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^{2}H$(p, pp)n$ reaction as a probe of the short-range nuclear force

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We examine the feasibility of using the ^{2}H$(p, pp)n$ reaction as a means of extracting information about the short-range behavior of the nuclear force not obtainable from N-N scattering experiments. To do this we use several separable potentials and examine the predicted cross section in various regions of phase space and for beam energies between 14 and 65 MeV. The questions that we address are likely to be insensitive to Coulomb effects. Both the form factor and the energy dependence of the potentials have been modified from the usual Yamaguchi form. The form of the energy dependence is chosen to obtain phase-shift equivalence for two different form factors while guaranteeing a unitary two-body scattering amplitude. The sensitivity of breakup results to the on-shell and off-shell aspects of the nuclear force is examined and discussed. Significant on-shell sensitivity occurs for breakup amplitudes in all states and for cross sections over all regions of phase space. Off-shell sensitivity appears only in the $S=\frac{1}{2}$, $L=0$ breakup amplitudes, with all $S=\frac{3}{2}$ and all $L>0$ amplitudes exhibiting negligible off-shell dependence. This result leads to only a very small ($\leq 5\%$) off-shell sensitivity for quasifree scattering. However, cross sections far from quasifree scattering, and in particular cross sections in the final-state interaction region of phase space, exhibit as much as a 50% variation for phase-shift-equivalent potentials. This sensitivity is small at low beam energy and increases with increasing energy. The energy dependence at negative energies of one potential is also altered to adjust the triton binding energy. This enables us to compare phase-shift-equivalent potentials differing off shell but predicting the same triton binding energy. The energy dependence of this potential is somewhat unconventional. Fixing of the triton binding energy reduces the off-shell sensitivity appreciably only for $E \lesssim 20$ MeV.

[NUCLEAR REACTIONS ^{2}H$(p,pp)n$, E=14.4, 23, 26, 30.3, 39.5, 44.9, 65 MeV; Faddeev calculation $\sigma(\theta_1, \theta_2, E)$. Four potentials.]

I. INTRODUCTION

Ever since the work of Faddeev,[1] many have looked upon the three-nucleon problem as a potentially powerful instrument to learn details of the two-nucleon (N-N) force unobtainable from two-nucleon experiments. The two-nucleon elastic scattering data can be reproduced by a multitude of potentials. For many of these potentials the two-body wave functions at short distances can be radically different although the asymptotic wave functions, which depend only on the phase shifts, are the same.[2] This is especially true when one considers the plethora of possible local and nonlocal forms. The basic problem is to find constraints on the possible forms of the wave function.

The short-range part of the two-nucleon wave function shows up in cases in which one is evaluating the transition (T) matrix elements off the energy shell. In two-body elastic scattering, the two nucleons interact on the energy shell, having the same energy before and after their interaction. In many processes, however, the presence of a third body allows the two nucleons to be interacting strongly, but to have quite different energies before and after the interaction. This leads to a situation in which changes in the two-body wave functions at short distances can lead to dramatically different predictions in many-body and nuclear matter calculations.[2,3]

The off-shell effects can only be investigated in systems involving three or more components. This paper discusses the N-d breakup reaction as a tool for investigating such effects. Originally, proton-proton bremstrahlung (p-$p\gamma$) was looked upon as the ideal process for examining off-shell behavior.[4] However, predictions of p-$p\gamma$ cross sections are generally insensitive to off-shell effects in the kinematic regions explored experimentally.[4,5] The connection between p-$p\gamma$ results and off-shell matrix elements is much more transparent,[6] at least in the single scattering approximation, than it is in the three-body problem. However, difficulties in defining the electromagnetic current, due to relativistic, velocity-dependent, and exchange effects,[7,8] becloud comparisons between theory and experiment.

Off-energy-shell two-body T matrix elements show up directly in the Faddeev equations for the three-body system. Calculations of the binding energy (E_T) and electromagnetic form factors of the trinucleon bound states have shown some sensitivity to the off-shell behavior of the low energy N-N transition (T) matrix.[9-12] However, direct off-shell information is unobtainable from the electromagnetic form factors because, as in p-$p\gamma$, there are uncertainties in meson-exchange current corrections.[13] It is not yet evident whether the static properties of the bound states (such as E_T) are sufficient to lead to clear restrictions on the possible off-shell behavior.

The N-d breakup problem is potentially a rich source of off-shell information. The three-body final states cover a very large region of phase space, some of which may be very sensitive to off-shell effects. Furthermore, we can look at many different energies to obtain the most favorable situations for examining different off-shell properties. While the corrections to the current S-wave N-d scattering theories (higher partial waves, Coulomb force) are difficult to calculate,[14,15] at least they are not ambiguous as in the electromagnetic processes (p-$p\gamma$;^{3}H, ^{3}He form factors). Even if off-shell information is eventually available from electromagnetic properties of the two-nucleon system (polarization measurements in e-d elastic scattering seems the best candidate)[16] the measurement of off-shell properties from three-nucleon systems would still be desirable. Any discrepancy in the measurement of off-shell behavior between the two- and three-nucleon systems could be indicative of the presence of three-nucleon forces. For that matter, discrepancies between the measurements of off-shell behavior for different three-body observables could also be indicative of three-body forces.

Calculations of off-shell effects in the N-d scattering problem above breakup threshold are just now in the beginning phase. The work of Kloet and Tjon[17] has already indicated that some regions of phase space may be favorable for distinguishing the presence or absence of a repulsive core in the N-N interaction. Strictly speaking, the presence or absence of a repulsive core affects both the on-shell and off-shell features of the force. Therefore, the differences in breakup results reported by Kloet and Tjon could be partially attributable to changes in the N-N scattering predictions. The issue of off-shell behavior addresses more the presence of nonlocality (or velocity dependence) at short distances rather than merely the shape of the core. On the other hand, calculations by Brayshaw,[18] which employ exact on-shell equivalence, have indicated that once one fixes the on-shell behavior and restricts the off-shell behavior to give the experimental E_T, breakup results at 14.4 MeV are insensitive to a residual off-shell dependence.

This paper examines the dependence of ^{2}H(p, pp)n reaction predictions for several potentials at energies between 14.4 and 65 MeV. The main focus is possible off-shell effects, i.e., effects not also associated with differences in two-nucleon scattering predictions. We perform our calculations with the separable potential methods of Ebenhöh,[19] but we consider several alternate separable potentials to the Amado[20] model Yamaguchi[21] potential. These potentials differ mainly in their momentum dependence, i.e., they have different form factors. Also, an energy-dependent modification of the two-nucleon (N-N) T matrix, described in earlier papers,[22,23] is employed to make some of the potentials nearly or exactly phase-shift-equivalent for two-nucleon scattering, which enables us to distinguish off-shell from on-shell effects. The different potentials correspond to changes in the total nuclear force, consistent with unchanged N-N scattering, in regions of three-body configuration space where both off-shell effects and three-body forces act. We then are really investigating the sensitivity to three-body effects. We consider the quasifree scattering (QFS) and final-state-interaction (FSI) regions of phase space as well as regions away from both of these processes. Our calculations suggest that measurements of the FSI angular distributions between 20 and 50 MeV are the most favorable for studying the off-shell N-N interactions. A portion of our conclusions has been reported and discussed in previous publications.[23-26]

II. POTENTIALS STUDIED

The primary aspect of the N-N force that we would like to examine is the role of off-shell behavior. This enters through the form factors $g(k)$ of the potential in separable models. However, when one changes $g(k)$ one not only changes the off-shell T matrix elements, $[\langle k|T(E)|k'\rangle]$ but one also changes the on-shell elements $[\langle k|T(k^2)|k\rangle]$. To isolate off-shell effects, one must consider potentials that give the same on-shell, or two-body scattering results. Another factor has been pointed out by Brayshaw.[18] He presents the hypothesis that deuteron breakup cannot lead to any off-shell information that is not already implicit in the N-d doublet scattering length ^2a or the triton binding energy E_T. In order to examine his hypothesis, we include potentials that differ off shell but give

TABLE I. Potential parameters.

Potential	HA1-8.3	HA2-8.3	HB2-8.3	HB2-11
k_c^2 (fm^{-2})	4.00	4.00	−2.89	−2.89
β_0 (fm^{-2})	3.318 06	3.318 06	1.541 30	1.541 30
β_1^{n-p} (fm^{-1})	2.246 63	2.246 63	1.374 49	1.374 49
β_1^{p-p} (fm^{-1})	2.286 94	2.286 94	1.390 32	1.390 32
γ (MeV^{-1})	0.333 333		0.50	0.50
E_0 (MeV)	25.0		−34.0	−40.0
E_1 (MeV)			−70.0	0.0
Δ_0	−0.098 1			
Δ_1^{n-p}	0.827 4			
Δ_1^{p-p}	0.707 3			
E_T (MeV)	8.3	8.3	8.3	10.9
^2a (fm)	1.314	1.294	1.020	−0.503
$\lambda_0 N_0^2$ (fm^{-3})	36.315	36.315	0.371 5	0.371 5
$\lambda_1 N_1^2$ $(n-p)$ (fm^{-3})	3.349 5	3.349 5	0.214 5	0.124 5
$\lambda_1 N_1^2$ $(p-p)$ (fm^{-3})	3.318 0	3.318 0	0.117 9	0.117 9

the same triton binding energy. In this way we are able to separate the dependence of breakup results on the off-shell behavior, the on-shell behavior, and the triton characteristics.

For separable potentials the momentum-space matrix elements are given by

$$V(k,k') \equiv \langle k | V | k' \rangle = -\lambda g(k) g(k') , \qquad (1)$$

where λ is the potential "strength" and $g(k)$ is the "form factor." Here we have suppressed angular momentum (we consider only S waves), spin and isospin labels. The T matrix is then

$$T(k,k';E) = \langle k | T(E) | k' \rangle = g(k) g(k') \tau(E) , \qquad (2)$$

where

$$\tau(E) = -\left[\lambda^{-1} + 4\pi \int_0^\infty \frac{q^2 dq g^2(k)}{(E - q^2 + i\epsilon)} \right]^{-1} .$$

In our investigations we employ two form factors of the type

$$g(k) = N(k_c^2 - k^2)/(k^2 + \beta^2)^2 \qquad (3)$$

which differ in their k_c^2 values: A form factor labeled HA which has $k_c^2 = 4$ fm^{-2}, and a form factor HB which has $k_c^2 = -2.89$ fm^{-2}. The strength parameter (λN^2) and the range parameter (β), which appear in Table I, are determined, in each spin isospin state, by the N-N effective range parameters.

