

Doubly radiative np capture

H. C. Lee and F. C. Khanna

Atomic Energy of Canada Limited, Chalk River Nuclear Laboratories, Chalk River, Ontario, Canada K0J 1J0

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The theory of the reaction $n + p \rightarrow d + \gamma + \gamma$ is reviewed with emphasis on the dominant, or $(E1, E1)$, mode. The differences between the length and the gradient electric dipole operators are studied in detail and the two-photon cross section ($\sigma_{2\gamma}$) is calculated using both operators. The length operator yields a very reliable value of $\sigma_{2\gamma} = 0.118 \mu\text{b}$ ($\pm 1\%$). The result obtained with the gradient operator is $\sim 20\%$ smaller and has large uncertainties. Results for other two-photon modes are also presented.

[NUCLEAR REACTIONS ${}^1\text{H}(n, \gamma\gamma)$, thermal n , calculated $\sigma_{2\gamma}$.]

I. INTRODUCTION

The doubly radiative np capture reaction $p(n, \gamma\gamma)d$ has received considerable attention since Dress *et al.*¹ reported a very large capture cross section of $\sigma_{2\gamma} \sim 350 \mu\text{b}$ or a branching ratio $\sigma_{2\gamma}/\sigma_{1\gamma}$ of 10^{-3} . Subsequent measurements by Earle *et al.*² and by Wüst *et al.*³ did not support the early result. Meanwhile Alburger⁴ suggested that the result of Dress *et al.*¹ could be instrumental. This was later substantiated by Monte Carlo calculations by Lee and Earle.⁵ The most recent measurements by Earle, McDonald, and Lone⁶ resulted in a value of $\sigma_{2\gamma} = -3 \pm 8 \mu\text{b}$ for photon energies in the range $600 \text{ keV} < \omega < 1620 \text{ keV}$. We refer to Ref. 6 for a more detailed review of the experimental aspect of the subject.

Adler,⁷ allowing the possibility of nonorthogonality⁸ between the wave functions of the triplet continuum and the deuteron ground state, obtained a value of $42 \mu\text{b}$ for $\sigma_{2\gamma}$. (This was the motivation for the measurements by Dress *et al.*¹) In normal quantum theory orthogonality between states with different energies is related to the Hermiticity of the Hamiltonian or the realness of the energy spectrum. In such theories apparent complex energy (e.g., that of a resonance) and non-Hermiticity of the Hamiltonian (e.g., its explicit energy dependence⁹) can arise only as a result of not treating explicitly all states involved in a reaction.¹⁰ Experimentally there is very strong evidence against the existence of any significant amount of non-orthogonality in general. In fact, strong hindrance of some transition processes requires orthogonality between the wave functions at different energies. For example the strong hindrance of the radiative capture of neutrons by deuterons is a direct consequence of the orthogonality between the wave functions of the nd continuum and the triton.¹¹ Certainly there is no experimental evidence that requires nonorthogonality.¹² In this paper we shall

accept the normal quantum theory and require the wave functions of the triplet continuum and the deuteron to be orthogonal.

The formalism for two-photon processes in nuclei has been examined in detail by Grechukhin.¹³ Since then measurements have been made for two-photon branching ratios in several $0^+ \rightarrow 0^+$ transitions.¹⁴⁻¹⁶ Typically these branching ratios are of the order of $2-5 \times 10^{-4}$. However, calculation of two-photon decay rates in such transitions is complicated by uncertainties in nuclear structure.¹⁷ In contrast, a reliable prediction of the ratio $\sigma_{2\gamma}/\sigma_{1\gamma}$ in np capture is feasible because of the simplicity of the two-nucleon system.

Several calculations of $\sigma_{2\gamma}$ have been reported.^{7, 18-23} Grechukhin¹⁸ pointed out that the $(E1, E1)$ mode dominates the two-photon capture reaction. There is general agreement among the various calculations that $\sigma_{2\gamma} \sim 0.1 \mu\text{b}$. However, the two dipole operators [the length operator $e[H, \vec{r} \cdot \vec{\epsilon}]$ and the gradient operator $(ie/M)\vec{\nabla} \cdot \vec{\epsilon}$] used in these calculations are not equivalent; in our earlier paper²² we pointed out that the length operator is the one that ought to be used. These two operators are equivalent for interacting particles only when the interaction is local or momentum independent. It was also pointed out that even if the two operators were equivalent, only the length operator would give an accurate value²⁴ for $\sigma_{2\gamma}$, because only it is insensitive to details of the wave function at short distances about which our information is incomplete. In this paper we elaborate these points and present the calculations in more detail. As well, we review the whole theoretical aspect of two-photon process relating to np capture.

In Secs. II and III the basic theory relating to the two-photon matrix element and the capture cross section is reviewed. In Sec. IV, $\sigma_{2\gamma}$ is calculated using the length operator. We show that the cross section is completely determined by the low ener-

gy data of the triplet np system. In Sec. V we calculate $\sigma_{2\gamma}$ using the gradient operator and show that the result is very sensitive to refinements in the wave function. In Sec. VI results of two-photon capture modes other than the $(E1, E1)$ mode are presented. Finally in Sec. VII the present position is summarized.

II. BASIC THEORY

A. Electromagnetic transition operators

For convenience we choose a coordinate frame such that the z axis is along the direction of the photon momentum $\vec{\omega}$ (we use units such that $\hbar = c = 1$). The vector potential in the Lorentz gauge, normalized to a field intensity of one photon per second per unit area, is¹³

$$\begin{aligned}\vec{A}_q(\vec{\omega}) &= \left(\frac{2\pi}{\omega}\right)^{1/2} e^{i\vec{\omega}\cdot\vec{r}} \hat{\epsilon}_q \\ &= -\left(\frac{\pi}{\omega}\right)^{1/2} \sum_{\lambda=1}^{\infty} [q \vec{\mathfrak{M}}_{\lambda q}(\omega) + \vec{\mathcal{G}}_{\lambda q}(\omega)], \quad q = \pm 1,\end{aligned}\quad (1)$$

where $\hat{\epsilon}_{\pm 1}$ are the unit polar vectors perpendicular to $\vec{\omega}$. The electric and magnetic fields are, respectively, given by

$$\vec{\mathcal{E}}_{\lambda\mu}(\omega) = \frac{1}{\omega} \frac{i^{1+\lambda}}{[\lambda(\lambda+1)]^{1/2}} \vec{\nabla} \times \vec{L} \phi_{\lambda\mu}(\omega), \quad (2)$$

$$\vec{\mathfrak{M}}_{\lambda\mu}(\omega) = \frac{i^\lambda}{[\lambda(\lambda+1)]^{1/2}} \vec{L} \phi_{\lambda\mu}(\omega), \quad (3)$$

where $\phi_{\lambda\mu}(\omega) = [4\pi(2\lambda+1)]^{1/2} i^\lambda j_\lambda(\omega r) Y_{\lambda\mu}(\hat{r})$ and $\vec{L} = \vec{r} \times \vec{p}$. For photons with wavelength which is long compared to the size of the nucleus, only terms which are lowest order in ω need be kept. In this case

$$\vec{\mathcal{E}}_{\lambda\mu}(\omega) \simeq \frac{i}{\omega} \left(\frac{\lambda+1}{\lambda}\right)^{1/2} \vec{\nabla} \phi_{\lambda\mu}(\omega). \quad (4)$$

