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# Pion-gamma angular correlations following inelastic scattering

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Angular correlations between inelastically scattered pions and deexcitation $\gamma$ rays depend upon the spin density matrix of the residual nucleus and thus provide additional information not available in the differential cross sections. The correlation function is calculated for the $0^+ \to 2^+ \to 0^+$ transition in $^{12}C$ and the resulting symmetry angles are found to be quite similar to those obtained in the adiabatic limit. The model sensitivity is tested for medium energy pions by comparing the Kisslinger potential results with those using a local Laplacian potential. Some differences, especially near cross section minima, are found between the different potential models.

[NUCLEAR REACTIONS $^{12}C(\pi, \pi'\gamma)$, $E=30$–$150$ MeV; calculated pion-$\gamma$ angular correlation.]

## I. INTRODUCTION

The elastic and inelastic scattering of pions by nuclei has recently attracted much interest both experimentally[1] and theoretically. Differential cross sections for the inelastic scattering of pions by $^{12}C$ have been measured for incident pion energies between 120 and 280 MeV.[1] Such measurements, however, yield only a limited amount of information about the nuclear processes under investigation. This arises from the fact that the set of scattering amplitudes $T(J,M)$, which describe the inelastic scattering process, appear in the cross section only as a sum of absolute squares. Thus, the cross section is independent of the relative phases of the substate transition amplitudes and depends only on the overall incoherent sum. Other bilinear combinations of transition amplitudes do appear in the polarization of the excited nucleus following inelastic scattering. The nonrandom spin orientation of the product nucleus may be determined by observing deexcitation $\gamma$ rays. Angular correlation experiments with pions are thus sensitive to the relative phases of the complex transition amplitudes and should provide additional information not available in the differential cross sections.

The study of pion-$\gamma$ angular correlations is similar to that of $\alpha$-$\gamma$ angular correlations since both pions and $\alpha$ particles have zero spin. Experimental measurements of the correlation between inelastically scattered $\alpha$ particles and the deexcitation $\gamma$ rays have been carried out for several nuclei at different $\alpha$ energies.[2-8] The plane-wave Born[9] and adiabatic[10] approximations were shown to be inadequate for the description of the $(\alpha, \alpha'\gamma)$ angular correlation, even though both of these models are fairly accurate in their prediction of inelastic scattering cross sections. The distorted wave Born approximation (DWBA), however, was found to give a qualitative description of the data.

A measurement of the pion-$\gamma$ angular correlation has been proposed for a meson factory experiment.[11] A theoretical description of the process will give some idea of what type of information, in regard to both the nuclear structure and the nature of the pion-nucleus interaction, might be obtainable from such an experiment. It is the purpose of this paper to present model calculations appropriate for the experiments likely to be performed in the near future.

As a framework for estimating the pion-nucleus inelastic scattering we use a derived optical potential including a deformed nuclear density distribution.[12] This is used to calculate the transition amplitude for the inelastic scattering process as outlined in Sec. II. The $(\pi, \pi'\gamma)$ angular correlation is then discussed in the same manner as in Rybicki, Tamura, and Satchler[13] for the specialized case of spinless projectiles and spinless targets. An expression for the correlation function in terms of the transition amplitudes is obtained in Sec. III. Finally, in Sec. IV we present the results of some sample calculations for pions inelastically scattered by $^{12}C$.

## II. PION-NUCLEUS INELASTIC SCATTERING

In order to describe inelastic pion scattering we employ the Kisslinger optical potential[14] which is derivable from pion-nucleon phase shifts using the Watson multiple-scattering formalism.[15] This potential gave good fits[16] to $\pi^-$-$^{12}$C elastic scattering data[1] and a collective model generalization[12,17] yielded $2^+$ and $3^-$ inelastic scattering cross sections in agreement with the data without adjustable parameters (the extracted deformation parameters were found to be energy independent and in agreement with those obtained by other techniques). It should be noted that other theories of elastic and inelastic pion-$^{12}$C scattering also give reasonable fits to the Binon data.[1] For example, a WKB-Glauber approach,[18,19] a local (i.e., Laplacian) model,[20] a deformed black nucleus model,[21] and a field theoretic Low-equation approach[22] all work quite adequately in the region where the data exist. A measurement of the pion-$\gamma$ angular correlation function may provide a means of discriminating between these theories. For the sake of comparison we also perform calculations using the local potential mentioned above which is quite similar to the Kisslinger potential. These potentials are both convenient for treating spin and Coulomb effects and should give reasonable estimates of what structure information might be deducible from experiment.

