Nuclear rotational current and velocity fields, the cranking model, and transverse electroexcitation of the first excited state of ^{12}C†

Mark Radomski
Cyclotron Laboratory, Physics Department, Michigan State University, East Lansing, Michigan 48824
(Received 19 July 1976)

We examine current and velocity fields of rotating deformed nuclei. Although the harmonic oscillator cranking model gives rigid moments of inertia, the velocity field is not a rigid rotation in the examples studied. Such fields are partially observable via electroexcitation, and we compare ^{12}C to experiment. A slight improvement in experimental precision would allow different models to be distinguished.

[NUCLEAR REACTIONS Inelastic electron scattering, deformed nuclei; transverse electroexcitation probability, rotational model; ^{12}C$(e,e')^{12}$C*(4.4 MeV).]

The nature of nuclear rotation has received much attention and interest in nuclear physics. Initial recognition of the phenomenon was hindered by a preconception in favor of rigid-body rotation. The fluid drop picture with irrotational flow[1] has also been used extensively. The actual nature of the flow need follow no assumption based on a classical ideal, but is dynamically determined by the interplay of collective and intrinsic degrees of freedom, in a manner as yet not understood.

I shall present some dynamically calculated velocity fields for rotation of idealized versions of ^{8}Be and ^{12}C, compare them with classical expectations, and show how they manifest their nature in the transverse electroexcitation of the rotational states. My model for carbon may already be marginally in conflict with experiments on excitation of the 2^+ (4.44 MeV) state. Thus a modest increase of experimental precision may enable this and other models to be distinguished.

The cranking model will be used to generate current density fields from which velocity fields will be deduced. An axially symmetric potential well is rotated with constant angular velocity ω about the x axis, the symmetry axis starting in the z direction at time $t=0$. Time-dependent wave functions for the motion of a particle are found, through first order in ω, in, e.g., Ref. 2. The extension to many particles is straightforward. Current density fields are arrived at by taking expectation values, using these wave functions, of a local current operator. The time dependence of the current field, and of other local fields such as the density, consists of rotation to follow the potential. It is thus sufficient to regard the fields obtained at $t=0$ and to treat them as being related to body-fixed axes.

For independent particles one thus obtains the current[3]

$$\vec{j}(\vec{x}) = 2\,\mathrm{Re} \sum_{\mu\ \mathrm{occupied}}^{A} \sum_{\nu\neq\mu} \langle\mu|\vec{j}_{op}(\vec{x})|\nu\rangle\langle\nu|-\omega j_x|\mu\rangle \times(\epsilon_\mu-\epsilon_\nu)^{-1}+O(\omega^3), \qquad (1)$$

where $\vec{j}_{op}(\vec{x})$ is the local one-body current density operator. The current at $\omega=0$ vanishes for a time-reversal invariant intrinsic structure such as is appropriate to the ground-state band of even-even nuclei. The density field $\rho(\vec{x})$ is equal to that for $\omega=0$, except in order ω^2, under the same conditions.

Equation (1) contains much more information than just the moment of inertia, for which the cranking model result has long been known.[4] But if the mass-convection current is used for $\vec{j}_{op}$, and $\vec{x}\times\vec{j}(\vec{x})$ is integrated over space, one finds $l_x=\omega\mathcal{I}_{\mathrm{crank}}$, where $\mathcal{I}_{\mathrm{crank}}$ is given by Inglis's result.

In a fluid drop picture, the current is a product of density and velocity. It is natural to define a velocity field by

$$\vec{v}(\vec{x})=\vec{j}(\vec{x})/\rho(\vec{x}). \qquad (2)$$

Rigid-body flow has $\vec{v}=\vec{\omega}\times\vec{x}$. Irrotational flow has a $\vec{v}$ field with zero curl. In general, different current and density operators (e.g., electromagnetic vs baryon number currents and densities) produce different velocities. For definiteness, let us take the electromagnetic convection current and charge; for proton states

$$\langle\mu|\vec{j}_{op}(\vec{x})|\nu\rangle=\frac{1}{2im}\{\psi_\mu(\vec{x})^\dagger\vec{\nabla}\psi_\nu(\vec{x})-[\vec{\nabla}\psi_\mu(\vec{x})]^\dagger\psi_\nu(\vec{x})\}.$$

Then $\vec{v}(\vec{x})$ is the average velocity field of the protons. For the doubly even, self-conjugate nuclei explicitly to be considered, the neutron velocity field is the same. If the cranked Hamiltonian is furthermore spin independent, the magnetic moment contribution to the current vanishes, and the convection current is the whole electromagnetic current (ignoring finite nucleon size and meson exchange effects).

