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On the use of $\alpha+\alpha$ scattering to study the nucleon-nucleon interaction*

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An analysis of δ_4 for $\alpha+\alpha$ scattering below 15 MeV (c.m.) has been made to study whether or not useful information about the direct part V_d of the nucleon-nucleon interaction can be obtained. It was expected that the peripheral nature of the $l=4$ interaction would allow a relatively simple analysis to be made. However, it was found that such an analysis does not exclude any of the commonly used nucleon-nucleon potentials as being inconsistent with $\alpha+\alpha$ scattering, but that proper handling of the short-range part of V_d might well result in such an exclusion.

[NUCLEAR REACTIONS Analysis of $\alpha+\alpha$ scattering to study direct part of nucleon-nucleon potential; $E_{\text{c.m.}}<15$ MeV.]

It has been suggested[1] that a study of the interaction between two α particles can be used to obtain information on the direct part (spin-isospin independent part) V_d of the nucleon-nucleon potential. The $\alpha+\alpha$ system has several characteristics which make it especially useful for such an investigation: (i) the first reaction threshold occurs at a c.m. energy of 17.35 MeV, and therefore, an analysis of $\alpha+\alpha$ scattering in terms of a real potential $V_{\alpha\alpha}$ can be carried out over a broad energy range. (ii) Fully antisymmetrized calculations[2] have shown that the nonlocal exchange contributions to $V_{\alpha\alpha}$, as opposed to the local direct contribution V_D, are of minor importance in states with $l\geq 4$. (iii) Because the α particle has an isospin I of 0 and a spin-parity J^π of 0^+, V_D arises mainly from V_d, and (iv) the $l=4$ phase shift increases rapidly in the c.m. energy range 9 to 14 MeV[3] and is therefore sensitive to $V_{\alpha\alpha}$ in this energy range. Furthermore, one would hope that the peripheral nature of the $l=4$ interaction would allow one to neglect complications arising from the short-range part of V_d. These characteristics suggest the possibility that a simple potential-model analysis of the $\alpha+\alpha$, $l=4$ phase shift δ_4 might be used to extract information on V_d. In fact, it would be of great interest if $\alpha+\alpha$ scattering could be used to impose constraints on the nucleon-nucleon interaction in addition to those obtained from direct studies of the two-nucleon system.

Because of item (ii) above, we analyze δ_4 by use of the direct contribution V_D to $V_{\alpha\alpha}$ and neglect exchange contributions. Thus the potential V for $\alpha+\alpha$ scattering in states with $l\geq 4$ is written as $V=V_D+V_C$, where V_C is the Coulomb potential given by Eq. (11) of Ref. 3 and V_D is obtained by using a double-folding procedure [see Eq. (14) of Ref. 3], i.e.,

$$V_D(r)=\int \rho(r_1)\rho(r_2)V_d(s)\delta(\vec{r}_2-\vec{r}_1-\vec{s}+\vec{r})\times d\vec{r}_1 d\vec{r}_2 d\vec{s}\,. \qquad (1)$$

In Eq. (1) the α-particle matter density ρ is taken to be of Gaussian form and is given by Eq. (16) of Ref. 3.

We choose to use for V_d, the direct part of the nucleon-nucleon potential, a form deduced from a recent[4] investigation of the nucleon-nucleon interaction using a one-boson-exchange model. Only exchange of mesons having $I=0$ and $J^\pi=0^+$ or 1^- contributes to V_d, and furthermore, Ref. 4 shows that the coupling strengths of the ϵ ($I=0$, $J^\pi=0^+$) and the ω ($I=0, J^\pi=1^-$) to the nucleon are much greater than those of other mesons having the appropriate quantum numbers. Therefore, we consider only ϵ and ω exchange as contributing to V_d. The contribution $V_{d\omega}$ to V_d from ω exchange is given by[5]

