Structure of low-lying positive-parity states of ^{18}O

R. D. Lawson and F. J. D. Serduke*
Argonne National Laboratory, Argonne, Illinois 60439

H. T. Fortune†
Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania 19174
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Data on single-nucleon transfer, (t, p) cross sections, and static and dynamic electromagnetic properties place severe limitations on the structure of the low-lying positive-parity states of ^{18}O. We deduce wave functions that best reproduce these data by use of a model space that includes one collective state each of spin 0^+, 2^+, and 4^+ together with all basis states formed from a closed ^{16}O core plus two $d_{5/2}$, $s_{1/2}$ neutrons and up to one $d_{3/2}$ neutron. There is need for only a small $d_{3/2}$ admixture. The 0^+ collective configuration is the dominant constituent of the 0_2^+ state at 3.63 MeV, and the 2^+ collective configuration dominates the 2_3^+ state at 5.25 MeV. Overall, the data favor a small negative value ($Q/e \simeq -5$ fm^2) for the quadrupole moment of the first 2^+ level. The matrix elements of the Hamiltonian that reproduce both the observed energies and transition rates are constructed. Several experiments are suggested to further probe the structure of this nucleus.

[NUCLEAR STRUCTURE Deduce wave functions of ^{18}O low-lying states that best fit single-nucleon transfer, (t,p) data, and static and dynamic electromagnetic properties. Construct Hamiltonian that leads to these eigenfunctions.]

I. INTRODUCTION

In recent years it has become clear[1] that the low-lying states of ^{18}O cannot be adequately described as two nucleons confined to the $(1d, 2s)$ shell that move outside an inert ^{16}O core. In order to explain both the number and the properties of positive-parity states, four-particle–two-hole excitations must be taken into account. The influence of core excitation on the theoretical properties of ^{18}O has been examined by many authors. There are two procedures that have been employed in these studies. In the first, a specific form for the residual nucleon-nucleon interaction is chosen and the spectrum and transition rates are calculated in a straightforward manner using some truncation of the $(1d, 2s)^2$ and $(1d, 2s)^4(1p)^{-2}$ model space.[2] In the second, it is assumed that the $I^\pi = 0^+$, 2^+, and 4^+ collective components may be introduced into the model by projecting[3] wave functions with good angular momentum from a single intrinsic deformed state in which two $p_{1/2}$ protons are excited to the lowest $(K = \pm\frac{1}{2})$ $(1d, 2s)$ Nilsson orbit. Within this model all off-diagonal matrix elements between the collective and ordinary shell-model states are proportional to one linear combination of two-body interaction energies. Federman and Talmi[4] have exploited this approach and carried out a least-squares fit to the energy levels in ^{18}O, ^{19}O, and ^{20}O in terms of this parameter and the usual two-body matrix elements that characterize the residual interaction in the $(1d_{5/2}, 2s_{1/2})$ model space.

A variant of this last procedure is to adjust the wave functions to fit a set of experimental data other than the energies, and this has recently been done[5] for the three lowest 0^+ levels in ^{18}O. In that study it was shown that there are two possible sets of eigenfunctions that describe the single-neutron-transfer experiments involving these states. When these data were combined with the (t,p) experiments it was found that only one of these sets could simultaneously explain the (t,p), (d,p), and (d,t) data. One can, of course, extend this idea to other than the 0^+ levels and in this paper we extract the ^{18}O wave functions of the three lowest 0^+ levels, the three lowest 2^+ states, and the two lowest 4^+ levels that best reproduce the observed properties. The data we fit are the ratios of single-nucleon-transfer spectroscopic factors, ratios of (t,p) cross sections, electromagnetic transition rates, and static electric and magnetic multipole moments.

Once the wave functions have been obtained the problem can be inverted in the sense that the Hamiltonian that gives rise to these eigenfunctions and observed energies can be deduced. It is the matrix elements of this Hamiltonian that a fundamental theory must strive to reproduce.

II. THEORY

The states of ^{18}O whose properties we wish to describe are eight in number—the three lowest 0^+ levels (the ground state and the 3.63- and 5.33-MeV excited states), the three lowest 2^+ levels

(those at 1.98-, 3.92-, and 5.25-MeV excitation energy), and the two lowest 4^+ levels which are observed 3.55 and 7.11 MeV above the ground state. We first discuss the basis states relevant to our desciption of these levels. Our reference energy is the binding energy of the $1d_{5/2}$ neutron to the ^{16}O core. From the spectrum[6] of ^{17}O it follows that, relative to this level, the energies of the $2s_{1/2}$ and $1d_{3/2}$ single-particle states are

$$\epsilon_{1/2} = 0.871 \text{ MeV},$$

$$\epsilon_{3/2} = 5.08 \text{ MeV}$$

and the energy of the ^{18}O ground state relative to twice the $1d_{5/2}$ binding energy is[7]

$$E_0 = -3.915 \text{ MeV}. \quad (1)$$

If one includes only the diagonal part of the residual nucleon-nucleon interaction, the unperturbed energy of the state $(j_1 j_2)_I$ is given by

$$E_I(j_1 j_2) = \langle j_1 j_2 | H | j_1 j_2 \rangle_I = \epsilon_{j_1} + \epsilon_{j_2} + \langle j_1 j_2 | V | j_1 j_2 \rangle_I,$$

where $\langle j_1 j_2 | V | j_1 j_2 \rangle_I$ is the matrix element of the residual nucleon-nucleon interaction in the angular momentum state I. Using Kuo's values[8] for these matrix elements, the predicted positions of the three 0^+ levels that can arise from putting two particles into the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ single-particle levels are

$$E_0(\tfrac{5}{2}, \tfrac{5}{2}) = -2.44 \text{ MeV},$$

$$E_0(\tfrac{1}{2}, \tfrac{1}{2}) = -0.21 \text{ MeV},$$

$$E_0(\tfrac{3}{2}, \tfrac{3}{2}) = 9.35 \text{ MeV}.$$

Thus the unperturbed $(1d_{3/2})^2_{I=0}$ configuration lies about 11.8 MeV above the unperturbed $(1d_{5/2})^2_{I=0}$ configuration and about 13.3 MeV above the ^{18}O ground-state energy, Eq. (1). Clearly, if one is to explain the existence of three 0^+ states below 6 MeV, a core-excited level is needed. That a low-lying core-excited 0^+ state is likely to occur in ^{18}O may be seen from the following arguments: In Fig. 1 we show the most likely structure of the ground state and 6.05-MeV excited 0^+ state in ^{16}O. We assume the sd-shell nucleon configuration of this excited 0^+ is the same as in the ground state of ^{20}Ne, and that the p-shell configuration is the same as in the ground state of ^{12}C. The energy of this excited level relative to the ground state is

$$\Delta E(^{16}\text{O}) = \text{B.E.}(^{20}\text{Ne}) + \text{B.E.}(^{12}\text{C}) - 2\text{B.E.}(^{16}\text{O}) + 16\bar{E}$$

where B.E.(A) stands for the ground-state binding energy of the nucleus A and $\bar{E}$ is the average interaction energy between a ds-shell particle and a p-shell hole. From the binding energy tables of Wapstra and Gove[7] one concludes that

$$\bar{E} = 0.226 \text{ MeV}$$

if $\Delta E(^{16}\text{O})$ is to be 6.05 MeV. Once $\bar{E}$ is known one can estimate the excitation energy of the 0^+ core-excited state in ^{18}O. The probable structure of this state is also illustrated in Fig. 1 and one would estimate its excitation energy to be

$$\Delta E(^{18}\text{O}) = \text{B.E.}(^{20}\text{Ne}) + \text{B.E.}(^{14}\text{C}) - \text{B.E.}(^{16}\text{O}) - \text{B.E.}(^{18}\text{O}) + 8\bar{E} = 3.31 \text{ MeV}. \quad (2)$$

In a similar manner one can examine the position of the possible 2^+ levels that arise from the configuration $(ds)^2$. By use of Kuo's matrix elements we obtain

$$E_2(\tfrac{5}{2}, \tfrac{5}{2}) = -1.04 \text{ MeV},$$

$$E_2(\tfrac{5}{2}, \tfrac{1}{2}) = -0.42 \text{ MeV},$$

$$E_2(\tfrac{5}{2}, \tfrac{3}{2}) = 4.88 \text{ MeV},$$

$$E_2(\tfrac{3}{2}, \tfrac{1}{2}) = 5.62 \text{ MeV},$$

$$E_2(\tfrac{3}{2}, \tfrac{3}{2}) = 10.24 \text{ MeV}.$$

Thus for $I^\pi = 2^+$, once more there are two low-lying levels. According to the above estimate the $(d_{5/2} d_{3/2})_{I=2}$ state should lie about 6 MeV above the lowest 2^+—i.e., at about 8-MeV excitation energy. Since three 2^+ states are seen below 5.3 MeV it follows that a core-excited 2^+ state is required. Since the 2^+ member of the ^{16}O excited-

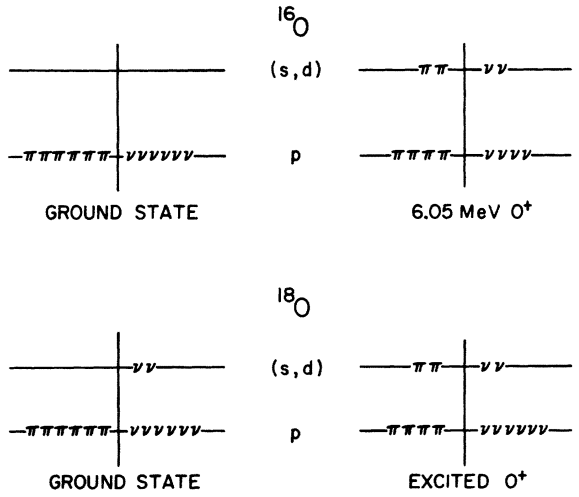

FIG. 1. The upper part of the figure illustrates the dominant structure of the ground and 6.05-MeV excited 0^+ level in ^{16}O. The lower part shows the ground state of ^{18}O and the assumed structure of the core-excited 0^+. In each case the four nucleons in the (sd) shell are assumed to have the structure of the ^{20}Ne ground state.

state rotational band lies 0.87 MeV above[6] the 6.05-MeV level, one would estimate, by use of Eq. (2), that the 2^+ core-excited state might be expected to lie at about 4.2 MeV. Alternatively, if one uses the energy of the first[9] 2^+ level in ^{20}Ne (1.63 MeV), one would estimate this energy to be 4.9 MeV. In any case, it is clear that a core-excited 2^+ level is likely to occur in the region of 4–5-MeV excitation energy.

