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Communications

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Critique of explanations of the anomalous large-angle scattering of alpha particles*

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Several alternative analyses of anomalous large-angle scattering data for elastic scattering of α particles from nuclei are illustrated. Statistically equivalent fits to the data are achieved by various parametrizations of the optical potential. These fits though statistically insignificant in the sense that $\chi^2/N \gg 1$ are comparable to or better than those produced by different representations of the potential. We therefore question the ability to reach detailed conclusions about any microscopic model of the processes giving rise to anomalous large-angle scattering.

[NUCLEAR REACTION Anomalous large-angle elastic scattering, α particles $E = 15–30$ Mev. Optical model analyses.]

A problem of some 10 years standing in the study of the elastic scattering of α particles is the so-called anomalous large-angle scattering (ALAS).[1,2] The problem exists not because of lack of solutions but because of a surfeit of reasonable ones and a lack of precise, complete solutions. This comment illustrates several new fits, or fits equivalent to earlier work, to several sets of α scattering data. It utilizes a parametrization of the exchange phenomena thought to be present in ALAS.[3] This parametrization and the derived fits lead to some improvement upon earlier analyses; however, it is just the fact that fits equivalent to earlier analyses can be made without meeting an adequate criterion that leads to the inability to decide on the basic underlying processes behind ALAS.

To illustrate this criticism use is made of a simple modification to a usual optical model code to include a representation of exchange effects. Greenlees and Tang[4] suggested that exchange effects could be simulated by the introduction of a term in the potential with the exchange properties of a Majorana exchange operator which "space exchanges" the incident particle and the target nucleus. The basis of this suggestion was the work of Thompson and Tang[5] on nucleon-α-particle scattering. A significant step was made by Kondō *et al.*[6] with the introduction of an additional term, to the usual four or six parameter representation of the optical model, to represent exchange processes. They argued that such processes could be represented by adding a term which was surface peaked and had a simple two-body Majorana exchange term $(-1)^L V_{ex}$. Such a term is only a convenient representation of exchange processes as shown in Refs. 5 and 6. As such it does not represent all exchange processes to all orders. As in any optical model analysis it is therefore possible that the higher order processes can be represented by an imaginary term. We therefore modify the exchange representation of Ref. 6 by making it complex and of the form;

$$V = V_N + V_c + V_{ex}, \tag{1}$$

$$-V_N = V_{0c} f(x_R) + iW_{0c} f(x_I), \tag{2}$$

$$-V_{ex} = (-1)^L (V_{0x} + iW_{0x}) \begin{Bmatrix} f(x_x) \\ \text{or} \\ g(x_x) \end{Bmatrix}, \tag{3}$$

$$\begin{aligned} V_C &= \left(3 - \frac{r^2}{R_c^2}\right) \frac{Zze^2}{2R_c}, \quad r \le R_c \\ &= \frac{Zze^2}{r}, \quad r \ge R_c, \end{aligned} \tag{4}$$

where

$$f(x_i) = \frac{1}{1+e^{x_i}};$$

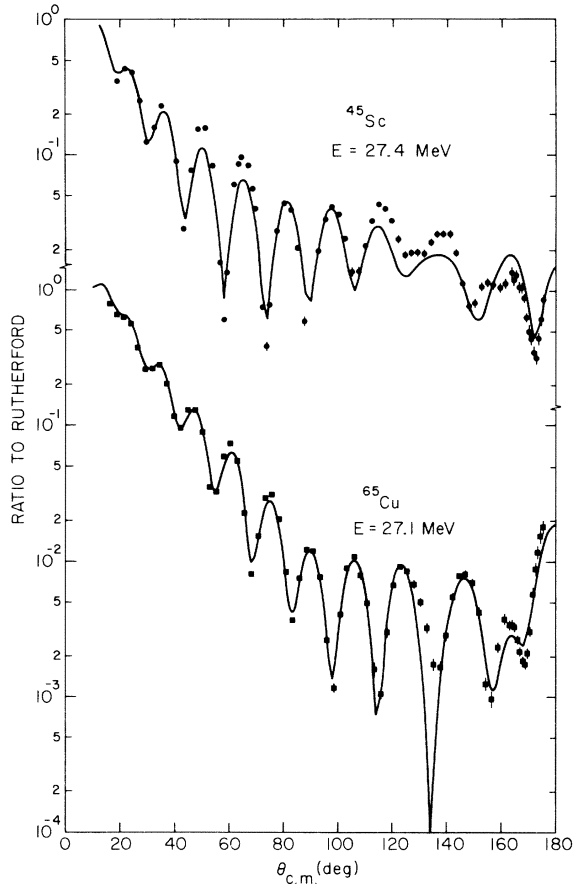

FIG. 1. The best fit prediction and comparison with the 27 MeV elastic α-particle scattering experimental data of Ref. 8 for ^{45}Sc and ^{65}Cu using the optical model as formulated by Eqs. (1)–(4) with the parameters shown in Table I.