$$V_{d\omega}=g_\omega^2\hbar c\left(1+\frac{m_\omega^2}{4M^2}+\frac{m_\omega f_\omega}{2Mg_\omega}\right)^2 r^{-1}\exp(-m_\omega c r/\hbar)\,, \qquad (2)$$

with $m_\omega c^2=783.9$ MeV, $Mc^2=938.905$ MeV, and[4] $f_\omega/g_\omega=0.637$. The status of the ϵ meson is discussed in Refs. 3, 4, and 6. Briefly, the ϵ seems to appear in the $\pi\pi$ interaction as an $I=0$, s-wave resonance which is several hundred MeV broad. Thus the expression for $V_{d\epsilon}$, the contribution to V_d from ϵ exchange, involves an integration over the ϵ mass distribution $J(m)$. We take[7]

$$V_{d\epsilon}=-g_\epsilon^2\hbar c\int_{2\mu c^2}^{\infty}J(m)r^{-1}\exp(-mcr/\hbar)d(mc^2)\,, \quad (3)$$

with $2\mu c^2=269.929$ MeV (μ is the pion mass) and

$$J(m)=\frac{2m_\epsilon c^2\Gamma}{\pi}mc^2f/[(m^2c^4-m_\epsilon^2c^4)^2+m_\epsilon^2c^4\Gamma^2f^2]\,,$$

with (4)

$$f=\left(\frac{m^2-4\mu^2}{m_\epsilon^2-4\mu^2}\right)^{1/2}.$$

In Ref. 4 the mass and width of the ϵ were taken as $m_\epsilon c^2=670$ MeV and $\Gamma=500$ MeV. On taking $V_d=V_{d\omega}+V_{d\epsilon}$ we obtain from Eq. (1) $V_D=V_{D\omega}+V_{D\epsilon}$, where $V_{D\omega}$ is given by Eq. (18) of Ref. 3, wherein the appropriate expressions for V_0 and β can be deduced by comparing Eq. (2) above with Eq. (17) of Ref. 3. The expression for $V_{D\epsilon}$ is

$$V_{D\epsilon}=-16g_\epsilon^2\hbar c\int_{2\mu c^2}^{\infty}J(m)r^{-1}\exp[-\beta r+3\beta^2/(8\alpha)]\times\tfrac{1}{2}\{1+\Phi(\lambda_-)-\exp(2\beta r)\times[1-\Phi(\lambda_+)]\}d(mc^2)\,, \quad (5)$$

where $\beta=mc/\hbar$, $\alpha=0.514$ fm^{-2} and is the α-particle size parameter deduced[8] from electron-scattering data, Φ is the error function [Eq. (12) of Ref. 3], and λ_+ and λ_- are given by Eq. (20) of Ref. 3. In the fitting, $V_{D\epsilon}$ is calculated numerically.

To fit δ_4 vs c.m. energy E, values for m_ϵ and Γ were fixed, and the coupling constants g_ω^2 and g_ϵ^2 were varied to minimize χ^2. Very good fits were obtained, and a typical such fit to δ_4 is shown in Fig. 1 along with the result for δ_6 using the potential which produced the best fit to δ_4. Figure 2 illustrates several best-fit values for g_ω^2 and g_ϵ^2. The circles are obtained from the present analysis with several different combinations of m_ϵ and Γ. The point which is encircled gives the result found when the m_ϵ and Γ values

