Dependence of the width of a doorway state on the ratio of the numbers of the correlating and open channels*

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The width $\langle \Gamma_d \rangle$ of a doorway state, the escape width $\langle \Gamma_d \uparrow \rangle$, the damping width $\langle \Gamma_d \downarrow \rangle$, and the interaction $\langle V_d \rangle$ which causes transitions from the doorway state to the more complicated states are discussed as a function of the ratio (n_d/n) between the numbers of the correlating and open channels. The results show that the widths $\langle \Gamma_d \rangle$, $\langle \Gamma_d \uparrow \rangle$, $\langle \Gamma_d \downarrow \rangle$, and the interaction $\langle V_d \rangle$ depend strongly on the square of this ratio.

[NUCLEAR REACTIONS Discussed $\langle \Gamma_d \rangle$, $\langle \Gamma_d \uparrow \rangle$, $\langle \Gamma_d \downarrow \rangle$, $\langle V_d \rangle$.]

The relation between the escape width $\langle \Gamma_d \uparrow \rangle$ from a doorway state and the ratio n_d/n of the numbers of correlating and open channels has been discussed by Hsu,[1] and the relationship has been also confirmed experimentally by Hsu *et al.*[2,3] Due to the agreement between theoretical and experimental results, the ratio $(n_d/n)_{\text{expt}}$ which is determined experimentally by a limited number of the open channels can exactly take the place of the theoretical ratio $(n_d/n)_{\text{theor}}$ in the formulation of the relation between the escape width and the ratio. In the present paper, I derive the relationships between $(n_d/n)_{\text{expt}}$ and the width of a doorway state $\langle \Gamma_d \rangle$, the damping width $\langle \Gamma_d \downarrow \rangle$, and the interaction V_d which causes transitions from the doorway state to the more complicated states. Finally, I also discuss those quantities as functions of various variables.

By assuming (i) the compound nuclear reaction proceeds in a high-excitation region where many levels are overlapping, (ii) the channel correlation is in fact a result of a doorway state, that is, the ratio $(n_d/n)_{\text{expt}} \neq 0$, I derived[1] the following:

$$|A_{\mu d}|^2 = \frac{2D/\pi \langle \Gamma_\mu \rangle}{(n_d/n)^2_{\text{expt}} \, C_{cc'}{}^2} , \tag{1}$$

where (i) $|A_{\mu d}|^2$ is the probability that the doorway state is present in a compound state at excitation energy E_μ, (ii) D is the average level spacing, (iii) $\langle \Gamma_\mu \rangle$ is the average total level width, (iv) $C_{cc'}{}^2$ is the channel correlation coefficient, and (v) (n_d/n) is the ratio of the numbers of the correlating and open channels. Suppose the wave function for the doorway state Ψ_d is present with some amplitude $A_{\mu d}$ in the compound state ψ_μ, then

$$\psi_\mu = A_{\mu d} \Psi_d + \sum_q B_{\mu q} \psi_q , \tag{2}$$

where ψ_q is the wave function describing all more complicated excited states. Assuming that the interactions V_{dq} which cause transitions from Ψ_d to the ψ_q are independent of q, Lemmer[4] finds that the widths of the compound states are given by

$$\Gamma_\mu = |A_{\mu d}|^2 \Gamma_d \uparrow , \tag{3}$$

where the $\Gamma_d \uparrow$ is the escape width of the doorway state Ψ_d, and

$$|A_{\mu d}|^2 = \frac{|V_d|^2}{(E_\mu - E_d)^2 + |V_d|^2 + (\pi |V_d|^2/D)^2} . \tag{4}$$