The matrix element for the electric transition from the state $|i\rangle$ to the state $|f\rangle$ is

$$\begin{aligned}\langle \mathcal{E}_{\lambda q}(\omega) \rangle_{fi} &= -\frac{1}{\omega} \left(\frac{\pi(\lambda+1)}{\omega\lambda}\right)^{1/2} \int \vec{J}_{fi} \cdot \vec{\mathcal{E}}_{\lambda q}(\omega) d^3r \\ &= \frac{i}{\omega} \left(\frac{\lambda+1}{\lambda}\right)^{1/2} \int \vec{J}_{fi} \cdot \vec{\nabla} \phi_{\lambda q} d^3r,\end{aligned}\quad (5)$$

where \vec{J}_{fi} is the current density between the states $|i\rangle$ and $|f\rangle$. Integrating by parts, and using the equation of motion

$$\vec{\nabla} \cdot \vec{J} = -ie[H, \rho],$$

where ρ is the density operator, Eq. (5) becomes^{25, 26} (Siebert's theorem)

$$\langle \mathcal{E}_{\lambda q}(\omega) \rangle_{fi} = e \frac{E_f - E_i}{\omega} \left(\frac{\pi(\lambda+1)}{\omega\lambda}\right)^{1/2} \langle \phi_{\lambda q}(\omega) \rangle_{fi}. \quad (6)$$

The advantage of (6) as compared to (5) is that no explicit knowledge of the current \vec{J}_{fi} is needed to compute the electric transition matrix element for long wavelength photons; only the wave functions of $|i\rangle$ and $|f\rangle$ are needed. The direction of the photon is implicitly expressed only in the coordinates we have chosen. The computation of the two-photon matrix element will be made easier with the following relation:

$$\begin{aligned}\langle \mathcal{E}_{\lambda q}(\omega) \rangle_{fi} &= (-i)^\lambda e \frac{E_f - E_i}{\omega} \left(\frac{(\lambda+1)(2\lambda+1)}{4\omega\lambda}\right)^{1/2} \\ &\quad \times \int d\Omega_\omega Y_{\lambda q}(\hat{\omega}) \langle e^{i\vec{\omega}\cdot\vec{r}} \rangle_{fi}.\end{aligned}\quad (7)$$

When both $|i\rangle$ and $|f\rangle$ are bound states $\langle \vec{\omega} \cdot \vec{r} \rangle_{fi} \ll 1$, (7) may be expanded in powers of ωr and we get the familiar form

$$\langle \mathcal{E}_{1q}(\omega) \rangle_{fi} \approx e \frac{E_f - E_i}{\omega} \left(\frac{2\pi}{\omega}\right)^{1/2} \omega \langle \hat{\epsilon}_q \cdot \vec{r} \rangle_{fi}. \quad (8)$$

The magnetic transition matrix element depends explicitly on the current \vec{J} :

$$\vec{J} = \frac{e}{2M} (-ig\vec{\nabla} + \mu\vec{\nabla} \times \vec{\sigma}) + \vec{J}^{\text{exch}}, \quad (9)$$

where $g_p = 1$, $g_n = 0$; and $\mu_p = 2.79$ and $\mu_n = -1.91$ are, respectively, the magnetic moments of the proton and the neutron in nuclear magnetons. The meson exchange current, \vec{J}^{exch} makes a correction of about 5%^{27, 28} to \vec{J} . Not including the effects of \vec{J}^{exch} , the magnetic transition matrix element is

$$\begin{aligned}\langle \mathfrak{M}_{\lambda q}(\omega) \rangle_{fi} &= (-i)^{\lambda-1} e \frac{\omega}{2M} \left(\frac{\lambda+1}{4\omega}\right)^{1/2} q \\ &\quad \times \int d\Omega_\omega [Y_{\lambda-1}(\hat{\omega}) \times \hat{\epsilon}]_{\lambda q} \\ &\quad \times \left\langle e^{i\vec{\omega}\cdot\vec{r}} \left(g \frac{\vec{1}}{\lambda+1} + \mu\vec{\sigma} \right) \right\rangle_{fi}.\end{aligned}\quad (10)$$

In the long wavelength approximation, the dipole matrix element reduces to the familiar form

$$\langle \mathfrak{M}_{1q}(\omega) \rangle_{fi} = e \frac{\omega}{M} \left(\frac{\pi}{2\omega}\right)^{1/2} q \hat{\epsilon}_q \cdot \left\langle \frac{1}{2} g \vec{1} + \mu\vec{\sigma} \right\rangle_{fi}. \quad (11)$$

B. Two-photon transition matrix element

In general, a two-photon transition matrix element has the form [the only exception is that of the contact or $(e^2/2M)\vec{A} \cdot \vec{A}$ term]

$$\begin{aligned} & \left\langle \Theta(\omega_1) \frac{1}{E_i - H} \Theta(\omega_2) + (1 \rightleftharpoons 2) \right\rangle_{fi} \\ &= \sum_n \langle \Theta(\omega_1) \rangle_{fn} \frac{1}{E_i - E_n - \omega_2} \langle \Theta(\omega_2) \rangle_{ni} + (1 \rightleftharpoons 2), \end{aligned} \quad (12)$$

where the second term is similar to the first term except that the two photons with respective energies ω_1 and ω_2 are interchanged. The summation is over a complete set of intermediate states subscribed by n . The symbol Θ stands for \mathfrak{M}_{λ_q} or