A simple example is the axisymmetric harmonic oscillator with $\omega_1 = \omega_2 \neq \omega_3$. In this case the intermediate state sum Eq. (1) can be closed by explicit construction of an operator whose commutator with the unperturbed Hamiltonian is j_x:

$$-im[(\omega_3^2 + \omega_2^2)yz - 2\nabla_y\nabla_z/m^2]/(\omega_3^2 - \omega_2^2).$$

One finds

$$\vec{j}(\vec{x})/\omega = [(\omega_3^2 + \omega_2^2)\vec{j}_1(\vec{x}) + 2\omega_2\omega_3\vec{j}_2(\vec{x})]/(\omega_3^2 - \omega_2^2), \quad (3)$$

where

$$\vec{j}_1(\vec{x}) = \rho(\vec{x})\vec{\nabla}yz, \quad (3a)$$

$$\vec{j}_2(\vec{x}) = \mathrm{Re}\sum_{\mu}^{Z} \{[\vec{\nabla}\psi_\mu^\dagger(\vec{x})][\nabla_y\nabla_z\psi_\mu(\vec{x})] - \psi_\mu^\dagger(\vec{x})\vec{\nabla}\nabla_y\nabla_z\psi_\mu(\vec{x})\}/(m^2\omega_2\omega_3), \quad (3b)$$

$$\rho(\vec{x}) = \sum_{\mu}^{Z} \psi_\mu^\dagger(\vec{x})\psi_\mu(\vec{x}), \quad (4)$$

where ψ_μ are normalized harmonic oscillator eigenfunctions and the sum over quantum numbers μ includes all occupied proton states.

For a filled major shell, one finds $\vec{j}_1 = -\vec{j}_2$ and, remarkably, the velocity field is irrotational. This property has been surmized on the basis of $\mathcal{I}_{\text{crank}}$ alone,[5] but the present result is needed to verify detailed irrotational flow. The physical relevance is moot, however, since closed-shell nuclei are spherical.

When the potential is self-consistently deformed (the oscillator frequencies ω_i being chosen inversely proportional to $3A/2$ plus the total number of quanta of excitation in that direction), $\mathcal{I}_{\text{crank}}$ takes the rigid-body value. The same happens for arbitrary fixed deformations in the ground-state configuration as $A \to \infty$. These results have been used to argue for rigid-body motion as the consequence of self-consistent independent-particle dynamics.[5,6] Examination of the currents does not support this interpretation. It is true that $\vec{v}$ in the $A \to \infty$ limit is rigid, if certain improprieties are permitted in passing to the limit.[3] However, self-consistent deformation does not produce rigid $\vec{v}$ for finite A.

Straightforward models of ^{8}Be and ^{12}C as rotational nuclei can be constructed in this harmonic oscillator cranking model. The configuration for ^{8}Be is $(0\,0\,0)^4(0\,0\,1)^4$ and, using Eqs. (2)–(4),

$$\rho_8(\vec{x}) = 2(1 + 2m\omega_3 z^2)(m^3\omega_1\omega_2\omega_3/\pi^3)^{1/2} \times \exp\left(-\sum_{i=1}^{3} m\omega_i x_i^2\right), \quad (5)$$

$$\vec{v}_8(\vec{x}) = \frac{\omega_3 - \omega_2}{\omega_3 + \omega_2}\,\omega\vec{\nabla}yz - \frac{4\omega_2\omega_3}{\omega_3^2 - \omega_2^2}(1 + 2m\omega_3 z^2)^{-1}\vec{\omega} \times \vec{x}.$$