Although the unperturbed positions of the $(d_{5/2}d_{3/2})_{I=2}$ and $(d_{3/2}s_{1/2})_{I=2}$ configurations are quite far removed from the lowest two 2^+ levels, we have found that they give a marked effect on the magnetic dipole properties of the 2^+ levels. The magnetic moment of the lowest 2^+ state is particularly sensitive to small admixtures of the former configuration because of the rather large value of

$$\langle (d_{5/2})^2_{2,M=2} | \sigma_z | (d_{5/2}d_{3/2})_{2,M=2} \rangle = [-\tfrac{1}{5}(4\sqrt{2})\mu_n]\mu_N,$$

where μ_N is the nuclear magneton and $\mu_n = -1.91$ for the free neutron. Thus a 1% admixture of this configuration into the lowest 2^+ state (which is about the mixing one would estimate by perturbation theory) will produce a change in the computed g factor of

$$\Delta g(2_1^+) = 2\{[-\tfrac{1}{5}(2\sqrt{2})\mu_n]\mu_N\}(0.1) = 0.216\,\mu_N. \quad (3)$$

Since the measured value[10] of this quantity is

$$g(2_1^+) = (-0.35 \pm 0.04)\mu_N,$$

it is clear that this small admixture produces an important change. Admixtures of the $d_{3/2}$ single-particle level also affect some of the predicted electric multipole properties. In particular, $B(E2; 0_3^+ \to 2_1^+)$ is small and a 1% $d_{3/2}$ admixture leads to a 20% change in the theoretical value of this $B(E2)$. Consequently, at least in perturbation theory, admixtures of these configurations must be considered.

The same sort of analysis can be carried out for the 4^+ levels. The unperturbed positions of the two $(sd)^2$ configurations are

$$E_4(\tfrac{5}{2}, \tfrac{5}{2}) = -0.05 \text{ MeV},$$

$$E_4(\tfrac{5}{2}, \tfrac{3}{2}) = 3.42 \text{ MeV}.$$

Thus one would estimate that the $(d_{5/2}d_{3/2})_{I=4}$ state should occur at around 7-MeV excitation energy and consequently there would appear to be no need for a collective 4^+. On the other hand, both the ^{16}O and ^{20}Ne spectra exhibit a collective 4^+ at about 4.3 MeV. By use of Eq. (2) one would estimate that a core-excited 4^+ state should occur at about 8 MeV in ^{18}O. Since this expected state is close to $(d_{5/2}d_{3/2})_{I=4}$, it would be inconsistent to consider one without the other.

Consequently we arrive at the model space we shall use in these calculations:

$I=0$: $(d_{5/2})^2$, $(s_{1/2})^2$, and a collective state Ψ_0;

$I=2$: $(d_{5/2})^2$, $(d_{5/2}s_{1/2})$, $(d_{5/2}d_{3/2})$, $(d_{3/2}s_{1/2})$, and a collective state Ψ_2;

$I=4$: $(d_{5/2})^2$, $(d_{5/2}d_{3/2})$, and a collective state Ψ_4.

Since we shall fit the electromagnetic properties of ^{18}O we must know the matrix elements of the $M1$ and $E2$ operators for both the collective and single-particle states. To determine the latter we may use the data on ^{17}O. Since the magnetic moment of the $d_{5/2}$ ground state is[11] $-1.8937\,\mu_N$, close to the free particle value, we shall assume the $M1$ γ-decay operator to be used in the $(1d, 2s)^2$ configuration is

$$(M1)_z = \left[\left(\frac{3}{4\pi}\right)^{1/2}\mu_n\right]\mu_N \sum_i \sigma_z(i) \quad (4)$$

with $\mu_n = -1.91$. The magnetic moment operator is, of course, $(4\pi/3)^{1/2}(M1)_z$.

One may use the measured quadrupole moment to determine the $d_{5/2}$ $E2$ matrix element. Since

$$Q/e = \left(\frac{16\pi}{5}\right)^{1/2}\langle \chi_{5/2,m=5/2} | (E2)_0 | \chi_{5/2,m=5/2}\rangle = \left(\frac{16\pi}{5}\right)^{1/2}(\tfrac{5}{2}2\tfrac{5}{2}0\,|\,\tfrac{5}{2}\tfrac{5}{2})\langle \chi_{5/2} \| E2 \| \chi_{5/2}\rangle,$$

it follows that

$$q_{dd} = \langle \chi_{5/2} \| E2 \| \chi_{5/2}\rangle = -1.37 \pm 0.16\ e\ \text{fm}^2 \quad (5)$$

when the experimental value[11]

$$Q/e = -2.6 \pm 0.3 \text{ fm}^2$$

is used. The second $E2$ matrix element $\langle \chi_{5/2} \| E2 \| \chi_{1/2}\rangle$ can be extracted from the mean lifetime of the 871-keV $\frac{1}{2}^+$ level[6]

$$\tau_m = 258.6 \pm 2.6 \text{ psec}.$$

This leads to the reduced matrix element

$$q_{ds} = \langle \chi_{5/2} \| E2 \| \chi_{1/2}\rangle = -1.447 \pm 0.007\ e\ \text{fm}^2, \quad (6)$$

where the sign of q_{ds} has been chosen to be the same as one would deduce theoretically if the $2s_{1/2}$ radial wave function is positive at the origin.

To incorporate the effect of the $d_{3/2}$ orbital on computed $E2$ properties we make use of the $r^2 Y_\mu^2(\theta, \phi)$ form of the $E2$ operator, so that

$$\langle \chi_{j'm'} | r^2 Y_\mu^2 | \chi_{jm} \rangle = (-1)^{l+l'+j-j'} \left(\frac{5}{4\pi}\right)^{1/2} \left(\frac{2j+1}{2j'+1}\right)^{1/2} (j2\tfrac{1}{2}0 | j'\tfrac{1}{2})(j2m\mu | j'm') \int R_{j'} r^2 R_j r^2 dr,$$

where R_j is the radial wave function of the state j and the coupling order $\vec{j} = \vec{l} + \vec{s}$ has been used. We assume that the radial wave function and the effective charge associated with the $d_{3/2}$ orbit are the same as for the $d_{5/2}$ level. Consequently

$$\langle \chi_{j'} \| E2 \| \chi_j \rangle = (-1)^{j-j'} \left(\frac{2j+1}{2j'+1}\right)^{1/2} \frac{(j2\frac{1}{2}0 | j'\frac{1}{2})}{(\frac{5}{2}2\frac{1}{2}0 | \frac{5}{2}\frac{1}{2})} q_{dd}$$

$$= (-1)^{j+j'} \left(\frac{35}{8}\right)^{1/2} \left(\frac{2j+1}{2j'+1}\right)^{1/2} (j2\tfrac{1}{2}0 | j'\tfrac{1}{2}) q_{dd} \quad (7)$$

when j and $j' = \frac{3}{2}$ or $\frac{5}{2}$. Similarly for the $d \rightleftarrows s$ transition

$$\langle \chi_{j'} \| E2 \| \chi_j \rangle = (-1)^{j-j'} \sqrt{3} \left(\frac{2j+1}{2j'+1}\right)^{1/2} \frac{(j2\frac{1}{2}0 | j'\frac{1}{2})}{(\frac{1}{2}2\frac{1}{2}0 | \frac{5}{2}\frac{1}{2})} q_{ds}$$

$$= (-1)^{j-j'} \sqrt{5} \left(\frac{2j+1}{2j'+1}\right)^{1/2} (j2\tfrac{1}{2}0 | j'\tfrac{1}{2}) q_{ds} \quad (8)$$

when j or $j' = \frac{1}{2}$.

To obtain magnitudes for the electromagnetic properties of the core-excited states we look at two cases which should exhibit similar matrix elements—the ground-state rotational band in ^{20}Ne and the band built on the 6.05-MeV 0^+ state in ^{16}O. In ^{20}Ne, the quadrupole moment of the 1.63-MeV 2^+ state has the value[11]

$$Q/e = -24 \pm 3 \text{ fm}^2$$

from which it follows that

$$q_{22}(^{20}\text{Ne}) = \langle \Psi_2(^{20}\text{Ne}) \| E2 \| \Psi_2(^{20}\text{Ne}) \rangle = -14.2 \pm 1.8 \, e \text{ fm}^2. \quad (9)$$

There is no information on q_{22} from ^{16}O since the quadrupole moment of the 6.92-MeV 2^+ state is not known.

The mean γ-ray lifetime[9] for the 1.63-MeV 2^+ level in ^{20}Ne is 1.2 ± 0.15 psec which leads to the off-diagonal $E2$ matrix element

$$q_{02}(^{20}\text{Ne}) = \langle \Psi_0(^{20}\text{Ne}) \| E2 \| \Psi_2(^{20}\text{Ne}) \rangle = -17.06 \pm 1.07 \, e \text{ fm}^2. \quad (10a)$$

The lifetime, of course, does not determine the sign of the matrix element and our choice of minus for the phase implies a relative sign for Ψ_0 and Ψ_2 which, as we shall see, fixes the signs of the off-diagonal Hamiltonian matrix elements $\langle (j_1 j_2)_I | V | \Psi_I \rangle$.

The lifetime of the 6.92-MeV 2^+ state in ^{16}O is known to be[6] 6.8 ± 0.4 fsec. The level decays mainly to the ground state with a small branch, $(2.7 \pm 0.3 \times 10^{-2})\%$, to the 6.05-MeV 0^+. To obtain an estimate for the ^{16}O analog of q_{02}, we assume that the ^{16}O ground state has the structure[12]

$$|^{16}\text{O g.s.}\rangle = (1-\beta^2)^{1/2} \Phi_0 + \beta \Psi_0,$$

where Φ_0 is the closed p-shell wave function and Ψ_0 is the four-particle–four-hole state. We take the 6.05-MeV level to be the orthogonal linear combination of these two configurations. If we assume that the 6.92-MeV state is purely four-particle–four-hole and that the transition is induced by a sum of one-body operators it follows that

$$\frac{1-\beta^2}{\beta^2} \left(\frac{0.869}{6.919}\right)^5 = 2.7 \times 10^{-4}.$$

Thus

$$\beta^2 = 0.104.$$

When this result is combined with the measured lifetime one finds

$$q_{02}(^{16}\text{O}) = \langle \Psi_0(^{16}\text{O}) \| E2 \| \Psi_2(^{16}\text{O}) \rangle = -19.08 \pm 0.56 \, e \text{ fm}^2. \quad (10b)$$

The lifetime[9] of the 4.25-MeV 4^+ level in ^{20}Ne for decay to the 2^+ state is

$$\tau_m = 93 \pm 9 \text{ fsec}$$

which leads to the value

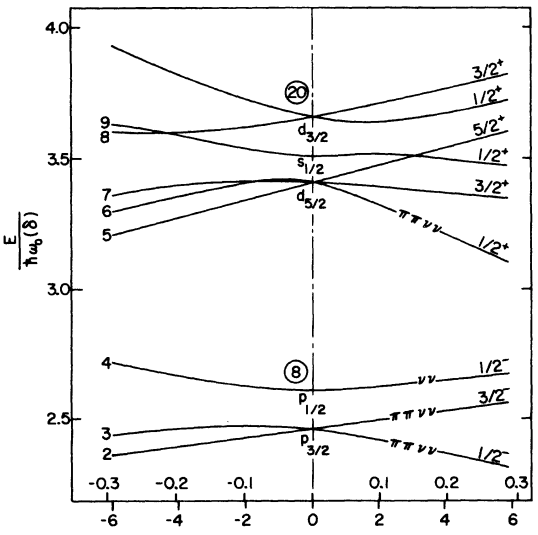

FIG. 2. Nilsson diagram for the $1p$ and $(1d, 2s)$ shells. The intrinsic state from which the core-excited wave functions are generated is shown.

$$q_{24}(^{20}\mathrm{Ne}) = \langle \Psi_2(^{20}\mathrm{Ne}) \| E2 \| \Psi_4(^{20}\mathrm{Ne}) \rangle = -11.36 \pm 0.55\ e\,\mathrm{fm}^2, \tag{11a}$$

where again our choice of a minus sign implies a specific phase for the wave functions involved.