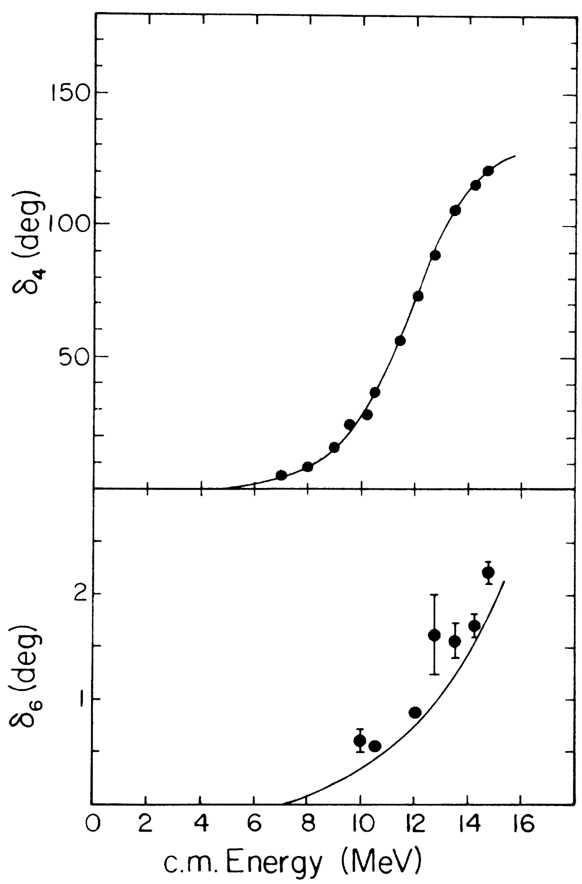

FIG. 1. The curves show the $l=4$ and 6 $\alpha+\alpha$ phase shifts obtained from the $\alpha+\alpha$ potential yielded by a least-χ^2 fit to the empirical $l=4$ phase shifts. The empirical phases (points) used are the same as mentioned in Ref. 3, and not all of these are shown here.

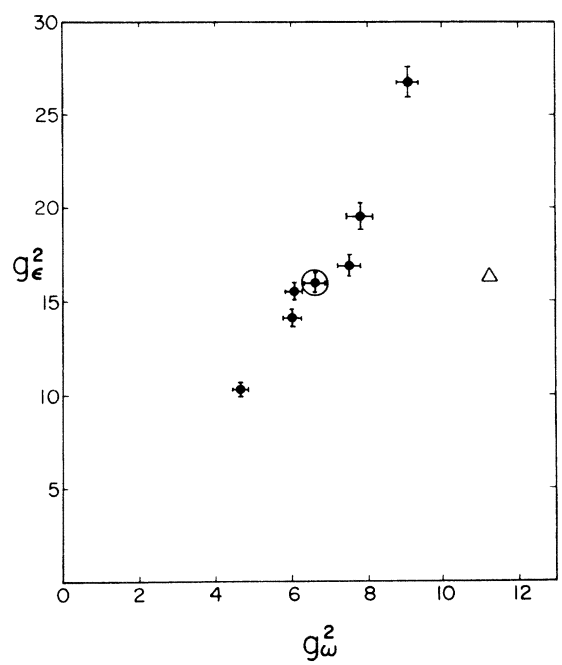

FIG. 2. The circles show a sequence of ωNN and ϵNN coupling constants obtained from least-χ^2 fits to the $l=4$ $\alpha+\alpha$ phase shifts. Different members of the sequence result from the use of different values for m_ϵ and Γ [Eq. (4)]. The bars correspond to coupling constant changes which yield a doubling of the best-fit χ^2 value. The encircled point corresponds to the use of the m_ϵ and Γ values employed in the nucleon-nucleon study of Ref. 4, and the triangle shows the values of the coupling constants obtained in Ref. 4.

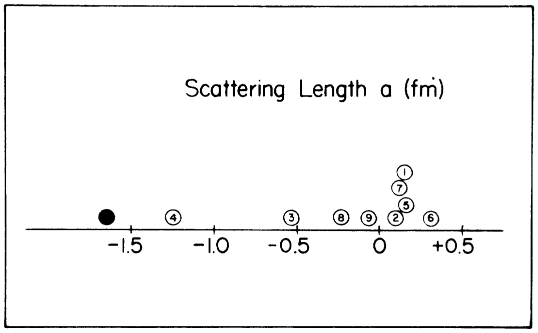

FIG. 3. Nucleon-nucleon s-wave scattering lengths a calculated from the direct parts only of nucleon-nucleon potentials. The solid circle refers to the present potential corresponding to the encircled point of Fig. 2. The labels 1–5 refer to the potentials of Refs. 4 and 9–12, respectively; the labels 5 and 6 refer to the hard-core and soft-core potentials, respectively, of Ref. 13; and the labels 8 and 9 refer to the potentials of Refs. 14 and 15, respectively.

