

I. INTRODUCTION

Nearly all analyses of transfer reactions have involved the use of the first-order distorted-wave Born approximation. Such an approximation entails the evaluation of the transition amplitude matrix element

$$T_{if} = \int\int \chi_f^* \langle \varphi_f | V | \varphi_i \rangle \chi_i d^3\vec{r}_i d^3\vec{r}_f ,$$

using for χ_i and χ_f optical model solutions to the scattering problem. An exact calculation of T_{if} necessitates the evaluation of a six-dimensional integral. Therefore, in order to reduce the six-dimensional integral to a relatively tractable three-dimensional one, the motion of the transferred nucleons and the core of the projectile is usually treated as a δ function. This zero-range approximation is generally valid when the transferred cluster is in an S state in the projectile and the size of the projectile is small compared to the target and residual nuclei. Several authors[1,2] have pointed out the need for using approximations other than zero range when the above criteria are not satisfied.

A natural extension of the zero-range approximation is the fixed-range approximation[3] in which the transferred cluster and the core of the projectile are considered to be at some fixed separation and in which the two cores and the transferred cluster are assumed to be collinear (see Fig. 1). This approximation suffers from two shortcomings: (a) the structure of the projectile is not taken into account in the reaction and (b), more significantly, the angular momentum selection rules are still the same as for the "no-recoil" result. As will be shown later, these approximations lead to angular distributions significantly different then the exact calculation.

Another technique that has been applied extensively for an approximate evaluation of the full six-dimensional finite-range integral for single-nucleon transfer is the expansion of the distorted waves in a Taylor series about $\vec{r}_b = \alpha\vec{r}_a$, where $\vec{r}_a$ and $\vec{r}_b$ are the radii indicated in Fig. 1. Such an expansion has been shown to converge[4] for single-nucleon transfer in certain cases. It does not appear, however, to be valid for α-particle transfer on light nuclei.

Several methods have been devised to tackle the problem of an exact-finite-range treatment. Perhaps one of the earliest was the use of plane waves[2,5] instead of the usual distorted waves to describe the scattering of the nuclei in the initial and final systems. In such a method the transition amplitude is factorable into the product of two three-dimensional integrals and, hence, is easily evaluated. Because of the neglect of nuclear distortions, however, the plane-wave method has limited applicability. Another technique has been the expansion of the distorted waves into a complete plane-wave basis[6] where the six-dimensional integral can be factored as discussed above. This technique has the advantage that the series expansion is rapidly convergent. One of the more widely used techniques for evaluating the six-dimensional integral is the bipolar expansion of Austern *et al.*[5] Several codes now make use of this procedure including that of LOLA,[7] SATURN-MARS,[8] and the one to be discussed below, FRIMP.

In order to better understand the motivation for FRIMP it is helpful to examine some basic assumptions in the distorted-wave Born-approximation (DWBA) theory.[9] First, the transfer is assumed to occur directly from the elastic channel to the final state. Only the transferred particles participate in the reaction while all other nucleons, designated the core, remain inert. Second, the DWBA assumes the optical model provides good wave functions in the region where the transfer takes