The 4^+ member of the ^{16}O rotational band is the 10.341-MeV state. The decay of this state to the 6.92-MeV 2^+ level has recently been remeasured,[13] $\Gamma_\gamma = (58 \pm 7) \times 10^{-3}$ eV, so that

$$q_{24}(^{16}\mathrm{O}) = \langle \Psi_2(^{16}\mathrm{O}) \| E2 \| \Psi_4(^{16}\mathrm{O}) \rangle = -16.6 \pm 1.0\ e\,\mathrm{fm}^2. \tag{11b}$$

In order that a fit to the experimental data be acceptable we shall require that the $E2$ core matrix elements be similar to the values given by Eqs. (9)–(11).

The magnetic properties of the collective states Ψ_I are assumed to be described in terms of one parameter, the g factor. We assume this to be state independent and to have the value

$$g_c = \tfrac{1}{2}\mu_N. \tag{12}$$

This is consistent with the value $(Z/A)\mu_N$ used in the collective model and also with that deduced from a specific form of the core-excited states which we shall now discuss.

As far as the collective wave functions Ψ_0, Ψ_2, and Ψ_4 are concerned we need make no assumption about their specific form and in this respect our calculation is different from that of other authors. However, as already stated our selection of the signs associated with the $E2$ matrix elements, Eqs. (10) and (11), imply a phase for the collective wave functions. Once this phase has been assumed the signs of the off-diagonal matrix elements of the effective interaction

$$W_I(j_1 j_2) = \langle (j_1 j_2)_{IM} (\pi p_{1/2})^2_0 (\nu p_{1/2})^2_0 (\pi p_{3/2})^4_0 (\nu p_{3/2})^4_0 \,|\, V \,|\, \Psi_{IM} \rangle \tag{13}$$

follow. In order to determine what these signs should be we present, in Fig. 2, the Nilsson diagram[14] for the $1p$ and $(1d, 2s)$ shells. On the basis of the rotational model, the core-excited levels would arise from the Nilsson intrinsic state χ_0 illustrated in this figure—i.e., from exciting two protons out of level No. 4 ($K^\pi = \frac{1}{2}^-$) to No. 6 ($K^\pi = \frac{1}{2}^+$). We shall assume that this is true also in the shell-model calculation, but instead of using the rotational form for the collective states we project states of good angular momentum from the intrinsic state. Thus

$$\Psi_{IM} = (2I+1) N_I \int dR\, D^I_{M0}(R) \chi_0(Rx),$$

where $D^I_{MK}(R)$ is the rotation matrix and N_I is a normalization constant. Since we shall never be concerned with the explicit form of the Ψ_{IM} but merely on the sign constraints imposed upon the $W_I(j_1 j_2)$, we shall make the simplifying assumption that the $K = \frac{1}{2}^-$ level is purely $p_{1/2}$ and that the $K = \frac{1}{2}^+$ state is that appropriate to infinite positive deformation. This is precisely the SU_3 limit for the (ds)-shell nucleons and the restriction to only $p_{1/2}$ holes is the same as made by Brown and Green[15] and Federman and Talmi.[4] Under this assumption the Nilsson wave function for orbit No. 6 is

$$\varphi_{K=1/2^+} = \sqrt{\tfrac{2}{5}}\, \chi_{5/2,1/2} - \frac{2}{\sqrt{15}}\, \chi_{3/2,1/2} - \frac{1}{\sqrt{3}}\, \chi_{1/2,1/2}.$$

It is now straightforward,[3] but tedious, to derive the explicit form for Ψ_{IM}. If we define

$$\begin{aligned}
\phi_0 &= \frac{1}{3\sqrt{5}} [2\sqrt{3}\,(d_{5/2})^2_0 + 2\sqrt{2}\,(d_{3/2})^2_0 + 5(s_{1/2})^2_0], \\
\phi_2 &= \frac{1}{15} [2\sqrt{6}\,(d_{5/2})^2_2 + \sqrt{14}\,(d_{3/2})^2_2 + 2\sqrt{3}\,(d_{5/2} d_{3/2})_2 + \sqrt{105}\,(d_{5/2} s_{1/2})_2 - \sqrt{70}\,(d_{3/2} s_{1/2})_2], \\
\phi_4 &= \frac{1}{\sqrt{5}} [(d_{5/2})^2_4 + 2(d_{5/2} d_{3/2})_4],
\end{aligned} \tag{14}$$

then the collective states are given by

$$\Psi_{00} = \tfrac{1}{35}\{21[\phi_0(\pi) \times \phi_0(\nu)]_{00} + 12\sqrt{5}[\phi_2(\pi) \times \phi_2(\nu)]_{00} + 8[\phi_4(\pi) \times \phi_4(\nu)]_{00}\} (\nu p_{1/2})^2_0 (\nu p_{3/2})^4_0 (\pi p_{3/2})^4_0, \tag{15a}$$

$$\begin{aligned}
\Psi_{2M} = \frac{1}{35\sqrt{14}} &(21\sqrt{11}\{[\phi_0(\pi) \times \phi_2(\nu)]_{2M} + [\phi_2(\pi) \times \phi_0(\nu)]_{2M}\} + 12\sqrt{11}\{[\phi_2(\pi) \times \phi_4(\nu)]_{2M} + [\phi_4(\pi) \times \phi_2(\nu)]_{2M}\} \\
&+ 6\sqrt{110}[\phi_2(\pi) \times \phi_2(\nu)]_{2M} + 8\sqrt{5}[\phi_4(\pi) \times \phi_4(\nu)]_{2M}) (\nu p_{1/2})^2_0 (\nu p_{3/2})^4_0 (\pi p_{3/2})^4_0,
\end{aligned} \tag{15b}$$

$$\begin{aligned}
\Psi_{4M} = \frac{1}{35\sqrt{42}} &(7\sqrt{143}\{[\phi_0(\pi) \times \phi_4(\nu)]_{4M} + [\phi_4(\pi) \times \phi_0(\nu)]_{4M}\} + 20\sqrt{13}\{[\phi_2(\pi) \times \phi_4(\nu)]_{4M} + [\phi_4(\pi) \times \phi_2(\nu)]_{4M}\} \\
&+ 6\sqrt{715}[\phi_2(\pi) \times \phi_2(\nu)]_{4M} + 36[\phi_4(\pi) \times \phi_4(\nu)]_{4M}) (\nu p_{1/2})^2_0 (\nu p_{3/2})^4_0 (\pi p_{3/2})^4_0,
\end{aligned} \tag{15c}$$

where $\phi_I(\pi)/\phi_I(\nu)$ stands for the two-proton/two-neutron states given by Eqs. (14) and $[\ \times\ ]_{IM}$ implies vector coupling to total angular momentum (I,M). With these wave functions one would calculate $E2$ matrix elements with the signs given by Eqs. (9)–(11), provided the signs of the single-particle matrix elements are those given by Eqs. (5)–(8).

Since $(\pi p_{1/2})^2$ can couple only to angular momentum zero it follows that all the $M1$ properties of the intruder state are determined by the $(ds)^4$ wave function. In the limit of infinite deformation this has $S=T=0$ and $I=L$, where S and L are the total spin and orbital angular momentum and T is the isospin. Because the magnetic moment operator can be written as

$$\mu_{op}/\mu_N = \mu_n \sum_i \sigma_z(i)\tfrac{1}{2}[1-\tau_z(i)] + \sum_i [\mu_p\sigma_z(i) + l_z(i)]\tfrac{1}{2}[1+\tau_z(i)]$$

$$= (\mu_n+\mu_p)S_z + \tfrac{1}{2}L_z + \tfrac{1}{2}(\mu_p-\mu_n)\sum_i \sigma_z(i)\tau_z(i) + \tfrac{1}{2}\sum_i l_z(i)\tau_z(i)$$

it follows that only the term $\frac{1}{2}L_z$ gives a contribution to the magnetic properties of the core. Thus

$$\langle \Psi_{IM} | \mu_{op} | \Psi_{I'M} \rangle = \tfrac{1}{2}M\mu_N\delta_{II'},$$

and consequently one would predict the g factor of the core to be $\frac{1}{2}\mu_N$ in agreement with our assumed value Eq. (12).

A second consequence of the fact that $(\pi p_{1/2})^2$ has $I=0$ is that only the first term in each of the collective wave functions is involved in $W_I(j_1j_2)$; that is, all matrix elements are proportional to

$$w = \langle (\pi p_{1/2})^2{}_0 | V | \phi_0(\pi)\rangle$$

$$= \frac{1}{3\sqrt{5}}[2\sqrt{3}\langle(\pi p_{1/2})_0 | V | (\pi d_{5/2})^2{}_0\rangle + 2\sqrt{2}\langle(\pi p_{1/2})^2{}_0 | V | (\pi d_{3/2})^2{}_0\rangle + 5\langle(\pi p_{1/2})^2{}_0 | V | (\pi s_{1/2})^2{}_0\rangle]. \tag{16}$$

The number that multiplies w in calculating $W_I(j_1j_2)$ is the coefficient of $(j_1j_2)_I$ in Eqs. (14) times the coefficient of $\phi_0(\pi)$ that occurs in Ψ_{IM}, Eq. (15).

For an attractive δ function potential $V=-V_0\delta(r_1-r_2)$ the $T=1$ $I=0$ interaction energy has the form

$$\langle (j)^2{}_0 | V | (j_1)^2{}_0\rangle = -\frac{V_0}{8\pi}(-1)^{l+l_1}[(2j+1)(2j_1+1)]^{1/2}\int R_j{}^2 R_{j_1}{}^2 r^2 dr, \tag{17}$$

where l and l_1 are the orbital angular momenta associated with j and j_1, respectively. Thus when the parity of the interacting orbits differs, the off-diagonal $I=0$ matrix element is positive. The same result holds for the Hamada-Johnston potential[15] and has also been found by McGrory and Wildenthal[16] in their matrix element fit to the data near $A=20$. Thus w, as defined by Eq. (16), is positive; from Eqs. (14) and (15) it follows that all $W_I(j_1j_2)$ except $W_2(d_{3/2}s_{1/2})$ should be positive.

III. DATA

The data we choose to fit are listed below.

(1) *Ratios* of spectroscopic factors for $^{17}O(d,p)^{18}O$ leading to the eight states of interest. There are seven such ratios: the $l=2$ spectroscopic factors for the 0_2^+ and 0_3^+ states relative to that for the 0_1^+ ground state (g.s.); the $l=0$ S for the 2_2^+ and 2_3^+ states relative to that for the 2_1^+ state at 1.98 MeV; the $l=2$ S for the 4_2^+ state relative to that for the 4_1^+ state at 3.55 MeV; the ratio of the $l=2$ S for the first two 2^+ levels; and the ratio of $S_{l=0}$ to $S_{l=2}$ for the first 2^+ state. Two sets of experimental spectroscopic factors exist: those from a distorted-wave Born-approximation (DWBA) analysis[17] of the data of Wiza, Middleton, and Hewka[18] and those from the more recent work of Li *et al.*[19] We use the averages of the two sets of numbers.

