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"Unique" energy-independent Woods-Saxon optical potential for $^{16}O + ^{28}Si$ elastic scattering

J. G. Cramer
Nuclear Physics Laboratory, University of Washington, Seattle, Washington 98195*

R. M. DeVries
Nuclear Structure Research Laboratory,† University of Rochester, Rochester, New York 14627

D. A. Goldberg*
Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742

M. S. Zisman and C. F. Maguire†
Lawrence Berkeley Laboratory, Berkeley, California 94720*

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We have obtained new $^{16}O + ^{28}Si$ elastic scattering data at 215.2 MeV laboratory bombarding energy which show no evidence of strong rainbow scattering effects. We found that it is possible to simultaneously fit low and high energy data with the same energy-independent Woods-Saxon optical potential. We find that the imaginary potential for such fits is greater than or equal to the real potential in the surface region and that a marked preference is found for a real well depth of about 10 MeV, as determined in the region of the nuclear surface.

[NUCLEAR REACTIONS $^{28}Si(^{16}O, ^{16}O)$, $E_L = 215.2$ MeV measured $d\sigma/d\Omega\,(\theta)$; optical model analysis; deduced W/V and energy-independent optical model parameters.]

Despite a rather large amount of effort at many laboratories neither the parameters for, nor the validity of, an optical model description of heavy-ion interactions has been established. This is because only a small region of the potential tail has been sampled with the existing low energy data. Therefore, potentials with central real depths ranging from 15 to hundreds of MeV and similarly large variations in geometry have been employed. These ambiguities are similar to those found in the analysis of light-ion ($A \leq 4$) scattering. A powerful technique for resolving these ambiguities in the case of α particles, for example, has been to perform elastic scattering measurements at high energies where the data at large angles exhibit the structureless falloff characteristic of a nuclear rainbow.[1] In this paper we show that, although the absence of similar behavior in high energy heavy-ion scattering makes this method inadequate, a *simultaneous* analysis of high and low energy data is possible with an energy-independent potential. Such analysis leads to an optical potential which is very strongly absorbing and has a real well depth of about 10 MeV, as determined in the nuclear surface region.

We have used existing $^{16}O + ^{28}Si$ elastic scattering differential cross sections in the energy region 33–142.5 MeV (Ref. 2) and combined them with measurements that we have made at 215.2 MeV. The latter were performed using the LBL 88 inch cyclotron $^{16}O^{5+}$ beam. We employed an array of Si(Li) detectors spaced 2° apart in the lab system. Each detector subtended a lab angle of 0.27°. Zone refined isotopically separated targets of approximately 500 $\mu g/cm^2$ were used. The total accumulation of data is shown in Fig. 1. The energies given in Fig. 1 are incident beam energies. Target energy loss only significantly affected the three lowest energies. The target center energies used in the optical model calculations were 32.7, 35.7, and 37.7 MeV.

The most striking feature of the high energy data is the absence of a structureless falloff characteristic of nuclear rainbow scattering.[1] As mentioned this feature *is* observed in light-ion scattering and not only allows the determination of the real depth, but also indicates that light-ion optical potentials have a central imaginary well depth $\frac{1}{3}-\frac{1}{6}$ of the real depth. Usual "strongly absorbing" heavy-ion optical potentials which yield good fits to low energy data have a W_0/V_0 of about $\frac{1}{4}-\frac{1}{2}$.[3] Such standard optical potentials are exemplified by the third curve in Fig. 1 which predicts nuclear rainbow behavior at higher energies which is not seen in the experimental data. The absence of rainbow scattering implies very strong absorption and/or a very shallow real depth.

We have analyzed these data using a heavy-ion

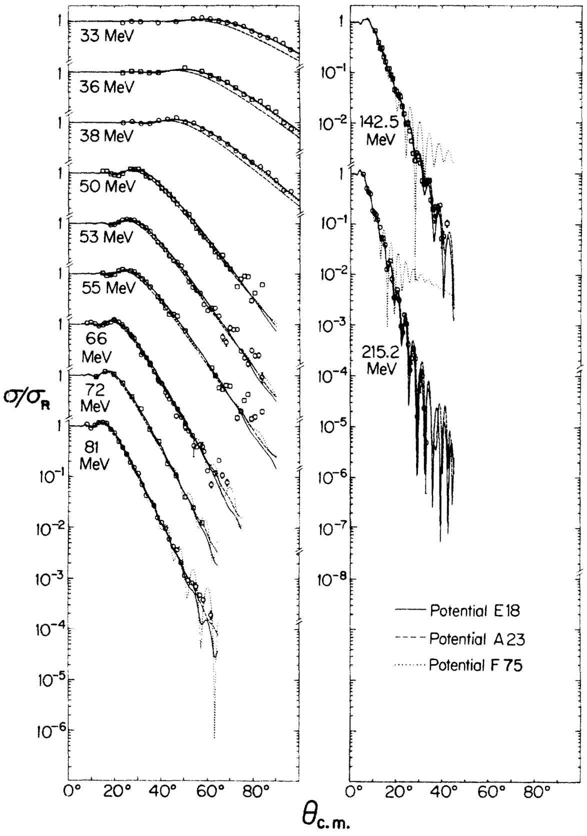

FIG. 1. $^{16}O+^{28}Si$ scattering at the labeled incident energies. The lines are optical model calculations using the parameter sets indicated and listed in Table I. The potential F75 (Ref. 3) was derived from fitting the low energy portion of this data set ($E \leq 81$ MeV).