\mathcal{E}_{λ_q} . A very interesting point arises when at least one electric transition is involved. In this case, from (6), factors of either $(E_f - E_n)$ or $(E_n - E_i)$ or both will appear in the numerator in (12). This would indicate a slow convergence in the summation over n . It was pointed out by Grechukhin¹³ that this undesirable feature is removed when the two terms in (12) are explicitly combined. If we use the notation $\langle \tilde{\mathcal{E}}_{\lambda_q}(\omega) \rangle_{fi}$ for the right hand side of (6) excluding the factor $(E_f - E_i)/\omega$, and for the moment suppress the indices λq , then it can be shown that

$$\begin{aligned} \left\langle \mathcal{E}(\omega_1) \frac{1}{E_i - H} \mathcal{E}(\omega_2) + (1 \rightleftharpoons 2) \right\rangle_{fi} &= \left\langle \tilde{\mathcal{E}}(\omega_1) \frac{1}{E_i - H} \tilde{\mathcal{E}}(\omega_2) + (1 \rightleftharpoons 2) \right\rangle_{fi} \\ &+ \frac{1}{\omega_1 \omega_2} \langle [\tilde{\mathcal{E}}(\omega_1), [H, \tilde{\mathcal{E}}(\omega_2)]] \rangle_{fi} - \frac{1}{\omega_1} \langle [\tilde{\mathcal{E}}(\omega_1), \tilde{\mathcal{E}}(\omega_2)] \rangle_{fi}. \end{aligned} \quad (13)$$

The last term on the right hand side in (13) vanishes, since the commutator is zero. The second term can be estimated by assuming that the momentum-dependent part of H is given by $p^2/2M^*$, where $M^* \sim 0.5M$ is the effective mass of the nucleon. It then follows that

$$\langle [\tilde{\mathcal{E}}(\omega_1), [p^2/2M^*, \tilde{\mathcal{E}}(\omega_2)]] \rangle_{fi} = - \frac{\vec{\omega}_1 \cdot \vec{\omega}_2}{M^*} \langle \tilde{\mathcal{E}}(\omega_1) \tilde{\mathcal{E}}(\omega_2) \rangle_{fi}. \quad (14)$$

Noting that the leading term in the last matrix element vanishes we conclude that the second term in (13) is of order $\omega/M^*(\omega R)^2$ compared to the first term, and can be justly ignored.

Similarly it can be shown that

$$\begin{aligned} \left\langle \mathcal{E}(\omega_1) \frac{1}{E_i - H} \mathfrak{M}(\omega_2) + \mathfrak{M}(\omega_1) \frac{1}{E_i - H} \mathcal{E}(\omega_2) + (1 \rightleftharpoons 2) \right\rangle_{fi} \\ = - \left\langle \tilde{\mathcal{E}}(\omega_1) \frac{1}{E_i - H} \mathfrak{M}(\omega_2) + \mathfrak{M}(\omega_1) \frac{1}{E_i - H} \tilde{\mathcal{E}}(\omega_2) + (1 \rightleftharpoons 2) \right\rangle_{fi} + \Theta(\omega R). \end{aligned} \quad (15)$$

III. GENERAL EXPRESSION FOR THE DOUBLY RADIATIVE CAPTURE CROSS SECTION

The total doubly radiative cross section is

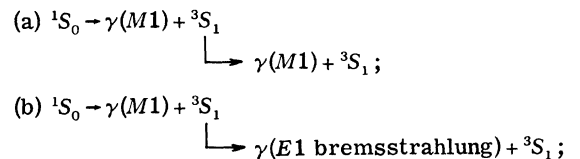
$$\sigma_{2\gamma} = \frac{1}{2} \frac{2J_i + 1}{4} v_n^{-1} \int \frac{d^3\omega_1}{(2\pi)^3} \frac{d^3\omega_2}{(2\pi)^3} \frac{d^3p_f}{(2\pi)^3} (2\pi)^4 \delta(E_i - E_f - \omega_1 - \omega_2) \delta^3(\vec{p}_i - \vec{p}_f - \vec{\omega}_1 - \vec{\omega}_2) \sum' |M_{2\gamma}|^2, \quad (16)$$

where the factor $\frac{1}{2}$ normalizes the two-photon final state; J_i is the spin of the initial state; the factor $\frac{1}{4}(2J_i + 1)$ is the spin-statistical weight factor; $v_n^{-1} = 1.37 \times 10^5 c^{-1}$ is the inverse velocity for thermal neutrons; \sum' sums over the polarization of the photons and magnetic substates of the final state and averages over those of the angular momentum-coupled initial state; and $M_{2\gamma}$ is the two-photon matrix element discussed in the last section. The differential cross section is

$$\frac{d^2\sigma_{2\gamma}}{d\omega_1 d\cos\theta} = \frac{2J_i + 1}{32\pi^3} v_n^{-1} \sum' |M_{2\gamma}|^2 \omega_1^2 (\omega - \omega_1)^2, \quad (17)$$

where θ is the angle between the two photons and $\omega = E_i - E_f = \omega_1 + \omega_2$.

In thermal np capture the initial np system can be in the spin-singlet continuum 1S_0 or the triplet continuum $^3S_1'$. The final np system, or the deuteron, is predominantly (>93%) in the triplet S state 3S_1 . Thus we have the following leading 2γ transitions:



$$(c) \ ^3S_1' - \gamma(M1) + \ ^1S_0$$

$$\quad \quad \quad \searrow \rightarrow \gamma(M1) + \ ^3S_1;$$

$$(d) \ ^3S_1' - \gamma(E1) + \ ^3P$$

$$\quad \quad \quad \searrow \rightarrow \gamma(E1) + \ ^3S_1.$$

It has been pointed out^{13,19} that the cross section for mode (d) or the $(E1, E1)$ mode is by far the largest among these reaction modes. Several authors^{13,19,23} have used the $E1$ operator $(e/M)\vec{p} \cdot \vec{\epsilon}$, which will be called the "gradient operator," as opposed to the "length operator" given in (7) for this mode. Earlier we have shown that Siegert's theorem, leading to the length operator (7), holds as long as the wavelength of the emitted photon is large compared to nuclear sizes. No assumption was made about the nuclear current other than that it satisfies the equation of continuity. Using the gradient operator, on the other hand, assumes explicitly that the nuclear current (for electric transitions) is the convection current $(e/M)\vec{p}$. This is true when the nuclear interaction is momentum independent, since in this case we have

$$e \int \psi_f^* [H, e^{i\vec{\omega} \cdot \vec{r}}] \psi_i d^3r$$

$$= -\frac{e}{2M} \int \psi_f^* [\nabla^2, e^{i\vec{\omega} \cdot \vec{r}}] \psi_i d^3r$$

or

$$e \frac{(E_f - E_i)}{\omega} \langle e^{i\vec{\omega} \cdot \vec{r}} \rangle_{fi} = i \frac{e}{M} \langle \vec{\nabla} \cdot \vec{\epsilon} e^{i\vec{\omega} \cdot \vec{r}} \rangle_{fi}, \quad (18)$$

whereby $(e/M)\vec{p}$ can be identified as the nuclear current. However, even then the equality in (18) will hold only if E_f and E_i and ψ_f and ψ_i are eigenvalues and eigenfunctions of the same Hamiltonian. This self-consistent condition is, of course, not always met in actual computations; it more often happens that experimental energies and approximate wave functions are used. In the next two sections consequences of using these two dipole operators are examined in detail. Results for the capture modes (a), (b), and (c) are presented in a later section.