(2) The ratio of spectroscopic factors for $^{18}O(d,t)^{17}O$ to the $\frac{5}{2}^+$ g.s. and $\frac{1}{2}^+$ first-excited state of ^{17}O. These are taken from a DWBA analysis[17] of the data of Armstrong and Quisenberry.[20] For this part of the analysis, the ground and first-excited states of ^{17}O were taken to be pure $1d_{5/2}$ and $2s_{1/2}$, respectively.

(3) Ratios of (t,p) cross sections from the data of Middleton and Pullen.[21] For the 0^+ states, these are the ratios of the two excited 0^+ states relative to the ground state, evaluated at 5° in the laboratory coordinate system. For the 2^+ states, the ratios of the 2_2^+ and 2_3^+ relative to that of the 2_1^+ state were evaluated at the peak of the cross section. The theoretical predictions for these cross sections were computed by use of the two-particle-transfer option of the code DWUCK,[22] with the optical-model parameters quoted in Ref. 5. The theoretical values are based on the assumption

that ^{16}O is an inert closed shell.

(4) The magnitude and sign of the g factor of the 1.98-MeV 2_1^+ state.[10]

(5) The magnitude of the g factor[23] of the 3.55-MeV 4^+ state. Since this state is dominantly $(d_{5/2})^2_{I=4}$, we *assume* this quantity to be negative.

(6) The quadrupole moment of the 1.98-MeV level given by Christy and Häusser[24] as $Q/e = -11 \pm 5$ fm^2. We discuss later the implications of fitting the more recent value,[25] which is considerably larger.

(7) The $B(E2)$ values for the first 2^+ and 4^+ states, evaluated from the measured mean lifetimes[26]

$$\tau_m(2_1^+) = 3.58 \pm 0.18 \text{ psec},$$

$$\tau_m(4_1^+) = 24.8 \pm 1.3 \text{ psec}.$$

(8) The lifetime of the second 2^+ level at 3.92 MeV. Because of the large variation in measurements of the lifetime of this state, we initially did not fit this quantity. But we always arrived at a lifetime that was consistent with the Brookhaven result[27] and consequently we included their value in our final fit.

(9) The branching ratio $(2_2^+ \to 0_1^+)/[(2_2^+ \to 2_1^+) + (2_2^+ \to 0_1^+)]$ and the $E2/M1$ mixing ratio $\delta(2_2^+ \to 2_1^+)$, both taken from the work of Olness, Warburton, and Becker.[27]

(10) The $B(E2;\ 0_3^+ \to 2_1^+)$ taken from Ref. 27.

(11) The branching ratio $(2_3^+ \to 0_1^+)/[(2_3^+ \to 2_1^+) + (2_3^+ \to 0_1^+)]$.[28,29]

(12) The branching ratios of the 4_2^+ level at 7.11 MeV, whose decay is predominantly to the 4_1^+ level at 3.55 MeV. Because of uncertainty[28,30] in the possibility of weak branches to negative-parity states, we fit only the observed[30] ratios $4_2^+ \to 4_1^+/4_2^+ \to 2_1^+$ and $4_2^+ \to 2_1^+/4_2^+ \to 2_2^+$.

(13) The $B(E2)$ value for $0_2^+ \to 2_1^+$. Measurements of this transition probability give markedly different results. The recent Brookhaven experiment[27] yields

$$B(E2; 0_2^+ \to 2_1^+) = 47.5 \pm 4.8\ e^2\,\text{fm}^4 \tag{18a}$$

while the average of earlier experiments[31,32] is

$$B(E2; 0_2^+ \to 2_1^+) = 26.1 \pm 4.6\ e^2\,\text{fm}^4. \tag{18b}$$

The use of the two different values in the fit produce very similar wave functions, but require quite different $E2$ matrix elements for the core-excited basis states. The two possibilities are discussed separately below.

(14) The $E2/M1$ mixing ratio $\delta(2_3^+ \to 2_1^+)$. Here, also we perform the fit for two different values

$$\delta(2_3^+ \to 2_1^+) = 0.14 \pm 0.04 \quad \text{(Ref. 29)} \tag{19a}$$

and

$$\delta(2_3^+ \to 2_1^+) = 0.2 \pm 0.1 \quad \text{(Ref. 33)}. \tag{19b}$$

IV. FITTING PROCEDURE

We are interested in finding the physical states Φ_{IM} of good angular momentum I and z projection M as a linear combination of basis configurations u_{IM}. The Φ and u are related by the orthogonal transformation U_I

$$\Phi_{IM}(i) = \sum_k (U_I)_{ik} u_{IM}(k) \tag{20}$$

that is subject to the orthonormality conditions

$$\sum_k (U_I)_{ik}(U_I)_{jk} = \delta_{ij}.$$

This orthogonal transformation is adjusted so that the physical states Φ best fit the experimental data. For an N-dimensional problem, the orthonormality conditions tell us there are $\frac{1}{2}[N(N-1)]$ independent parameters in the transformation U_I. For the purposes of our fit, we find that the most convenient representation of an N-dimensional orthogonal matrix is as a successive product of $\frac{1}{2}[N(N-1)]$ simple rotations

$$U_I = \prod_{\substack{i,j=1 \\ i<j}}^{N} R_{ij}(\theta_{ij}).$$

In three dimensions, for example,

$$U_I = \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} \\ 0 & 1 & 0 \\ -\sin\theta_{13} & 0 & \cos\theta_{13} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{pmatrix}. \tag{21}$$

The extension to $N > 3$ is straightforward.

If it were not for the sign constraints imposed on the off-diagonal Hamiltonian matrix elements (see Sec. II) the procedure to be adopted would be to vary the θ_{ij} independently so as to minimize

$$\chi^2 = \sum_i \left(\frac{\text{expt}(i) - \text{theory}(i)}{\Delta\,\text{expt}(i)} \right)^2, \tag{22}$$

where expt(i), Δexpt(i), and theory(i) are the experimental value, its uncertainty, and the theoretical prediction for the ith quantity to be fitted. In multidimensional spaces many local minima exist and unless the initial guess for the parameters θ_{ij} is close to the one dictated by the sign constraints, discussed in Sec. II, a nonphysical

result may be obtained. To be sure that the "correct" minimum is obtained it is simplest to modify χ^2 so as to encompass these constraints. To do this, we note that once the U_I have been obtained one may recover the Hamiltonian matrix H_I, whose eigenvectors are the rows of U_I, from the spectral representation

$$H_I = \tilde{U}_I E_I U_I, \tag{23}$$

where E_I is a diagonal matrix whose elements are the observed energies of the states and $\tilde{U}_I$ is the transpose of U_I.

The matrix elements of H_I contain both $W_I(j_1 j_2)$, Eq. (13), and the off-diagonal elements within the $(ds)^2$ configuration. Provided none of these has an anomalously small absolute value, their signs are known—the former by the considerations discussed at the end of Sec. II and the latter from Kuo's calculation.[8] To insure that these conditions are fulfilled we therefore add to χ^2 [Eq. 22] the term

$$\chi_1^2 = \sum_j \left(\frac{W_j - H_j}{\Delta W_j}\right)^{2p}, \tag{24}$$

where the W_j are what we consider to be reasonable values for the above matrix elements and H_j are the values that emerge by use of our fitted wave functions and the energy eigenvalues Eq. (23). Although the uncertainty in W_j, ΔW_j, should be taken to be large it should be chosen so that $W_j \pm \Delta W_j$ has the expected sign. Consequently if p is taken to be >1 (in our calculations p was chosen equal to 4) there is little contribution of χ_1^2 to the total $\bar{\chi}^2 = \chi^2 + \chi_1^2$, provided $|W_j - H_j| < \Delta W_j$. However, once this inequality is violated a severe penalty is imposed upon the fit. It is by minimizing

$$\bar{\chi}^2 = \chi^2 + \chi_1^2 \tag{25}$$

that we have fitted the θ_{ij} to the experimental data. Thus, although we appear to have a rather large number of parameters, a severe limitation is imposed upon them by the added "pseudoexperimental" data of Eq. (24) and our fit is, therefore, meaningful.

In carrying out fits to the data we include, in addition to one collective state of each spin, the effect of having one valence neutron in the $d_{3/2}$ orbit. As we shall see, the admixtures of $(d_{5/2}, d_{3/2})_2$ and $(d_{3/2}, s_{1/2})_2$ are small and could be treated by perturbation theory. Their major effects are to reduce $|g(2_1^+)|$ [if they were neglected, the least-squares fit to the data gives a preference for $g(2_1^+) \approx -0.5\mu_N$] and to reduce $B(E2; 0_3^+ \to 2_1^+)$ by about 20% [within the $(d_{5/2})^2{}_2$, $(d_{5/2}, s_{1/2})_2$, and Ψ_2 model space there is a cancellation in this matrix element and consequently these small admixtures have a substantial effect]. In order to obtain 2^+ wave functions that are orthogonal and normalized we have chosen explicitly to retain these small components in the fit instead of treating their effects by perturbation theory.

V. RESULTS

As we have already discussed, estimated magnitudes for the core $E2$ matrix elements can be obtained from the data on ^{16}O and ^{20}Ne [Eqs. (9)–(11)]. We shall first examine a fit to the experimental data in which these core matrix elements are constrained to lie within the limits imposed by ^{16}O and ^{20}Ne. Such a fit to the data will be labeled "constrained" [since there is a large discrepancy between the recent value[27] of $B(E2; 0_2^+ \to 2_1^+)$ and earlier experiments,[31,32] two fits labeled "constrained I" and "constrained II" are considered]. As an alternative, we have also carried out a fit in which the core $E2$ matrix elements were allowed to vary without restriction. This fit has been labeled "unconstrained." In all cases the single-particle $E2$ matrix elements q_{dd} and q_{ds} were held within the limits imposed by the ^{17}O data, the magnetic properties were computed by use of Eq. (4), and g_c was taken to be $\frac{1}{2}$.

A. Constrained I

For this fit, the effective electromagnetic operators are constrained as discussed above. We take the Brookhaven value for $B(E2; 0_2^+ \to 2_1^+)$ [Eq. (18a)] and the average value of $\delta(2_3^+ \to 2_1^+)$

$$\delta(2_3^+ \to 2_1^+) = 0.15 \pm 0.04. \tag{26}$$

In column 3 of Table I we tabulate this fit to the data and in column 4 we give the difference between theory and experiment divided by the experimental uncertainty—that is the number of standard deviations by which the fit misses each data point. The values of the $E2$ matrix elements and the various contributions to χ^2 are summarized in column 2 of Table II. Except for $B(E2; 0_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$, the data are quite reasonably fitted. In this fit, as in all others, the major contribution to χ^2 (two-nucleon transfer) comes from the small $0_2^+/0_1^+$ cross-section ratio. Because the 0_2^+ cross section is so small, the experimental value represents an upper limit to the direct two-nucleon-transfer strength since, undoubtedly, nondirect processes are present for such a weak transition. Also, since the small calculated cross section arises from an almost complete cancellation, the addition of a small $(d_{3/2})^2$ component (which has been ignored throughout) could easily double the predicted value.

