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Intrashell quartet states

L. Satpathy*
International Centre for Theoretical Physics, Trieste, Italy

K. W. Schmid, S. Krewald, and Amand Faessler
Institut für Kernphysik der Kernforschungsanlage Jülich, D-5170 Jülich, West Germany

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The intrashell quartet states proposed earlier on the basis of intrinsic multiconfiguration Hartree-Fock calculations are reexamined for ^{20}Ne and ^{24}Mg in angular momentum projection formalism and supports in favor of the existence of such states are obtained. Quartet correlation in excited states of ^{22}Ne has also been investigated.

[NUCLEAR STRUCTURE Intrashell quartet states, investigated in angular momentum projection formalism.]

I. INTRODUCTION

The heavy ion reactions in which two protons and two neutrons are transferred give rise to selective excitation[1] of nuclear states. The quartet model[2,3] was proposed to explain the occurrence of such states. In 1970, Arima, Gillet, and Ginocchio[4] proposed a class of quartet states in which two protons and two neutrons are assumed to be excited in a correlated manner from one major shell to another (referred as intershell quartet state). In 1972, we proposed[5] that, in addition to the existence of intershell quartet states, there can be another class of states originating from the excitation of two protons and two neutrons from one deformed single-particle orbit to another deformed orbit in the same major shell, which would be termed as intrashell quartet state. This hypothesis was based on the consideration of energetics and also on our intrinsic multiconfiguration Hartree-Fock (MCHF) calculations of s-d shell nuclei. We had concluded that in $4n$ nuclei, within 12 MeV two intrashell quartet states might exist. We had expressed the hope that angular momentum projection would not alter our conclusion. Nuclear ground state wave functions are expected to the simpler in structure. During the last decade it has been amply demonstrated that nuclear ground state can be well represented by a single-determinantal Hartree-Fock (HF) wave function. However at several MeV of excitation, configuration mixing is expected to be large, and existence of $J=0$ state with similar simple structure is rather unexpected. Hence it is worthwhile to reexamine the genuineness of intrashell quartet states in angular momentum projection formalism. With this view we have performed the following series of calculations for ^{20}Ne and ^{24}Mg. We have also performed similar calculations for one of the $N=Z+2$ even-even nucleus like ^{22}Ne.

II. MULTICONFIGURATION HARTREE-FOCK (MCHF) CALCULATION

A very brief outline of the method[6-8] is given here only. In this scheme, one has taken as a trial wave function $|\Phi\rangle$ a superposition of Slater determinants $|I\rangle$, i.e.,

$$|\Phi\rangle = \sum_I C_I |I\rangle. \tag{1}$$

The configurations $|I\rangle$ are built from single-particle states $|k\rangle$, expanded in an oscillator basis

$$|k\rangle = \sum_a A_{ak} |a\rangle. \tag{2}$$

The mixing and the expansion coefficients C_I and A_{ak}, respectively, are determined by minimizing the energy $\langle\Phi|H|\Phi\rangle$. The configurations $|I\rangle$ are chosen by making particle-hole excitation with respect to that reference Slater determinant which has the largest overlap with the HF ground state. It has been ensured that for $4n$ nuclei the $|\Phi\rangle$ has good isospin $T=0$ ($T=1$ for $N=Z+2$ even-even nuclei) and total angular momentum projection $K=0$.

III. PROJECTION OF ANGULAR MOMENTUM AFTER MCHF CALCULATION (PMCHF)

In this scheme, good angular momentum basis states are obtained by projecting out angular momentum from a set of intrinsic states constructed with the MCHF solution. Then the Hamiltonian matrix is set up in this basis space and diagonalized to obtain the eigenstates.

IV. VARIATION AFTER PROJECTION FROM MCHF STATE MCHFP

Here, a good angular momentum state Ψ^J is used as the variational wave function which is obtained from the MCHF state $|\Phi\rangle$ as

$$\Psi^J = P^J|\Phi\rangle,$$

where P^J is the usual angular momentum projection operator. Then one minimizes the expectation value of the Hamiltonian with respect to the mixing coefficient C and the expansion coefficient A,

$$\delta_{C,A}(\langle\Psi^J|H|\Psi^J\rangle - E^J\langle\Psi^J|\Psi^J\rangle) = 0,$$

where E^J is the Lagrangian multiplier to be identified as the energy of the state.