IV. RESULTS WITH THE LENGTH OPERATOR

A. Using asymptotic wave functions

The length operator is insensitive to the details of the wave function at short distances. Therefore we can expect a good first order estimate of the cross section by using asymptotic wave functions, which are completely determined by low energy scattering data. Improvements to the wave functions will be treated later. For the initial triplet radial wave function we have

$$u_t(r) = \frac{\sin(kr + \delta_t)}{kr}, \quad (19)$$

where δ_t is the triplet (S -wave) phase shift; for low energy $\cot \delta_t = -1/a_t k$, where $a_t = 5.41$ fm is the triplet scattering length. For the deuteron

$$u_d = \frac{N}{\sqrt{4\pi}} \frac{e^{-\kappa r}}{r}, \quad (20)$$

where $\kappa = (M\omega_d)^{1/2}$, $\omega_d = 2.23$ MeV is the deuteron binding energy, and N is the normalization constant. Experimentally N^2 is determined to within¹¹ 1% to be

$$N^2 = 2\kappa/(1 - r_{0t}\kappa) = 3.37\kappa, \quad (21)$$

where $r_{0t} = 1.75$ fm is the triplet effective range. This is to be compared with $N^2 = 2\kappa$ when u_d given in (20) is normalized to unity. For the intermediate state, as a first order approximation, we use a complete set of plane waves.

From (7) and (13), the two-photon matrix element is

$$M_{2\gamma} = \alpha \frac{2\pi}{(\omega_1\omega_2)^{1/2}} \frac{3}{4\pi} \sum_{\nu} \int d\Omega_1 Y_{1q_1}(\Omega_1)$$

$$\times \int d\Omega_2 Y_{1\nu}(\Omega_2) \mathfrak{D}_{\nu q_2}^{-1}(R)$$

$$\times \tilde{M}_{2\gamma}(\omega_1, \omega_2) + (1 \leftrightarrow 2), \quad (22)$$

$$\tilde{M}_{2\gamma}(\vec{\omega}_1, \vec{\omega}_2) = \frac{M}{(2\pi)^3} \int \frac{d^3p}{p^2 + M\omega_1} I_d(\vec{p}) I_t(\vec{p}), \quad (23)$$

where

$$I_d(\vec{p}) \equiv \langle \ ^3S_1 | e^{i\vec{\omega}_2 \cdot \vec{r}/2} | \vec{p} \rangle$$

$$= \frac{N\sqrt{4\pi}}{(\vec{p} + \vec{\omega}_2/2)^2 + \kappa^2}, \quad (24)$$

$$I_t(\vec{p}) \equiv \langle \vec{p} | e^{i\vec{\omega}_1 \cdot \vec{r}/2} | \ ^3S_1' \rangle$$

$$= (2\pi)^3 \delta(\vec{p} - \vec{\omega}_1/2) - 4\pi \frac{a_t}{(\vec{p} - \vec{\omega}_1/2)^2}, \quad (25)$$

and ω_i and q_i are, respectively, the energy and polarization of the two photons; R is the rotation bringing the direction of the second photon to that of the first photon; $\mathfrak{D}(R)$ is the rotation matrix; and $|\vec{p}\rangle$ is the plane wave intermediate state. The rotation R is necessary because in (7) we have defined the z axis to be along the direction of the first photon. Because of the projection operator in (22) we will only pick out the term proportional

to $\vec{\omega}_1 \cdot \vec{\omega}_2$ in (23). Writing

$$\vec{M}_{2\gamma}(\vec{\omega}_1, \vec{\omega}_2) = \dots f_{2\gamma} \left(\frac{\omega_1}{\omega_d} \right) \vec{\omega}_1 \cdot \vec{\omega}_2 + \dots \quad (26)$$

and substituting into (22) we have

$$d^2\sigma_{2\gamma} = \frac{3}{8} \bar{\sigma}_{2\gamma} \left[\frac{2 + \sqrt{x}}{\sqrt{x}(1 + \sqrt{x})^2} + \frac{2 + (1-x)^{1/2}}{(1-x)^{1/2}[1 + (1-x)^{1/2}]^2} - \frac{3}{\eta(1-x)x} \right]^2 x^3(1-x)^3(1 + \cos^2\theta) dx d\cos\theta, \quad (28)$$

where $\eta = a_t\kappa$, $x = \omega_1/\omega$, and $\bar{\sigma}_{2\gamma} = (N^2/9\kappa)\alpha^2 v_n^{-1} a_t^2 \times (\omega_d/M)^{5/2} = 0.219 \mu\text{b}$, with $N^2 = 3.37\kappa$. The total cross section is

$$\sigma_{2\gamma} = \bar{\sigma}_{2\gamma} \left(\frac{191}{10} + \pi - 32 \ln 2 - \frac{3}{5\eta} + \frac{3}{2\eta^3} \right) = 0.118 \mu\text{b}. \quad (29)$$

$$d\sigma_2^D = \frac{45}{16} \xi^2 \bar{\sigma}_{2\gamma} \left[\frac{2 + 2\sqrt{x} + x/3}{(1 + \sqrt{x})^2 \sqrt{x}} + \frac{2 + 2(1-x)^{1/2} + (1-x)/3}{[1 + (1-x)^{1/2}]^2 (1-x)^{1/2}} - \frac{3}{\eta(1-x)x} \right]^2 x^3(1-x)^3 \left[\frac{7}{10}(1 + \cos^2\theta) + \frac{3}{5}\sin^2\theta \right] dx d\cos\theta \quad (30)$$

where¹¹ $\xi = 0.027$ is the asymptotic D to S ratio in the deuteron wave function. This gives a cross section of $\sigma_{2\gamma}^D = 5.3 \times 10^{-4} \mu\text{b}$ which is a 0.44% correction to the result of (29).