The different off-shell momentum dependence of these form factors are illustrated in Fig. 1, which shows the Noyes-Kowalski half-shell functions[27] $f(p,k)$ for $k=0$ for each of the form factors. The half-shell function is merely the ratio of the half-shell T matrix element to the on-shell element, i.e., $f(p,k) = T(p,k;k^2+i\epsilon)/T(k,k;k^2+i\epsilon)$. This function is normalized to one at the on-shell point and is a useful indication of the off-shell behavior of any potential independent of the on-shell behavior. For separable potentials $f(p,k) \sim g(p)$. For comparison, we also include $f(p,k=0)$ for the Reid-soft-core (RSC)[28] and Yamaguchi (Y) potentials.[21] Note that the HB form factor is very similar to the Yamaguchi form factor off energy shell, while HA is similar to RSC. There is some devia-

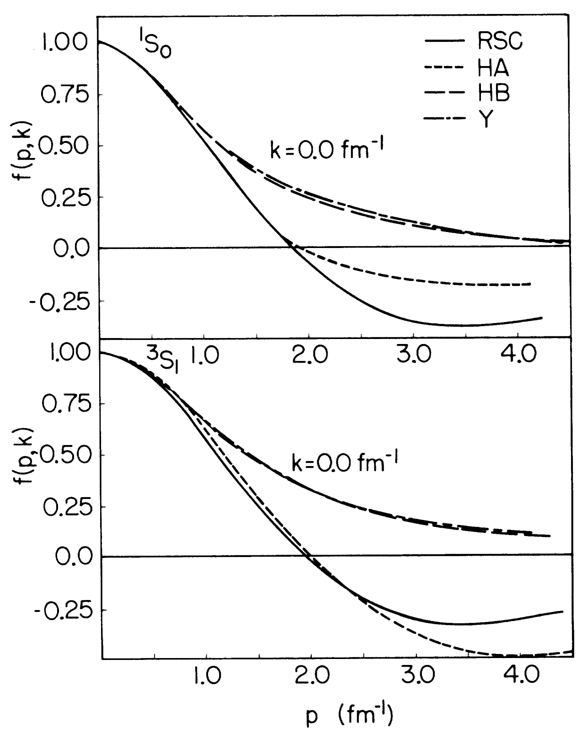

FIG. 1. The zero-energy Noyes-Kowalski half-shell function $f(p,k)$ for the HA, HB, and Yamaguchi (Y) form factors and for the Reid soft core potential (RSC). A comparison of cross sections using the HA and HB forms indicates the off-shell sensitivity.

tion between HA and RSC in the $^3S_1$ state even for $p \lesssim 1.5$ fm^{-1}; this deviation is due to the influence of the strong tensor force of the Reid potential on the $^3S_1$ T matrix. On the whole, however, the HA form factor gives off-shell behavior similar to that of potentials with strong short-range repulsion, while HB gives off-shell behavior like that of potentials with "softer" nonlocal short-range repulsion.

To accomplish phase-shift equivalence, we employ the methods of Ref. 22 by replacing $\tau(E)$ in Eq. (2) by

$$\tau'(E) = \rho(E)\tau(E)\{1 - 2i\pi^2 E^{1/2} T(E)[1 - \rho(E)]\}^{-1}, \tag{4}$$

where $T(E)$ is the on-shell T matrix element calculated with $g(k)$, and $\rho(E)$ is a real function of energy. The presence of the denominator in Eq. (4) means that the modification of Eq. (2) corresponds to multiplying the tangent of the phase shift $[\tan\delta(E)]$ by $\rho(E)$. Hence, the phase shift remains real and a unitary two-body amplitude is retained. To fit the on-shell T matrix to a given set of on-shell elements (or equivalently, phase shifts) one need only choose the appropriate $\rho(E)$. The freedom exists in Eq. (4) to extend this energy dependent modification to negative energies. Hence, we may use $\rho(E)$ at negative energies to alter the energy dependence of the negative energy T matrix and nuclear properties that depend on it, such as the triton binding energy.

We consider four potentials labeled HA1-8.3, HA2-8.3, HB2-8.3, and HB2-11 with the following characteristics (also see Table I)[29]:

1. For potential HA2-8.3, $\rho(E) = 1$. This potential is unchanged by $\rho(E)$ so that the predicted triton binding energy and two-body phase shifts are those given by the unmodified HA form factor.
2. For potentials HB2-8.3, HB2-11;

$$\rho(E) = 1 + \left(\frac{\tan\delta_{\mathrm{HA}}(E)}{\tan\delta_{\mathrm{HB}}(E)} - 1\right)(1 + e^{-\gamma(E-E_0)})^{-1} \quad \text{for } E > E_1 \tag{5}$$

$$= 1 \quad \text{for } E < E_1,$$

where $\delta_{\mathrm{HA}}(E)$ and $\delta_{\mathrm{HB}}(E)$ are the phase shifts predicted by the unmodified HA and HB form factors when $\rho(E) = 1$. For $\gamma(E - E_0) \gg 1$ (which occurs for all positive energies for the γ and E_0 parameters of Table I) this $\rho(E)$ gives potentials HB2-8.3 and HB2-11 virtually the same phase shifts as predicted by the unmodified HA form factor potential (HA2-8.3). The different γ, E_0, and E_1 parameters of HB2-8.3 and HB2-11 affect the negative energy T matrix [here $\delta(E)$ is the analytic continuation of the phase-shift function] and change the predicted triton binding energy. Furthermore, potentials HA2-8.3 and HB2-8.3 give (approximately) the same triton binding energy while having different form factors and off-shell behavior. In view of Brayshaw's hypothesis[18] we feel that it is important to examine whether fixing E_T necessarily eliminates all off-shell effects at energies above 14 MeV.

3. For potential HA1-8.3

$$\rho(E) = 1 + \left(\frac{E + \kappa^2}{E_0 + \kappa^2}\right)\Delta(1 + e^{-\gamma E})^{-1}, \tag{6}$$

where κ is the parameter of Ebenhöh[19] and is determined by the deuteron binding energy (in the triplet case $|E_d| = M\kappa/\hbar^2$) or the effective range parameters (in the singlet case). The choice of the Δ, γ, and E_0 parameters in Table I leads to p-p and n-p 90° (c.m.) cross sections in good agreement with experiment[30] up to 70 MeV (lab) (see Fig. 2).

All potentials are fitted to the same low energy scattering parameters as the Ebenhöh potential.[19] Therefore, $\rho(0) = 1$ and $\rho(-\kappa^2) = 1$ [except $\rho(0) \approx 1$ for potential HA1-8.3].

Our potential nomenclature can be summarized

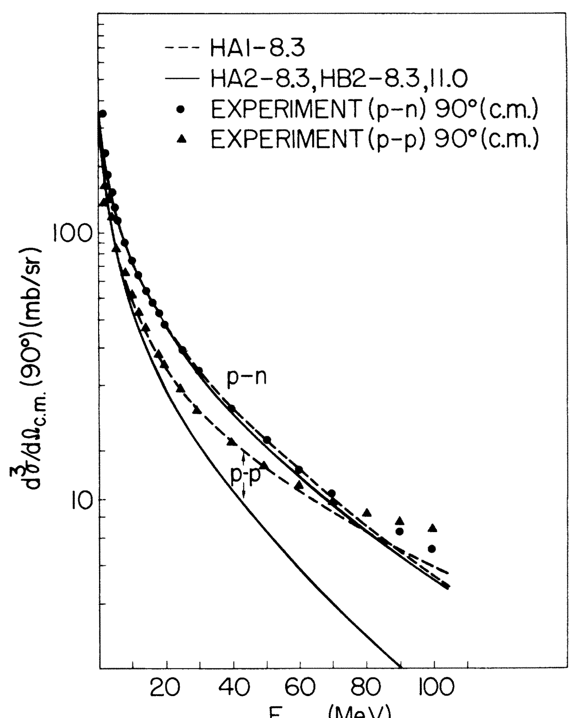

FIG. 2. The 90° c.m. p-p and p-n differential cross sections for the different separable potentials. The experimental results are calculated from the phase-shift analysis of MacGregor, Arndt, and Wright (Ref. 30). A comparison of cross sections using HA1 and HA2 checks on-shell sensitivity.

as follows: 1. The first letters indicate the form factor (HA or HB). 2. The first number indicates the set of phase shifts; 1 denotes experimental phase shifts; 2 denotes phase shifts predicted by HA form factor (i.e., HA2-8.3 potential). 3. The number after the hyphen indicates the approximate triton binding energy (E_T). Table I also lists E_T and ^2a for each potential.[31]

These potentials enable us to isolate various effects in the following ways. A comparison using the potentials HA1-8.3 and HA2-8.3 corresponds formally to only an on-shell change in the potential. However, on-shell effects are long-range effects on the wave functions and in this way are similar to other long-range effects such as higher partial waves, and Coulomb forces. Therefore, when calculated cross sections in a given portion of phase space are sensitive to the change from HA1-8.3 to HA2-8.3, they are likely to be sensitive to other long-range effects. Contrariwise, if certain cross sections are not sensitive to HA1 vs HA2, then it is probable that these cross sections are less sensitive to long-range effects.

With our potentials there are really two types of off-shell comparisons. The comparison between HA2-8.3 and HB2-11 corresponds mainly to a change in the momentum dependence of the T matrix. The $\rho(E)$ used in HB2-11 is quite mild and leads to a smooth energy dependence of the T matrix at both positive and negative energies (aside from the deuteron pole and the $^1S_0$ resonance). Consequently HA2-8.3 and HB2-11 differ off-shell in a way similar to other previously studied phase-shift-equivalent potentials[2,3,9,10,12] in that the principal change is in the momentum dependence of the T matrix. As with conventional off-shell changes in the N-N potential,[12] a change in the momentum dependence from that of HA2-8.3 to that of HB2-11 increases E_T. Comparisons between HA2-8.3 and HB2-11 can point out the types of breakup observables sensitive to conventional off-shell variations.