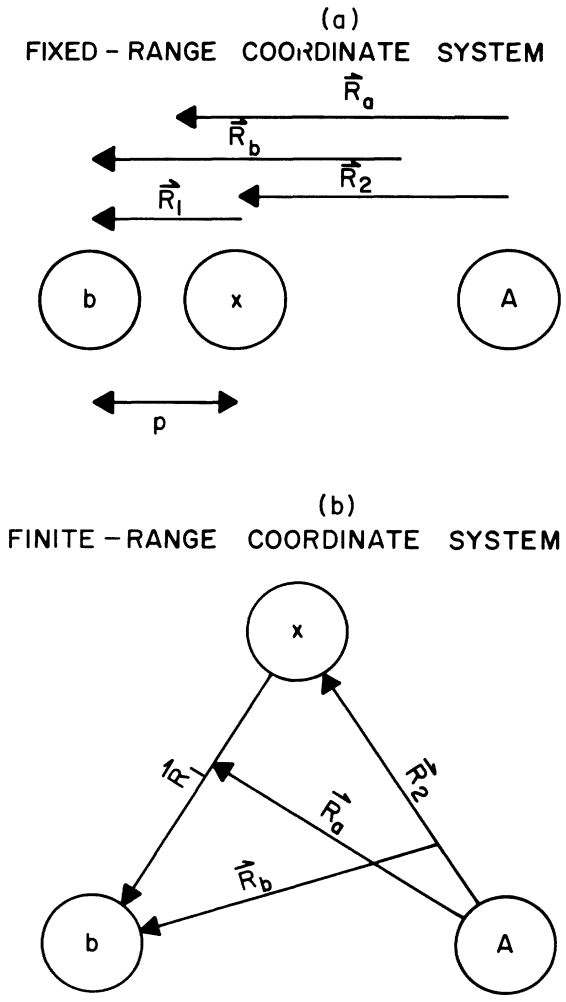

FIG. 1. The coordinate systems used to describe the reaction $A(a,b)B$, where $a=b+x$ and $B=A+x$. The vectors $\vec{R}_1$ and $\vec{R}_2$ describe the internal motion of a and B, respectively. (a) Fixed-range coordinate system. Here the scalar p denotes the fixed separation of b and x within a. (b) Finite-range coordinate system.

place—usually at the nuclear surface. Third, it assumes the reaction process is sufficiently weak so that it can be treated in first order.

Ascuitto and Glendenning[9] have given examples where the first two assumptions cannot be made. In particular, it may be that certain states of the final nucleus do not have any parentage with the ground state of the target but do with an excited state of the target. Then the reaction can only proceed through two-step processes including core excitation and direct transfer. The DWBA cannot hope to treat transfers to such states. The second assumption is violated if there are strongly enhanced inelastic transitions in either the target or final nucleus. The optical model potential, chosen to fit the elastic scattering, gives the correct relative motion wave function for the external region, but the interior wave function (or at least the wave function at the nuclear surface) is needed to calculate the transfer amplitude. If the coupling of inelastic transitions to the elastic channel is sufficiently large, then the elastic optical model will break down.

Thus, it was felt that because of the strong inelastic scattering of the projectiles typically used in α-particle-transfer reactions in light nuclei, it would be desirable to include inelastic effects in the analysis of these reactions. A reaction code has, therefore, been written which treats inelastic effects to all orders in the included channels and treats the transfer in an exact finite-range evaluation of the DWBA transition amplitude.

The code uses as its basis the zero-range coupled-channels Born-approximation (CCBA) code LISA[10] which employs the source term method of Ascuitto and Glendenning[9] to solve the problem of coupling the channels in the entrance and exit systems. (It should be pointed out that this is not the only way the coupled-channels problem can be solved. There is at least one other code[8] which uses the transition amplitude technique[11] to arrive at a solution.) The code LISA was modified to include a new source term program which treats the transfer in exact finite range. The formalism for the source term closely followed that outlined by Austern *et al.*[5] Extensive checks of the accuracy of the code were made against the zero-range coupled-channels code LISA[10] and the finite-range DWBA code LOLA.[7] It was found to give good agreement with both these codes. Section II of this paper will provide a brief derivation of the coupled equations and source term, and Sec. III will discuss the results of these calculations.

II. THEORY

Only an outline of the technique needed in calculating the coupled equations and source term will be presented here. For a complete derivation see Refs. 9, 12, and 13. We use the usual notation for a stripping reaction:

$$A(a,b)B, \quad a=b+x, \quad B=A+x.$$

The coordinates used in this presentation are shown in Fig. 1. The result for pickup reactions is of essentially the same form as for stripping, apart from the usual statistical factors.