We begin with the Kisslinger optical potential which for a spherical target is given by[14]

$$U_K^{(0)}(r) = \frac{(\hbar c)^2 A}{2E}\left\{-b_0 k^2\rho(r) + b_1\vec{\nabla}\cdot[\rho(r)\vec{\nabla}]\right\}, \quad (1)$$

where $A$ is the mass of the target, $k$ and $E$ are the lab momentum and total lab energy of the incident pion, respectively, $\rho(r)$ is the nucleon density normalized to unity, and $b_0$ and $b_1$ are directly related to the pion-nucleon phase shifts as given elsewhere.[23] A different off-shell extrapolation of the pion-nucleon scattering amplitude results in a potential which contains no gradient terms on the right. This is referred to as the local Laplacian potential and is given by

$$U_L^{(0)}(\vec{r}) = \frac{(\hbar c)^2 A}{2E}\left[-(b_0+b_1)k^2\rho(r) - \tfrac{1}{2}(b_1)\nabla^2\rho(r)\right]. \quad (2)$$

For a deformed target, the density $\rho(r)$ is generalized to be functionally dependent upon the body-fixed polar angle $\theta'$.[12] This gives rise to a deformed optical potential which may be written as

$$U(r) = U^{(0)}(r) + U^{(1)}(r,\theta'), \quad (3)$$

where $U^{(0)}$ is the usual spherical term as in either Eq. (1) or Eq. (2). For the Kisslinger potential [Eq. (1)] we have

$$U_K^{(1)}(r,\theta') = \beta_2\frac{(\hbar c)^2 A}{2E}\left\{-b_0 k^2 F(r) Y_0^2(\theta') + b_1\vec{\nabla}\cdot[F(r)Y_0^2(\theta')\vec{\nabla}]\right\}, \quad (4)$$

where

$$F(r) = R_0 \left.\frac{\partial\rho}{\partial R}\right|_{R=R_0}.$$

Only quadrupole excitations $0^+\to 2^+$ are considered in this paper.

In the distorted wave Born approximation, the transition amplitude for inelastic scattering from a $0^+$ ground state to an excited state of angular momentum $J$ and projection $M$ is

$$T(J,M) = \int d^3r\,\chi_f^{(-)*}(\vec{k}_f,\vec{r})\langle J,M|U^{(1)}|00\rangle\chi_i^{(+)}(\vec{k}_i,\vec{r}), \quad (5)$$

where $\chi_i$ and $\chi_f$ are the distorted waves representing the elastic scattering of a pion of incident momentum $\vec{k}_i$ and final momentum $\vec{k}_f$, respectively.

Expanding the distorted waves in multipoles and performing some angular momentum algebra yields the results given by Edwards and Rost[12] for the Kisslinger potential [Eq. (1)]. A similar result with different algebraic terms is obtained when the local Laplacian potential Eq. (2) is used instead.

### III. ANGULAR CORRELATION FORMALISM

We consider a $(\pi,\pi')$ reaction leading to an excited state of definite spin and parity, which then decays by the emission of a single $\gamma$ ray back to the ground state $A(\pi,\pi')A^*(\gamma)A$. Rybicki *et al.*[13] have discussed the more general case for the reaction $A(a,b)B(\gamma)C$. Using their formalism we find the double differential cross section for the emission of a pion along $\vec{k}_f$ and a $\gamma$ ray along $\vec{k}_\gamma$ when the final nuclear polarization is not detected to be

$$\frac{d^2\sigma}{d\Omega_\gamma d\Omega_\pi} = \frac{W(\theta_\gamma,\phi_\gamma;\theta_\pi)}{4\pi}\frac{d\sigma}{d\Omega_\pi}(\theta_\pi). \quad (6)$$