Comparisons employing HA2-8.3 and HB2-8.3 involve off-shell changes with the constraint of a fixed E_T. To force $E_T = 8.3$ MeV with the HB form factor, we had to change the negative energy dependence of HB2-8.3 from the smooth behavior of HB2-11. Our separable potential HB2-8.3 has been criticized[25] on the grounds that the energy dependence of the T matrix is unlike that obtained from conventional energy-independent potentials and leads to unusual analytic properties of the T matrix for $E<0$. Figure 3 illustrates the energy dependence of $T(0,0;E)$ for potentials HA2-8.3, HB2-11, and HB2-8.3 (in the $^3S_1$ state). The curve labeled PSE shows the off-shell matrix elements for HB2-8.3 if $\rho(E)$ did not have the exponential term or cutoff in Eq. (5), i.e., as if HA2-8.3 and

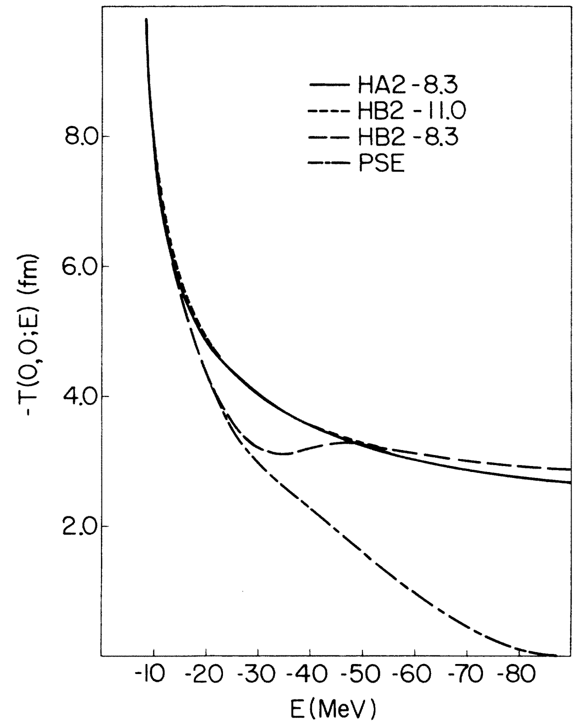

FIG. 3. The negative energy T matrix element $T(0,0;E)$ as a function of energy for the different separable potentials. The curve labeled PSE would be the result for potential HB2-8.3 if $\rho(E)$ in Eq. (5) did not contain the exponential factor or the cutoff at $E=E_1$.

HB2-8.3 were phase shift equivalent at all negative energies. It is the difference in the energy dependence of HB2-11 and HB2-8.3 that leads to the variation in triton binding energy from 10.9 to 8.3 MeV. This choice of procedure gives a test, alternative to Brayshaw's,[18,26] as to whether there is an explicit dependence of breakup results on E_T.

All curves in Fig. 3, except HB2-8.3, are smooth and analytic in that portion of the complex plane corresponding to the real values of the energy plotted.[32] As pointed out by Brayshaw,[25] the exponential in Eq. (5) leads to a series of poles in $T(k,k',E)$ for $\mathrm{Re}E = E_0 = -34.0$ MeV, $\mathrm{Im}E \neq 0$. In Fig. 3 this nonanalyticity shows up in the "dip" in the region where the HB2-8.3 curve connects the PSE and HB2-11 curves. The present calculation employs matrix inversion so that the T matrix is calculated at a finite number of energies along the real axis. It is possible to connect the T matrix at these values by an analytic function. The "dip" in HB2-8.3 is then really a feature of the unconventionality of our model. The issue is not the analyticity of our model but its unconventionality.

What physical significance does the unusual energy dependence of the potential HB2-8.3 have? This can be examined through the defect wave function $\chi_{\vec{k},E}(\vec{r})$ which describes the interaction of two particles under the influence of the nuclear medium,[2,33] or in the three-body case, the presence of the third particle. The defect wave function is defined as

$$\chi_{\vec{k},E}(\vec{r}) = \Psi_{\vec{k},E}(\vec{r}) - \varphi_{\vec{k}}(\vec{r}) , \tag{7}$$

where $\varphi_{\vec{k}}(\vec{r})$ is a plane-wave function with momentum $\vec{k}$, and $\Psi_{\vec{k},E}$ is a solution to the inhomogenous equation

$$(E - H_0)\Psi_{\vec{k},E} = (E - k^2)\varphi_{\vec{k}} + V\Psi_{\vec{k},E} . \tag{8}$$

The defect wave function is directly related to the completely off-shell T matrix by

$$\chi_{\vec{k},E}(\vec{r}) = \int \frac{d^3q T(\vec{q},\vec{k};E)}{E - q^2}\, \varphi_{\vec{q}}(\vec{r}) . \tag{9}$$

Unusual (or nonanalytic) behavior in the energy dependence of the T matrix will then lead to an unusual energy dependence in χ. For $E < 0$, $\chi_{\vec{k},E}(\vec{r})$ decays like $\sim e^{-\alpha r}$ where $\alpha = [(-M/\hbar^2)E]^{1/2}$. There are contributions that decay like $e^{-\beta r}$ from the form factor, but β is greater than α in the region of the unusual energy dependence observed in Fig. 3. Since the unusual energy dependence starts at $E \approx -30$ MeV (Fig. 3), $\alpha = 0.92$ fm^{-1}, and $\chi_{\vec{k},E}(\vec{r})$ is confined to approximately 1.2 fm. The unconventional energy dependence of the T matrix is, therefore, only relevant to the total interaction when two particles are close (less than ~1.2 fm) and the third particle is interacting with at least one of the pair, i.e., in the "interior" region described by Noyes.[34] Because of the likely presence of three-body forces,[12,35,36] the interaction in the interior may be qualitatively unlike that predicted by conventional two-body forces.

Returning to Fig. 3, it is evident that the short-range correlations [e.g., the short-range behavior of $\chi_{\vec{k},E}(\vec{r})$] of HB2-8.3 differ appreciably from those of a more conventional two-body force (HB2-11) only for $20 < -E < 50$ MeV. Comparisons of ^{3}He and ^{3}H charge form factor calculations with the data[9,12,36] indicate that the trinucleon wave functions have stronger short-range correlations than those predicted by two-body potentials that fit E_T. The origin of these stronger short-range correlations are believed to be three-body forces. Realistic three-body forces result mainly from processes involving two pion exchange and nuclear isobar [Δ(1236)] states.[35] The energy dependence of such processes are not as radical at the onset as is HB2-8.3 nor are they limited to a small energy region (like −20 to −50 MeV). They do, however, lead to different correlations than do two-body potentials. We speculate that perhaps the unusual behavior of HB2-8.3, if averaged over a large enough energy region, simulates the effects of a three-body force.

Potentials HA2-8.3 and HB2-11 differ in a way characteristic of off-shell changes in two-body potentials—a change in the off-shell momentum dependence leads to a change in triton binding energy.[11,12] Potentials HA2-8.3 and HB2-8.3 differ in a more exotic way—a change in the off-shell momentum dependence leads to no change in the triton binding energy. This behavior may well simulate a combination of two-body off-shell changes and three-body forces. The comparison of HB2-8.3 and HB2-11 does not correspond to a change in the form factor and hence these potentials do not differ in a way characteristic of conventional off-shell variations. In some sense, therefore, this comparison may give an estimate of the sensitivity of cross sections to three-body forces.

The main numerical advantage of our approach is that we consider separable potentials of only one term. Therefore, the currently available breakup codes, like Ebenhöh's,[19] can be very easily applied to such potentials. By choosing the appropriate $\rho(E)$ we can fix on-shell matrix elements and can isolate the effects of off-shell matrix elements in breakup calculations. The explicit effects of off-shell matrix elements in breakup calculations have not previously been examined within the context of potential models.

The potentials we propose differ in their interaction in regions of three-body coordinate space relevant to three-body effects (off-shell behavior and three-body forces), yet they are consistent with unchanged two-body scattering results. Any differences in calculated breakup amplitudes and cross sections obtained using these potentials indicate regions of phase space which are sensitive to changes in the short-range nuclear potential undetectable by two-body scattering.

III. BREAKUP AMPLITUDES

Before considering the effects these different potentials have on ^{2}H$(p,pp)n$ cross sections, we study their effects on breakup amplitudes. The breakup amplitudes $T_L^S(q_f\sigma_f, q_i\sigma_i)$ satisfy, for S-wave separable potentials, the integral equation[19]

$$T_L^S(q_f\sigma_f, q_i\sigma_i) = \frac{3\sqrt{3}}{2} k_L^S(q_i\sigma_i, q_f\sigma_f)\tau_{\sigma_f}(E - q_f^2) g_{\sigma_f}(p_f) + 4\pi \sum_{\sigma'} \int q'^2 dq' k_L^S(q_i\sigma_i, q'\sigma')\tau_{\sigma'}(E - q'^2) T_L^S(q_f\sigma_f, q'\sigma') , \tag{10}$$

where L is the total orbital angular momentum, S the total spin, and σ represents the spin-isospin quantum numbers of the two-nucleon subsystems ($\sigma=0\to{}^3S_1, \sigma=1\to{}^1S_0$). The total c.m. energy of the three-body system is E, with $p_f{}^2=E-q_f{}^2$. The kernel of the integral equation is given by

$$k_L^S(q\sigma,q'\sigma')=\frac{2\Lambda^S_{\sigma'\sigma}}{\sqrt{3}}\int_{-1}^{1}dx\frac{g_\sigma(q^2/3+4q'^2/3+4qq'x/3)g_{\sigma'}(4q^2/3+q'^2/3+4qq'x/3)P_L(x)}{3E/4-q^2-q'^2-qq'x}, \tag{11}$$

where the $\Lambda^S_{\sigma'\sigma}$ are the three-body spin-isospin recoupling coefficients. Breakup cross sections are given by[37]

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}=(\text{kinematic factors})\times[|M_{D_1}|^2+|M_{D_2}|^2+|M_Q|^2], \tag{12a}$$

where

$$\begin{aligned} M_{D1}&=\frac{\sqrt{3}}{12}T^{1/2}_{1_{np}}(1)-\frac{\sqrt{3}}{12}T^{1/2}_{1_{np}}(2)+\frac{\sqrt{3}}{12}T^{1/2}_{0_{np}}(1)-\frac{\sqrt{3}}{12}T^{1/2}_{0_{np}}(2),\\ M_{D2}&=\tfrac{1}{3}[T^{1/2}_{1_{pp}}(3)+\tfrac{1}{4}T^{1/2}_{1_{np}}(1)+\tfrac{1}{4}T^{1/2}_{1_{np}}(2)-\tfrac{3}{4}T^{1/2}_{0_{np}}(1)-\tfrac{3}{4}T^{1/2}_{0_{np}}(2)],\\ M_Q&=\frac{1}{\sqrt{6}}[T^{3/2}_{0_{np}}(1)-T^{3/2}_{0_{np}}(2)]. \end{aligned} \tag{12b}$$

The M_{D1}, M_{D2}, and M_Q amplitudes describe the scattering from the initial state to a final state where the total spins and p-p total spins are $(\frac{1}{2},1)$, $(\frac{1}{2},0)$, and $(\frac{3}{2},1)$, respectively. The Faddeev amplitudes $T^S_\sigma(j)$ are defined as

$$T^S_{\sigma_f}(j)=\sum_L \tfrac{1}{2}(2L+1)T^S_L(q_{jf}\sigma_f,q_i\sigma_i)P_L(\hat{q}_{jf}\cdot\hat{q}_i), \tag{13}$$

where

$$q_i=(E+\kappa_0^2)^{1/2} \text{ and } \sigma_i=0.$$

The n-p or p-p label in Eq. (12) describes the final interaction pair for each Faddeev amplitude. Both of the doublet p-p spin amplitudes (M_{D1},M_{D2}) have contributions corresponding to the final interaction being n-p singlet $(T^{1/2}_{1_{np}})$ and n-p triplet $(T^{1/2}_{0_{np}})$ since the p-p final spin states can be expressed as linear combinations of n-p final spin states. Later, when we refer to n-p and p-p Faddeev amplitudes[22] (Sec. IV) we simply mean $T^S_{\sigma_{np}}$ and $T^S_{\sigma_{pp}}$.