The configuration of ^{12}C is $(0\,0\,0)^4(1\,0\,0)^4(0\,1\,0)^4$ and

$$\rho_{12}(\vec{x}) = 2(1 + 2m\omega_1 x^2 + 2m\omega_2 y^2)(m^3\omega_1\omega_2\omega_3/\pi^3)^{1/2} \times \exp\left(-\sum_{i=1}^{3} m\omega_i x_i^2\right), \quad (6)$$

$$\vec{v}_{12}(\vec{x}) = \frac{\omega_3 - \omega_2}{\omega_3 + \omega_2}\,\omega\vec{\nabla}yz + \frac{4\omega_2\omega_3}{\omega_3^2 - \omega_2^2}(1 + 2m\omega_1 x^2 + 2m\omega_2 y^2)^{-1}\vec{\omega} \times \vec{x}.$$

Neither Eqs. (5) nor Eqs. (6) become rigid at the respective self-consistent deformations $\omega_1 = \omega_2 = 2\omega_3$, $\omega_1 = \omega_2 = 0.6\omega_3$.

The fields of Eq. (6), for self-consistent deformation $\omega_1 = \omega_2 = 0.6\omega_3$, are depicted in Figs. 1–3 *with the velocity* $\vec{\omega} \times \vec{x}$ *subtracted* for ease of comparison with rigid-body motion. There are three vortices, one rotating faster than rigid in the center and two counterrotating in the limbs. The net angular momentum in this *relative* flow field vanishes, since $\mathcal{I}_{\text{crank}} = \mathcal{I}_{\text{rigid}}$ for self-consistent deformation.

It may be that, in larger nuclei with occupied states having more nodes, the vortices in the relative velocity field grow in number and density, yielding a flow which resembles rigid-body flow when averaged over a distance fairly small compared to nuclear dimensions. Preliminary calculations of ^{20}Ne support such a supposition. The connection between self-consistent independent particle dynamics and rigid flow may not have been entirely lost, but it is surely not microscopic, and is clearly wrong for light nuclei.

Models in which a spherical core does not participate in the rotation are not supported by these results. In both nuclei, the central region rotates *more rapidly* than rigid.

The currents and velocities have been presented in lowest order in ω, whence they are linear in ω. *A priori*, this approximation need not be very good for even the lowest states of light nuclei. The exact solution of the harmonic oscillator cranking may be carried out.[7] For ^{12}C, the exact $\langle l_x \rangle$ is 1.9 when the linear approximation is 2. The approximation may be good for the first excited state. It is certainly quite bad for the next state, for which the corresponding figures are 3.0 and 4.

The nuclear current fields determine the trans-

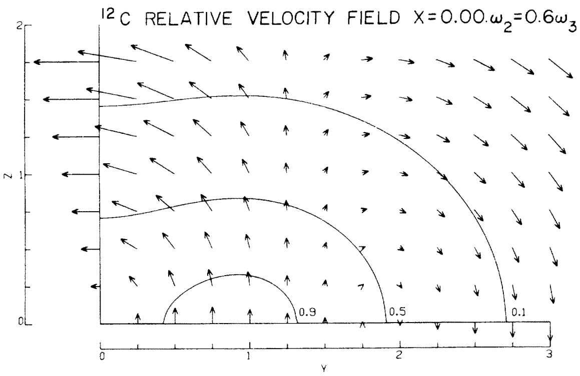

FIG. 1. Velocity field for self-consistently deformed ^{12}C with rigid-body velocity $\vec{\omega}\times\vec{x}$ subtracted, is shown in the midplane of rotation $x=0$. The figure uses units $\hbar=m=\omega_3=1$; for $\hbar\omega_3=22.4$ MeV the unit distance is 1.36 fm. The angular velocity is $\omega\hat{e}_x$ with $\omega=0.1$; $\vec{v}$ is linear in ω to the approximation considered. The arrows represent $\vec{v}(\vec{x})$ for $\vec{x}$ at the origin of the arrow; $\vec{v}$ has no x component. The other quadrants are symmetrically related to the displayed first quadrant. The isodensity contours are drawn at 0.9, 0.5, and 0.1 of the overall maximum charge density.