In all our calculations, we have taken ^{16}O as the inert core and used only $1s$-$0d$ shell basis states for the construction of the states $|k\rangle$ of Eq. (2). Axial symmetry and charge conjugation symmetry are imposed. Only $J=0$ states are considered. In the MCHF calculation, the study[5] of the quartet structure of a state is rather easy. In this calculation, the basic states $|I\rangle$ of Eq. (1) are orthogonal. So the square of the mixing coefficient C_I immediately provides the strength of the configuration $|I\rangle$ in the state Φ. The coefficient C_I is equal to the overlap O_I,

$$O_I = \langle\Phi|I\rangle. \tag{3}$$

However, in the PMCHF (projection after variation) and MCHFP (projection before variation) calculation the basic states are not orthogonal and hence the expansion coefficients C_I would not provide a measure of the strength of the configuration $|I\rangle$ in Ψ^J and as such are not meaningful. To find out the strength of the configuration $|I\rangle$ in Ψ^J one has to calculate the overlap O_J,

$$O_J = \langle\Psi^J|P^J|I\rangle. \tag{4}$$

V. RESULTS

In the case of ^{20}Ne, our multiconfiguration wave function contains 60, 36, and 36 Slater determinants for MCHF, PMCHF, and MCHFP calcula-

TABLE I. Results of the calculations in the various schemes (see text) for the 0^+ states. For each scheme, the first and second columns, respectively, give the energy of the eigenstates and the structure of the quartet component in it. The third column gives the overlap [Eqs. (3) and (4)] of the quartet component with the eigenstate.

MCHF			PMCHF			MCHFP		
Energy (MeV)	Quartet component	Overlap O_I	Energy (MeV)	Quartet component	Overlap O_J	Energy (MeV)	Quartet component	Overlap O_J
				^{20}Ne (MSDI)				
−36.09 (g.s.)	$(\frac{1}{2})^4$	0.99	−39.32 (g.s.)	$(\frac{1}{2})^4$	0.99	−39.74 (g.s.)	$(\frac{1}{2})^4$	0.99
−31.76	$(\frac{5}{2})^4$	0.99	−32.54	$(\frac{5}{2})^4$	−0.70	−32.39	$(\frac{5}{2})^4$	−0.69
				$(\frac{3}{2})^4$	−0.69		$(\frac{3}{2})^4$	−0.66
−30.70	$(\frac{3}{2})^4$	0.99						
−23.20	$(\frac{1}{2}')^4$	0.94	−27.51	$(\frac{1}{2}')^4$	−0.49	−24.86	$(\frac{1}{2}')^4$	−0.48
				^{20}Ne (Kuo)				
−37.86 (g.s.)	$(\frac{1}{2})^4$	0.99	−41.20 (g.s.)	$(\frac{1}{2})^4$	0.99			
−29.25	$(\frac{5}{2})^4$	0.99	−31.82	$(\frac{5}{2})^4$	0.64			
				$(\frac{3}{2})^4$	−0.64			
−27.15	$(\frac{1}{2}')^4$	0.99						
−26.37	$(\frac{3}{2})^4$	0.99						
				^{24}Mg (Kuo)				
−84.94 (g.s.)	$(\frac{1}{2})^4(\frac{3}{2})^4$	0.92	−88.04 (g.s.)	$(\frac{1}{2})^4(\frac{3}{2})^4$	0.90			
−72.43	$(\frac{1}{2})^4(\frac{1}{2}')^4$	0.95	−77.06	$(\frac{1}{2})^4(\frac{1}{2}')^4$	0.88			
−67.46	$(\frac{1}{2})^4(\frac{1}{2})^4$	0.90	−72.35	$(\frac{1}{2})^4(\frac{1}{2}')^4$	−0.65			

tions, respectively, out of which six determinants are exclusively of 4p-4h quartet[9] type in each case. For ^{24}Mg, in the MCHF and PMCHF calculations, multiconfiguration wave functions having 81 and 36 Slater determinants with 9 and 6 determinants of 4p-4h quartet types, respectively, were taken. In order to study the interaction dependence of our result we have used two interactions, i.e., the modified surface delta interaction[10] (MSDI) and the effective interaction calculated by Kuo.[11] Since the calculations are very involved, we have only performed MCHF and PMCHF calculations with Kuo interaction for ^{24}Mg. However, for ^{20}Ne we have used both the interactions, but MCHFP calculations have been done only with the MSDI interaction. The states having relatively pure structure obtained in these calculations are presented in Table I.

From an analysis of the table, it is evident that the excitation energies of the quartet states obtained in the MCHF calculation are not drastically changed in the angular momentum projection schemes like PMCHF and MCHFP calculations. This corroborates our earlier conclusion[5] based on energetical considerations and MCHF study. We further notice from the values of O_I that the overlap of the quartet configuration with the MCHF eigenstate is close to unity. However, of the corresponding overlap O^J in the PMCHF and MCHFP calculation though comparatively smaller, still in most of the states, the magnitude is as high as ~0.65 and even higher. This suggests that though due to the projection of angular momentum, the strength of the quartet configuration in the eigenstate is reduced, still it is large enough to qualify such states as intrashell quartet states.

The structure of these states was investigated by Dhar, Kulkarni, and Bhatt[12] in ^{20}Ne, ^{24}Mg, and ^{28}Si in a very restricted projected Hartree-Fock scheme. They had concluded that these states are not well defined and would lie very high in energy. This calculation was very inadequate in the sense that they had used a linear superposition of only three Slater determinants obtained from Hartree-Fock solutions as the multiconfiguration wave function. The basis is too small, so much so even in intrinsic space, the Hamiltonian cannot connect them as they differ by 4p-4h or even more number of particle-hole excitation. Some 2p-2h excitation should have been included to get the full impact of the Hamiltonian. Further from the magnitude of the mixing coefficient C, the strength of the quartet configuration in the eigenstate has been inferred which is not proper as discussed earlier.