The normalization of the correlation function $W$ is chosen so that the integral over the solid angle $d\Omega_\gamma$ is equal to $4\pi$. This function has the form

$$W(\theta_\gamma,\phi_\gamma;\theta_\pi) = (2J+1)^{1/2}\sum_{\substack{K\,\text{even}\\ Q}}\frac{\rho_{KQ}}{\rho_{00}}\left(\frac{4\pi}{2K+1}\right)^{1/2} \times Y_Q^K(\theta_\gamma,\phi_\gamma)(JJ-11|K0)(-)^{1-J}, \quad (7)$$

where $J$ is the angular momentum of the excited nucleus. The statistical tensor $\rho_{KQ}(J)$ is related to the density matrix $\rho_{MM'}$ by

$$\rho_{KQ}(J) = \sum_{MM'}\rho_{MM'}(-)^{K-J-M}(JJ-MM'|KQ), \quad (8)$$

with the density matrix given by

$$\rho_{MM'} = T(J,M)T^*(J,M') , \tag{9}$$

where $T(J,M)$ is defined in Eq. (5). The differential cross section is given by

$$\frac{d\sigma}{d\Omega_\pi} = \left(\frac{2\pi}{\hbar c}\right)^4 E_f E_i \frac{k_f}{k_i} \sum_M |T(J,M)|^2 . \tag{10}$$

Two special choices of coordinate systems lead to simpler forms for $W$.[13] If the $z$ axis is chosen to be perpendicular to the scattering plane, then only even values of $Q$ in Eq. (8) contribute, and the correlation function takes on a much simplified form. Another useful coordinate system places the $z$ axis along the incident beam and the $y$ axis perpendicular to the scattering plane. With this choice of axes the reflection symmetry theorem of Bohr[24] leads to the requirement that

$$T(J,-M) = (-)^M T(J,M) .$$

We choose to work with the second set of axes and consider only transitions to the $J = 2^+$ excited state. The simpler expression[8] for the in-reaction-plane ($\phi_\gamma = 0$) correlation function for the $0^+ \to 2^+ \to 0^+$ spin sequence is then

$$W(\theta_\gamma) = A + B \sin^2 2(\theta_\gamma - \theta_0) . \tag{11}$$

This result follows from Eq. (7) for $J = 2$. Explicit expressions for $A$, $B$, and $\theta_0$ in terms of the transition amplitudes $T(J=2,M)$ are given by Banerjee and Levinson.[25]

## IV. APPLICATIONS

We use a modified version of the distorted wave program DWUCK[17] to calculate the transition amplitudes for inelastic scattering of pions by $^{12}$C. The calculations are performed using a modified Gaussian density for $^{12}$C:

$$\rho(r) = \rho_0[1 + 4/3(r/a)^2]e^{-r^2/a^2} ,$$

$a = 1.5$ fm as is appropriate for a shell model description of $^{12}$C in a harmonic oscillator potential. The optical model parameters are derived from pion-nucleon phase shifts and averaged over the Fermi motion of the target nucleons.[12] The inelastic differential cross section for the scattering of negative pions at 90 MeV is shown in Fig. 1. The results are given for both the Kisslinger and local Laplacian potential forms. At forward angles the predictions of both potentials are quite similar although at larger angles the local Laplacian potential tends to give larger cross sections.