There are certain quantities for which only experimental limits exist. These are listed at the end of Table I and in all cases—except the sign of $\delta(4_2^+ \to 4_1^+)$—the theoretical predictions lie within

TABLE I. Comparison of the fitted experimental data and the theoretical values for different models. All values of C^2S are from the $^{17}O(d,p)^{18}O$ data except $C^2S(0_1^+)$ $l=0/l=2$ which comes from the $^{18}O(d,t)^{17}O$ experiment. The notation $(t,p)I_m^+/I_1^+$ is the ratio of differential cross sections at 5° for the 0^+ levels and at their peak for $I\neq 0$. B.R.$(2_m^+\rightarrow 0_1^+)$ gives the branching ratio in percent for decay of the 2_m^+ level to the ground state (in both cases the other observed branch for decay is to the 2_1^+ level). Further information on the data fitted and references to the experimental papers are given in Sec. III. In the columns headed constrained I we give the theoretical best fit to the data when the $E2$ matrix elements of column 2 Table II are used. $\sqrt{\chi^2}$ is the absolute value of the difference between theory and experiment divided by the experimental uncertainty. The theoretical results listed under unconstrained are the best fit to the data when the $E2$ matrix elements of column 4 Table II are used. In constrained II we give the values obtained when a second possible set of experimental data is fitted (see text) and the $E2$ matrix elements are given in column 5 Table II.

Quantity		Experimental value and uncertainty	Constrained I Theory	$\sqrt{\chi^2}$	Unconstrained Theory	$\sqrt{\chi^2}$	Constrained II Theory	$\sqrt{\chi^2}$
		Ratios of $^{17}O(d,p)^{18}O$ stripping strengths						
$C^2S(l=2)$	$0_2^+/0_1^+$	0.21 ±0.05	0.319	2.17	0.202	0.16	0.295	1.70
	$0_3^+/0_1^+$	0.10 ±0.035	0.093	0.21	0.122	0.61	0.095	0.15
$C^2S(l=0)$	$2_2^+/2_1^+$	1.90 ±0.40	1.473	1.07	1.694	0.51	1.553	0.87
	$2_3^+/2_1^+$	1.56 ±0.30	1.621	0.20	1.494	0.22	1.536	0.08
$C^2S(l=2)$	$2_1^+/2_2^+$	1.26 ±0.30	1.468	0.69	1.606	1.15	1.576	1.05
$C^2S(2_1^+)$	$l=0/l=2$	0.20 ±0.04	0.203	0.08	0.192	0.20	0.196	0.10
$C^2S(l=2)$	$4_2^+/4_1^+$	0.103±0.02	0.094	0.44	0.108	0.25	0.092	0.53
		Ratios of $^{18}O(d,t)^{17}O$ pickup strengths						
$C^2S(0_1^+)$	$l=0/l=2$	0.17 ±0.04	0.273	2.58	0.245	1.86	0.267	2.43
		Ratios of $^{16}O(t,p)^{18}O$ transfer strengths						
(t,p)	$0_2^+/0_1^+$	0.0324±0.01	0.0161	1.63	0.0170	1.54	0.0165	1.59
	$0_3^+/0_1^+$	0.433 ±0.12	0.581	1.23	0.567	1.12	0.481	0.40
	$2_2^+/2_1^+$	0.298 ±0.09	0.243	0.61	0.265	0.37	0.252	0.51
	$2_3^+/2_1^+$	0.702 ±0.21	0.637	0.31	0.571	0.62	0.709	0.03
		Static moments						
$g(2_1^+)$	μ_N	−0.35 ±0.04	−0.315	0.87	−0.320	0.75	−0.320	0.75
$g(4_1^+)$	μ_N	−0.62 ±0.10	−0.614	0.06	−0.615	0.05	−0.632	0.12
$Q(2_1^+)$	$e\,\mathrm{fm}^2$	−11± 5	−4.968	1.21	−3.895	1.42	−4.774	1.25
		Electromagnetic transition data						
$B(E2;\ 2_1^+\rightarrow 0_1^+)$	$e^2\,\mathrm{fm}^4$	7.42 ±0.37	7.128	0.79	7.242	0.48	7.052	0.99
$B(E2;\ 4_1^+\rightarrow 2_1^+)$	$e^2\,\mathrm{fm}^4$	3.43 ±0.18	3.415	0.09	3.435	0.03	3.415	0.09
$\tau(2_2^+)$	psec	0.024±0.01	0.028	0.40	0.030	0.58	0.030	0.58
B.R. $(2_2^+\rightarrow 0_1^+)$	(%)	13± 3	8.194	1.60	9.303	1.23	7.747	1.75
$\delta(2_2^+\rightarrow 2_1^+)$		0.12 ±0.04	0.110	0.25	0.092	0.71	0.114	0.16
$B(E2;\ 0_3^+\rightarrow 2_1^+)$	$e^2\,\mathrm{fm}^4$	5.2 ±1.16	5.168	0.03	6.258	0.91	5.164	0.03
B.R. $(2_3^+\rightarrow 0_1^+)$	(%)	32± 3	23.77	2.74	32.21	0.07	30.03	0.66
$(4_2^+\rightarrow 4_1^+)/(4_2^+\rightarrow 2_1^+)$		2.7 ±0.3	2.774	0.25	2.708	0.03	2.783	0.28
$(4_2^+\rightarrow 2_1^+)/(4_2^+\rightarrow 2_2^+)$		6.5 ±3.3	5.686	0.25	4.922	0.48	5.304	0.36
$B(E2;\ 0_2^+\rightarrow 2_1^+)$	$e^2\,\mathrm{fm}^4$	47.5±4.75	24.76	4.78	46.42	0.22		
$\delta(2_3^+\rightarrow 2_1^+)$		0.15±0.04	0.303	3.83	0.155	0.13		
$B(E2;\ 0_2^+\rightarrow 2_1^+)$	$e^2\,\mathrm{fm}^4$	26.1±4.6					22.98	0.68
$\delta(2_3^+\rightarrow 2_1^+)$		0.2± 0.1					0.369	1.69
		Other data not in fits						
$C^2S(l=2)$	$2_3^+/2_1^+$	Small	0.002		0.004		0.004	
(t,p)	$4_2^+/4_1^+$	$<\frac{1}{2}$	0.23		0.28		0.26	
$\tau(2_3^+)$	psec	≤ 0.035	0.030		0.030		0.044	
$B(M1;\ 4_2^+\rightarrow 4_1^+)$	μ_N^2	>0.05	0.312		0.328		0.296	
$\delta(4_2^+\rightarrow 4_1^+)$		−0.035± 0.035	0.043		0.041		0.043	

TABLE II. Matrix elements of the $E2$ operator (in $e\,\mathrm{fm}^2$), collective state energies (in MeV), and contributions to the χ^2 for the fits discussed in Secs. V and VI of the text.

$B(E2;0_2^+ \to 2_1^+)$ ($e^2\,\mathrm{fm}^4$) $\delta(2_3^+ \to 2_1^+)$	Fits I 47.5 ±4.8 0.14±0.04			Fits II 26.1±4.6 0.2±0.1		Fits III 47.5 ±4.8 0.14±0.04	26.1±4.6 0.2±0.1
	Constrained	Free q_{02}	Unconstrained	Constrained	Unconstrained	Unconstrained	Unconstrained
$q_{dd} = \langle \chi_{5/2} \Vert E2 \Vert \chi_{5/2} \rangle$	−1.44	−1.49	−1.49	−1.53	−1.53	−1.28	−1.53
$q_{ds} = \langle \chi_{5/2} \Vert E2 \Vert \chi_{1/2} \rangle$	−1.447	−1.447	−1.447	−1.447	−1.447	−1.447	−1.447
$q_{22} = \langle \Psi_2 \Vert E2 \Vert \Psi_2 \rangle$	−12.39	−12.39	−9.05	−12.39	−11.09	−14.86	−17.78
$q_{02} = \langle \Psi_0 \Vert E2 \Vert \Psi_2 \rangle$	−19.64	−28.20	−26.37	−19.64	−23.08	−24.40	−22.25
$q_{22} = \langle \Psi_2 \Vert E2 \Vert \Psi_2 \rangle$	−17.60	−17.60	−20.35	−17.60	−20.27	−17.96	−17.79
E_0(coll)	3.68	3.72	3.72	3.72	3.74	3.67	3.69
$\Delta E_2 = E_2$(coll) − E_0(coll)	0.89	0.96	0.90	0.82	0.89	0.95	0.90
$\Delta E_4 = E_4$(coll) − E_0(coll)	3.93	3.76	3.53	3.90	3.95	3.56	3.55
χ^2 (one-neutron transfer)	13.24	9.64	5.61	10.94	8.11	10.61	12.05
χ^2 (two-neutron transfer)	4.65	4.59	4.14	2.95	2.72	6.52	4.71
χ^2 [$B(E2;0_2^+ \to 0_1^+)$]	22.88	0.11	0.05	0.46	0.36	0.01	0.71
χ^2 [$\delta(2_3^+ \to 2_1^+)$]	14.69	2.27	0.02	2.87	0.06	19.54	8.02
χ^2 (remaining data)	13.31	5.83	6.24	7.16	5.18	19.59	7.54
$\sum \chi^2$	68.77	22.44	16.06	24.38	16.43	56.27	33.03

the experimental limits.

In order to calculate this latter quantity we must know a value for $\langle \Psi_4 \Vert E2 \Vert \Psi_4 \rangle$. To estimate this we use the core wave functions Eqs. (14) and (15). The value of the single-particle $E2$ matrix elements involving $d \rightleftarrows d$ neutron transitions is taken from Eqs. (5) and (7) and for $s \rightleftarrows d$ from Eqs. (6) and (8). We assume an isoscalar polarization charge so that the analog for q_{dd} (which requires an effective neutron charge of $0.49e$ when oscillator wave functions are used) is, for protons

$$q_{dd}(\pi) = -4.17\ e\,\mathrm{fm}^2 \tag{27}$$

and q_{ds} (which implies an effective neutron charge for $s \rightleftarrows d$ transitions of $0.61e$) is

$$q_{ds}(\pi) = -3.81\ e\,\mathrm{fm}^2 . \tag{28}$$

From this it follows that

$$\langle \Psi_4 \Vert E2 \Vert \Psi_4 \rangle = -8.53\ e\,\mathrm{fm}^2 . \tag{29}$$

The constraints imposed on the signs of off-diagonal energy matrix elements (see the discussion at the end of Sec. II) always lead us to a positive value for $\delta(4_2^+ \to 4_1^+)$. When Eq. (29) is used we predict

$$\delta(4_2^+ \to 4_1^+) = +0.043 . \tag{30}$$

Thus, in agreement with experiment, we find a small mixing ratio. The fact that the experimental value is small and negative should not be considered a serious conflict with our theoretical prediction since a small misalignment of the experimental setup could easily change the sign of this quantity.