and

$$x_i = \frac{r - R_i}{a_i}, \quad g(x_i) = \frac{e^{x_i}}{[1+e^{x_i}]^2},$$

$$R_i = r_{0i} A^{1/3}, \quad R_c = r_c A^{1/3}.$$

In this case $L\hbar$ is the incident α-particle nucleus orbital angular momentum, and i indicates the real R, imaginary I, or the exchange X potential.

Earlier work of Eberhard[7] used an L-dependent optical potential to represent a density of states effect. He argued that nuclei may not contain a sufficient density of levels, for the grazing angular momenta present in α-particle interactions, to use the usual Woods-Saxon representation of the absorption.

Also within the framework of modified optical model approaches is the work by Mailandt, Lilley, and Greenlees[8] who achieved a rather good fit to the data using a real optical well derived from folding an effective α-nucleon potential with the nucleon density factor of the form, $f(x)$, incorporating mean square radii and rounding parameter a, as determined by muonic x rays. They then treated these parameters as phenomenological and searched on them to further improve the quality of the fit to the data of Ref. 8 for ^{45}Sc and ^{65}Cu. The quality of our fits, shown in Fig. 1, is comparable to that in Ref. 8. This statement is based on the values in Table I of the reduced χ^2, γ^2:

$$\gamma^2 = \chi^2/N = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{\sigma_{\exp}(\theta_i) - \sigma_{\rm calc}(\theta_i)}{\Delta\sigma(\theta_i)}\right]^2 \tag{5}$$

with N equal to the total number of experimental points. Table I lists the parameters of the best fit potential as well as several other attempts which

TABLE I. Various optical model parameters in the notation of Eqs. (1)–(4) used in the calculations discussed in the text producing the fits to the data illustrated in Figs. 1 and 3. All energies are in MeV and all lengths in fm (10^{-13} cm).

	^{65}Cu Surface peaked	Vol. term	^{45}Sc Surface	Surface	^{40}Ca Volume	Surface
E_α	27.094	27.094	27.398	29.00	29.0	29.0
V_{0c}	136.08	142.00	150.1	154.25	155.13	154.36
r_{0R}	1.51	1.477	1.406	1.424	1.526	1.424
a_R	0.523	0.543	0.632	0.587	0.487	0.588
W_{0c}	28.28	21.44	15.75	20.48	19.56	20.45
r_{0I}	1.649	1.578	1.783	1.479	1.699	1.480
a_I	0.269	0.43	0.361	0.149	0.115	0.144
V_{0x}	132.6	15.23	107.3	29.18	42.54	30.39
W_{0x}	−129.6	1.67	8.86	0.092	8.77	0.0
r_{0x}	1.475	0.908	1.398	1.488	1.411	1.484
a_x	0.199	0.0459	0.327	0.182	0.40	0.187
r_c	1.116	1.116	1.113	1.116	1.116	1.116
γ^2	14.8	44.9	42.7	333	933	327
$J/4A$	527.3	518.0	505.5	532.9	626.2	533.6