A quantity which has proved to be useful in the study of $\alpha$ particle inelastic scattering is the symmetry angle $\theta_0$.[7,8] For a $0^+ \to 2^+ \to 0^+$ transition it can be calculated in terms of the transition

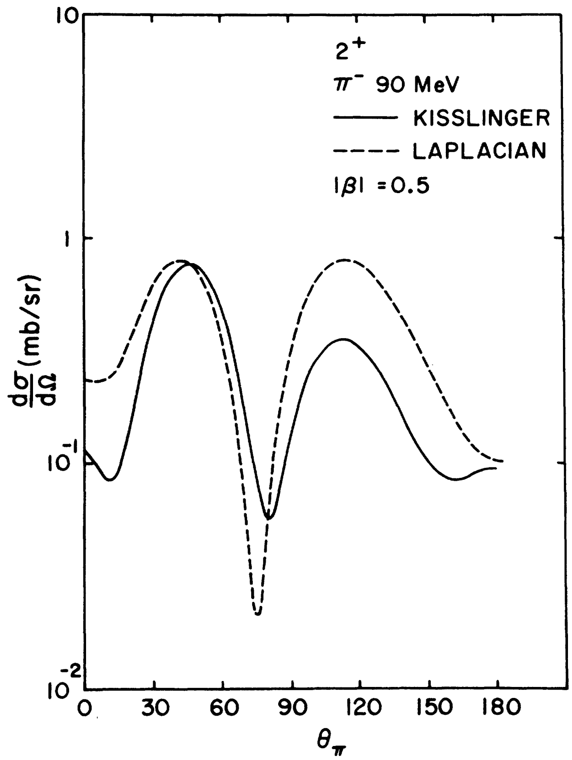


FIG. 1. $^{12}$C$(\pi^-,\pi^{-\prime})^{12}$C* differential cross section for the $2^+$ level (4.44 MeV) at $T_\pi = 90$ MeV. The solid curve is for the Kisslinger potential and the dashed curve for a local Laplacian potential.

amplitudes $T(2,M)$ [Eq. (5)] for different values of the pion center-of-mass scattering angle $\theta_\pi$. In the adiabatic approximation the reaction is symmetric with respect to the interchange of entrance and exit channels, and the impulse delivered to the target nucleus is constrained to lie along the adiabatic recoil direction making an angle of $\frac{1}{2}(\pi - \theta_\pi)$ with the incident beam.[8] This direction then defines an axis of symmetry, both for the elastic amplitude and for the density matrix that parameterizes the $\gamma$ ray correlation. The variation of $\theta_0$ with $\theta_\pi$ is shown in Fig. 2 for several different values of the incident pion energy. The straight line is obtained using the adiabatic approximation where the excitation energy is taken to zero. Our calculations are seen to be quite similar to the adiabatic results. The largest discrepancies occur near minima in the inelastic angular distribution (Fig. 1). Near maxima the calculation coincides with the adiabatic predictions. A calculation of the symmetry angle for 30 MeV incident pions was performed using the best fit parameters of Marshall, Nordberg, and Burman.[26] The results were found to still coincide with the adiabatic predictions. This is somewhat interesting since the adiabatic approximation is

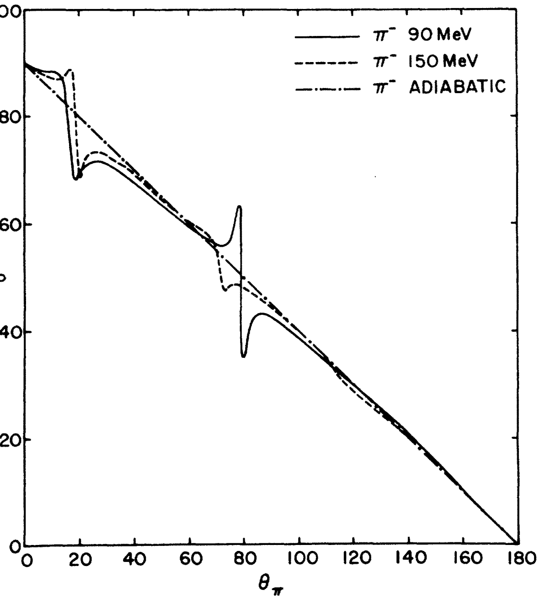


FIG. 2. The symmetry angle $\theta_0$ as a function of the pion c.m. scattering angle $\theta_\pi$. The solid curve is for $T_\pi = 90$ MeV and the dashed curve for $T_\pi = 150$ MeV. The straight line (dot-dashed curve) is the adiabatic approximation.

usually justified in the high-energy limit. Another criterion for applicability of the adiabatic approximation is that the change in the nuclear wave function be slight during the time pion "feels" the nuclear potential. Near 90 MeV the interaction begins to enter the resonance region where this condition may no longer be valid. From the figure we see that a larger deviation from the adiabatic results occurs for the 90 MeV case than for the 30 MeV case. As the energy is increased even further (150 MeV) the interaction approaches the resonance but at the same time the excitation energy becomes negligible compared to the energy of the incident pion and the adiabatic limit is approached one more.