A further check on the overall consistency of our model is provided by the energies of the collective states. One may use the fitted wave functions given in Tables III–V together with the experimental energies of the 0^+, 2^+, and 4^+ levels to determine the collective energies. If one takes 3.63 and 5.33 MeV for the excitation energies of the 0^+ states one can use Eq. (23) to extract the unperturbed position of the core 0^+ relative to the ^{18}O ground state. This turns out to be E_0(collective) = 3.68 MeV, in quite reasonable agreement with the estimate obtained from the binding energy data Eq. (2).

Similarly one can estimate the unperturbed position of the 2^+ collective state. However, in this case one must know the positions of the lowest five 2^+ levels in ^{18}O. If one assumes the first five observed 2^+ levels (at 1.98, 3.92, 5.25, 8.21, and 11.39 MeV) are indeed the lowest, one concludes that $\Delta E_2 = E_2$(collective) − E_0(collective) = 0.89 MeV. This value is remarkably close to the 2^+-0^+ splitting in ^{16}O, 6.92−6.05 = 0.87 MeV. Moreover, since

TABLE III. 0^+ wave functions for the three different fits listed in Table I. 0_1^+, 0_2^+, and 0_3^+ refer to the ground state, the 3.63-MeV level, and the 5.33-MeV state, respectively.

		$(d_{5/2})^2_0$	$(s_{1/2})^2_0$	Ψ_0
	Constrained I	0.842	0.440	−0.313
0_1^+	Unconstrained	0.869	0.430	−0.244
	Constrained II	0.848	0.438	−0.297
	Constrained I	0.475	−0.329	0.816
0_2^+	Unconstrained	0.391	−0.294	0.872
	Constrained II	0.461	−0.335	0.822
	Constrained I	0.256	−0.836	−0.486
0_3^+	Unconstrained	0.303	−0.854	−0.424
	Constrained II	0.261	−0.834	−0.486

our calculation indicates there is little probability for Ψ_2 in the fourth and fifth 2^+ states (0.28% and 0.88%, respectively), the estimated position is insensitive to whether a 2^+ level below 11.39 MeV has been missed or not.

The three lowest observed 4^+ levels in ^{18}O are the 3.55-, 7.11-, and 10.29-MeV states. With these values one finds $\Delta E_4 = E_4(\text{collective}) - E_0(\text{collective}) = 3.93$ MeV in satisfactory agreement with the collective splitting, $10.341 - 6.05 = 4.291$ MeV, observed in ^{16}O. This estimate, however, is quite sensitive to the position of the third 4^+ state since according to our calculations there is a 16% probability of finding Ψ_4 in the third 4^+ state. We shall return to a discussion of this point later.

Although the vast majority of the data have been satisfactorily fitted, there are two pieces of $E2$ data, $B(E2; 0_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$ that do not fit into the scheme. From a comparison of the $E2$ matrix elements for this fit with Eqs. (9)–(11) it is clear that the core matrix elements are at their limiting values, and since these dominate the calculated $E2$ properties it is interesting to see if a small change in them would improve the overall fit to the data.

B. Unconstrained

We now relax the constraint conditions on q_{22}, q_{02}, and q_{24} and consider fitting the same data as in the previous calculation. Freeing q_{02}, the $2^+ \to 0^+$ core matrix element, is by far the most important in improving the fit. If one frees this quantity, leaving the others still constrained, one reduces χ^2 by almost 50. The resultant $E2$ matrix elements, collective configuration energies, and χ^2 contributions are in column 3 of Table II. Clearly the main disagreeable features of the preceding fit have disappeared. Again since q_{22} and q_{24} are bumping up against their limiting values some additional improvement can be obtained by freeing them. In columns 5 and 6 of Table I and column 4 of Table II we give the results of letting all the core $E2$ matrix elements vary freely.

Now, the single piece of data that gives the largest contribution to χ^2 is the ratio of the $l=0$ to $l=2$ spectroscopic factors for the ^{18}O ground state as determined by (d, t) experiments.[20] However, even in this case theory and experiment differ by less than 2 standard deviations.

In the rows labeled Unconstrained of Tables III–V we list the wave functions for this fit and it is clear that only minor differences occur between them and the previously calculated ones. For the three 0^+ levels the worst overlap is 0.994; for the 2^+'s, 0.998; and the 4^+'s, 0.999. Thus the experimental data determine the wave functions within rather narrow limits and it is only through changes in the core $E2$ operators that a fit to $B(E2; 0_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$ can be achieved.

As found in the previous fit, our predicted values also agree with quantities which have only experimental limits. Moreover, the collective energies, listed in column 4 of Table II, are almost the same

TABLE IV. 2^+ wave functions for the three different fits listed in Table I. 2_1^+, 2_2^+, and 2_3^+ refer to the 1.98-, 3.92-, and 5.25-MeV states, respectively.

		$(d_{5/2})^2_2$	$(d_{5/2}s_{1/2})_2$	Ψ_2	$(d_{5/2}d_{3/2})_2$	$(d_{3/2}s_{1/2})_2$
	Constrained I	−0.764	−0.488	0.358	−0.049	0.219
2_1^+	Unconstrained	−0.771	−0.479	0.322	−0.054	0.262
	Constrained II	−0.774	−0.485	0.347	−0.056	0.204
	Constrained I	−0.631	0.592	−0.497	0.028	−0.065
2_2^+	Unconstrained	−0.609	0.623	−0.487	0.032	−0.048
	Constrained II	−0.617	0.605	−0.501	0.035	−0.042
	Constrained I	0.033	−0.621	−0.783	0.008	0.017
2_3^+	Unconstrained	0.044	−0.585	−0.807	0.039	0.059
	Constrained II	0.046	−0.602	−0.793	−0.021	0.085

TABLE V. 4^+ wave functions for the three different fits listed in Table I. 4_1^+ and 4_2^+ refer to the 3.55- and 7.11-MeV states, respectively.

		$(d_{5/2})^2_4$	Ψ_4	$(d_{5/2}d_{3/2})_4$
	Constrained I	0.983	−0.070	0.169
4_1^+	Unconstrained	0.983	−0.062	0.170
	Constrained II	0.986	−0.065	0.151
	Constrained I	0.132	0.913	−0.387
4_2^+	Unconstrained	0.130	0.897	−0.422
	Constrained II	0.120	0.912	−0.392
	Constrained I	−0.127	0.403	0.906
4_3^+	Unconstrained	−0.126	0.437	0.891
	Constrained II	−0.113	0.404	0.908

as before. The value of q_{22} needed in this fit is about 25% smaller than that implied by the ^{20}Ne 2^+ quadrupole moment. On the other hand, q_{02} and q_{24} are more negative than their corresponding values in ^{16}O by about 35% and 15%, respectively. One can, of course, correctly argue that the collective state in ^{18}O is not exactly the same as that in ^{16}O or ^{20}Ne, so that variations in the core matrix elements are likely to occur. However, one would feel more comfortable with a value of q_{02} closer to that found empirically for the $A = 16$ and 20 systems.

An alternative way of estimating magnitudes for the core matrix elements is by use of the wave functions Eqs. (14) and (15) and the values of q_{dd}, q_{ds}, $q_{dd}(\pi)$, and $q_{ds}(\pi)$ given by Eqs. (5), (6), (27), and (28), respectively. This calculation gives the result

$$\begin{aligned} q_{22}(\text{def}) &= \langle \Psi_2 \| E2 \| \Psi_2 \rangle = -9.1\ e\,\text{fm}^2 , \\ q_{02}(\text{def}) &= \langle \Psi_0 \| E2 \| \Psi_2 \rangle = -16.9\ e\,\text{fm}^2 , \\ q_{24}(\text{def}) &= \langle \Psi_2 \| E2 \| \Psi_4 \rangle = -11.4\ e\,\text{fm}^2 . \end{aligned} \tag{31}$$

The value of q_{22} obtained in this way is in excellent agreement with that needed to fit the data. However, q_{02} and q_{24} are smaller in absolute value than those of the unconstrained fit I and in fact are smaller than the values extracted from the ^{16}O data, Eqs. (10b) and (11b).

Thus one must conclude that if one used the values of $B(E2; 0_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$ given by Eqs. (18a) and (26), a fit to all of the experimental data using state-independent effective operators can be obtained, but the value of the core $E2$ matrix element $\langle \Psi_0 \| E2 \| \Psi_2 \rangle$ has to be taken uncomfortably large.

C. Constrained II

In the fit labeled Constrained I the theoretical value of $B(E2; 0_2^+ \to 2_1^+)$ was 24.76 $e^2\,\text{fm}^4$—in good agreement with the average of the Litherland *et al.*[32] and Lawson[31] values, Eq. (18b). In addition, the computed value of $\delta(2_3^+ \to 2_1^+) = 0.303$ is within one standard deviation of the result of Lopes *et al.*,[33] Eq. (19b). Consequently, it is clear that if these experimental values were used, a satisfactory fit to the data could be obtained by use of core $E2$ matrix elements consistent with the constraints imposed by the ^{16}O and ^{20}Ne data, Eqs. (9)–(11).

We have redone the calculations using the values given by Eqs. (18b) and (19b). The results that emerge when we demand that the core $E2$ matrix elements be consistent with the ^{16}O, ^{17}O, and ^{20}Ne data are listed under the heading Constrained II in Tables I, III, IV, and V, and under Fits II Constrained in Table II. Thus as had been anticipated, the major discrepancies have disappeared and only minor changes in the wave functions have resulted. With the exception of the 2_3^+ lifetime, the predicted quantities are within the experimental limits and as far as this lifetime is concerned the predicted value, $\tau(2_3^+) = 0.044$ psec, is not in violent disagreement with the limit $\tau(2_3^+) \leq 0.035$ psec.

The unperturbed energies of the core states are quite similar to the values obtained before and are listed in column 5 of Table II. Further, as in the unconstrained fit, the single piece of experimental data that contributes most to χ^2 ($\chi^2 = 5.88$) comes from the ratio of the $l=0$ to $l=2$ spectroscopic factors for the ^{18}O ground state. All other data are fitted to within two standard deviations.

If one relaxes the constraint on the $E2$ matrix elements the fit is, of course, improved but not as dramatically as before. When q_{22}, q_{02}, and q_{24} are all freed one obtains the results of column 6 Table II. As in the previous unconstrained fit, q_{02} is the most important of the core matrix elements. It goes to a more negative value than the limit set by the ^{16}O data, Eq. (10b); however, in this case only about 20% larger. q_{22} and q_{24} are not appreciably outside their limiting values, Eqs. (9) and (11).

In this case the 2_3^+ lifetime is 0.031 psec and hence is within the limits quoted in Table I. As in every fit, the quantity that seems worst reproduced is the $l=0$ to $l=2$ ratio for the ^{18}O ground state which contributes 4.69 to the χ^2. Thus although a slight improvement is obtained when one frees the core $E2$ matrix elements, the overall reproduction of the data is quite similar to the constrained fit and the wave functions are virtually unchanged.

VI. DISCUSSION

From inspection of Tables III–V it is clear that the experimental data pin down the wave functions within rather narrow limits. Any change in the

TABLE VI. Comparison of the spectroscopic factors for single-nucleon transfer. The results listed in column 4 are taken from Ref. 35 and those of column 5 from Ref. 2. The Federman-Talmi values were calculated from the wave functions given in Ref. 4.