Figures 4(a)–(e) illustrate breakup amplitudes at 44.9 MeV incident proton energy for different potentials in various spin and angular momentum states.[38] In these figures p_f is the relative momentum of a two-body substate (referred to as d or d^*, depending on the spin coupling) in the final state. By conservation of energy, $p_f{}^2+q_f{}^2=E_{\text{c.m.}}$. The total spin is S, and L is the N-d (or N-d^*) relative orbital angular momentum, which, in the case of S-wave potentials, is the total orbital angular momentum. These plots exhibit two significant trends. The first concerns off-shell behavior while the second concerns on-shell behavior. First with regards to off-shell behavior, only the $S=\frac{1}{2}$, $L=0$ amplitudes vary appreciably if the potentials are phase shift equivalent (compare results for HA2-8.3, HB2-8.3, and HB2-11.0). The imaginary parts of the $S=\frac{1}{2}$, $L=0$ amplitudes are appreciably more sensitive than the real parts except for $p\approx 0$, where the real part is sensitive as well. The $p\approx 0$ amplitudes are important in the final-state-interaction (FSI) region of phase space.[39] We note that for $S=\frac{1}{2}$, $L=0$, fixing E_T as well as the phase shifts eliminates only about 30% of the off-shell differences between the HA and HB form factors (compare amplitudes of HA2-8.3 with HB2-8.3 and with HB2-11.0). Further calculations indicate that fixing E_T has greater effect in eliminating off-shell differences at lower energies (this eliminates about 60% of the off-shell effect at 14 MeV).

One can understand the relative sensitivity of breakup amplitudes in various spin and angular momentum states as follows. In the quartet state ($S=\frac{3}{2}$) the protons spins must couple to one (triplet); thus the wave function is in a spin-isospin symmetric state under proton exchange. Hence, the three-nucleon final state spatial wave function must be antisymmetric under proton exchange. The protons then have a very low probability of approaching closely to each other. The Pauli principle here prevents a situation where all three nucleons can be simultaneously close, thus excluding configurations where off-shell effects should be strongest. For $S=\frac{3}{2}$, this Pauli "repulsion" occurs not only in the final state, but also in intermediate state scattering since our interactions conserve total spin thus restricting the p-p system to always have spin one. The above Pauli principle arguments also apply to final states where the p-p spin is one but $S=\frac{1}{2}$; here, however, intermediate scatterings to p-p spin zero are able to occur since n-p interactions, while conserving total spin, can connect three-body states with different p-p spin. The Pauli principle arguments, therefore, only partially apply to amplitudes describing breakup to the $S=\frac{1}{2}$ p-p spin one final state (i.e., the M_{D1} amplitudes, which are certain linear combinations of $S=\frac{1}{2}$ amplitudes of Fig. 4). The Pauli principle arguments do not apply to the

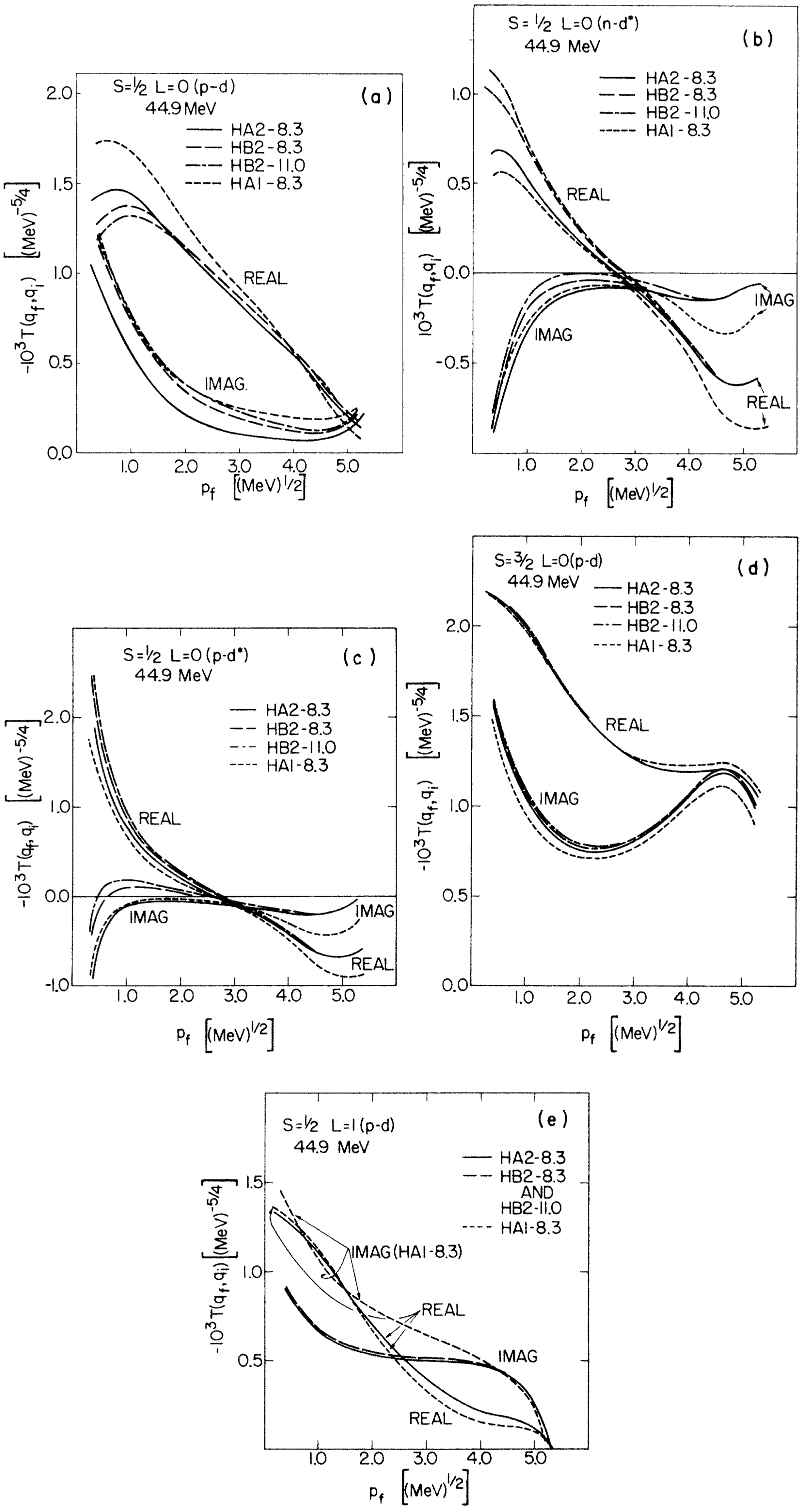

FIG. 4. (a)–(e) Breakup amplitudes in various spin and angular momentum states. S is the total spin, while the label (N-d) or (N-d^*) indicates, respectively, the spectator particle and the reference two-body subsystem (d if n-p triplet, d^* if n-p or p-p singlet). The relative momentum of the two-body subsystem in the final state is p_f while L is the N-d or (N-d^*) orbital angular momentum.

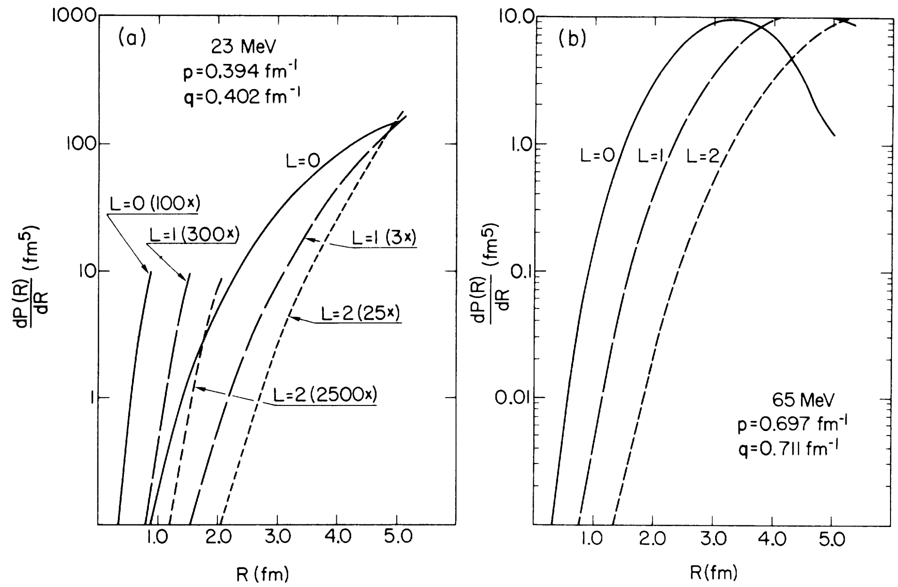

FIG. 5. The differential probability of three particles having a hyperspherical radius R, as given by Eq. (14), for unperturbed partial waves with different N-d angular momenta (L). Figure 5(a) is for 23 MeV and Fig. 5(b) is for 65 MeV.

$S=\frac{1}{2}$ p-p spin zero final state, which is described by the M_{D2} amplitude. Later in Sec. IV, we illustrate the M_{D1}, M_{D2}, and M_Q ($S=\frac{3}{2}$) contributions to breakup cross sections for various potentials. We find the M_{D2} amplitude sensitive to off-shell effects in those regions of phase space where it dominates the cross section. Sizable sensitivity occurs for the M_{D1} amplitude over the FSI region of phase space. The M_Q amplitude is not affected by off-shell changes, thus reflecting the operation of the Pauli principle in both final and intermediate states.