We begin by solving the entrance channel equations

$$(H-E)\chi_a^{(+)}=0 \tag{1}$$

subject to the boundary condition that there be incoming waves only in those channels where the

nuclei are in the ground state. Here

$$H = H(\text{internal}) + T(\text{relative}) + V(\text{relative}), \quad (2)$$

where V is the optical model interaction between the nuclei in the entrance system. The exit system is similar to the entrance system except that it is coupled in first order to the entrance system via the usual stripping interaction V_s:

$$(\bar{H} - E)\Psi_b^{(+)} = V_s \chi_a^{(+)}, \quad (3)$$

where we have the additional boundary condition that $\Psi_b^{(+)}$ contains only outgoing waves. For a discussion of V_s and its relationship to the complete Hamiltonian, see Ref. 14.

We now expand the solution of Eq. (1) on a limited number of basis states:

$$\chi_{a\pi I}^{(+)M} = \frac{1}{R_a} \sum_{a'} u_{a'}^{a\pi I}(R_a) \Phi_{a'\pi I}^{M}(\hat{R}_a, \vec{a}, \vec{A}), \quad (4)$$

where the notation is similar to Ref. 9, and in particular, $\vec{a}$ and $\vec{A}$ represent the internal coordinates in particles a and A. Where both a and a' occur in an equation, a refers to the channel containing the ground state. Substituting (4) into (1) yields a set of coupled equations:

$$[T_{a'} + V_{a'a'}^{\pi I}(R_a) - E_{a'}] u_{a'}^{a\pi I}(R_a) = - \sum_{a'' \neq a'} V_{a'a''}^{\pi I}(R_a) u_{a''}^{a\pi I}(R_a), \quad (5)$$

where

$$V_{aa'}^{\pi I}(R_a) = \langle \Phi_{a\pi I}^{M}(\hat{R}_a, \vec{a}, \vec{A}) \,|\, V(\vec{a}, \vec{A}) \,|\, \Phi_{a'\pi I}^{M}(\hat{R}_a, \vec{a}, \vec{A}) \rangle. \quad (6)$$

Similarly for the exit system, we have

$$\Psi_{b\pi I}^{(+)M} = \frac{1}{R_b} \sum_{b'} w_{b'}^{a\pi I}(R_b) \Phi_{b'\pi I}^{M}(\hat{R}_b, \vec{b}, \vec{B}) \quad (7)$$

and

$$(T_{b'} + V_{b'b'}^{\pi I}(R_b) - E_{b'}) w_{b'}^{a\pi I}(R_b) = - \sum_{b'' \neq b'} V_{b'b''}^{\pi I}(R_b) w_{b''}^{a\pi I}(R_b) - \rho_{b'}^{a\pi I}(R_b), \quad (8)$$

where

$$\rho_{b'}^{a\pi I} = R_b \sum_{a'} \langle \Phi_{b'\pi I}^{M}(\hat{R}_b, \vec{b}, \vec{B}) \,|\, V_s \,|\, \Phi_{a'\pi I}^{M}(R_a, \vec{a}, \vec{A}) u_{a'}^{a\pi I}/R_a \rangle. \quad (9)$$

Note that the solutions to (8) are labeled with the entrance channel quantum numbers a, since the source term depends on these as well as on those of the exit system.

Before more explicit evaluation of either $V_{aa'}^{\pi I}$ or the source term $\rho_{b'}^{a\pi I}$ can be made, a model must be chosen for the inelastic scattering, the nuclear structure, and the stripping mechanism. It was felt that for the nuclei considered in this analysis the rotational model of Bohr and Mottelson would be appropriate. For even-even nuclei the simple adiabatic wave function

$$|JMK=0\rangle = \left(\frac{2J+1}{8\pi^2}\right)^{1/2} |0\rangle D_{M0}^{J} \quad (10)$$

was chosen. It is straightforward to substitute this form of the wave function into (6) and evaluate the resulting matrix element using an expansion of V about a spherical distribution. The procedure is shown in detail in Refs. 12 and 15 and the results will not be presented here.