version of the optical model search code GENOA,[4] assuming no energy dependence in the optical model parameters. By simultaneously fitting high (215.2 MeV) and low (typically 38 MeV) energy data we expect to sample a much broader portion of the potential than can be observed when analyzing data taken at only one incident energy. A conventional six-parameter Woods-Saxon well was used. The real well depth was gridded on, $V = 100$, 40, 15, 10, and 5 MeV, and for each real depth, all of the remaining five parameters were free.

The resulting optical potentials are given in Table I. Figure 1 shows a comparison of the predictions of two of these potentials ($V = 10, 100$ MeV) with the complete set of data. The χ^2 values in Table I indicate that good fits to both high and low energy data sets are not possible unless $V_0 < 40$ MeV. Notice that for the derived potentials W_0/V_0 ranges from 1.5 to 2.7. The large value of this ratio is essential to fit the high energy data. In particular it appears that $V_0 = 10$ MeV (Set E18) yields the best fits, although they are only slightly better than $V_0 = 5$ MeV (Set D23) or $V_0 = 15$ MeV (Set C17). Next, the two data sets were searched individually to obtain the best possible fit to each data set. The χ^2 values obtained were χ^2/N (215.2 MeV) = 4.7 and χ^2/N (38.0 MeV) = 1.2. Thus the potential E18, which was obtained from a search on the combined data sets, yields fits which are almost as good as fits obtained from individual searches. Figure 1 also shows the predictions of potential E18 for all the rest of the existing $^{16}O+^{28}Si$ data. Excellent fits are obtained at all energies except for some large angle data in the region 50 MeV $\leq E \leq$ 66 MeV, which is discussed below. Moreover, individual "best fits" to these other data sets, obtained using the E18 parameters as initial values, yield potentials which do not differ significantly from potential E18. We would like to suggest, therefore, that the assumption that the $^{16}O+^{28}Si$ optical potential is independent of energy is consistent with the data and leads to a potential which has a real depth of about 10 MeV with other parameters as given in Table I. Notch perturbation tests[5] on potential E18 indicate that the region of sensitivity ranges from about 10.5 fm for the low energy data to about 6.5 fm for the 215.2 MeV data [$1.35(28^{1/3}+16^{1/3}) = 7.5$ fm]. Our investigation of the potential, therefore, samples a region of roughly 4 fm which

TABLE I. Derived optical model potentials. In this table $R = r_0(16^{1/3}+28^{1/3})$; $r_{\mathrm{Coul}} = 1.0$ fm [this value yields a Coulomb potential reasonably close to that obtained from a double Fermi folded potential—see R. M. DeVries and M. R. Clover, Nucl. Phys. A243, 528 (1975)].

Label	V_0	r_0	a_0	W_0(volume)	r_I	a_I	χ^2/N(215.2 MeV)	χ^2/N(38.0 MeV)
E18	10	1.35	0.618	23.4	1.23	0.552	4.9	1.2
A23	100	0.932	0.797	165	0.890	0.764	8.6	15
B17	40	1.09	0.754	61.8	1.05	0.723	6.9	10
C17	15	1.27	0.671	34.6	1.16	0.611	5.1	2.0
D23	5	1.43	0.600	13.7	1.29	0.571	14	1.9
F75[a]	100	1.06	0.64	42	1.06	0.64	1.3×10^5	2.0

[a] Y. D. Chan *et al.*, Ref. 3.

spans the regions of the nuclear surface and tail. We cannot, of course, rule out non-Woods-Saxon potentials which might, for example, continue to increase rapidly in the central region of the nucleus or rapidly decrease due to the presence of a repulsive core, provided such behavior takes place at radii of less than 6.5 fm. Over the investigated radial region W/V changes from 15.1 MeV/8.4 MeV at 6.5 fm to 0.030 MeV/0.077 MeV at 10.5 fm. The large value of this ratio in the surface region justifies our use of the term "very strongly absorbing." Moreover, if this were converted to a four-parameter Woods-Saxon potential ($R_v = R_w$ and $a_v = a_w$) the above variation in W/V would lead to a large energy dependence of W_0/V_0 similar to that which has recently been proposed by Satchler.[6]