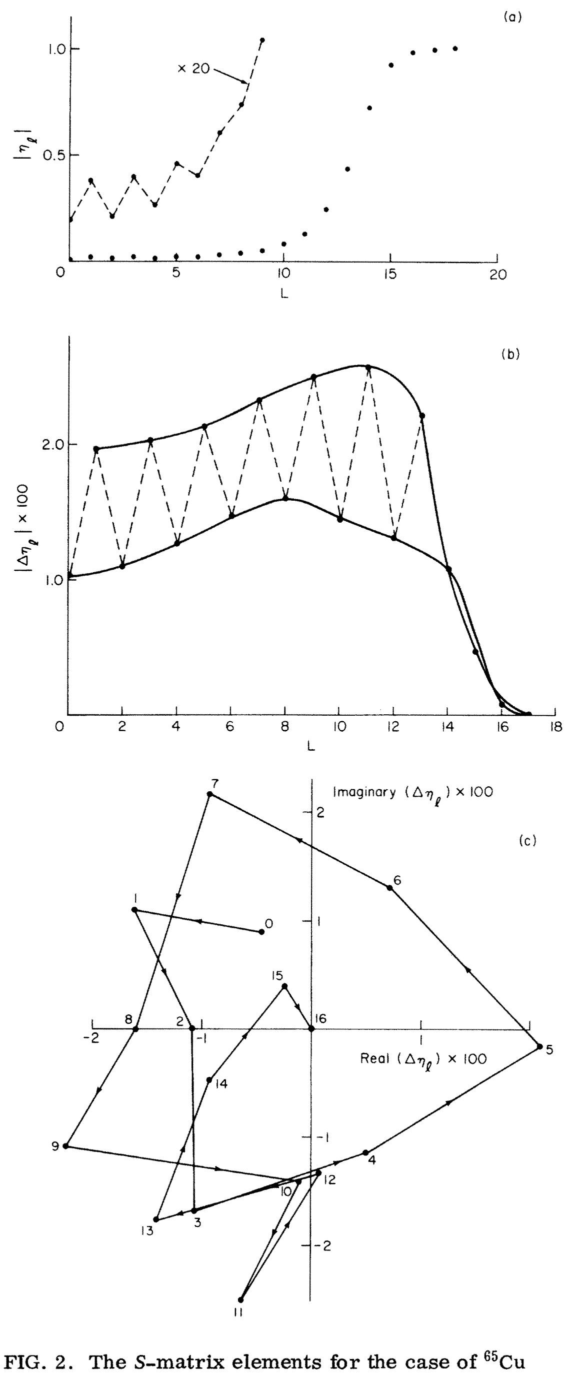

FIG. 2. The S-matrix elements for the case of ^{65}Cu shown in Fig. 1. (a) The S-matrix element as a function of l, the incident particle partial wave. (b) The $|\Delta_l|$ where $\Delta\eta_l$ is defined by Eq. (6) of the text. (c) The Argand plot of $\Delta\eta_l$.

are of interest. We emphasize that, as can be seen from Table I for ^{45}Sc and ^{65}Cu, this form of the potential achieves a γ^2 which, while still quite far from unity, is of the same quality as the results of a folding calculation.

The magnitude of the imaginary part of the exchange potential is seen to be much smaller for ^{45}Sc than ^{65}Cu and the large angle scattering is seen to have a smaller back angle increase. This, as well as the negative sign (sourcelike rather than absorptive) of the imaginary part of the exchange potential for ^{65}Cu, suggests that the usual formulation of the optical potential, as expressed in the first two terms of Eq. (1) serve to represent the absorption of the incident particle, but that we are explicitly taking exchange processes into account by allowing for a source term. While such a remark is speculative since there is no basic derivation of either an optical model representation of the exchange process (or for that matter of the optical model parameters associated with the so-called direct term), the surface peaked nature of this term is reasonable from the argument of Agassi and Wall[3] where an explicit calculation of a particular exchange amplitude was made. Figure 2(a) shows the S-matrix elements $\eta_l \equiv e^{2i\delta_l}$ for the best fit complex surface derivative potential used in fitting the ^{65}Cu data. If should be noted that even with the source term the values of $|\eta_l|$ do not exceed 1, thereby manifestly maintaining unitarity.

A closer examination of η_l for ^{65}Cu cases calculated with and without V_{ex} contains a somewhat surprising result. In Figs. 2(b) and 2(c) we show the quantity $\Delta\eta_l$ where

$$\Delta\eta_l = n_l^{(V_{ex})} - \eta_l^{(\text{no } V_{ex})}. \tag{6}$$

That is, $\Delta\eta_l$ is the difference of the S-matrix elements for the same potential of Eqs. (2) and (4) with and without the presence of the l-dependent term, Eq. (3), representing exchange processes. Figure 2(b), showing $|\Delta\eta_l|$ vs l, shows the pronounced alternation which might be expected from the $(-1)^L$ factor. Similar behavior has been seen in low partial waves without exchange, but this does not extend to the surface partial waves, nor is the alternation with respect to adjacent partial waves. Figure 2(c) has a somewhat surprising result. The particular point to be noted is that the sense of rotation of $\Delta\eta_l$ in the complex plane starts off counterclockwise, but at l values near the surface, l, L_0 ($L_0 \sim 10$ in this case) the rotation reverses direction. This is quite different from the result calculated in distorted wave Born approximation (DWBA) in Ref. 3 in which the exchange contribution rotated clockwise in a manner which was similar to that of the Regge poles introduced by Cowley and Heymann[9] and analyzed very thoroughly by McVoy.[10] Another difference is that the low partial waves, as well as those in the vicinity of L_0, have similar magnitudes $|\Delta\eta_l|$; that is there seems to be no particular spiking of $|\Delta\eta_l|$ near L_0. It has been shown by Albinski[11] that the