The rapid variation of $\theta_0$ with scattering angle seen in $\alpha$ inelastic scattering[7,8] is not present in our pion results. Evidently the kinematic effect of the small pion mass prevents appreciable departures from the classical recoil symmetry axis.

Since it appears that little information is obtainable from the single symmetry angle parameter, we need to examine the correlation function, Eq. (7), in detail. This is shown in Fig. 3 as a function of the angle of the scattered pions $\theta_\pi$ for various positions of the $\gamma$ counter in the reaction plane.

From Eq. (11) we have $W(\theta_\gamma + \frac{1}{2}\pi) = W(\theta_\gamma)$ so the curves for $\theta_\gamma > \frac{1}{2}\pi$ need not be shown. The results shown in Fig. 3 were obtained using the

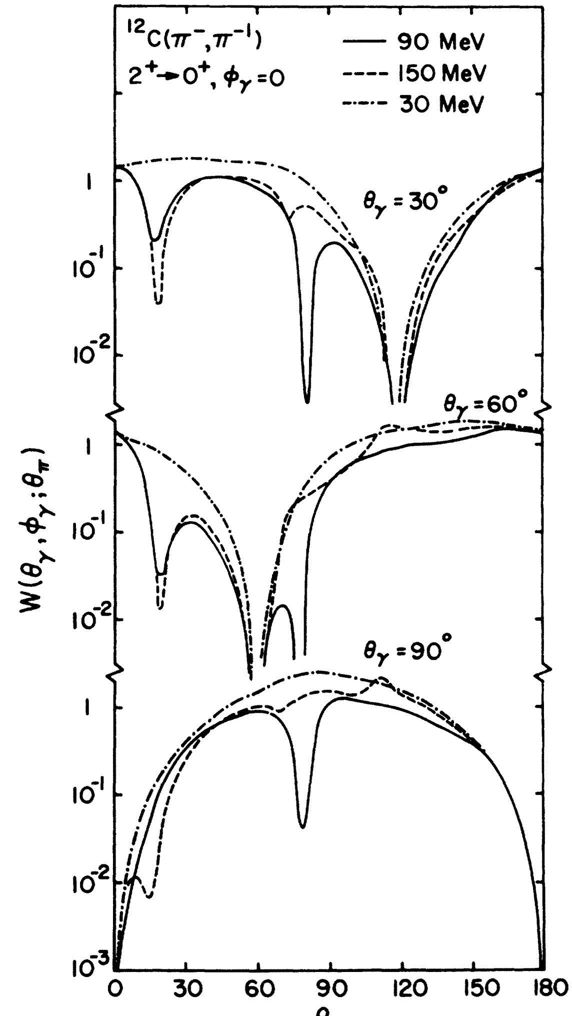


FIG. 3. The correlation function $W(\theta_\gamma, \phi_\gamma; \theta_\pi)$ as a function of the pion angle $\theta_\pi$ for various positions of the $\gamma$ counter in the reaction plane.