With respect to orbital angular momentum, the centrifugal barrier tends to prevent a nucleon from approaching the center of mass of the final d (or d^*) for states with $L\neq 0$; so in these states it is less probable that all three particles can be near each other. The centrifugal barrier operates in intermediate states as well as final-state scattering since, for S-wave interactions, L is the total orbital angular momentum and is conserved by central forces. Our above arguments and calculations indicate that breakup amplitudes are sensitive to off-shell effects only in states where all three particles can approach each other, i.e., the $S=\frac{1}{2}$, $L=0$ states.[40]

An illustration of the role of different angular momentum states is obtained by calculating the mutual closeness of the three particles as a function of L. A coordinate that measures this "mutual closeness" is the hyperspherical radius R where

$$R^2=\frac{2}{3}\sum_{i<j}(x_i-x_j)^2.$$

For unperturbed wave functions, the probability that the three particles with angular momentum L have a hyperspherical radius between R and $R+dR$ is given by

$$\frac{dP(R)}{dR}\sim\frac{1}{4}\int_0^{\pi/2}R^5 j_0^{\,2}(pR\cos\Phi)j_L^{\,2}(qR\sin\Phi)\sin^2 2\Phi\, d\Phi\,. \tag{14}$$

Figure 5 illustrates $dP(R)/dR$ of the $L=0,1,2$ unperturbed partial wave functions for $p\approx q$ at 23 and 65 MeV. As seen in Fig. 5, wave functions with $L=0$ are much more likely to have all three particles very close together ($R\lesssim 2$ fm). These are also the states where three-body forces should be the strongest,[36] which may suggest an intrinsic difficulty in disentangling the effects of off-shell behavior and three-body forces in breakup.

The second trend is that all amplitudes, including quartet and $L>0$ are sensitive to on-shell changes. (Compare HA1-8.3 and HA2-8.3.) Since on-shell changes involve the long-range internucleon wave functions, the arguments limiting off-shell sensitivity for quartet and $L>0$ are inapplicable. The results we show in this paper suggest that on-shell differences between potentials should affect breakup cross sections in all regions of phase space while off-shell differences would only show up in

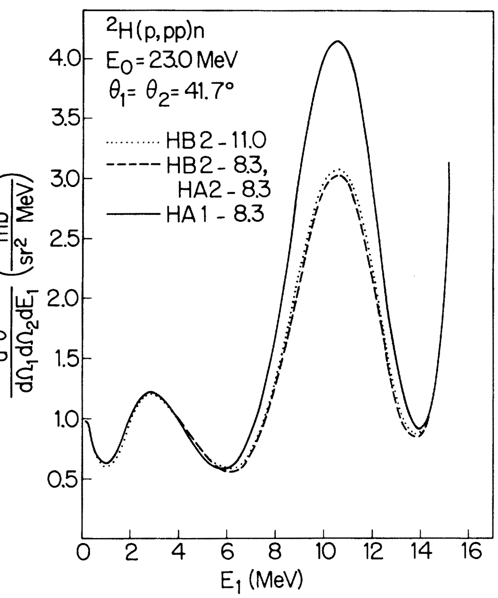

FIG. 6. The symmetric angle quasifree spectrum at 23 MeV as predicted by the different separable potentials. The out-of-plane angle $\varphi = 180°$ for all spectra unless indicated otherwise.

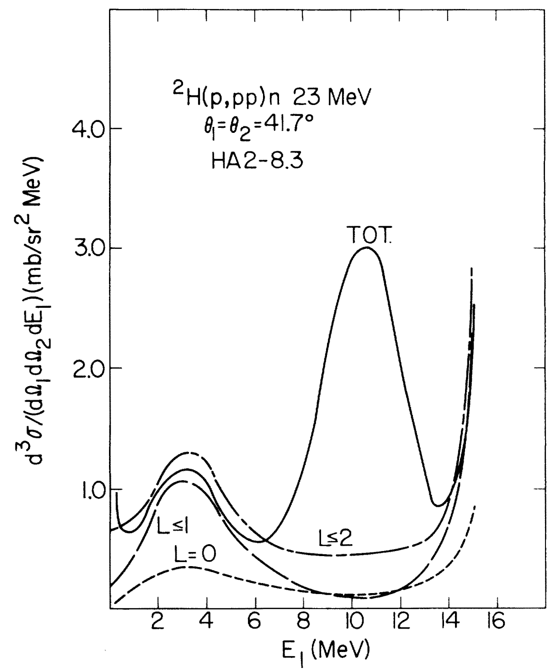

FIG. 7. Symmetric angle QFS spectrum at 23 MeV as calculated retaining only the $L=0$, $L\leq 1$, $L\leq 2$, and all (TOT) Faddeev amplitudes for potential HA2-8.3.

restricted regions (i.e., where $S=\frac{1}{2}$, $L=0$ is important).

IV. BREAKUP CROSS SECTIONS

What effects do changes in the amplitudes have on calculated cross sections? We first look at quasifree scattering (QFS) since this region of phase space has been thoroughly examined experimentally. Figure 6 illustrates the potential dependence of the ^{2}H$(p,pp)n$ spectrum for $\theta_1=\theta_2 = 41.7°$ at 23.0 MeV. The spectrum is independent (to within about 2%) of potential as long as the potentials differ only off shell (e.g., compare HA2-8.3, HB2-8.3, and HB2-11.0). On-shell sensitivity is larger (compare HA1-8.3 and HA2-8.3). The sensitivity (or lack thereof) displayed in Fig. 6 also occurs for all QFS spectra we have investigated regardless of the angle or projectile energy.

We can understand the lack of off-shell sensitivity as follows. We have shown that the only breakup amplitudes sensitive to off-shell effects are the doublet $(S=\frac{1}{2})$, $L=0$ amplitudes. Now, the quasifree enhancement, when viewed in terms of N-d (or N-d^*) partial waves, occurs because of the coherent contributions of many partial waves. This statement essentially reiterates the belief that QFS is a "peripheral" process. Figure 7, which illustrates the $L\leq 0,1,2$ contributions for HA2-8.3 to the QFS 41.7-41.7 cross sections at 23.0 MeV, demonstrates just how insignificant the $L=0$ amplitude is for QFS. With the $L=0$ contribution so small for QFS, not surprisingly, QFS cross sections are very insensitive to off-shell changes.

On-shell changes affect all partial waves and thus have large effects in QFS. The importance of on-shell behavior can be seen in Fig. 8, which illustrates the beam energy dependence of QFS for potentials HA1-8.3 and HA2-8.3. Potential HA1-8.3 has an energy dependence that nearly predicts the free two-nucleon 90° scattering, although remaining only S wave, while HA2-8.3 predicts a considerably smaller proton-proton cross section. Most of the data shown are for the nearly quasifree angles 43°-43°.[41] At 14 MeV, the predictions for this angle pair are 65% of those at the pure quasifree angles 39.4°-39.4°.

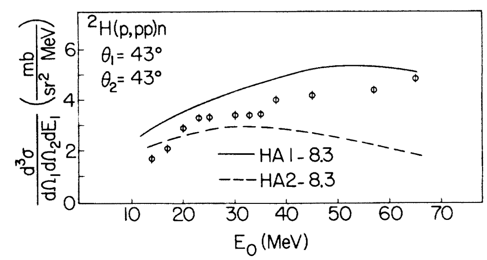

FIG. 8. The peak cross section at symmetric angles plotted as a function of energy for two different separable potentials. The experimental points are from Ref. 41.

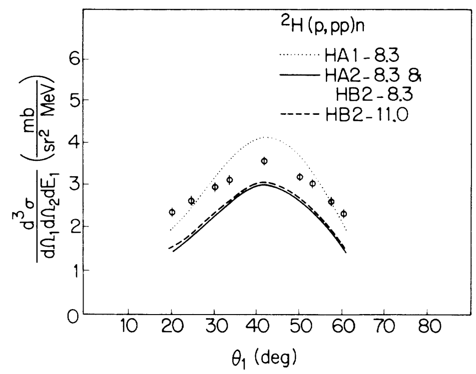

FIG. 9. The peak cross section at quasifree angles at 23 MeV. The experimental points are from Ref. 22.

In a previous publication,[22] the authors noted that S-wave Yamaguchi potentials give QFS angular distributions whose shapes were potential-independent but not in agreement with experiment. One suggestion was to consider off-shell variations by varying the form factor. Figure 9 shows that the QFS angular distribution is form-factor-independent. This is not surprising in view of the insensitivity of QFS spectra. All of our S-wave separable potentials, regardless of off-shell or on-shell behavior, predict about the same shape for the QFS angular distribution, which is in disagreement with experiment. The lack of sensitivity to off-shell changes, both in magnitude and in shape, and the sensitivity to on-shell or long-range changes in N-N wave functions, indicate that this particular phase of deuteron breakup needs the

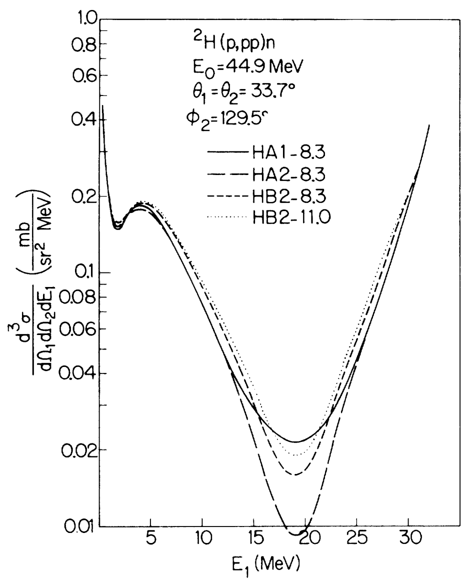

FIG. 10. The predicted spectrum for several separable potentials at an out-of-plane angle sensitive to the M_{D2} amplitude.

inclusion of higher partial waves and other long-range effects for its description. The comparison of theory and experiment in Fig. 9 indicates that these additional effects may be angle-dependent. The on-shell variation between potentials HA1 and HA2 is probably large enough so that the angle-dependent part of the nuclear interaction, which is mainly an on-shell effect, will not produce cross sections grossly outside their range. The sensi-