Quantity	Experimental value	Constrained II	Benson and Irvine	Ellis and Engeland	Federman and Talmi Case I	Federman and Talmi Case II
$C^2S(l=2)$ $0_2^+/0_1^+$	0.21 ± 0.05	0.295	0.263	0.074	0.285	0.140
$0_3^+/0_1^+$	0.10 ± 0.035	0.095	0.087	0.104	0.001	0.064
$C^2S(0_1^+)\,l=0/l=2$	0.17 ± 0.04	0.267	0.107	0.104	0.124	0.076
$C^2S(l=0)$ $2_2^+/2_1^+$	1.90 ± 0.40	1.553	1.411	1.586	11.456	11.032
$2_3^+/2_1^+$	1.56 ± 0.30	1.536	1.066	0.172	2.367	0.797
$C^2S(l=2)$ $2_1^+/2_2^+$	1.26 ± 0.30	1.576	1.066	1.619	7.482	7.772
$C^2S(2_1^+)\,l=0/l=2$	0.20 ± 0.04	0.196	0.272	0.284	0.039	0.046
$C^2S(l=2)$ $4_2^+/4_1^+$	0.103 ± 0.02	0.092	0.165	0.104	•••	•••

theoretical predictions is due predominantly to changes in the core $E2$ matrix elements. The overall fit implies approximately a 5–10% probability of core excitation in the ^{18}O ground state and this is in agreement with the ^{18}O(^{3}He, d)^{19}F($\frac{1}{2}^-$) data which puts a limit of between 5 and 15% on this component.[34] The data require that the intruder 0^+ state be concentrated (≅65%) in the 3.63-MeV first-excited 0^+ level. This result is in agreement with the findings of Ellis and Engeland[2] and Benson and Irvine[35] but disagrees with the results of Federman and Talmi.[4] As to the intruder 2^+, all calculations predict it to be the main component (≅65%) of the 2_3^+ level at 5.25 MeV. We predict the collective 4^+ to be the main constituent of the 7.11-MeV state and to be only a small component ($\sim\frac{1}{2}$%) of the 3.55-MeV level.

Although our wave functions are similar to the Ellis-Engeland and Benson-Irvine eigenfunctions there are some important differences. In order to illustrate the differences in the noncollective parts, we have compared, in Table VI, the single-nucleon transfer spectroscopic factors that emerge from our constrained II calculation with those of various other works. (Although all our results give similar spectroscopic factors, we have chosen to present the results for constrained II because within the $E2$ constraints imposed by ^{16}O, ^{17}O, and ^{20}Ne the data are best fitted by this calculation.) Clearly the Federman-Talmi wave functions are inappropriate for describing single-nucleon transfer. The Ellis-Engeland eigenfunctions for the 2^+ states have too much $(d_{5/2}s_{1/2})$ in 2_1^+ and not enough in 2_3^+. Moreover, both the Ellis-Engeland and Benson-Irvine ground-state wave functions have less $(s_{1/2})^2{}_0$ than does ours. Overall, our wave functions are most similar to those of Benson and Irvine. One major difference, though, is the amount of $(s_{1/2})^2{}_0$ in the 3.63-MeV level. In our case the probability of this configuration is 11.5% whereas in theirs it is only (3×10^{-2})%. This has the consequence that Benson and Irvine predict a (t,p) cross section to 0_2^+ approximately 10 times stronger than observed.

Within the assumed model space (no more than one particle in the $d_{3/2}$ orbit plus one collective state of spin 0, 2, and 4) it is now possible, by use of Eq. (23), to give the Hamiltonian matrix which best reproduces *not only the energies but also the transition rates*. In order to do this we must know the position of the first three 0^+ and 4^+ levels and the first five 2^+ states. If one uses the existing data[9] on ^{18}O one would conclude that the appropriate energies are

$$E_0 = (-3.915),\ 3.63,\ 5.33\ \text{MeV},$$
$$E_2 = 1.98,\ 3.92,\ 5.25,\ 8.21,\ 11.39\ \text{MeV}, \quad (32)$$
$$E_4 = 3.55,\ 7.11,\ 10.29\ \text{MeV},$$

where all energies are excitation energies except the bracketed binding energy of the ground state.

Since all states in ^{18}O up to 6 MeV excitation have definite spin assignments it is clear that the $I=0$ energies of Eq. (32) represent the lowest three 0^+ states. Moreover, since a detailed fit to all the properties of these three states has been made, the Hamiltonian matrix which best describes them within the $(d_{5/2})^2{}_0$, $(s_{1/2})^2{}_0$, and Ψ_0 model space is completely determined.

As far as the $I=2$ states are concerned, the properties of the first three are dominated by the configurations $(d_{5/2})^2{}_2$, $(d_{5/2}s_{1/2})_2$, and Ψ_2. Because of this, the Hamiltonian within this model space should be well determined. On the other hand, matrix elements involving a $d_{3/2}$ nucleon cannot be reliably extracted for two reasons:

(a) No dynamic properties of the fourth and fifth 2^+ states have been fitted and consequently the interplay between $(d_{5/2}d_{3/2})_2$ and $(d_{3/2}s_{1/2})_2$ is not known.

TABLE VII. Values of the shell-model matrix elements that give the best fit wave functions. These are listed for two different possible sets of excitation energies. Those of column 3 rely on Eq. (32) and those of column 4 use the energies of Eq. (33). It is assumed that the $s_{1/2}$ and $d_{3/2}$ single-particle states lie 0.87 and 5.08 MeV, respectively, above the $d_{5/2}$ orbit. The Kuo matrix elements are taken from Ref. 8.

$\langle(j_1j_2)_I\|V\|(j_3j_4)_I\rangle$	I	Constrained II Energies Eq. (32)	Constrained II Energies Eq. (33)	Kuo
$\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2}$	0	−2.78	−2.78	−2.44
$\frac{5}{2}\frac{5}{2};\frac{1}{2}\frac{1}{2}$	0	−1.72	−1.72	−0.97
$\frac{1}{2}\frac{1}{2};\frac{1}{2}\frac{1}{2}$	0	−1.54	−1.54	−1.95
$\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2}$	2	−1.02	−1.04	−1.04
$\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{1}{2}$	2	−0.59	−0.61	−0.85
$\frac{5}{2}\frac{1}{2};\frac{5}{2}\frac{1}{2}$	2	−0.59	−0.62	−1.29
$\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{5}{2}$	4	−0.22	−0.26	−0.05
$\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{3}{2}$	2	−0.11	−0.13	−0.40
$\frac{5}{2}\frac{5}{2};\frac{3}{2}\frac{1}{2}$	2	1.25	1.14	0.84
$\frac{5}{2}\frac{1}{2};\frac{5}{2}\frac{3}{2}$	2	−0.21	−0.23	−0.22
$\frac{5}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2}$	2	1.40	1.25	1.55
$\frac{5}{2}\frac{3}{2};\frac{5}{2}\frac{3}{2}$	2	−0.72	−0.75	−0.20
$\frac{5}{2}\frac{3}{2};\frac{3}{2}\frac{1}{2}$	2	0.58	0.44	0.77
$\frac{3}{2}\frac{1}{2};\frac{3}{2}\frac{1}{2}$	2	1.01	0.18	−0.33
$\frac{5}{2}\frac{5}{2};\frac{5}{2}\frac{3}{2}$	4	−0.86	−0.61	−1.36
$\frac{5}{2}\frac{3}{2};\frac{5}{2}\frac{3}{2}$	4	0.66	−1.36	−1.66

(b) There may be a 2^+ level in the neighborhood of 10.5-MeV excitation energy in ^{18}O that has so far escaped detection.

Finally since the probability that Ψ_4 and $(d_{5/2}d_{3/2})_4$ are contained in the lowest two 4^+ levels is only about 80% and 20%, respectively, it is clear that only the diagonal $(d_{5/2})^2_4$ energy matrix element can be accurately obtained from the data.

In column 2 of Table VII we list the shell-model matrix elements that do not involve the core states. These have been extracted by use of the constrained II eigenfunctions, the energies given in Eq. (32), and are based on the assumption that the single-particle energies of the $s_{1/2}$ and $d_{3/2}$ levels relative to $d_{5/2}$ are 871 keV and 5.08 MeV, respectively. The table is divided into two parts—the upper portion involves only the $d_{5/2}s_{1/2}$ configurations, and as we shall show these matrix elements are well determined by our calculation. Thus, provided the core $E2$ properties and unperturbed energies are those selected for constrained fit II, column 5 of Table II, any "first-principles" attempt to fit the experimental data using this model space must give rise to these matrix elements.

In Table VIII column 2 the diagonal and off-diagonal matrix elements that involve the collective states are listed. These are determined using the energies of Eq. (32). As stated in our previous discussion only the first six of these are well determined.

In the last column of Table VII we list the predictions of Kuo[8] for the shell-model matrix elements. For the seven "well-determined" values the rms difference between his predictions and our best fit is 450 keV. In particular, it is seen that the Kuo diagonal $(d_{5/2})^2_I$ energies are in excellent agreement with experiment. However, matrix elements involving an $s_{1/2}$ nucleon are not as well reproduced. This may be due to the shortcomings of the oscillator wave functions for describing loosely bound single-particle states with $l=0$.[36]

As we have already stated, the matrix elements of Table VII that involve the $d_{3/2}$ level may be poorly determined for two reasons. On the other hand, because the trace of a matrix is invariant, the sum of the diagonal matrix elements is determined only by the assumed positions of the levels, Eq. (32). With this in mind, one is immediately struck by the large discrepancy between the Kuo $(d_{5/2}d_{3/2})_{I=4}$ diagonal matrix element, −1.66 MeV, and the value 0.66 MeV that arises when the third 4^+ level is assumed to lie at 10.29 MeV. Thus it appears likely that an ^{18}O 4^+ level in the vicinity of 8-MeV excitation energy has been missed. As to the 2^+ states, the situation is less compelling. The sum of the Kuo diagonal $(d_{5/2}d_{3/2})_{I=2}$ and $(d_{3/2}s_{1/2})_{I=2}$ matrix elements is −530 keV, whereas our empirical value, based on Eq. (32), is 290 keV. Since even the "well fitted" have an rms deviation of 450 keV, it is clear that since two matrix elements are involved the difference between theory and experiment is not much outside the expected error.

To illustrate the sensitivity of the matrix elements to changes in the energies we tabulate in column 3 of Tables VII and VIII the values that would arise if we choose

$$E_0 = (-3.915),\ 3.63,\ 5.33\ \text{MeV},$$
$$E_2 = 1.98,\ 3.92,\ 5.25,\ 8.21,\ \underline{10.5\ \text{MeV}}, \qquad (33)$$
$$E_4 = 3.55,\ 7.11,\ \underline{7.85\ \text{MeV}},$$

where the underlined values differ from those of the previous discussion, Eq. (32). As anticipated, those in the upper part of each table are unchanged. Our choice of 10.5 MeV for the excitation energy of the fifth 2^+ state was based only on the desire to have the sum of the diagonal $(d_{5/2}d_{3/2})_{I=2}$ and

TABLE VIII. Energy matrix elements that involve the core-excited configurations Ψ_I. We present both the diagonal matrix elements, $\langle\Psi_I|H|\Psi_I\rangle$ and off-diagonal matrix elements $\langle(j_1j_2)_I|V|\Psi_I\rangle$. These are listed for the two sets of excitation energies given by Eqs. (32) and (33). The Federman-Talmi values are from Ref. 4.