We can proceed with the evaluation of the source term if we assume that we are treating either single-nucleon transfer or the transfer of a single cluster. We then have:

$$\rho_{b'}^{a\pi I} = R_b \sum_{a'} \langle \{[Y_{l_b}(\hat{R}_b)\Phi_{s_b}(\vec{\xi}_b)]_{j_b}, \Phi_{J_B}(\vec{\xi}_A, \vec{r}_2)\}_I^M \,|\, V_s \,|\, \{[Y_{l_a}(\hat{R}_a)\Phi_{s_a}(\vec{\xi}_a, \vec{r}_1)]_{j_a}, \Phi_{J_A}(\vec{\xi}_A)\}_I^M u^{a\pi I}(R_a)/R_a \rangle. \quad (11)$$

We now make a parentage expansion of the two bound state wave functions:

$$\Phi_J(\vec{\xi}, \vec{r}) = \sum_{l s J'} \gamma_l(J', J) \{[\Phi_s(\vec{\xi}_s)\chi_l(\vec{r})]_j \Phi_{J'}(\vec{\xi}_{J'})\}_J, \quad (12)$$

where s is the spin of the transferred cluster and χ_l is a relative motion wave function. These wave functions are put into (11), and several recoupling transformations are used to reduce the matrix element. The final expression for the source term is

$$\rho_{b'}^{a\pi I} = R_b \sum_{a'} \sum_{g_1 g_2 l_1 l_2 s} \gamma_{l_1}\gamma_{l_2}(2g_1+1)(2g_2+1)[(2J_B+1)(2S_a+1)(2j_a+1)(2j_b+1)(2j_1+1)(2j_2+1)]^{1/2}$$

$$\times (-)^{I+J_A+3(s_a+s_b+g_1)+j_1+j_2+l_2+s} \begin{Bmatrix} j_b & j_2 & j_a \\ J_A & I & J_B \end{Bmatrix} \begin{Bmatrix} g_1 & g_2 & j_a \\ l_a & s_a & l_1 \end{Bmatrix} \begin{Bmatrix} s & l_1 & j_1 \\ s_a & s_b & g_1 \end{Bmatrix} \begin{Bmatrix} s_b & s & g_1 \\ l_b & l_2 & g_2 \\ j_b & j_2 & j_a \end{Bmatrix}$$

$$\times \sum_{l} (-)^{l}(2l+1) I_{l l_a l_b}(R_b) \begin{Bmatrix} l_b & l_2 & g_2 \\ l_1 & l_a & l \end{Bmatrix},$$

where the subscripts 1 and 2 refer to the quantum numbers in the incident projectile and final nucleus respectively, the g's are the intermediate angular momenta from the recoupling transformations, and $I_{l_1 a l_b}(R_a)$ is the following integral:

$$I_{l_1 l_b l_a}(R_b) = J \int_0^\infty R_a^2 dR_a H_{l_1 l_b l_a}(R_b, R_a) u_{a'}^{a\pi I}(R_a)/R_a, \tag{13}$$

where J is the Jacobian of the $\vec{r}_1, \vec{r}_2$ to $\vec{R}_a, \vec{R}_b$ coordinate transformation and $H_{l_1 l_b l_a}$ is the bipolar expansion coefficient of Austern *et al.*[5] This coefficient in itself is a summation, weighted by l-coupling coefficients, of integrals of the form:

$$g_K(R_b, R_a) = \int_{-1}^{+1} d\mu [u_1(r_1)/r_1^{l_1}][u_2(r_2)/r_2^{l_2}] \times V_i(r_i) P_K(\mu), \tag{14}$$

$$\mu \equiv \hat{R}_a \cdot \hat{R}_b,$$

where the u_i are the radial parts of the bound state wave functions and $V_i = V_1$ or V_2 depending on whether the post or prior approximation is used. All nuclear structure information is contained in the $H_{l_1 l_b l_a}$ coefficients. Since no approximations are made vis à vis the coordinate system in the evaluation of the integral $I_{l_1 l_b l_a}$, recoil is taken into account exactly.