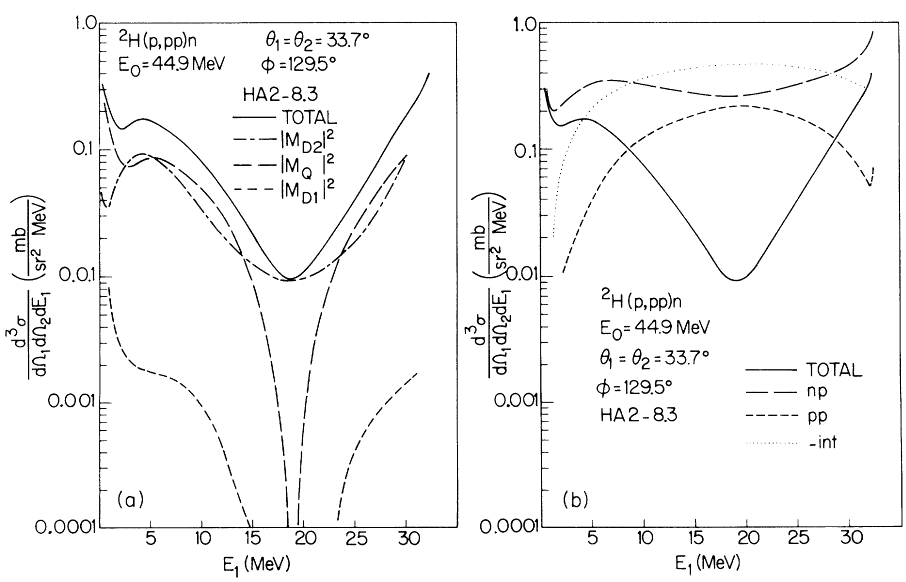

FIG. 11. The spectrum of Fig. 10 decomposed: (a) according to M_{D1} M_{D2}, and M_Q contributions; (b) according to n-p, p-p, and interference contributions.

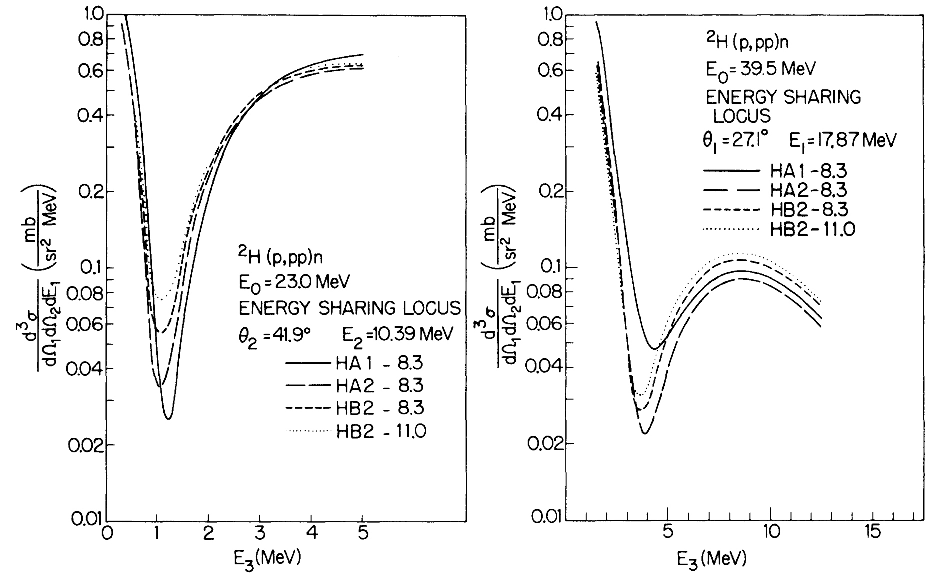

FIG. 12. (a), (b) Predicted cross sections along the energy sharing locus suggested by Jain, Rogers, and Saylor (Ref. 42). Figures 10 and 12 show the deep interference minimum.

tivity of this region to on-shell changes indicates the reason why the original Ebenhöh code calculations gave poor agreement with quasifree scattering data. The Ebenhöh potential is close to the HB2-11 potential, which does not predict the free p-p cross section.

Since it has been hypothesized that the $L=0$, $S=\frac{1}{2}$, M_{D_2} breakup amplitudes are sensitive to off-shell effects,[42] one should investigate regions of phase space where these amplitudes dominate the cross section. In Fig. 10 we plot predictions of ^{2}H$(p,pp)n$ cross sections in one of the regions suggested by Kloet and Tjon.[17] Figure 11(a) illustrates the contributions of the M_{D_1}, M_{D_2}, and M_Q amplitudes to the cross sections for potentials HA2-8.3 and HB2-8.3 Figure 11(b), which depicts the contributions to the cross sections from Faddeev amplitudes corresponding to final p-p and n-p rescattering and interferences between the two, indicates the interference nature of the minimum.

Although the M_{D_2} amplitude dominates at the minimum, the cross section is determined by both on-shell and off-shell properties of the potential. The cross section variation is larger for isolated on-shell changes (HA2-8.3 vs HA1-8.3). Similar sensitivity to on-shell variations is evident in Fig. 12 which shows ^{2}H$(p,pp)n$ cross sections at 23 and 39.5 MeV on the energy sharing locus of Jain, Rogers, and Saylor.[42] These loci again examine the interference region dominated by the M_{D_2} amplitude. A spectrum from a region in which the $L=0$ doublet amplitude is important, but away from the interference region is shown in Fig. 13. The contributions of the M_{D1}, M_{D2}, and M_Q amplitudes are shown in Fig. 14(a), while Fig. 14(b) shows the contributions to the cross section of n-p and p-p Faddeev amplitudes and interference. Figure 14(c) illustrates the cross section obtained from the $L\leq 0$, 1, and 2 amplitudes for HA2-8.3 and indicates the relative importance of the $L=0$ amplitude in this spectrum.

Figures 10, 12, and 13 all show regions where relatively large sensitivity to off-shell changes occur. Unfortunately, the cross sections are small, and careful observation of the differences would require extensive beam time. In fact, for the $L=0$ amplitude to dominate, the cross section most probably must be small since one would be

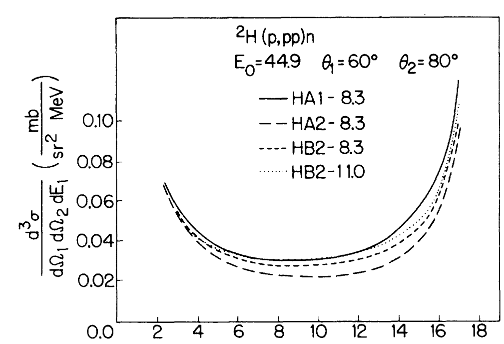

FIG. 13. Predicted cross section for $\theta_1 = 60°$, $\theta_2 = 80°$, and $E_0 = 44.9$ MeV. This is in a part of phase space away from FSI and QFS peaking and also away from the interference minimum.

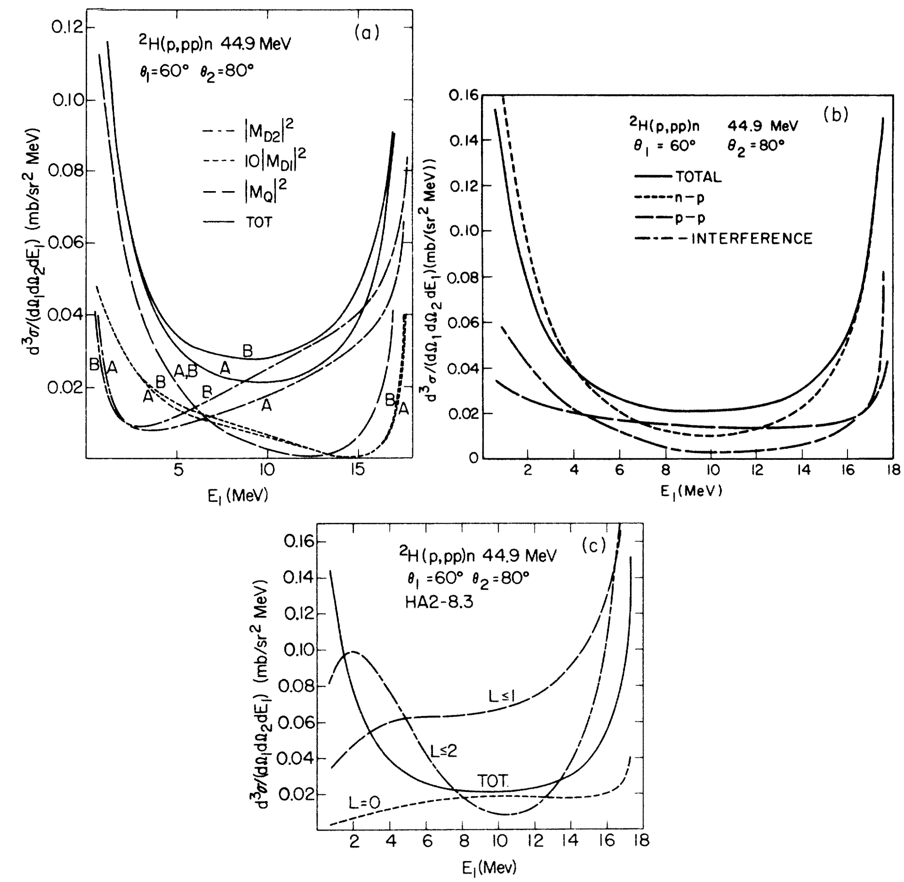

FIG. 14. Various decompositions of the calculated spectrum for $\theta_1 = 60°$, and $\theta_2 = 80°$ at 44.9 MeV. Where curves A and B are indicated, A refers to potential HA2-8.3 while B refers to HB2-8.3, otherwise the curves are for potential HA2-8.3. (a) Decomposition by p-p spin amplitudes M_{D_1}, M_{D_2}, and M_Q; (b) Decomposition by final N-N interaction amplitude (n-p and p-p) and interference; (c) cross sections as a function of the maximum L value retained.

automatically ruling out the mechanism that accounts for the QFS enhancement (i.e., the coherent contributions of many partial waves). These regions are also very sensitive to changes in the on-shell portion of the potential as one can see by comparing potentials HA1-8.3 and HA2-8.3 in Figs. 12(a) and 12(b). These factors make any unambiguous decisions about off-shell potentials difficult if one does not have an accurate on-shell description. Furthermore, the inclusion of higher partial wave N-N interaction effects and Coulomb forces, even if relatively small over most of phase space, could have a large percentage effect on a small cross section. This would be especially true in the interference region where a small relative change in the n-p or p-p amplitudes [Fig. 11(b)] could greatly alter the shape of the minimum. Conceivably, higher partial waves could fill in the minimum for any potential thus largely negating the advantage of this type of experiment. Away from the interference region, higher partial waves might not be so critical, but the featurelessness of the spectrum (Fig. 13) subjects such experiments to both statistical and normalization problems.[43] Therefore, at present it would be very difficult to get any definitive off-shell information from these regions of phase space.