The success of an energy-independent potential in fitting the $^{16}O + ^{28}Si$ data set leads to a more detailed consideration of the energy dependence of the real part of nuclear potentials. On general theoretical grounds one would expect the heavy-ion-nucleus potential to have some degree of nonlocality, as do the nucleon-nucleon and nucleon-nucleus potentials, and thus to exhibit a negative energy dependence (i.e., to decrease with energy). Experimentally the energy dependence of the real well depth seems to be less for α particles than for protons.[7] Recently, Jackson and Johnson[8] have shown that this reduced energy dependence of the α real potential can be derived from a folding model; furthermore, they show that the nonlocality and energy dependence for heavy ions should be even less, decreasing as $1/A_{\text{proj}}$. In contrast to these theoretical expectations, experimentalists have shown a marked preference for a strong *positive* energy dependence for the real potential in optical model analyses of data (usually over an energy variation of less than a factor of 2), as exemplified by the work of Siemssen and co-workers.[9] Such an unexpected energy dependence may be the result of the choice for the Woods-Saxon geometry parameters used in their analysis. In particular, the diffuseness of 0.49 fm used throughout Siemssen's analysis is markedly different from the value of approximately 0.6 fm which is essential for the fitting of the low energy data in the present work. This value of $a_0 \simeq 0.6$ is in good agreement with the work of Satchler[6] for other heavy-ion systems.

The "shallow" 10 MeV potential which is strongly preferred for the energy-independent fits to the present data is reminiscent of other "shallow" potentials found in previous investigations of heavy-ion scattering, particularly those of Siemssen and co-workers[9] used for the analysis of oxygen scattering from various targets and the potentials of Maher *et al.*[10] and Gobbi *et al.*[11] used for the analysis of the very puzzling $^{16}O + ^{16}O$ system which exhibits prominent "structures" in the 90° excitation function. The Siemssen potentials cannot, in the strictest sense, be considered "shallow" because they have an exceptionally strong energy dependence. For example, Siemssen's $^{16}O + ^{24,26}Mg$ potential would be 115 MeV deep at 215.2 MeV, the energy of the data presented here, and would predict very strong rainbow scattering effects. The Maher-Gobbi potentials, on the other hand, are "shallow" at all energies, but unfortunately do not fit more recent high energy data for the $^{16}O + ^{16}O$ system, as has been demonstrated by the Oak Ridge group.[12] Thus any preference of the $^{16}O + ^{16}O$ data for "shallow" potentials is probably not a result of rainbow scattering effects such as those discussed above. It should be further pointed out that both the Siemssen and the Maher-Gobbi potential families are energy dependent and neither is capable of fitting the $^{16}O + ^{28}Si$ data set.

Theoretical estimates of the potential depth for heavy-ion scattering have varied very widely, depending on the model used. Most folding calculations predict rather deep potentials, but it is difficult to relate these to the present results because they in general have distinctly non-Woods-Saxon shapes. Folding calculations are likely to provide overestimates of the real potential in the central region because of the neglect of saturation effects in the interior. Possibly this is the reason that folding potentials fail to fit α elastic data in angular regions where nuclear rainbow scattering effects are present.[7] On the other hand, calculations which are heavily influenced by surface energy considerations indicate that the real depth should be on the order of 12 MeV.[13] We also note that potential E18 has $r_i < r_0$ and $a_i < a_0$, a result which others have obtained and for which some theoretical justification has been advanced.[14] Some of these studies also showed that data could be fitted with a potential having $W_0/V_0 \geq 1$. Finally, we mention that potential E18, used in distorted-wave Born-approximation calculations, is capable of fitting angular distributions for the reaction $^{26}Mg(^{16}O, ^{14}C)^{28}Si$ at 128 MeV.[15] We have not, however, tested reaction data sensitivity to the potentials presented here in any detail.

As mentioned above, potential E18 is not capable of reproducing the large angle elastic data in the energy region 50 MeV $\leq E \leq$ 66 MeV. While this is disturbing, it has been found that *no* simple optical potential is capable of fitting this data region.[16] This problem is under further study. Another possible difficulty with potential E18 is that the energy required to separate ^{44}Ti into $^{16}O + ^{28}Si$ is 11.48 MeV. Therefore, in the zero incident energy limit the bound-state potential for this system would be considerably in excess of 10 MeV. Finally, strong coupling to the first excited state in ^{28}Si could conceivably so dominate the ^{16}O + ^{28}Si interaction that a simple optical model prediction would be invalid. Preliminary coupled-channels calculations indicate that this is not the case.[17]

It should be emphasized that implicit in this investigation are the assumptions that (1) the potential is energy independent, (2) the radial shape of the potential is well described by a Woods-Saxon function, and (3) that no statement is made about the potential depth in the central region of the nucleus but only in the "inner surface" region. The "uniqueness" and the "shallowness" of the potential presented here are only in the limited context of these assumptions. Even with these qualifications, the derived $V_0 = 10$ MeV value of the ^{16}O + ^{28}Si potential represents a striking departure from the monotonic increase in real well depths found for light ions, i.e., 50 MeV for nucleons, 80–100 MeV for deuterons, and 110–130 MeV for α particles (as determined from rainbow scattering data[1]).

It is clear that more theoretical work on the potentials appropriate to heavy-ion interactions is indicated. It would also appear that more experimental data are needed on the elastic scattering at high and low energies of projectiles in the mass region $4 < A < 16$ so that optical potentials in this critical transition region can be determined.

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‡Present address: Vanderbilt University, Nashville, Tennessee 37325.

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