Kisslinger potential for incident pion energies of 30, 90, and 150 MeV. The 30 MeV curves appear to be smoothly varying functions of the pion scattering angle $\theta_\pi$ with deep minima when $\theta_\gamma$ corresponds to the appropriate symmetry angle for pions scattered by $\theta_\pi$ (for example, the very deep minimum which occurs at $\theta_\pi = 120°$ in the $\theta_\gamma = 30°$ curve corresponds to the fact that the symmetry angle for pion scattered by 120° is 30°. The deep minima in the other curves have a similar explanation.) The 90 MeV results all exhibit an additional minimum at a pion scattering angle of about 80°. Upon examining the inelastic differential cross section at this energy (Fig. 1) we see that it also passes through a minimum at this point, corresponding to the fact that the transition amplitudes are rapidly varying functions of the pion

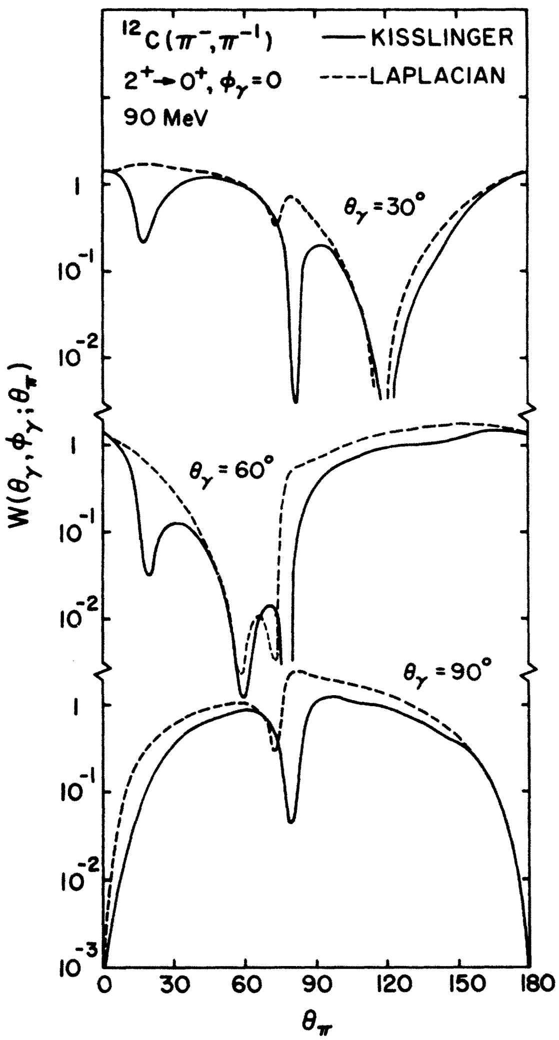


FIG. 4. The correlation function $W(\theta_\gamma, \phi_\gamma;\ \theta_\pi)$ as a function of the pion angle $\theta_\pi$ for an incident pion energy of 90 MeV. The solid curve is the result of the Kisslinger potential and the dashed curve the local Laplacian potential.

scattering angle near here. Both the 90 and 150 MeV curves also possess minima at $\theta_\pi \approx 18°$ where, again, the differential cross sections pass through minima. The small bumps present in the 150 MeV curves at $\theta_\pi \approx 110°$ also correspond to a minimum in the inelastic differential cross section (not shown) which is present at this angle.

In Fig. 4 we compare the correlation pattern predicted by the Kisslinger potential with that predicted by a local Laplacian potential for 90 MeV incident pion energy. The largest discrepancies between the two occur near minima in the inelastic differential cross sections (Fig. 1). A minimum is predicted by the Kisslinger potential at 18° which is not observed inthe local Laplacian version. Unfortunately, this point is near the minimum in the differential cross section and thus might be difficult to measure as a means of discriminating between the two potentials.

## V. CONCLUSIONS

Our pion-$\gamma$ angular correlation results show the symmetry angles calculated for the $0^+ \to 2^+ \to 0^+$ spin sequence to be quite similar to those obtained in the adiabatic limit. There is very little variation of the symmetry angle $\theta_0$ with scattering angle $\theta_\pi$ in contrast to $(\alpha, \alpha'\gamma)$ results from the same nucleus. It would seem that pion-$\gamma$ angular correlation measurements may not prove to be as useful in the determination of the pion-nucleus scattering amplitudes as $\alpha$-$\gamma$ correlation experiments have been in the investigation of the scattering amplitudes for $\alpha$-nucleus scattering. Nonetheless, there is still some information available from pion-$\gamma$ angular correlation measurements and accurate experiments near cross section minima would be useful.

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