# Model independent estimate of the *dnp* coupling constant

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Analyticity of scattering amplitude has been exploited for the determination of the *dnp* vertex from very low energy proton deuteron scattering data. The result compares favorably with previous estimates, from other consideration.

[NUCLEAR REACTIONS Spectroscopic factor $g$ for *dnp* system. Reaction chosen *pd*-elastic scattering at 1.5 to 3 MeV. Calculations, analyticity method of Cutkosky.]

In modern nuclear physics one of the fundamental principles is practical exploitation of analyticity. Taking the nuclei as structureless[1,2] elementary objects one can try to estimate the particle particle-nucleus or nucleus nucleus-particle coupling from the extrapolation of the low energy scattering data. On the other hand, the same quantity known as the spectroscopic factor can be obtained on the basis of specific nuclear models and can be compared.

The principal tool underlying the analytic extrapolation, is the information about the nearest singularities of the amplitude in the $Z=\cos\theta$ plane at fixed energy.[3,4] It was proposed that this method could be useful in nuclear problems also. Attempts worth mentioning are those of Kisslinger[5] and Dumbrois[6]: While the former extracts the $^3$He $\to p+d$ vertex from $p$-$^3$He scattering in the energy range 4–20 MeV, the latter considers several cases like $^3$He$^3$H$\pi$, $^3$H$dn$, $^4$He$dd$, etc. Incidentally, it can also be mentioned that Kisslinger[7] also considered the *dpn* case from elastic *nd* scattering. In this note we have reconsidered the problem of the *dpn* vertex. This was calculated at three different energies ranging from 1 to 3 MeV. In the following we briefly discuss the conformal mapping technique which successfully tackles the low energy problem.

The conformal mapping procedure of Cutkosky and Deo[8] and Ciulli[9] is used to handle the large distortions caused by the cuts which have previously prevented the extrapolation. The method, well known by this time, consists of mapping the entire cut $\cos\theta$ plane onto an unifocal ellipse in the $Z$ plane so that the data are placed in the interval $-1\le Z\le +1$, the poles are inside the ellipse, and the cuts are on the ellipse. Let us suppose that the data are known somewhere between $\cos\theta_1$ and $\cos\theta_2$ in the physical region. First one maps the $\cos\theta$ plane onto the $Y$ plane extending the region $(\cos\theta_1,\cos\theta_2)$ to $(-1,1)$ by

$$Y=(2\cos\theta-\cos\theta_1-\cos\theta_2)/(\cos\theta_2-\cos\theta_1). \quad (1)$$

Next, using

$$\begin{aligned}\omega&=(y-y_0)/(1-yy_0),\\ y_0&=(y_- - y_+)/(y_+y_- + Y_+Y_- -1),\end{aligned} \quad (2)$$

one symmetrizes the positions of the singularities so that they run along $(-\infty,\omega)$ and $(\omega,\infty)$. Here, $Y_\pm=(y_\pm{}^2-1)^{1/2}$ and $y_-$ ($y_+$) denotes the beginning of the nearest (to the physical region) left hand (right hand) cut, and

$$\omega=\frac{y_+Y_- + y_-Y_+}{Y_+ + Y_-}. \quad (3)$$

Finally, the entire $Z$ plane is mapped onto a unifocal ellipse by

$$Z=\sin U\left(\omega,\frac{1}{W}\right). \quad (4)$$

In order to apply this mapping technique to the present problem one must search for the singularities of the amplitude in the $\cos\theta$ plane. These