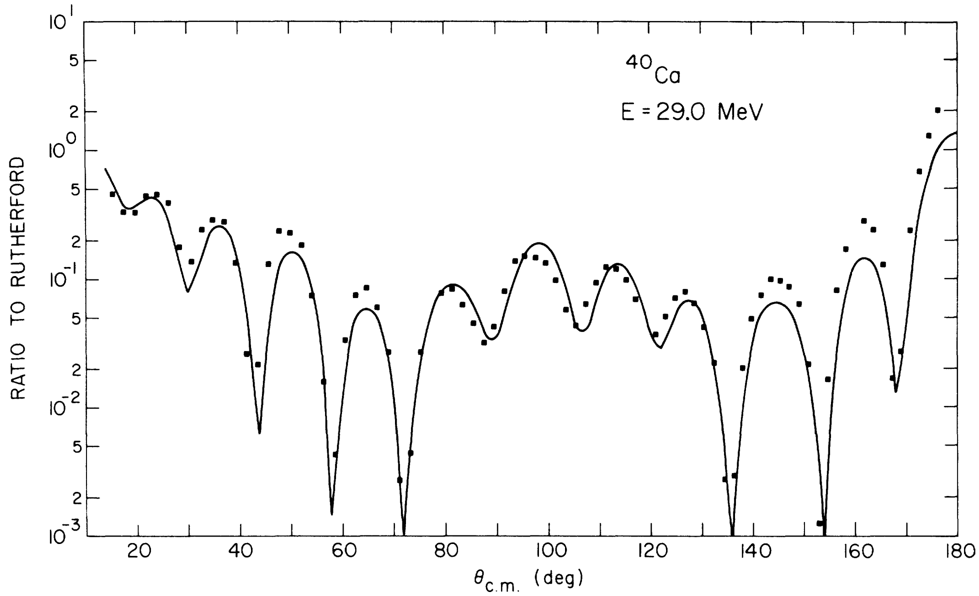

FIG. 3. The experimental data and best fit analysis for the scattering of 29 MeV α particles from ^{40}Ca using the data of Ref. 12 and the parameters of the best fit shown in Table I.

sort of exchange term proposed in Ref. 3 need not in general give rise to a "spike" in $|\eta_l|$ vs l.

In the absence of the direct term in the potential the scattering calculated by V_{ex} is some 5 orders of magnitude larger at back angles than when calculated with the direct term, Eq. (2), present. This suggests that, while a DWBA analysis for V_{ex} may be valid, in the absence of complete theoretical calculation, it is extremely difficult to associate the direct term of Thompson and Tang[5] with a parametrized representation such as Eq. (2) and therefore use Born approximation arguments to introduce an exchange process representation as given by Eq. (3). It also suggests that the double counting error inherent in the calculation of Ref. 3 may be more serious than they argued.

We have also attempted to fit the ^{65}Cu data with an exchange potential which did not have the derivative character of Eq. (3) but used a Woods-Saxon volume term $f(x_x)$. In this case γ^2 was more than twice the minimum γ^2 found with the derivative form (33.9 compared to 14.8). This was found for a negative W_{0x}; trying a positive W_{0x} produced even larger γ^2.

In the cases of ^{45}Sc and ^{65}Cu the ALAS is not as pronounced as in the archetypal case of ^{40}Ca. In this case, the observed cross section near 180° is larger than it is at the maximum near 65° and the ratio of the cross section to the Rutherford cross section near 180° ($\sigma/\sigma_R \sim 2$) is larger than any other angle.

We have attempted to use the parametrization of Eqs. (1)–(4) to fit the 29 MeV data of Schmeing[12] and the results are shown in Table I and Fig. 3. The function $\Delta\eta_l$ as defined by Eq. (6) is similar to that for the case of ^{65}Cu in that there is a reversal of the sense rotation in the Argand diagram. However, its sense of rotation for low partial waves is clockwise. $|\Delta\eta_l|$ does peak somewhat more than for ^{65}Cu but again does not look like the function calculated in Ref. 3.