In the most recent measurement by Earle *et al.*,⁶ the photon energy was restricted to be within the range $600 \text{ keV} \leq \omega_1 \leq 1630 \text{ keV}$. From (28), the integrated cross section over this range is $0.073 \mu\text{b}$.

B. Effects of the hard core

The asymptotic wave functions used in Sec. IV. A are not regular at $r=0$. One way of examining the effects of changes in the wave functions at small distances on the cross section is to introduce a cutoff in the wave functions (19) and (20) for $r \leq r_c$. The changes in the integrals $I_d(\vec{p})$ and $I_t(\vec{p})$ are

$$\delta I_d(\vec{p}) = -\frac{N\sqrt{4\pi}}{\kappa^2} \left\{ \frac{1}{2}\xi^2 - \frac{1}{3}\xi^3 + \frac{1}{24} [3 - (\vec{p} + \vec{\omega}_2/2)^2/\kappa^2] \xi^4 + O(\xi^5) \right\}, \quad (31a)$$

$$\delta I_t(\vec{p}) = -\frac{4\pi}{\kappa^3} \left\{ \frac{1}{2}\eta\xi^2 - \frac{1}{3}\xi^3 - \frac{1}{24} (\vec{p} - \vec{\omega}_1/2)^2 \xi^4/\kappa^2 + O(\xi^5) \right\}, \quad (31b)$$

where $\xi = \kappa r_c$. This results in a change in the ma-

$$M_{2\gamma} = 2\pi\alpha(\omega_1\omega_2)^{1/2} \left[f_{2\gamma} \left(\frac{\omega_1}{\omega_d} \right) + f_{2\gamma} \left(\frac{\omega_2}{\omega_d} \right) \right] \times \vec{\epsilon}_{a_1}(\omega_1) \cdot \vec{\epsilon}_{a_2}(\omega_2). \quad (27)$$

We point out that the same result is obtained if we use the approximation (8) instead of (7). Evaluating (23) and using (26) and (17), we have

In an earlier paper²² an expression identical to (28) was obtained, but the zeroth order normalization $N^2 = 2\kappa$ was used, giving a result of $0.069 \mu\text{b}$ for $\sigma_{2\gamma}$.

The D -state component in the deuteron contributes incoherently to the two-photon capture reaction. The differential cross section is

trix element

$$\frac{\delta M_{2\gamma}}{M_{2\gamma}} = -\frac{1}{12} \xi^4 = -2 \times 10^{-5} \quad (32)$$

when $r_c = 0.5 \text{ fm}$. Obviously this effect can be completely ignored. Similarly if we regularize, e.g., the deuteron wave function by introducing an extra term proportional to $e^{-\beta r}/r$, the resulting change in the matrix element is of order $(\kappa/\beta)^4$. Low energy data demand¹¹ $\beta \sim 6\kappa$, which leads to a correction of $|\delta M_{2\gamma}/M_{2\gamma}| < 10^{-3}$.

C. Effect of nucleon-nucleon interaction on the intermediate state

In Sec. IV A the intermediate states were represented by plane waves. Here we examine the effect of interaction on the intermediate state. The phase-shifted asymptotic P -wave radial function is

$$u_1(r) = \cos\delta_1 j_1(pr) - \sin\delta_1 n_1(pr), \quad (33)$$

where j_1 and n_1 are, respectively, the spherical Bessel and Neumann functions. For plane waves $\delta_1 = 0$ and $u_1 = j_1$. In order to get a rough estimate of this effect we assume

$$\tan\delta_1 = \Delta(p/\kappa)^3 \quad (34)$$

and δ_1 to be spin independent. The constant Δ can be determined from empirical data. Equation (34) is not realistic but it will *grossly exaggerate* the effect under examination. Taking a very generous value²⁹ of $\delta_1 = 0.25(\text{rad})$ at $E_{1ab} = 100 \text{ MeV}$ we have $|\Delta| \leq 3 \times 10^{-3}$. With (33) and (34) we find the change in the matrix element to be

$$\left| \frac{\delta M_{2\gamma}}{M_{2\gamma}} \right| \sim \Delta \leq 3 \times 10^{-3}, \quad (35)$$

which is again a very small effect.

V. RESULTS WITH THE GRADIENT OPERATOR

A. Using asymptotic wave functions

With the gradient dipole operator $(e/M)\vec{p} \cdot \vec{\epsilon} e^{i\omega \cdot \vec{r}}$ the two-photon matrix element becomes

$$M_{2\gamma} = \frac{2\pi\alpha}{M(\omega_1\omega_2)^{1/2}} \frac{4\pi}{3} \vec{\epsilon}_1 \cdot \vec{\epsilon}_2 [h_{2\gamma}(x) + h_{2\gamma}(1-x)], \quad (36)$$

where $x = \omega_1/\omega_d$ and

$$\begin{aligned} h_{2\gamma}(x) &= \frac{1}{(2\pi)^3} \int_0^\infty \frac{p^4 dp}{p^2 + x\kappa^2} \langle {}^3S_1 | \vec{p} \rangle \langle \vec{p} | {}^3S_1' \rangle \\ &= -\frac{N}{\sqrt{4\pi}} \frac{a_t}{\kappa} \frac{1}{1 + \sqrt{x}}. \end{aligned} \quad (37)$$

The differential cross section is

$$\begin{aligned} d^2\sigma_{2\gamma} &= \frac{3}{2}\bar{\sigma}_{2\gamma} \left(\frac{1}{1 + \sqrt{x}} + \frac{1}{1 + (1-x)^{1/2}} \right)^2 x(1-x) \\ &\quad \times (1 + \cos^2\theta) dx d\cos\theta, \end{aligned} \quad (38)$$

which is the result of Blomqvist and Ericson¹⁹ except that they used $N^2 = 2\kappa$ instead of $N^2 = 3.37\kappa$ in $\bar{\sigma}_{2\gamma}$. Note that if and only if we set $\eta = a_t\kappa = 1$, which is the condition that the *asymptotic* u_d and u_t be orthogonal, the result obtained with the length operator (28) is *identical* to (38). Therefore we identify $\eta = 1$ to be the consistency condition, in the present case, for the equivalence of the length and gradient operators. This condition introduces some ambiguity into the interpretation of (38); the resulting total cross sections are:

$$\sigma_{2\gamma} = \bar{\sigma}_{2\gamma} (20 + \pi - 32 \ln 2)$$

$$= \begin{cases} 0.210 \mu\text{b}, & \text{if } N^2 = 3.37\kappa \text{ and } a_t = 5.41 \text{ fm} \\ 0.125 \mu\text{b}, & \text{if } N^2 = 2\kappa \text{ and } a_t = 5.41 \text{ fm} \\ 0.079 \mu\text{b}, & \text{if } N^2 = 2\kappa \text{ and } a_t = \kappa^{-1} = 4.31 \text{ fm}. \end{cases} \quad (39a) \quad (39b) \quad (39c)$$

If the consistency condition is taken seriously, we have (39c). Blomqvist and Ericson¹⁹ obtained (39b).