The formalism used in this reaction analysis leads to a computer code which naturally divides into four sections. The coupled equations for the initial system are set up and solved in a manner identical to that of Ref. 9. The solutions to these equations are temporarily stored for use in constructing the source term, $\rho_{b'}^{a\pi I}$. Calculation of the source term is straightforward once the bipolar coefficients are evaluated. By far the largest amount of computer time is spent in evaluating these expansion coefficients, since one is required at each mesh point for every l transfer for every combination of entrance and exit channel. For a typical (^{7}Li, t) reaction calculation at tandem energies there may be 5000 mesh points, 25 partial waves, three l transfers, and a total of 21 channel combinations, thus requiring 7–8 million evaluations of $H_{l_1 l_b l_a}(R_b, R_a)$. Still such a calculation can be done in less than 25 minutes of central-processor time on the CDC 7600. These coefficients are generally kept on a permanent storage device, since they are independent of the optical models used for a given reaction. The source term is then used to construct the coupled equations for the final system, whose solution yields the S-matrix elements for the reaction process. These S-matrix elements are used to calculate the differential cross sections, which in the case of no coupled channels, i.e., DWBA, can then be compared to the experimental cross sections to determine the α-particle spectroscopic factors or reduced widths

$$\frac{(d\sigma/d\Omega)_{\text{exp}}}{(d\sigma/d\Omega)_{\text{calc}}} = S_\alpha(B) S_\alpha(a) = \langle B \,|A \otimes \alpha\rangle^2 \langle a \,|b \otimes \alpha\rangle^2.$$

III. DISCUSSION

A. Is finite range necessary?

As is readily evident an exact-finite-range DWBA is considerably more complex than the zero-range or fixed-range approximation, so that there may be some question of the need for using the exact result. This question was aptly answered by DeVries[1] for single-nucleon transfer in heavy-ion

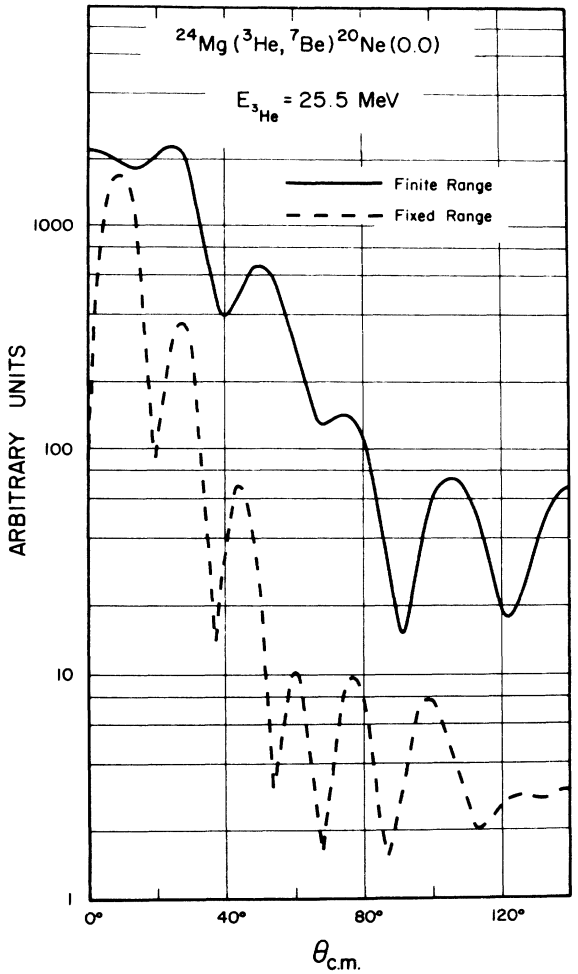

FIG. 2. A comparison of finite-range DWBA and fixed-range DWBA calculations for the ^{24}Mg(^{3}He, ^{7}Be$_{(0)}$)^{20}Ne (0.0 MeV) reaction. See text for a discussion of the differences in these calculations.