The investigation of another region of phase space has been suggested by Lambert *et al.*[44] He makes the intriguing point that there is another geometrically significant region besides the FSI and QFS regions. This is the small portion of phase space in which the three particles after the reaction are in a collinear condition; i.e., one particle is moving forward with the velocity of the center of mass, always between the two particles moving outward. It seems possible that in this geometry, three-body effects might be modified; for example, a "shielding" of meson exchange. The collinearity condition occurs only at one portion of the locus for an appropriate pair of angles, so that the experimentally observable effect would

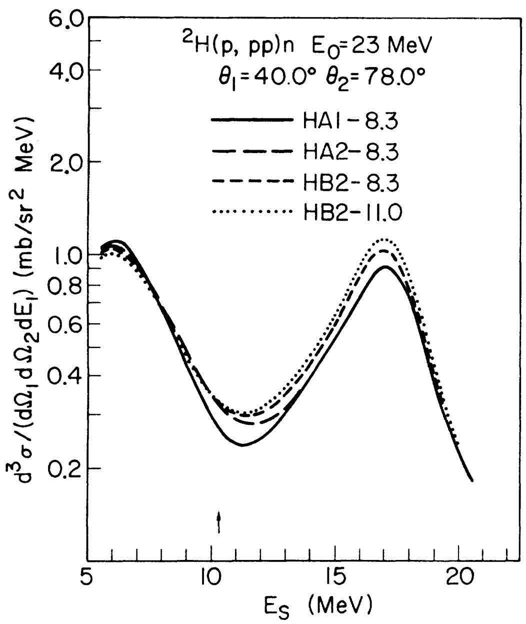

FIG. 15. Predicted cross section at a pair of angles suggested by Lambert (Ref. 44). In this case the spectrum is projected along the locus (E_s) rather than on the E_1 energy axis. The arrow indicates the kinematic situation at which the three emerging particles are collinear.

be a dimple in the experimental cross section. If one has two theoretical curves with differing three-body force contributions, the data might briefly shift from one of these curves to the other. We can use our potential to estimate the size of such an effect. If the colinearity condition occurs under a situation where HA2-8.3 and HB2-8.3 differ, then there could be an experimentally observable effect; while if there is no difference between these two potentials, the three nucleons do not get close enough to each other for three-body forces to be active. Figure 15 shows the calculated cross section for one such pair of angles, and the arrow indicates the region of the spectrum of interest. As in most regions of phase space there is on-shell sensitivity, but very little difference between the two potentials that are sensitive to three-body effects. This conclusion is consistent with the experimental results.[45]

There is a region of phase space where the $L=0$, $S=\frac{1}{2}$ amplitudes are important and the cross section is reasonably large. For the final-state interactions the enhancement in the cross section is due to the $^1S_0$ resonance in the n-p (or p-p) scattering amplitude at zero relative energy. Unlike QFS, this enhancement does not rely on the coherent contributions of many partial waves and possibly the $L=0$, $S=\frac{1}{2}$ contribution can be large. Furthermore, as we recall from Fig. 4, both the real and

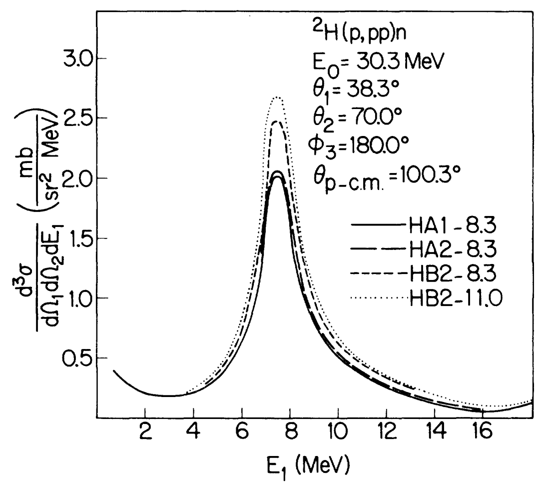

FIG. 16. The predicted spectrum at the final state interaction peak. Note the difference in predictions by the potentials that differ off-shell (HA2 vs HB2).

imaginary parts of the $S=\frac{1}{2}$, $L=0$ breakup amplitudes are sensitive to off-shell effects at $p\approx 0$.

Figure 16 illustrates results for a typical n-p FSI spectrum at 30.3 MeV. A large sensitivity occurs between the phase-shift-equivalent potentials HA2-8.3 and HB2-11 at the FSI peak. Since the relative importance of the $L=0$ amplitude varies with production angle, which is also the angle of the third particle in the c.m. system, both the

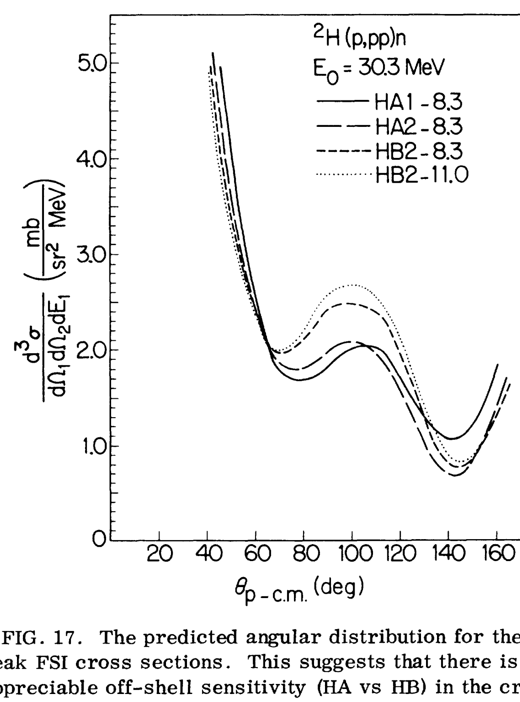

FIG. 17. The predicted angular distribution for the peak FSI cross sections. This suggests that there is appreciable off-shell sensitivity (HA vs HB) in the cross sections for the angles between 60° and 120°, but little on-shell sensitivity (HA1 vs HA2).

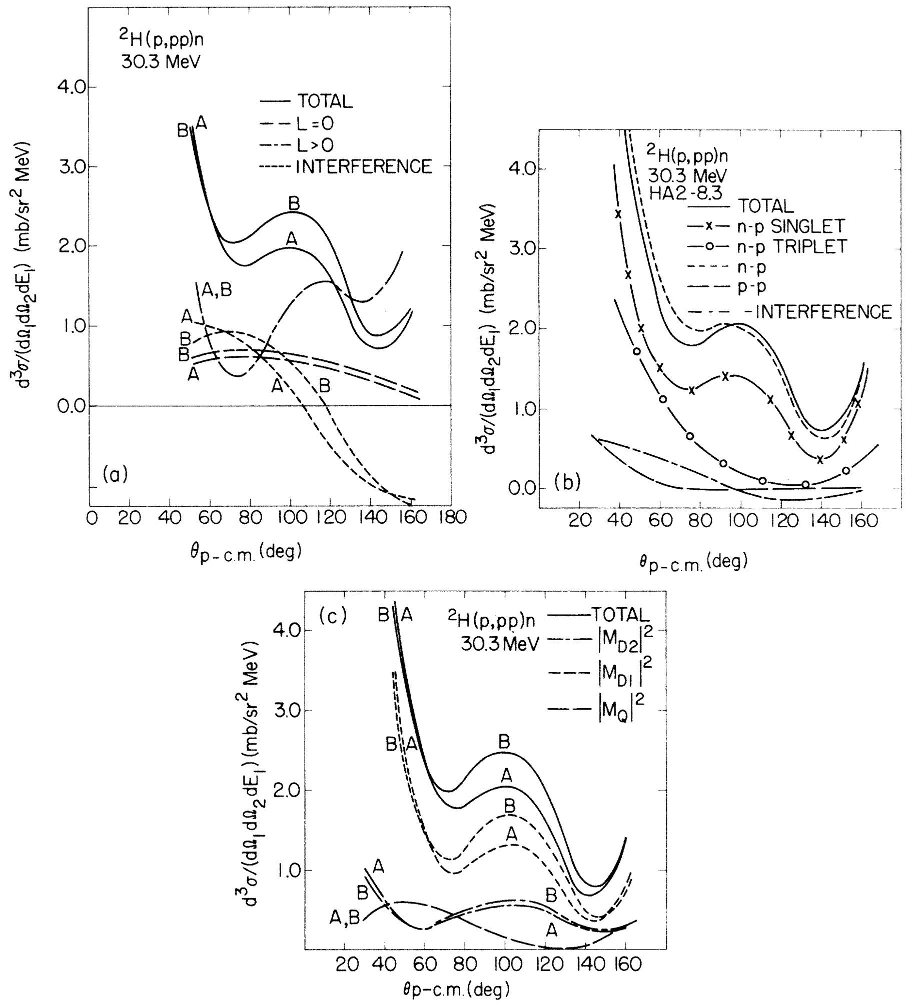

FIG. 18. Various decompositions of the calculated 30.3 MeV n-p FSI angular distributions. Curves A correspond to potential HA2-8.3, while curves B correspond to potential HB2-8.3. (a) Decomposition into cross section calculated for $L=0$ amplitudes only, $L>0$ amplitudes only, and interference. (b) Decomposition by final N-N interaction amplitude [n-p singlet, n-p triplet, n-p (singlet or triplet)], interference between n-p and p-p. (c) Decomposition by p-p spin amplitudes (M_{D1}, M_{D2}, M_Q).

magnitude and shape of FSI angular distributions should be sensitive to off-shell effects. Figure 17, which illustrates the FSI angular distribution at 30.3 MeV for various potentials, does indeed show changes in the magnitude and shape under off-shell changes in the force (HA2-8.3 vs HB2-11). These differences in magnitude largely remain even in the fixed E_T comparison (HA2-8.3 vs HB2-8.3). The reason for the sensitivity observed in Fig. 17 becomes apparent in Fig. 18(a), which shows the contributions of the $L=0$ and $L>0$ amplitudes and the interference between the two for potentials HA2-8.3 and HB2-8.3.