For the purpose of a direct comparison of the length and gradient operators, we use (39a). In the following section, we show that unlike the length operator, the result (39a) suffers large variations when other refinements are introduced into the calculation.²⁴

B. Effect of regularization

A typical soft-core realistic wave function for the deuteron may be expressed as

$$u_d = \frac{N}{\sqrt{4\pi}} \sum_{i=1}^n a_i \frac{e^{-\alpha_i r}}{r}, \quad N^2 = 3.37\kappa, \quad (40)$$

with $a_i = 1$, $\alpha_i = \kappa$, and $\sum_i a_i = 0$. Similarly we express the regularized u_t as

$$u_t = \frac{\sin(kr + \delta_t)}{kr} \sum_{i=1}^n b_i e^{-\beta_i r} \quad (41)$$

with $b_i = 1$, $\beta_i = 0$, and $\sum_i b_i = 0$.

Another kind of wave function is the hard-core type, e.g.,

$$\begin{aligned} u_d &= \frac{N}{\sqrt{4\pi}} \frac{e^{-\kappa r}}{r} (1 - e^{-\alpha(r-r_c)}), \\ u_t &= \frac{\sin(kr + \delta_t)}{kr} (1 - e^{-\beta(r-r_c)}), \quad \text{for } r \geq r_c, \end{aligned} \quad (42)$$

$u_d = u_t = 0$, for $r < r_c$.

In both cases u_d must be properly normalized: $\int u_d^2 d^3r = 1 - P_D$, where $P_D = 4-7\%$ is the D -state probability; u_d and u_t must be orthogonal: $\int u_d u_t d^3r = 0$. In Table I the computed total cross sections and their ratio to (39a) are given for four sets of wave functions. The two soft-core wave functions for deuteron are owing, respectively, to Hulthén¹¹ and to McGee.³⁰ The two hard-core wave functions have hard-core radii of 0.4 and 0.5 fm, respectively. Only for the realistic McGee-type wave functions are the parameters of u_t not completely determined by the orthogonality condition. In this case the ratio $\sigma_{2\gamma}/\sigma_{2\gamma}(\text{asym})$ are found to be insensitive to variations of these parameters provided that the effective range parameter r_{0t} is close to the experimental value of 1.75 fm. The param-

TABLE I. Effect of regularization on $\sigma_{2\gamma}$ obtained with the gradient operator.

Wave functions	$\sigma_{2\gamma}$ (μb)	$\sigma_{2\gamma}/\sigma_{2\gamma}(\text{asym})^a$
(i) Soft core (Hulthén)	0.092	0.44
(ii) Soft core (McGee)	0.090	0.43
(iii) Hard core ($r_c = 0.4 \text{ fm}$)	0.078	0.37
(iv) Hard core ($r_c = 0.5 \text{ fm}$)	0.076	0.36

^a $\sigma_{2\gamma}(\text{asym}) = 0.210 \mu\text{b}$.

eters of the wave functions are given in Table II. Adler *et al.*²³ also used Hulthén-type wave functions and found the cross section to be $9.0 \times 10^{-2} \mu\text{b}$, in good agreement with (i) and (ii) in Table I. It is seen that regularization drastically reduces the predicted cross section. The reduction increases with the hardness of the core. This is in sharp contrast with the calculation in Sec. IV when the length operator is used. There, regularization gave only a correction of $(\delta M_{2\gamma}/M_{2\gamma}) < 10^{-3}$. This difference can be understood by examining (23) and (37), where the intermediate states are integrated over for the two operators, respectively. In (23) the integrand behaves as $1/p^6$ for large p , whereas in (37) the integrand behaves as $1/p^2$ [this different behavior can be traced back to (13) where the factor $(p^2/M)^2$ is replaced by $\omega_1\omega_2$, when the length operator is used]. As a result intermediate states with energy, say above 50 MeV still con-

tribute about 16% to the integral in (37). The contribution from such high energy states is drastically reduced by the introduction of wave functions regular at the origin. In contrast the length operator is completely insensitive to the high energy tail of the spectrum of intermediate states.

C. Effect of the P -wave phase shift

So far the intermediate states have been taken to be plane waves. Here we examine the effect of P -wave phase shifts. Due to the nature of the gradient operator, we expect it to be sensitive to the P -wave phase shifts where the latter is large, i.e. in the region of 50–200 MeV; at the same time we expect the regularization of the wave function to dampen this sensitivity. Using (33) and the Hulthén wave function given in Table II, the differential cross section becomes

$$d^2\sigma_{2\gamma} = \frac{3}{2}\bar{\sigma}_{2\gamma} \left\{ \frac{1}{2}(1 + \cos^2\theta) [2H_0^2(x) + 3H_1^2(x) + 7H_2^2(x)] + 3\sin^2\theta [H_1^2(x) + H_2^2(x)] \right\} x(1-x) dx d\cos\theta, \quad (43)$$

where $x = \omega_1/\omega_d$ and

$$H_L(x) = \frac{\sqrt{4\pi}\kappa^2}{3N} \sum_{J=0}^2 (-)^J (2J+1) W(1111; JL) [h_J(x) + h_J(1-x)], \quad (44)$$

where $W(1111; JL)$ is a Racah coefficient and h_J is analogous to the function $h_{2\gamma}$ defined in (37):

$$h_J(x) = -\frac{N_t}{\pi^{3/2}\kappa} \int_0^\infty \frac{z^4 dz}{(z^2+x)} \left[\cos\delta_J \left(\frac{1}{1+z^2} - \frac{1}{\alpha_2^2+z^2} \right) + \sin\delta_J \left(\frac{1}{1+z^2} - \frac{\alpha_2}{\alpha_2^2+z^2} \right) \frac{1}{z} \right] \\ \times \left[\cos\delta_J \left(\frac{\eta}{z^2} - \frac{\eta(\beta_2^2+z^2) - 2\beta_2}{(\beta_2^2+z^2)^2} \right) + \sin\delta_J \left(\frac{1}{z^3} - \frac{\eta\beta_2(\beta_2^2+z^2) - \beta_2^2+z^2}{\eta(\beta_2^2+z^2)^2} \right) \right]. \quad (45)$$

The phase shifts δ_J ($p = \kappa z$) are those of the three triplet P -wave channels 3P_J , $J = 0, 1, 2$. Note that $H_1(\gamma) = H_2(\gamma) = 0$ when the phase shifts are J -independent. Using experimental phase shifts²⁹ and integrating numerically we have

$$\sigma_{2\gamma} = 4\bar{\sigma}_{2\gamma} \sum_{L=0,1,2} (2L+1) \int_0^1 H_L^2(x) x(1-x) dx \\ = 0.096 \mu\text{b} \quad (46)$$

which is an increase of 5% over the result [(i) in

TABLE II. Parameters for wave functions.