The γ^2 for this case is seen to be extremely large; however, the quality of the fit is better than that given by Kondō *et al.* in Ref. 6 (in that case we calculate γ^2 to be greater than 2×10^3). The value of γ^2 calculated here assumes no systematic errors or normalization uncertainties and uses the error, as given by Schmeing: 2% of the observed cross section at each point. We have also investigated the effect of a change in the normalization of the experimental data of ±10% which is twice the value cited by Ref. 12 for systematic errors. The value of γ^2 does not increase more than 15% and searching on the parameters produces small changes in most parameters and less than 10% changes in a_I, V_{0x}, and a_x. Again lacking theoretical guidance and with no significant change in γ^2, no significance is attributed to these variations. It is interesting to note that a large part of the contribution to γ^2 comes from the angular region forward of 60°. The intermediate angle region, 60° to 135°, is quite well fitted and at larger angles the best fit cross section is systematically below the experimental cross section as is the forward angle fit.

From Fig. 3 one sees that the quality of the fit to this data is quite good, especially since, in the past, few papers have even bothered to evaluate χ^2 for this case of ALAS. The present value $\chi^2 = 333$ is, however, not statistically significant. A serious attempt was therefore made to determine the uniqueness of the fit shown in Fig. 3 within the framework of the representation of Eqs. (1)–(4). While there may very well be a much deeper, narrow valley in the 10 dimensional parameter space, none has been found.

The smallness of W_{0x} in the case of ^{40}Ca suggests that it can just as easily be set equal to zero. In this case we are calculating in the same model as Kondō *et al.* in Ref. 6. We differ from Ref. 6 primarily in that the conventional optical model parameters were calculated for the case of ^{40}Ca in order to obtain the best possible fit to the data. The appropriate entry in Table I confirms that we can set $W_{0x}=0$. This smallness, or lack of an imaginary component to V_{ex} for ^{40}Ca is unexpected based on the larger ALAS. Relative to ^{65}Cu and ^{45}Sc, V_{0x} is also smaller. However, it should be noted that a_I is significantly smaller and V_{0c} is somewhat larger than the other cases. Again speculating, one might argue that we are incorporating the specific mechanisms giving rise to ALAS differently for ^{40}Ca because we are unable to treat them approximately as we did in the case of ^{65}Cu and ^{45}Sc. This only points up the need again for having theoretical guidance for all of the parameters of both the real and imaginary components to the optical potential if one chooses to calculate the elastic scattering in that framework. In this context it is possible that the low value of a_I and the somewhat deeper well depth V_{0c} may be simulating the true mechanisms present particularly in ALAS. However, it is difficult to explain such a low value for a_I on the basis of a folding model.[13] It may be that higher order virtual state excitations are being manifest in this way.[14]

The results of the present calculations illustrate the lack of uniqueness in the various optical model analyses possible for elastic α-particle scattering when ALAS is present. Since in the cases we have examined here χ^2 is much greater than unity, one may conclude that (supposing the experimental errors have not been underestimated) the present as well as previous parametrizations of the optical potential are inadequate. This is further confirmed by the nonphysical values particularly for the parameter a_I which is consistently lower than that expected from nuclear densities. The criterion of $\chi^2 \sim 1$ can be achieved in the analysis of elastic α-particle scattering as for example in the work of Goldberg.[15] By insisting on $\chi^2 \sim 1$ he was able to show that physically reasonable additions to the conventional six parameter representation of the optical potential were present. However, even in this case the interpretation of the derived results has been questioned.[16]

The energy dependence of ALAS has also been used to justify particular representations of ALAS.[3,17] The optical model is known to be energy dependent from both theoretical and experimental analyses. We therefore would extend the need for χ^2 to be of order of unity and further require that in the absence of demonstrable resonances the optical model parameters should at least vary smoothly with energy. Again to fully interpret such analyses one must have theoretical guidance as to the magnitude of the energy dependence. Reference 18 discusses some aspects of this problem.

Note added in proof. A recently published paper by Eberhard *et al.*,[19] suggests that for ^{40}Ca the backward enhancement arises primarily from a direct contribution whereas ^{44}Ca is showing resonant structure at energies just below 29.0 MeV. This would be consistent with incorporating any explanation of ALAS for ^{40}Ca within an optical model framework.

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