reactions, where recoil is manifested by the additional allowed l transfer. Such a question has not been answered for α-transfer reactions in which one of the projectiles is a light ion, e.g., (^{7}Li,t), (^{6}Li,d), (^{3}He,^{7}Be), and (d,^{6}Li). A comparison of two calculations can be seen in Fig. 2 for the ^{24}Mg(^{3}He,^{7}Be$_{(0)}$)^{20}Ne(0.0 MeV) reaction at a laboratory bombarding energy of 25.5 MeV. In this case the differences between the two calculations are not confused by additional l transfers allowed by recoil; since $l_2 = 0$, the exact calculation has the same l-transfer selection rule as the fixed-range one, i.e., $l = l_1$. Note the large differences in slope and peak-to-valley ratios of the two calculations. The reasons for the differences in the calculations can be seen in Fig. 3 where the kernel $g_0(R_b, R_a)$ is plotted for this reaction. It is this kernel, which when combined with the distorted waves, determines the regions of $\vec{R}_a, \vec{R}_b$ space that contribute to the reaction cross section. Note that there are significant contributions to the cross section away from the fixed-range line. It is for this reason that the full finite-range calculation gives a different result than fixed-range approximation.

B. Why and when of coupled channels

After examining the much more complex formulas that must be dealt with when including coupled channels in the DWBA analysis, it is fair to ask how necessary this is and how significant it is in the final result. Could the same effect be achieved with a careful selection of optical potentials? In general the answer is no. As was pointed out earlier, the one-channel optical model used in constructing the scattering state wave functions for the DWBA calculation breaks down when there is strong coupling to another channel. This is discussed by Glendenning[12] and by Perey and Satchler[16] for inelastic scattering, and their results can be readily carried over to transfer calculations. The coupled-channels analysis tries to explicitly take into account those excitation processes which are strongly coupled to the elastic channel. By including all strong channels, it is hoped that the effect of all neglected weak channels can be treated phenomenologically and that the resulting optical potential, and hence the scattering wave functions, will be as accurate as possible. The necessity for accurate scattering wave functions cannot be overly emphasized for the calculation of transfer cross sections, particularly where heavy ions are involved. In order to calculate transfer cross sections accurately and in a consistent fashion, it is necessary to include the effects of strong channels explicitly by analyzing elastic and inelastic scattering in a coupled-channel formalism.

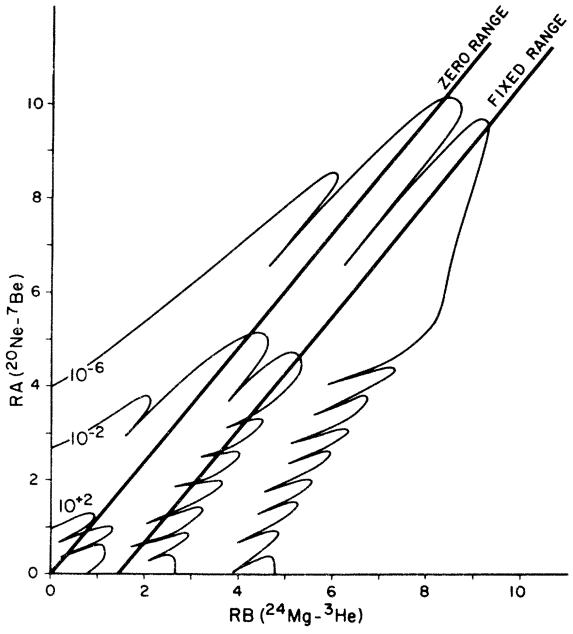

FIG. 3. The finite-range kernel $g_0(R_b, R_a)$ [Eq. (14)] for the ^{24}Mg(^{3}He,^{7}Be$_{(0)}$)^{20}Ne (0.0 MeV) reaction. Bound state wave functions are determined in Ref. 18. The contour lines reflect only the absolute value of the g_0 function.