Type	i^a	a_i	α_i (in units of κ)	b_i	$\beta_i(\kappa)$
(i) Hulthén	2	-1.0	5.73	-1.0	5.52
(ii) McGee	2	-0.64	5.73	-1.41	9.81
	3	-6.62	12.84	2.66	13.89
	4	15.22	17.33	66.22	15.38
	5	-8.96	19.64	-68.47	15.13
(iii) $H - C(r_c = 0.4 \text{ fm})$	2	-1.0	6.75	-1.0	12.07
(iv) $H - C(r_c = 0.5 \text{ fm})$	2	-1.0	7.74	-1.0	15.52

^a For all cases $a_1 = b_1 = 1$, $\alpha_1 = \kappa = 0.232 \text{ fm}^{-1}$, $\beta_1 = 0$.

Table I] with $\delta_J=0$. This essentially agrees with the result of Adler *et al.*²³ who found that the inclusion of phase shifts increases the cross section by $\sim 7\%$. The present result also agrees very well with the value $0.096 \mu\text{b}$ obtained by Grechukhin¹⁸ who used a potential model.

A very interesting aspect of the result of (46) is that it is still smaller than the result of $0.118 \mu\text{b}$ for the length operator by about 20%. Some of the discrepancy is undoubtedly due to approximations inherent in the wave functions. Another reason is that nuclear current exclusive of $(e/M)\vec{p}$ may have played an important role. For example if the strong J dependence in δ_J is due to a two-body spin-orbit force³¹ $V_{LS}(r)(\vec{r} \times \vec{p}) \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)$, then an additional piece of current $eV_{LS}(r)(\vec{r} \times \vec{\sigma})$ must be considered. However, because $V_{LS}(r)$ is very singular at small r the contribution of this term to $\sigma_{2\gamma}$ is extremely sensitive to the short range behavior of the wave functions and the result would therefore not be reliable. A detailed study of this problem is outside the scope of the present investigation.

VI. CROSS SECTION FROM OTHER TWO-PHOTON PROCESSES

A. Normal two-photon modes

The cross section from all other modes is expected to be much smaller than that of the $(E1, E1)$ mode. Table III lists the differential cross section for all the normal two-photon modes of np capture. Mode (a) describes the emission of an $M1$ photon causing the transition $^1S_0 \rightarrow ^3S_1$ followed by the emission of another $M1$ photon due to spin re-

orientation¹⁹ in the 3S_1 state. In mode (b) the emission of the second photon is due to $E1$ bremsstrahlung of the recoiling²¹ deuteron. The $M1$ operator is given in (11) and the $E1$ bremsstrahlung operator is

$$\frac{e}{M} \vec{p}_d \cdot \vec{\epsilon}_2 e^{i\vec{\omega}_2 \cdot \vec{r}} \approx -e \frac{\vec{\omega}_1 \cdot \vec{\epsilon}_2}{M},$$

where \vec{p}_d is recoil momentum of the deuteron. The differential cross section exhibits the characteristic "infrared catastrophe" at $x \approx 0$. Mode (c) describes the emission of two $M1$ photons through the 1S_0 continuum.¹⁹ The contact term $(\alpha/M)\vec{A} \cdot \vec{A}$ arising from minimal coupling in the nonrelativistic theory gives a one-step two-photon process distinct from the others. The integrated cross sections for these processes are given in Table IV.

B. Exchange currents and exotic modes

It is known that the pion exchange currents introduce a correction of $\sim 5\%$ to the isovector $M1$ matrix element²⁸ of the capture process $n+p \rightarrow d+\gamma$. Therefore, for the present case, we expect a correction of $\sim 5\%$ to each $M1$ vertex. This implies a correction of $\sim 20\%$ ($\sim 10\%$) to the cross section of the modes (a) and (c) [mode (b)]. In the dominant $(E1, E1)$ mode, there is no correction from exchange currents if the length operator is used.³²

A process not included in the above consideration involves the emission of two photons by the exchanged pion through the contact term (Fig. 1). The cross section for this process is

TABLE III. Differential cross section for other normal modes.

$$\begin{aligned} \sigma_{1\gamma} &= 2\pi \alpha v_n^{-1} \mu_V^2 \left(\frac{\omega_d}{M} \right)^{5/2} (\kappa^{-1} - a_s)^2 \approx 300 \text{ mb}; \quad a_s = -23.7 \text{ fm}; \\ \mu_V &= \mu_p - \mu_n, \quad \mu_s = \mu_p + \mu_n, \quad x = \omega_1/\omega_d, \quad \eta_s = a_s \kappa, \quad \eta = a_t \kappa; \\ \bar{\sigma}_{2\gamma} &= \frac{3.37}{9} \alpha^2 v_n^{-1} a_t^2 \left(\frac{\omega_d}{M} \right)^{5/2} = 0.219 \mu\text{b}. \end{aligned}$$

Mode (see Sec. III)	$d^2\sigma_{2\gamma}/[x(1-x)dxd\cos\theta]$
(a) $(M1, M1)$	$\frac{\alpha}{32\pi} \sigma_{1\gamma} \left(\frac{\omega_d}{M} \right)^2 \mu_s^2 (3 - \cos^2\theta)(1 - 2x)^2$
(b) $(M1, E1)$	$\frac{\alpha}{4\pi} \sigma_{1\gamma} \left(\frac{\omega_d}{M} \right)^2 \left[\frac{x^4}{(1-x)^2} + \frac{(1-x)^4}{x^2} \right]$
(c) $(M1, M1)$	$\frac{\alpha}{8\pi} \sigma_{1\gamma} \left(\frac{\omega_d}{M} \right)^2 \mu_V^2 [f^2(x) + f^2(1-x) + \frac{1}{2}(1 + \cos^2\theta)f(x)f(1-x)];$ $f(x) = \frac{1-x}{1-\eta_s}$
Contact term	$\frac{3}{32} \bar{\sigma}_{2\gamma} \left(\frac{\omega_d}{M} \right)^2 \left(1 - \frac{3}{\eta} \right)^2 [x^2 + (1-x)^2 + 2x(1-x)\cos\theta]^2 (1 + \cos^2\theta)$

TABLE IV. Summary of results.