verse amplitudes for inelastic electron scattering between members of a rotational band, just as do the charge distributions the longitudinal amplitudes. In the free rotation of a classical fluid drop with time-symmetric intrinsic state, the current distribution is, to lowest order, linear in $\vec{\omega}$ and follows the rotation of the drop. There is therefore an $\vec{\omega}$-independent local tensor field $\overleftrightarrow{T}(\vec{x})$ which defines the current through

$$\vec{j}(\vec{x},t)=\overleftrightarrow{R}_{\alpha\beta\gamma}\cdot[\overleftrightarrow{T}(\overleftrightarrow{R}_{\alpha\beta\gamma}{}^{-1}\cdot\vec{x})]\cdot(\vec{I}/\mathcal{J}), \tag{7}$$

where $\overleftrightarrow{R}_{\alpha\beta\gamma}$ is the 3×3 transformation matrix which actively maps the orthonormal triad of a fixed frame of reference onto the orthonormal triad of the body-fixed frame, through the Euler rotations by angles α, β, and γ. The angular momentum $\vec{I}$ is referred to body-fixed axes. In the terms previously used

$$\vec{j}(\vec{x})/\omega=\overleftrightarrow{T}(\vec{x})\cdot\hat{e}_x,$$

which relation, by axial symmetry, defines $\overleftrightarrow{T}$ completely. Equation (7) defines a current operator suitable for placement between $\mathfrak{D}$ function wave functions of the axisymmetric top.[8]

This procedure allows definition of all the intraband transition multipoles in terms of the intrinsic moments of $\vec{j}(\vec{x})/\omega$:

$$\langle I_f^\pi||\hat{T}_\lambda^{\mathrm{el}}(q)||I_i^\pi\rangle=\{[I_f(I_f+1)-I_i(I_i+1)]/2\mathcal{J}\}(-1)^\lambda \times[\lambda(\lambda+1)]^{-1/2}(2I_i+1)^{1/2} \times 2(\lambda 0,I_i 0|I_f 0)M_\lambda(q), \tag{8a}$$

$$M_\lambda(q)=-iq^{-1}[\lambda(\lambda+1)]^{-1/2} \times\int d^3x\{\vec{\nabla}\times[\vec{x}\times\vec{\nabla}j_\lambda(qx)Y_{\lambda 1}(\Omega)]\}\cdot\vec{j}(\vec{x})/\omega. \tag{8b}$$

The multipole notation is that of de Forest and Walecka.[9] An exactly similar result holds for magnetic multipoles T_λ^{mag}. However, only the even electric amplitudes $\langle I^+||\hat{T}_I^{\mathrm{el}}(q)||0^+\rangle$ are observable by inelastic electron scattering on even-even nuclei.

Equations (8) are quite general. It is not necessary to use the cranking model for $\vec{j}(\vec{x})/\omega$. For example, a rigid-body model $\vec{j}(\vec{x})/\omega=\hat{e}_x\times\vec{x}\rho(\vec{x})$ can be used.

Either Coriolis renormalization of the current operator[6] or Coriolis band mixing of the states[10,11] yields Eq. (8a) when carried out to first order for the even-even ground state band, if $M_\lambda(q)$ is chosen appropriately. If, furthermore, the cranking result for $\vec{j}(\vec{x})/\omega$ is used in Eq. (8b) to define $M_\lambda(q)$, the result is formally identical to that of calculating the renormalization or band-mixing parameters in terms of intrinsic states, if these last can

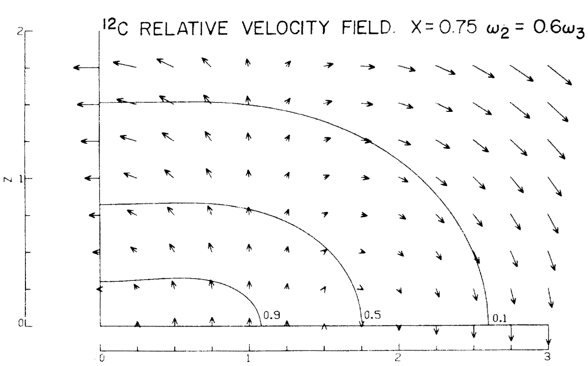

FIG. 2. Same as Fig. 1, for $x=0.75$.

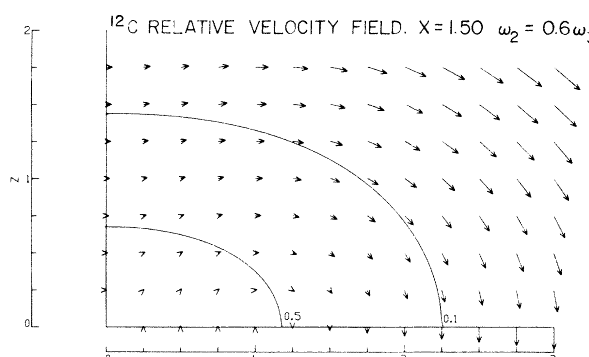

FIG. 3. Same as Fig. 1, for $x=1.50$. The 0.9 density surface does not intersect this plane.

be identified with unperturbed states of the cranking Hamiltonian.