The $L=0$ contributions alone are neither very large (about 20–30% of the total over most angles) nor very sensitive to off-shell changes. However, the interference between the $L=0$ and $L>0$ amplitudes is sensitive to off-shell effects. As one can observe from Fig. 4, and as evident in Fig. 18(a), off-shell changes in the potential primarily affect the phase of the $L=0$ amplitude in the FSI region ($p_f \approx 0$). The different interferences of the $L=0$ and $L>0$ amplitudes under off-shell changes accounts for the variations in magnitude and shape of the FSI angular distributions. In Fig. 18(b) we show the cross sections decomposed according to its n-p, p-p, and interference terms. In this case, unlike the case of Fig. 10, the interference term has a very small effect on the cross sections in the region of interest. The n-p singlet term [i.e., the contributions due to $T^{1/2}_{1np}$ terms in Eq. (12b)] dominates the region sensitive to off-shell changes.

One surprising aspect of the FSI results was the relative sensitivities of the M_{D1}, M_{D2}, and M_Q contributions to off-shell effects. These contributions appear in Fig. 18(c). Here the M_{D1} contribution is the largest one, and is also the amplitude most sensitive to off-shell effects. As argued in Sec. III, the Pauli principle operating on the final state described by the M_{D1} amplitude (p-p triplet) would tend to limit off-shell sensitivity; however, for $S=\frac{1}{2}$ intermediate transitions to p-p singlet are allowed for which off-shell sensitivity is not limited by the Pauli principle.

The above observations indicate that we have off-shell sensitivity when the two protons interact strongly in the intermediate state and a n-p pair interact strongly in the final state. For this situation to arise, all three particles must have been momentarily interacting strongly and the cross sections are sensitive to three-body effects.

Figure 19 illustrates FSI angular distributions at a number of energies.[46] The maximum off-shell sensitivity is quite large at the higher energies (compare HA2-8.3 with HB2-11). At 14 MeV the effect of a change of form factor is considerably reduced by a constraint on E_T. At higher energies, however, even the constraint on E_T still allows changes in magnitude and shape under off-shell changes. The maximum sensitivity occurs between

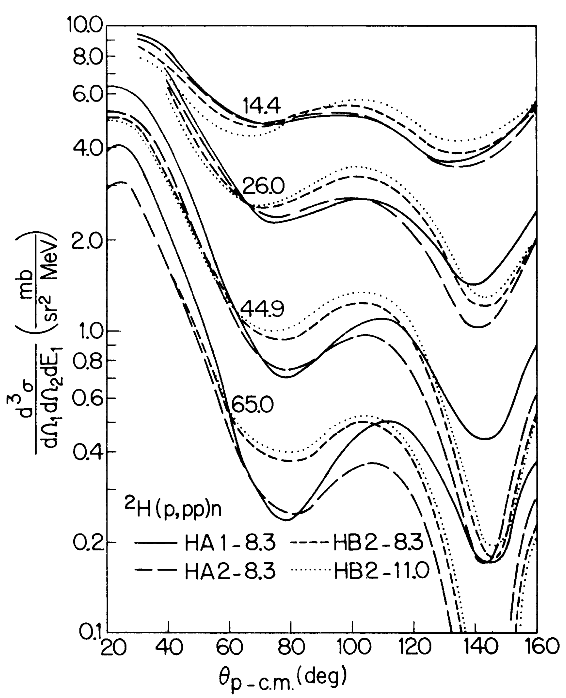

FIG. 19. The predicted angular distribution of the peak FSI cross section for several energies. The sensitivity to off-shell changes is low at low energies, but increases with increasing energy.

60° and 130° and is large enough, even for fixed E_T, that the difference in cross section is easily experimentally observable. The change in shape as well as magnitude improves the experimental distinguishability of the potentials. This result indicates that Brayshaw's remarks[18] on the connection between E_T and breakup results may apply only at very low projectile energies (compare the 45-MeV curve with the 14.4 MeV curve).

V. DISCUSSION

The N-d breakup problem is very similar to the N-N scattering problem. Low energy (<15 MeV) N-N scattering can be described very well in terms of the two-body binding energy (or scattering length) and effective range. It is only as one goes to higher energies that the true complexity of N-N scattering and of nuclear forces becomes evident. Similarly the N-d situation at low energies can apparently be described in terms of the triton binding energy (or ^2a) and the two-body phase shifts. It is only at energies above 20 MeV that the complicating aspects of the three-nucleon problem are approachable.

If one is to study the three-body problem in order to understand off-shell effects and three-body forces, one must approach the problem in several steps. First, one must find calculational methods that are capable of isolating the three-body effects from the predominant two-body effects, then one must find the appropriate experiments, perform them, and attempt to understand them.

According to Amado[47] the minimal approach to the three-body problem consistent with general principles is a separable potential with unit form factor. To go beyond the minimal approach discussed by Amado, one must specify some dynamical input. In separable potentials the dynamics are prescribed by the form factor, which is what we vary in our calculations. We thus examine in a simple way the effects of allowable two-body dynamical variations on the three-nucleon system.

The simplest three-body calculations with separable potentials are those that utilize only S-wave N-N interactions. These calculations are inexpensive yet reproduce the gross features of the two-body and three-body scattering data. As shown in the present work, S-wave calculations display the importance of fixing known features of the nuclear force (i.e., two-body cross sections, triton-binding energy). We found that the FSI region was the most favorable for investigating the off-shell N-N interaction on the basis of calculations with S-wave separable potentials.

Our model neglects higher N-N partial waves and takes no explicit account of Coulomb interactions, so that it is reasonable to expect some disagreement with experiment. The effect of the higher partial waves on the size of off-shell effects should be small. On the basis of the present calculations, off-shell effects do not show up except in states where the three nucleons can be mutually close together. Thus, in a more complete model,[14] we might expect any off-shell effects arising from two-body P and D waves to be suppressed by the centrifugal barrier between the two nucleons interacting at a given time. Since the N-N interaction is still mostly S wave at the energies considered, we would expect these higher partial waves not to grossly alter the size of the cross sections nor the off-shell differences observed in FSI angular distributions. The presence of the tensor force might have some effect on the size of off-shell effects. In the case of nuclear matter the nature of the tensor force is such as to actually magnify the off-shell differences between potentials.[3] Of necessity, our conclusions are tentative pending more complete three-body calculations. We argue that strong off-shell effects in the FSI region would persist in a more complete three-body model. S-wave calculations appear to be trustworthy enough to pick out sensitive regions of phase space and to guide immediate experimental investigations.

VI. CONCLUSIONS

This work was undertaken with the aim of examining deuteron breakup reactions with regard to the possibility of extracting information about the nucleon-nucleon interaction in the off shell energy region. From our calculations, this does indeed appear to be possible. Certain regions of phase space are sensitive to the off-shell nature of the interaction. With the additional off-shell constraint of a fixed triton binding energy, the off-shell sensitivity is only reduced significantly for projectile energies less than 20 MeV. However, since the fixed E_T result was obtained with a somewhat unconventional interaction, it should be checked by more conventional methods such as phase-shift-equivalent energy-independent potentials or Brayshaw's boundary condition approach. Comparisons should be carried out at energies above 14 MeV since even our calculations indicate little residual off-shell sensitivity beyond that resident in E_T for breakup cross sections at very low energies.

We have observed that breakup amplitudes are sensitive to the off-shell N-N interaction in only those states where all three nucleons can approach each other closely. Only the doublet $(S=\frac{1}{2})$ $L=0$ states meet this requirement. However, on-shell differences affect amplitudes in all states. As a result, the QFS region of phase space is insensitive to off-shell variations since the $L=0$ amplitudes are an unimportant contribution to the cross section. On-shell changes do have large effects on QFS spectra. Neither off-shell nor on-shell effects play any role in altering the shape of the QFS angular distributions as long as the potential is S wave; one should look to p-wave, tensor force, or Coulomb effects to correct disagreement with experiment here.

Far from QFS or FSI sensitivity may occur, but the cross sections are necessarily small and potentials would be hard to differentiate experimentally. Only in the FSI region does it seem that off-shell differences would be easily experimentally detectable. This paper suggests that measurements of the FSI angular distributions between 20 and 50 MeV would be a fruitful way to distinguish potentials that differ only off-shell.

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[30]We fit the parameters of Eq. (6) to a set of S-wave phase shifts that would be needed to predict the experimental 90° c.m. cross sections for p-p and n-p scattering. The experimental cross sections were calculated from the phase-shift analysis of MacGregor, Arndt, and Wright ["MAW"—M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. 169, 1128 (1968); 173, 1272 (1968); 182, 1714 (1969)] which included higher partial waves and Coulomb effects.
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[32]The energy-independent potentials only give singularities at the deuteron pole. Potential HB2-11 has a discontinuity at $E=0$, but this discontinuity is numerically minute. Miniscule discontinuities occur also for both HB2 potentials at $E=E_1$ [Eq. (5)]. Also, according to Eq. (5), with $E_0, E_1 \to -\infty$,

$$T_{\text{PSE}}(k,k';E)\ \frac{T_{\text{HA}}(\sqrt{E},\sqrt{E};E)}{T_{\text{HB}}(\sqrt{E},\sqrt{E};E)}\ T_{\text{HB}}(k,k';E).$$

Therefore, since $g_{\text{HA}}(-\beta^2) \to \infty$ T_{PSE} has a pole at $E = (-\hbar^2/M)\,\beta_{\text{HA}}{}^2 = -457$ MeV. This pole is not even remotely approached for HB2-8.3 due to the cutoff factors in Eq. (5).
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[38]The $T(q_f, q_i)$ of Fig. 4 are the on-shell breakup amplitudes

$$T_L^S\left[q_f\sigma_f;\ (E+K_0^2)^{1/2}\ \sigma=0\right],$$

where $T_L^S(q_f\sigma_f;\ q\sigma)=(3\sqrt{3}/8)T_{iL}(q,\sigma)$ of Eq. (17) of Ref. 19. Our $q_f\sigma_f$ is Ebenhöh's $q_i\sigma_i$.

[39]The Ebenhöh code explicitly calculates the amplitude at 22 values of $p_f^2(q_f^2=E-p_f^2)$ none of which corresponds exactly to $p_f^2=0$. The lowest positive value of p_f^2 is $\frac{9}{1600}E_{\mathrm{c.m.}}$, while the smallest negative value of p_f^2 is $-0.16\,E_d$. The amplitude is fitted to a function of q_f^2 with 12 parameters which determine the amplitude at any given p_f^2. Then the fitted function is used for the amplitudes in the cross-section calculations. Later in the paper we will show the cross sections at $p_f=0$.

[40]Since only the $L=0$ amplitudes have any off-shell sensitivity, we assume that the $L>0$ amplitudes for the HB2 potentials are exactly the same as those predicted by HA2-8.3 for the purpose of calculating cross sections in Sec. IV. This procedure saves a vast amount of computer time.

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