The distribution of $|A_{\mu d}|^2$ in the above equation is a Lorentzian distribution. It tells us that the width of the distribution, i.e. damping width $\Gamma_d \downarrow$, is given by the dominant term in the denominator of Eq. (4), either $|V_d|^2$ or $(\pi |V_d|^2/D)^2$. In the present case, $\langle \Gamma_\mu \rangle \gg D$, we expect the $(\pi |V_d|^2/D)^2$ term to dominate, the $\Gamma_d \downarrow$ is therefore

$$\Gamma_d \downarrow = 2\pi \frac{|V_d|^2}{D} . \tag{5}$$

Putting $E_\mu = E_d$ in Eq. (4) and $C_{cc'}{}^2 = 1$ in Eq. (1), and combining Eqs. (1), (3), (4), and (5), we obtain the following relations:

$$\langle V_d \rangle = \left[\left(\frac{n_d}{n}\right)^2_{\text{expt}} \frac{\pi \langle \Gamma_\mu \rangle}{2D} - 1 \right]^{1/2} \frac{D}{\pi} , \tag{6}$$

$$\langle \Gamma_d \downarrow \rangle = \left(\frac{n_d}{n}\right)^2_{\text{expt}} \langle \Gamma_\mu \rangle - \frac{2D}{\pi} , \tag{7}$$

$$\langle \Gamma_d \uparrow \rangle = \left(\frac{n_d}{n}\right)^2_{\text{expt}} \frac{\pi \langle \Gamma_\mu \rangle}{2D} \langle \Gamma_\mu \rangle . \tag{8}$$

From the above equations, it is clear that the widths $\langle \Gamma_d \rangle \equiv \langle \Gamma_d \uparrow \rangle + \langle \Gamma_d \downarrow \rangle$, $\langle \Gamma_d \uparrow \rangle$, $\langle \Gamma_d \downarrow \rangle$ and the interaction $\langle V_d \rangle$ depend strongly upon the ratio $(n_d/n)^2_{\text{expt}}$. Hence, an accurate value of the ratio (n_d/n) is needed to determine those quantities. However, it is experimentally not possible to measure the exact ratio (n_d/n), and normally the data

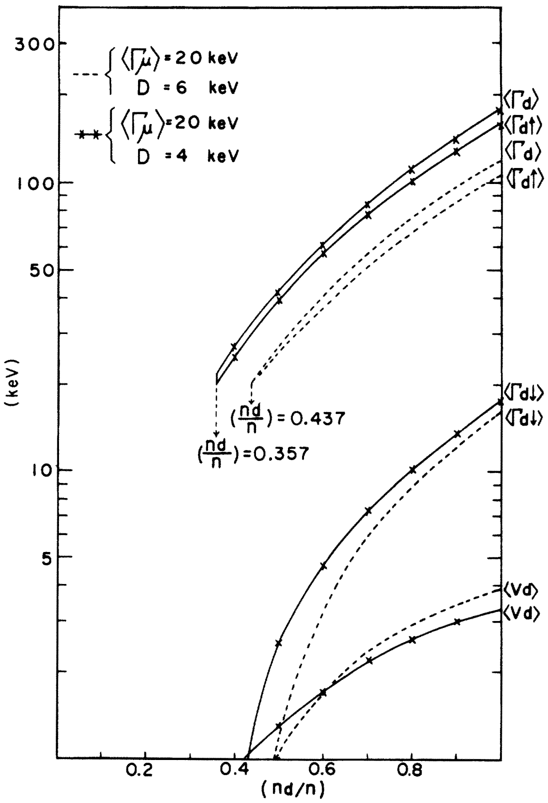

FIG. 1. The quantities $\langle\Gamma_d\rangle$, $\langle\Gamma_d\uparrow\rangle$, $\langle\Gamma_d\downarrow\rangle$, and $\langle V_d\rangle$ as a function of (n_d/n). The dashed and crossed curves are the cases of $\langle\Gamma_\mu\rangle = 20$ keV, $D=6$ keV and $\langle\Gamma_\mu\rangle$ $=20$ keV, $D=4$ keV, respectively.

are obtained only for a limited number of low-lying levels. An experiment with a large number of open channels should be performed in order to obtain an experimental ratio $(n_d/n)_{\rm expt}$ as close to $(n_d/n)_{\rm theor}$ as possible. Since Eq. (8) has been already confirmed experimentally by Hsu *et al.*,[2,3] the ratio $(n_d/n)_{\rm expt}$ can exactly take the place of the theoretical ratio $(n_d/n)_{\rm theor}$; Eqs. (7) and (6) can be used to calculate $\langle\Gamma_d\rangle$, $\langle\Gamma_d\downarrow\rangle$, and $\langle V_d\rangle$ if the $(n_d/n)_{\rm expt}$, $\langle\Gamma_\mu\rangle$, and D are determined.[5-7] Since $|A_{\mu d}|^2 \le 1$,[4] $n_d \le n$, and $C_{cc'}{}^2 \approx 1$[1] in Eq. (1), we obtain

$$(2D/\pi\langle\Gamma_\mu\rangle)^{1/2} \le (n_d/n)_{\rm expt} \le 1\,. \tag{9}$$