Current conservation, $\vec{\nabla}\cdot\vec{j}(\vec{x})=\vec{\omega}\cdot\vec{x}\times\vec{\nabla}\rho$, and integration by parts of Eq. (8), guarantees

$$M_\lambda(q)=M_{\lambda,\mathrm{LD}}(q)+\tfrac{1}{2}q^{-1}\int d^3x\, qxj_{\lambda+1}(qx)Y_{\lambda 0}(\Omega)\rho(\vec{x})$$
$$+i[\lambda(\lambda+1)]^{-1/2}\int d^3x\, j_\lambda(qx)Y_{\lambda 1}(\Omega)q\vec{x}\cdot[\vec{j}(\vec{x})/\omega], \tag{9}$$

where

$$M_{\lambda,\mathrm{LD}}(q)\equiv-\tfrac{1}{2}(\lambda+1)q^{-1}\int d^3x\, j_\lambda(qx)Y_{\lambda 0}(\Omega)\rho(\vec{x}). \tag{10}$$

LD stands for "liquid drop". Substitution of $M_{\lambda,\mathrm{LD}}$ for M_λ in Eq. (7) gives results equivalent to the liquid drop vibrational model for allowed single surfon transitions,[9]

$$\langle I^\pi||\hat{T}_I^{el}(q)||0^+\rangle_{\mathrm{LD}}=-\frac{E_f-E_i}{q}\left(\frac{I+1}{I}\right)^{1/2}\langle I^\pi||\hat{M}_I^{\mathrm{Coul}}(q)||0^+\rangle,$$

when the first factor in Eq. (8a) is recognized as the transition energy. The remaining terms in Eq. (9) are of relative order $(qR)^2$ at small q. The ratio of transverse to Coulomb electroexcitation is *always* correctly given by the liquid drop model in the long-wavelength limit.

Transverse electroexcitation may be the only measurement sensitive to the spatial details of the currents of nuclear rotation. Separation of this from longitudinal excitation, especially for heavy nuclei, is extremely difficult but worth considerable effort. As far as direct observation of the even-even rotational core is concerned, only the electric multipoles are available. Any purely magnetic current, such as that of rigidly rotating structures with symmetry about the rotation axis, has no effect in Born approximation on excitation of the ground-state band of doubly even nuclei.

Experimental data[12,13] exist for the transverse excitation of the 2^+ (4.44 MeV) state of ^{12}C. We need to fix the parameters $\omega_2(=\omega_1), \omega_3$. I have worked out two choices in detail. Choice A, $\omega_3=\frac{5}{3}\omega_2=22.4$ MeV, has self-consistent deformation, an rms radius (neglecting finite proton size) in agreement with experiment $\langle r^2\rangle^{1/2}=2.46$ fm,[15] and a $B(C2,0^+\to 2^+)$ in agreement with that measured in the low-q electroexcitation of Ref. 13, as reported in Ref. 14. But the moment of inertia is too large and gives $E_{2+}=2.97$ MeV. Choice B, $\omega_2=14.7$ MeV, $\omega_3=29.8$ MeV, has been chosen to fit the $C2$ transition radius of the low-q longitudinal excitation, and has $\Gamma_0^\gamma/(E_{2+})^5=10.9$ meV/(4.439 MeV)^5, $R_{\mathrm{tr}}{}^2=9.35$ fm^2, $\langle r^2\rangle^{1/2}=2.32$ fm, and $E_{2+}=4.33$ MeV. Experimentally[12,13] $\Gamma_0^\gamma=11.0\pm0.6$ meV, and $R_{\mathrm{tr}}{}^2=9.35$ fm^2. Figures (1–3) are not altered much by choice B. The central, super-rotating vortex shrinks slightly, to the benefit of the limb vortices.