Mode/Remark	Initial state ^a	$\sigma_{2\gamma}$ (μb)
(E1, E1), length operator	${}^3S'_1$	0.118($\pm 1\%$) ^b 0.073 ^c
(E1, E1), gradient operator	${}^3S'_1$	0.096
(M1, E1 bremsstrahlung)	1S_0	1.1×10^{-4}
(M1, M1)	${}^3S'_1$	8.7×10^{-5}
(M1, M1)	1S_0	1.7×10^{-5}
Contact term		
Off nucleon	${}^3S'_1$	3×10^{-8}
Off pion	${}^3S'_1$	$\sim 10^{-6}$
Off "light scalar boson"	${}^3S'_1$	$< 10^{-8}$

^a ${}^3S'_1$ is the np triplet continuum; 1S_0 is the singlet continuum.

^b The error allows for uncertainties in the thermal neutron spectrum and in the values of low energy np parameters.

^c Restricted to photon energies $600 \text{ keV} \leq \omega_1 \leq 1630 \text{ keV}$.

$$\sigma_{2\gamma}^{\pi} \sim \bar{\sigma}_{2\gamma} \left(\frac{g_{\pi}^2}{4\pi} \right)^2 \left(\frac{m_{\pi}}{M} \right)^2 \frac{\omega_d}{m_{\pi}} \sim 10^{-6} \mu\text{b}, \quad (47)$$

where the pion-nucleon coupling constant $g_{\pi}^2/4\pi$ is taken to be 0.08. It has been suggested that instead of the pion, a scalar meson³³ with a small mass ($m_{\phi} \ll m_{\pi}$) may be exchanged between the two nucleons; in this case the cross section corresponding to Fig. 1 would be

$$\sigma_{2\gamma}^{\phi} \sim \bar{\sigma}_{2\gamma} \left(\frac{g_{\phi}^2}{4\pi} \right)^2 \left(\frac{M}{m_{\phi}} \right)^2. \quad (48)$$

In comparing (47) with (48), the former has an extra factor of $(m_{\pi}/M)^4$ due to the pseudoscalar-

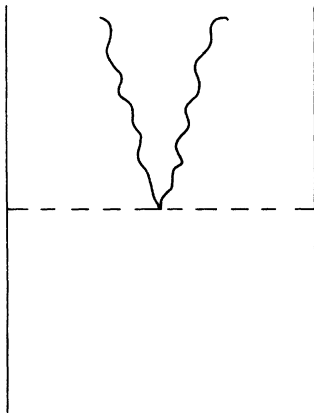


FIG. 1. Emission of two photons by the intermediate meson via the contact term $(\alpha/M) \vec{A} \cdot \vec{A}$.

coupling of the pions to the nucleons. The other factor (ω_d/m_{π}) in (47) arises from the radial integrals. In (48), it is advantageous if m_{ϕ} is small and $g_{\phi}^2/4\pi$ is large. However, accurate mesonic x-ray measurements^{34,35} in Pb and Ba put a severe limitation^{36,37} on the coupling constant: $g_{\phi}^2/4\pi < 10^{-7}$; and neutron-electron scattering³⁷ data imply a lower limit of $m_{\phi} > 0.7 \text{ MeV}$. Therefore we have $\sigma_{2\gamma}^{\phi} \leq 10^{-8} \mu\text{b}$.

VII. SUMMARY AND CONCLUSION

The doubly radiative capture of thermal neutrons by protons has been discussed in detail. The leading two-photon mode is ${}^3S'_1 \rightarrow {}^3S_1 + \gamma(E1) + \gamma(E1)$ for which the capture cross section $\sigma_{2\gamma}$ is of the order $0.1 \mu\text{b}$. The next leading mode is ${}^1S_0 \rightarrow {}^3S_1 + \gamma(M1) + \gamma(E1 \text{ bremsstrahlung})$, for which $\sigma_{2\gamma}$ is down by 3 orders of magnitude. Cross sections for other, including some exotic, modes are even smaller. A summary of the results is given in Table IV.

The cross section for the (E1, E1) mode was calculated in considerable detail. The calculations were divided into two parts. In the first part the theorems of Siegert²⁵ and Grechukhin¹³ were applied to reduce the dipole operator to essentially the usual length operator $e\omega(\vec{\epsilon} \cdot \vec{r})$. For low energy photons this operator includes the effect of all currents. It was shown that when this operator is used the result $\sigma_{2\gamma} = 0.118 \mu\text{b}$ (or $\sigma_{2\gamma} = 0.073 \mu\text{b}$ when the photon energy is restricted to $600 \text{ keV} \leq \omega_1 \leq 1630 \text{ keV}$) is very accurately determined by only three experimentally well known parameters, namely the binding energy ω_d of the deuteron, the triplet scattering length a_t , and the effective range r_{0t} . The cross section depends on the last parameter only through the overall normalization of the deuteron wave function. It has been shown that details such as the short range behavior of the np wave functions and the phase shifts of the P -wave intermediate states affect $\sigma_{2\gamma}$ by less than 1%. In the second part $(e/M)\vec{p}$ was explicitly assumed to be the current and the gradient operator $(e/M)\vec{p} \cdot \vec{\epsilon}$ was used in the calculation. In this case the first order result, obtained by using asymptotic wave functions only, is reduced by $\sim 60\%$ when the wave functions are regularized at the origin. This last result is then increased by about 5% when P -wave phase shifts are taken into account. The final result, $\sigma_{2\gamma} = 0.096 \mu\text{b}$, is about 20% less than the result obtained when the length operator is used. Furthermore, this result is not reliable because the effect of the exchange current has not been included.

In the last year we have seen the upper limit of the experimental value for $\sigma_{2\gamma}$, lowered by two orders of magnitude. Although the most recent

value ($-3 \pm 8 \mu\text{b}$) is still 50–100 times larger than the predicted value of $0.118 \mu\text{b}$, it is perhaps not being too optimistic to hope for experiment and theory on nuclear two-photon processes to meet on the same ground for the first time in the near

future.

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