In order to satisfy $C_{cc'}{}^2 \simeq 1$, Eq. (9) has to be satisfied by the condition of $\langle\Gamma_\mu\rangle \gtrsim D$ according to Feshbach, Kerman, and Lemmer.[8] From Eq. (9), it can be understood that $\langle\Gamma_\mu\rangle/D$ cannot be smaller than $2/\pi$.[1] If $\langle\Gamma_\mu\rangle/D$ is equal to $2/\pi$, then (n_d/n) must be 1 and the $\langle V_d\rangle$ and $\langle\Gamma_d\downarrow\rangle$ must vanish. The escape width $\langle\Gamma_d\uparrow\rangle$ would be equal to the total average level width $\langle\Gamma_\mu\rangle$, and $\langle\Gamma_d\rangle = \langle\Gamma_d\uparrow\rangle = \langle\Gamma_\mu\rangle$ if the doorway state exists in the excited energy region of the experiment. Combining Eqs. (6)–(8), we obtain

$$\frac{\langle\Gamma_d\uparrow\rangle}{\langle|V_d|^2\rangle} \approx \frac{\pi\langle\Gamma_\mu\rangle}{D^2}\,, \tag{10}$$

$$\frac{\langle\Gamma_d\uparrow\rangle}{\langle\Gamma_d\downarrow\rangle} \approx \frac{\pi\langle\Gamma_\mu\rangle}{2D}\,. \tag{11}$$

The above equations show that the escape width $\langle\Gamma_d\uparrow\rangle$ is much larger than the quantities of $\langle\Gamma_d\downarrow\rangle$ and $\langle V_d\rangle$, since $\Gamma_\mu > D$. These results are reasonable and consistent with those of Feshbach, Kerman, and Lemmer.[8] Otherwise the escape width could not be found in the experiments. In Fig. 1, we have plotted $\langle\Gamma_d\rangle$, $\langle\Gamma_d\uparrow\rangle$, $\langle\Gamma_d\downarrow\rangle$, and $\langle V_d\rangle$ as a function of (n_d/n). The dashed and crossed curves correspond to the cases of $\langle\Gamma_\mu\rangle = 20$ keV, $D=6$ keV and $\langle\Gamma_\mu\rangle = 20$ keV, $D=4$ keV, respectively. Since we require $(n_d/n) \ge (2D/\pi\langle\Gamma_\mu\rangle)^{1/2}$, dashed and crossed curves are cut off at $(n_d/n) = (0.437)$ and (0.357), respectively. In order to satisfy this condition, the decay width $\langle\Gamma_d\uparrow\rangle$ has to be greater than or equal to the total average width $\langle\Gamma_\mu\rangle$, i.e., $\langle\Gamma_d\downarrow\rangle$ and $\langle V_d\rangle$ have to be small, otherwise we could not distinguish the decay width from the result of the random fluctuation. Finally, I would conclude that (1) the widths $\langle\Gamma_d\rangle$, $\langle\Gamma_d\uparrow\rangle$, $\langle\Gamma_d\downarrow\rangle$, and $\langle V_d\rangle$ depend strongly on the (n_d/n), (2) the width $\langle\Gamma_d\rangle$ approaches $\langle\Gamma_d\uparrow\rangle$, i.e., $\langle\Gamma_d\downarrow\rangle$ and $\langle V_d\rangle$ become small, when the ratio $\langle\Gamma_\mu\rangle/D$ becomes large, and (3) the above results are valid only if $(2D/\pi\langle\Gamma_\mu\rangle)^{1/2} \le (n_d/n)_{\rm expt}$ ≤ 1 and $\langle\Gamma_\mu\rangle \gtrsim D$.

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