The DWBA also fails when the final state has a small parentage with the target ground state and when there are indirect routes to the final state which are of comparable magnitude to the direct one. If one such indirect route dominates, then it seems plausible that an optical model might be constructed which reproduces the observed angular distribution. On the other hand, if there are multiple routes of comparable magnitude, then the resulting interference among these routes cannot be simulated by any changes in the optical model. Consider the example shown in Fig. 4 where calculations for the ^{24}Mg(^{3}He,^{7}Be$_{(1)}$)-^{20}Ne(1.63 MeV) reaction are displayed. Because of the deformed nature of ^{24}Mg and ^{20}Ne this reaction is believed to have significant excitations in the entrance and exit systems. The curve labeled DWBA is the result of a non-coupled-channels transfer calculation using the optical potentials which fit the elastic scattering data, while the curve labeled CCBA is from a coupled-channels transfer calculation using optical potentials obtained by refitting the elastic and inelastic scattering data in a coupled-channels formalism. The ^{24}Mg target is viewed as a ^{20}Ne core coupled to an α particle. The DWBA cross section is proportional to the square of the overlap of the ^{24}Mg ground state with an α particle coupled to the ^{20}Ne(2$^+$) state, i.e., $\langle ^{24}\mathrm{Mg}(0^+) \,|\, ^{20}\mathrm{Ne}(2^+) \otimes \alpha \rangle^2$, and

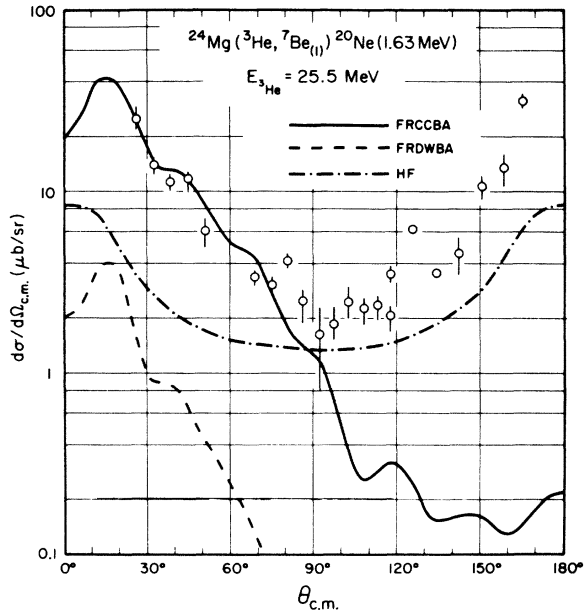

FIG. 4. A comparison of an exact-finite-range DWBA calculation with an exact-finite-range CCBA calculation for the ^{24}Mg(^{3}He, ^{7}Be$_{(1)}$)^{20}Ne (1.63 MeV) reaction at 25.5 MeV. The optical model parameters for the DWBA are those that fit the elastic scattering while those for the CCBA curve fit the elastic and inelastic scattering in a coupled-channels formulation. The optical model parameters and the bound state parameters are given in Ref. 18. The curve labeled HF is the result of a Hauser-Feshbach calculation and gives an estimate of the compound-nuclear contributions to the cross section. See Ref. 18 for details of this calculation.