The fractional difference between $\langle T^{\mathrm{el}}\rangle/\langle M^{\mathrm{Coul}}\rangle$ and the liquid drop model value thereof is, by Eq. (7), $[M_2(q)/M_{2,\mathrm{LD}}(q)-1]$. Equations (6), (9), and (10) determine this quantity, which has been numerically calculated by a power series expansion in q^2. Because of cancellation between numerator and denominator the lowest-order result remains valid to about 10% through 1 fm^{-2} for the range of ω_i considered here:

$$\frac{M_2(q)}{M_{2,\mathrm{LD}}(q)}-1=\frac{-q^2}{3m}\,\frac{\left[\frac{1}{7}(\frac{9}{2}\omega_3^{-2}+\frac{5}{2}\omega_3^{-1}\omega_2^{-1}-14\omega_2^{-2})+5\omega_2^{-1}\omega_3^{-1}(\omega_3-\omega_2)/(\omega_3+\omega_2)\right]}{(3\omega_3^{-1}-5\omega_2^{-1})}+O(q^4). \tag{11}$$

The numerical results at 0.6 fm^{-1} are -9.5% for choice A and -8.1% for choice B. Other reasonable choices for ω_i are between these extremes. If a rigid velocity is used for $\vec{v}_{12}$, the result (11) is modified by striking the term proportional to $(\omega_3-\omega_2)/(\omega_3+\omega_2)$ and gives -16% to -18% at 0.6 fm^{-1}. Thus our cranking model predicts a *cross-section* ratio for transverse/longitudinal excitation which is 16–20% below the liquid drop model at $q=0.6$ fm^{-1}, while the rigid-body model predicts ~35% less than LD.

According to Ref. 14, Ref. 13 finds no variation out to $q=0.6$ fm^{-1} from the liquid-drop ratio within experimental accuracy of about 20%. Reference (12) finds $(M_2/M_{2,\mathrm{LD}})^2=1.2\pm0.2$ at 0.27 fm^{-1} where we find 0.97. Values of $(M/M_{\mathrm{LD}})^2$ extracted from a Rosenbluth plot from Ref. (13) reproduced in Ref. (14) range from ~1.2 to ~1.5. Since our ratios are less than unity, the rigid-body model is in conflict with experiment and the cranking model may be marginally so. Thus, some information on the flows of nuclear rotation is already implicit in these five-year old experimental results. Perhaps more importantly, modest increase in experimental precision will really allow reasonable models to be distinguished.

ACKNOWLEDGMENT

Part of this work was done while the author was at Stanford University; the support of the Institute for Theoretical Physics is gratefully acknowledged. I thank J. D. Walecka for the suggestion which led to this work, and George Bertsch for useful discussions and criticism of the manuscript.

*Research sponsored in part by the National Science Foundation.

[1]A. Bohr and B. R. Mottelson, K. Dan. Vidensk. Selsk. Mat.–Fys. Medd. 27, no. 14, 1 (1952).

[2]A. de-Shalit and H. Feshbach, *Theoretical Nuclear Physics* (Wiley, New York, 1974), Vol. I.

[3]M. Radomski, Stanford University Ph.D. thesis, 1974 (unpublished); and (unpublished).

[4]D. R. Inglis, Phys. Rev. 97, 701 (1955).

[5]A. Bohr and B. R. Mottelson, K. Dan. Vidensk. Selsk. Mat.–Fys. Medd. 30, no. 1, 1 (1955).

[6]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, 1975), Vol. II.

[7]J. G. Valatin, Proc. Roy. Soc. London 238, 132 (1956).

[8]Provided (7) is symmetrized with respect to operator ordering, since $\vec{I}$ and $R_{\alpha\beta\gamma}$ do not commute.

[9]T. de Forest and J. D. Walecka, Adv. Phys. 15, 1 (1966).

[10]Yu. T. Grin and I. M. Pavlichenkow, Phys. Lett. 38B, 249 (1964).

[11]V. M. Mikhailov, Izv. Akad. Nauk SSSR Ser. Fiz 30, 1334 (1966) [Bull. Acad. Sci. USSR Phys. Ser. 30, 1392 (1966)].

[12]P. Strehl, Z. Phys. 234, 416 (1970).

[13]H. J. Halder and E. Spamer, cited in Ref. 14 as "to be published (1971)."

[14]H. Theissen, *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer-Verlag, New York, 1972), Vol. 65, p. 1.

[15]G. Fey, H. Frank, W. Schütz, and H. Theissen, Z. Phys. 265, 401 (1973).