of Ref. 4 were used, and the triangle shows the result of Ref. 4 from fitting nucleon-nucleon data. Because $V_{d\omega}$ is repulsive, we see from Fig. 2 that the V_d of the nucleon-nucleon potential of Ref. 4 is not as attractive as the V_d we determine here from $\alpha+\alpha$ scattering. If the coupling constants of Ref. 4 are used in the calculation of V_D, then a repulsive component is introduced which is large enough to cause δ_4 to attain only small positive values at the low energies and increasingly negative values at the high energies.

We also compare the present V_d with direct parts of other nucleon-nucleon potentials from the literature.[9-15] These comparisons are given in Fig. 3 where the s-wave scattering lengths for nucleon-nucleon scattering with interaction V_d are shown. The more negative the value of a, the more attractive is the potential (i.e., the larger is the low-energy s-wave phase shift yielded by V_d). The solid circle shows the present result and illustrates that the V_d we derive is more attractive than the V_d of any of the commonly used nucleon-nucleon potentials. Some corrections to the present formulation of the $\alpha+\alpha$ analysis have been discussed in Refs. 1 and 3, but these are expected to produce only minor changes in the present results. It is more likely that the fact that the $\alpha+\alpha$ analysis produces a too attractive V_d is associated with the existence of a short-range repulsive part in V_d (produced by ω exchange) and the lack of allowance for short-range correlations in Eq. (1). This lack results in the need for a too attractive V_d in Eq. (1) in order to yield sufficient attraction in V_D to properly reproduce δ_4 vs E. Of course for this effect to be important, some of the nucleons in one α particle must come close to some of those in the other during the collision, and one might at first suppose that the $l=4$ centrifugal barrier would prevent this from happening. However, it must be remembered that the present analysis covers an energy region where an $l=4$ resonance occurs, and therefore, the two α particles must overlap sufficiently to cause this resonance. By determining the spatial region of V_D which contributes significantly to δ_4, we have verified that significant overlap does occur.

It seems clear then that proper inclusion of short-range correlations in the analysis would bring the V_d deduced from the $\alpha+\alpha$ interaction more into agreement with that yielded by the nucleon-nucleon potentials in common use. We can therefore regard the present calculation as giving a lower limit to the value of the scattering length a which the direct part V_d of a nucleon-nucleon potential should yield in order that this V_d be consistent with $\alpha+\alpha$ scattering. From Fig. 3 it is seen that none of the nucleon-nucleon potentials considered contradict this constraint imposed by $\alpha+\alpha$ scattering. However, the above mentioned improvement in the calculation might well change this conclusion. To carry out such an improvement is not simple, but some of the possible difficulties are currently being studied.

Finally, in view of the complications introduced by the spatial overlap of the α particles during collision, it might seem that, because of the higher centrifugal barrier in the $l=6$ state, one should use the present procedure to fit δ_6 rather than δ_4. This is not so, however, because of several reasons. In the energy region below the first reaction threshold the δ_6 values are at most a few degrees,[3] and therefore, to obtain sufficient sensitivity of δ_6 to the $\alpha+\alpha$ potential one must apply the analysis in a higher-energy region where the $l=6$ resonance occurs. In that region, of course, the α particles will again overlap significantly during the collision. Furthermore, the additional complication is then present of there being open reaction channels so that a purely real $\alpha+\alpha$ potential cannot be used in the analysis. Therefore, we feel that future improvements to the present procedure should be directed toward fitting δ_4.

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