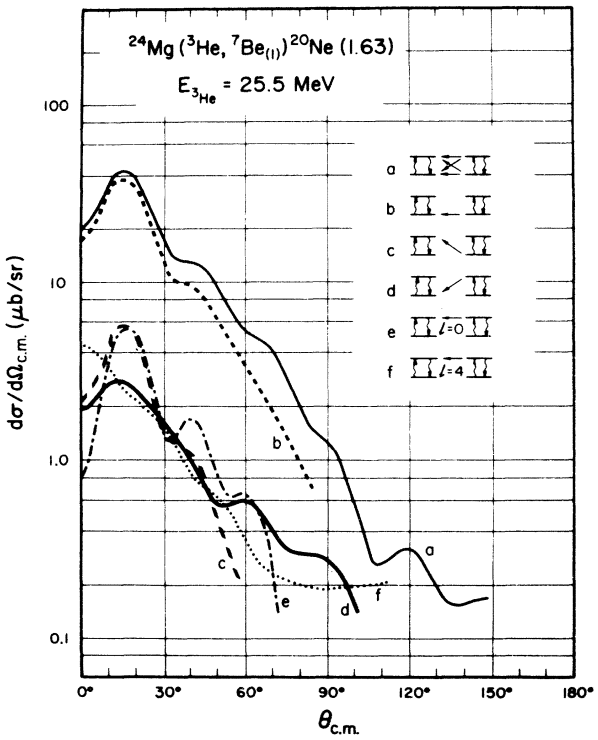

FIG. 5. Each of five separate routes which contribute to the full CCBA calculation for the ^{20}Ne (1.63 MeV) state in Fig. 4. The parentage factors for each route are determined by the SU_3 model (Ref. 17). The $l=2$ route between the 2^+ states is not shown as its magnitude is below the range of the figure.

the square of the ^{7}Be overlap $\langle {}^{7}\mathrm{Be}\,|\,{}^{3}\mathrm{He}\otimes\alpha\rangle^2$. If now, however, this first overlap is small, and the ^{24}Mg(0^+) state has a significant parentage based on the ^{20}Ne(0^+) state or the ^{24}Mg(2^+) state has parentage based on the ^{20}Ne(2^+) state (all of which are true according to the SU_3 model of these nuclei[17]), then one cannot hope to correctly calculate angular distributions to this state without also considering two-step transfers. This is illustrated in Fig. 5 where the different routes that contribute to the 2^+ cross section are plotted, weighted by the squares of their parentage coefficients as calculated within the SU_3 model. Note that the route connecting the ground states dominates this calculation of the 2^+ cross section. The ground-state to ground-state contribution to this transition is at least a factor of 5 greater than the direct route which would be calculated within the DWBA model. Note also that the magnitudes of the other contributing, two-step routes are comparable to the direct route. This can be understood in terms of (1) the fact that the square of the parentage factor for the ground-to-ground route is approximately eight times larger than that for the direct 2^+ route[17] and (2) the fact that the strength of the inelastic transition between the 0^+ and 2^+ states in the ^{20}Ne + ^{7}Be system is considerably stronger than the strength of the corresponding transition in the ^{24}Mg + ^{3}He system. In this case it is clearly necessary to include the two-step 0^+ to 0^+ to 2^+ route in any meaningful analysis of these data, and it is also clear that there will be other cases in which the inelastic transitions in the target system will be comparable to those in the residual system so that *all* of the routes leading to the 2^+ state will have comparable strength. Such a situation clearly requires the use of a complete coupled-channels treatment. A more complete discussion of the example presented above (including the extraction of spectroscopic factors) is given in Paper II[18] of this series.

While a single-channel optical model can be made to give adequate wave functions in the simple situation when the coupling to excited states is weak, and while such a simplified calculation can be made to simulate the effect of a single, isolated two-step transition, on the other hand it is clear that a full coupled-channels treatment is

necessary whenever the coupling is strong and whenever there are several competing routes leading to the final state.

IV. CONCLUSION

Utilizing existing techniques, we have developed a formalism for treating single-nucleon and cluster transfer reactions in an exact-finite-range DWBA including inelastic multistep processes in the initial and final systems to all orders in the included channels. We have demonstrated the importance of using an exact-finite-range evaluation of the DWBA transition amplitude for α-transfer reactions such as (^{3}He,^{7}Be) and (^{7}Li,t) and the importance of using the coupled-channels formalism to determine accurate scattering state wave functions when there is strong inelastic coupling involving either the initial or final states. We have also shown the necessity of calculating angular distributions using coupled channels when there are several competing and interfering routes to a single final state. The application of this formalism to the analysis of the (^{3}He,^{7}Be) and (^{7}Li,t) reactions on light nuclei is presented in Paper II[18] and Paper III[19] of this series.

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