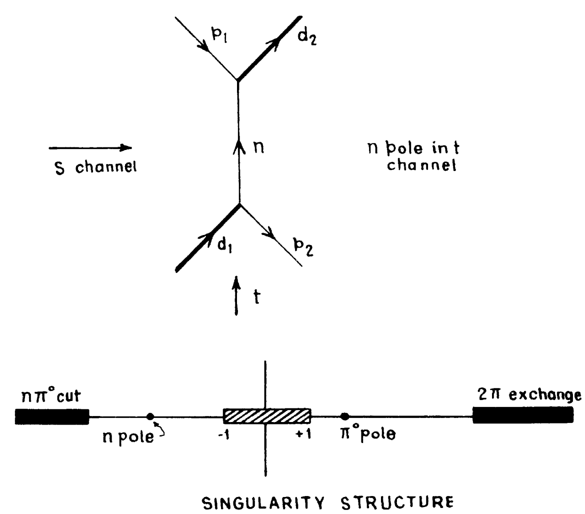


FIG. 1. Figure showing the neutron pole in *pd*-elastic scattering and analyticity structure of *pd*-scattering amplitude in $\cos\theta$ plane.

TABLE I. Various values of $g^2$ calculated from $pd$-elastic scattering data at $E$(MeV) = 2.08, 1.51. and 2.53, with different numbers of terms in the expansion (5). The first column gives the value of $g(dnp)$ as obtained in Ref. 9.

| $g^2$ (Ref. 9) | $E$ (MeV) | $n$ | $g^2$ (calculated) (fm) |
|---|---|---|---|
| | 2.08 | 3 | 0.220 |
| | | 4 | 0.284 |
| 0.398 fm | 1.51 | 2 | 0.146 |
| | | 3 | 0.170 |
| | | 4 | 0.270 |
| | 2.53 | 2 | 0.285 |
| | | 3 | 0.293 |
| | | 4 | 0.377 |

are best calculated with the help of Feynmann diagrams such as is shown in Fig. 1. These are then determined by the following formulas[10]:

$$X_{n\pi} = -\left[1 + \frac{(m_\pi + m_n)^2}{2k^2} - \frac{(m_p{}^2 - m_d{}^2)^2}{2k^2 s}\right],$$

$$X_n = -\left[1 + \frac{m_n{}^2}{2k^2} - \frac{(m_p{}^2 - m_d{}^2)^2}{2k^2 s}\right], \tag{5}$$

$$X_{2\pi} = 1 + \frac{2m_\pi{}^2}{k^2}.$$

The $\pi^0$ pole is not taken because of isospin conservation. The $n\pi^0$ represents the left hand cut.

We then use the following expansion;

$$(Z - Z_{\text{pole}})^2 \frac{d\sigma(z)}{d\Omega} = \sum_{n=1}^{M} A_n B_n T_n(Z), \tag{6}$$

where $T_n$'s are Tschebysheff polynomials and are known as functions of $R$, the sum of semiaxes of the ellipse. $A_n$ are to be obtained for the best fit. This specific form of expansion is used because it is seen that they accelerate maximally the rate of convergence of the expansion.

We have calculated $A_n$ up to $n = 4$ and have determined the right hand side and hence determined $(\text{Res}_{\text{pole}})^2$ through them:

$$\lim_{z \to z_{\text{pole}}} [Z(X) - Z(X_{\text{pole}})]^2 \frac{d\sigma(z)}{d\Omega}$$

$$= (\text{Res}_{\text{pole}})^2 = \sum_{n=1}^{4} A_n B_n T_n(Z_{\text{pole}}). \tag{7}$$

The connection between $(\text{Res}_{\text{pole}})^2$ and $g_{dnp}$ is

$$(\text{Res}_{\text{pole}})^2 = \frac{1}{16\pi^2 s} 7 m_n{}^6 (g_{dnp})^4 \left(\frac{dz}{dx}\right)_{z = z_{\text{pole}}}. \tag{8}$$

The results of our calculation at three different energies are exhibited in Table I. Since the values are converging and no experimental error in the data of Ref. 11 is available we have not done any $\chi^2$ estimate.

Lastly some comments are in order about our approach to the problem. As we have gone to a lower energy range than Kisslinger, Coulomb effects may be of some importance, which has already been considered in Ref. 7. Indeed as a first approximation we have neglected any such Coulomb corrections.

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