We would like to emphasize here that whether these states rigorously qualify to be termed as intrashell quartet states or not, this calculation shows that there are low-lying excited states in $4n$ nuclei whose major components are just single Slater determinants. Each single-particle level in the determinant is occupied by two protons and two neutrons. This may be due to some symmetry of nuclear dynamics which is not fully understood.

TABLE II. Results of the calculation in the various schemes (see text) for the 0^+ states. For each scheme the first and second column, respectively, give the energy of the eigenstate and the structure of the dominant component in it. The third column gives the overlap [Eqs. (3) and (4)] of the dominant component with the eigenstate.

MCHF			PMCHF			MCHFP		
Energy (MeV)	Dominant component	Overlap O_I	Energy (MeV)	Dominant component	Overlap O_J	Energy (MeV)	Dominant component	Overlap O_J
				^{22}Ne (MSDI)				
−54.46 (g.s.)	$(\frac{1}{2})^4(\frac{3}{2})^{2n}$	0.89	−57.31	$(\frac{1}{2})^4(\frac{3}{2})^{2n}$	0.83	−57.83	$(\frac{1}{2})^4(\frac{3}{2})^{2n}$	0.75
−50.13	$(\frac{1}{2})^4(\frac{5}{2})^{2n}$	0.64	−51.65	$(\frac{1}{2})^4(\frac{1}{2}')^{2n}$	0.63	−51.86	$(\frac{1}{2})^4(\frac{1}{2}')^{2n}$	0.50
−48.60	$(\frac{1}{2})^4(\frac{1}{2}')^{2n}$	−0.83	−50.20	$(\frac{1}{2})^4(\frac{5}{2})^{2n}$	−0.66	−50.64	$(\frac{1}{2})^4(\frac{5}{2})^{2n}$	−0.70
−46.40	$(\frac{5}{2})^4(\frac{3}{2})^{2n}$	0.68	−45.24	$(\frac{1}{2})^4(\frac{1}{2}'')^{2n}$	−0.85	−45.52	$(\frac{1}{2})^4(\frac{1}{2}'')^{2n}$	0.76
				^{22}Ne (Kuo)				
−57.11	$(\frac{1}{2})^4(\frac{3}{2})^{2n}$	0.85	−59.75	$(\frac{1}{2})^4(\frac{3}{2})^{2n}$	0.83	−59.95	$(\frac{1}{2})^4(\frac{3}{2})^{2n}$	0.85
−50.94	$(\frac{1}{2})^4(\frac{1}{2}'')^{2n}$	0.69	−53.60	$(\frac{1}{2})^4(\frac{1}{2}')^{2n}$	0.77	−53.94	$(\frac{1}{2})^4(\frac{1}{2}')^{2n}$	0.85
−49.37	$(\frac{1}{2})^4(\frac{5}{2})^{2n}$	0.76	−50.47	$(\frac{1}{2})^4(\frac{5}{2})^{2n}$	0.76	−50.98	$(\frac{1}{2})^4(\frac{5}{2})^{2n}$	0.79
−48.45	$(\frac{1}{2})^4(\frac{1}{2}')^{2n}$	0.77	−49.86	$(\frac{1}{2})^4(\frac{1}{2}'')^{2n}$	0.79	−49.88	$(\frac{1}{2})^4(\frac{1}{2}'')^{2n}$	0.85

From this point of view, it would be interesting to investigate the structure of some excited states in $N=Z+2$ even-even nuclei. We have chosen ^{22}Ne and have performed MCHF, PMCHF, and MCHFP calculations with the multiconfiguration wave function having 56, 30, 30 determinants, respectively, with both the MSDI and Kuo interactions. In the MCHF calculation 10 configurations and in each of the PMCHF and MCHFP calculations 8 configurations were exclusively 4p-4h quartet type. The low-lying states which have dominant one determinantal structure are presented in Table II. From the structure of these states it is clear that the pair of neutrons occupying the Fermi surface, $K=\frac{3}{2}$, in the usual HF spectrum are excited in a correlated manner from one single-particle level to another leaving the core consisting of two protons and two neutrons in the $K=\frac{1}{2}$ level. In most of the states presented in Table II, the values of O_I and O_J for the pair excited configuration are above 0.75 in magnitude, which is indicative of the high purity of such states. Low-lying quartet excited states were not observed. It appears that pair excitation is a characteristic of even-even $N=Z+2$ nuclei similar to quartet excitation in $4n$ nuclei. Purely from the HF study of ^{22}Ne, such features for its low-lying excited states were anticipated earlier and a model[13] was proposed. From the structure of the dominant configuration shown in Table II, it is quite clear that quartet correlation is also a dominant feature in these pair excited states.

Using two different interactions and the angular momentum projection formalism, the genuineness of intrashell quartet states and the quartet correlation in excited states of some representative s-d shell nuclei have been shown.

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*Present address: Institute of Physics, A/105, Saheed Nagar, Bhubaneswar-751007, India.

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