	Constrained II			
	Energies	Energies	Federman-Talmi	
Matrix element	Eq. (32)	Eq. (33)	Case I	Case II
$\langle(d_{5/2})^2{}_0\|V\|\Psi_0\rangle$	0.70	0.70	1.18	1.42
$\langle(s_{1/2})^2{}_0\|V\|\Psi_0\rangle$	1.16	1.16	1.24	0.75
$\langle\Psi_0\|H\|\Psi_0\rangle$	−0.20	−0.20		
$\langle(d_{5/2})^2{}_2\|V\|\Psi_2\rangle$	0.45	0.45	0.66	0.78
$\langle(d_{5/2}s_{1/2})_2\|V\|\Psi_2\rangle$	0.92	0.93	0.59	0.49
$\langle\Psi_2\|H\|\Psi_2\rangle$	0.62	0.61		
$\langle(d_{5/2}d_{3/2})_2\|V\|\Psi_2\rangle$	0.14	0.14		
$\langle(d_{3/2}s_{1/2})_2\|V\|\Psi_2\rangle$	−0.40	−0.38		
$\langle(d_{5/2})^2{}_4\|V\|\Psi_4\rangle$	0.08	0.19		
$\langle\Psi_4\|H\|\Psi_4\rangle$	3.70	3.30		
$\langle(d_{5/2}d_{3/2})_4\|V\|\Psi_4\rangle$	1.20	0.31		

$(d_{3/2}s_{1/2})_{I=2}$ energies close to the Kuo prediction. On the other hand, a 4^+ assignment for the 7.85-MeV state is consistent with the ^{12}C(^{7}Li, p)^{18}O total cross-section measurement[37] which indicates the spin of this level is $I=4\pm1$, with parity unknown. However, if the structure of the third 4^+ state is as indicated in Table IV one would expect it to be strongly populated in the ^{17}O(d, p)^{18}O reaction

$$C^2S(l=2)=0.85\,.$$

Since no strong $l=2$ has been seen in this region it may be that our predicted state is "dissolved" into other 4^+ levels.

The off-diagonal matrix elements involving the core states, Table VIII, are quite different from those of Federman and Talmi.[4] If the SU_3 limit is assumed for the core wave functions [Eqs. (14) and (15)] one can determine the value of w [Eq. (16)] that best reproduces the first four off-diagonal matrix elements in the table. The resulting value is

$$w=2.52\ \text{MeV}.$$

Despite the fact that we allowed these four matrix elements to have large deviations from the SU_3 limiting values, this single parameter leads to values for the first four off-diagonal elements in Table VIII of 0.78, 1.12, 0.44, and 0.91 MeV in excellent agreement with those determined empirically: 0.70, 1.16, 0.45, and 0.92 MeV. As already discussed in Sec. IV, we actually forced these off-diagonal matrix elements to have the signs given in Table VIII because these were the signs *predicted* by the special form of the core-excited wave functions, Eqs. (14) and (15). Since the four that are well determined were predicted to have fairly large values ($\gtrsim$0.5 MeV) it seems unlikely that any reasonable deviation from the SU_3 limit would change the signs.

The required value of w is quite similar to that determined by Brown and Green.[15] In their calculation they took

$$\langle(d_{5/2})^2{}_0|V|(p_{1/2})^2{}_0\rangle=3.37\ \text{MeV}\,,$$

$$\langle(s_{1/2})^2{}_0|V|(p_{1/2})^2{}_0\rangle=0.73\ \text{MeV}\,,$$

$$\langle(d_{3/2})^2{}_0|V|(p_{1/2})^2{}_0\rangle=2.10\ \text{MeV}\,,$$

which would lead to $w=3.17$ MeV. However, they then cut this contribution down by 25% to take into account lack of core overlap so that their effective value was

$$w_{\text{BG}}=0.75\times3.17$$
$$=2.38\ \text{MeV}\,.$$

So far we have said nothing about the 5.37-MeV 3_1^+ level. The ^{17}O(d, p)^{18}O data[18,19] tell us that this state is dominantly $(d_{5/2}s_{1/2})_3$ with at most a 15% admixture of $(d_{5/2}d_{3/2})_3$. In addition it is known[9] that the branching ratio for decay of this state to 2_1^+ and 2_2^+ is

$$\frac{3_1^+\to2_1^+}{3_1^+\to2_2^+}=\frac{88\pm3}{12\pm3}\,. \tag{34}$$

We may use these facts to check the consistency of our 2^+ wave functions. If we assume the 3_1^+ wave function has the form

$$|^{18}\text{O};3_1^+\rangle=(1-\alpha^2)^{1/2}(d_{5/2}s_{1/2})_{3^+}+\alpha(d_{5/2}d_{3/2})_{3^+}\,, \tag{35}$$

we may calculate the value of α needed to reproduce Eq. (34). On the assumption that the decays involved are $M1$'s and that the transition operator

is given by Eq. (4) we conclude that

$$\alpha = -0.27 \pm 0.06 \tag{36}$$

when the constrained II wave functions are used. Thus the fitted structure of the 2^+ states leads to a probability of $(d_{5/2}d_{3/2})_3$ in the 3_1^+ level of 5–10% which is consistent with the single-nucleon-transfer data.

In addition to implying strongly that a 4^+ state has been missed in the neighborhood of 8-MeV excitation energy, our calculations suggest some measurements that should be remade and some new experiments it would be interesting to carry out.

A. Measurement of $B(E2; 0_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$

As we have seen, the required values of the core $E2$ matrix elements depend sensitively on the magnitudes of these quantities. If the Brookhaven data Eq. (18a) and the small value of $\delta(2_3^+ \to 2_1^+)$ Eq. (26), are correct one is forced to an uncomfortably large value for $\langle \Psi_0 \| E2 \| \Psi_2 \rangle$—an absolute value about 35% larger than observed in ^{16}O and about 45% bigger than seen in ^{20}Ne. In addition the data also imply a value of $\langle \Psi_2 \| E2 \| \Psi_2 \rangle$ about 25% smaller than in ^{20}Ne; however, the fit is not so sensitive to this quantity.

On the other hand, if $B(E2; 0_2^+ \to 2_1^+)$ has the pre-Brookhaven value Eq. (18b) and $\delta(2_3^+ \to 2_1^+) \geq 0.2$, all the data can be adequately fitted with state-independent $E2$ matrix elements that are no larger than the limits imposed on them by ^{16}O, ^{17}O, and ^{20}Ne.

B. Measurement of $B(E2; 2_1^+ \to 0_1^+)$ and $Q(2_1^+)$

From Table I it is apparent that the better the overall experimental data are fitted, the smaller the 2_1^+ quadrupole moment becomes. It is, therefore, disturbing that a recent measurement[25] has led to an anomalously large value for this quantity. In that experiment, the reorientation effect in Coulomb excitation, the values that best fit the data are

$$\begin{aligned} &B(E2; 2_1^+ \to 0_1^+) = \tfrac{1}{5} B(E2; 0_1^+ \to 2_1^+) = 9.6\ e^2\,\mathrm{fm}^4\,, \\ &Q(2_1^+)/e = -19 \pm 2\ \mathrm{fm}^2\,. \end{aligned} \tag{37}$$

From the outset it should be noted that the ratio

$$\frac{Q}{[B(E2; 0_1^+ \to 2_1^+)]^{1/2}} = 2.74$$

is larger than observed in any other nucleus[24,25] and exceeds the rotational model limit

$$\frac{Q_{\mathrm{rot}}}{[B_{\mathrm{rot}}(E2; 0_1^+ \to 2_1^+)]^{1/2}} = \left(\frac{64\pi}{245}\right)^{1/2} = 0.91$$

by a factor of 3. Since in this case $B(E2; 0_1^+ \to 2_1^+)$ is not small it is clear that this is an anomalously large value of Q.

If one merely includes these values Eq. (37) in the fit only minor changes occur and the quadrupole moment remains small. On the other hand, one can force Q to have a large negative value by imposing a severe penalty on any fit that misses the moment by more than one standard deviation. To do this we make the replacement in χ^2

$$\left[\frac{Q(\mathrm{expt}) - Q(\mathrm{theory})}{\Delta Q(\mathrm{expt})}\right]^2 \to \left[\frac{Q(\mathrm{expt}) - Q(\mathrm{theory})}{\Delta Q(\mathrm{expt})}\right]^{10}.$$

When this is done, agreement between theory and experiment for other quantities completely disappears. For example when $B(E2; 0_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$ have the values given by Eqs. (18a) and (26) one can attain values of

$$\begin{aligned} &Q(2_1^+)/e = -16.2\ \mathrm{fm}^2\,, \\ &B(E2; 2_1^+ \to 0_1^+) = 9\ e^2\,\mathrm{fm}^4 \end{aligned}$$

at the expense of:

(a) Obtaining a χ^2 for single-nucleon transfer of 100.2.

(b) Reducing $g(2_1^+)$ to the value −0.044.

(c) Almost completely destroying the $2_2^+ \to 0_1^+$ and $2_3^+ \to 0_1^+$ γ branches. The values for these become 1.28% and 0.69%, respectively.

(d) Increasing the value of $\delta(2_3^+ \to 2_1^+)$ to 0.44.

(e) Concentrating the collective 2^+ state in 2_1^+. For this to happen the 2^+ collective state comes within 150 keV of the collective 0^+ level. [If $B(E2; 0_2^+ \to 2_1^+)$ and $\delta(2_3^+ \to 2_1^+)$ have the values given by Eqs. (18b) and (19b) the best fit leads to the collective 2^+ being 260 keV *below* the collective 0^+.] *Clearly if the experimental results of Kleinfeld et al.[25] are correct one cannot fit the data using the model we have studied.*

The value of $B(E2; 2_1^+ \to 0_1^+)$ obtained in the reorientation measurement is approximately 20% larger than found by Berant *et al.*[26] from a measurement of the lifetime of the 2_1^+ state. Further, as pointed out by Kleinfeld *et al.*,[25] the extracted value of $Q(2_1^+)$ is very sensitive to this quantity. If the average of the pre-1975 lifetime measurements[38] is taken

$$B(E2; 2_1^+ \to 0_1^+) = 7.88 \pm 0.23\ e^2\,\mathrm{fm}^4$$

and combined with the 175° inelastic cross-section measurement,[25] one gets

$$Q(2_1^+)/e = -7 \pm 1\ \mathrm{fm}^2\,.$$

We therefore attempt to fit these two values instead of those given in Table I. If the Brookhaven data for $B(E2; 0_2^+ \to 2_1^+)$ [Eq. (18a)] and the small value of $\delta(2_3^+ \to 2_1^+)$ [Eq. (26)] are used one obtains a

best fit value of

$$Q(2_1^+)/e = -5.9 \text{ fm}^2 ,$$

$$B(E2; 2_1^+ \rightarrow 0_1^+) = 7.89\ e^2 \text{fm}^4$$

when the $E2$ matrix elements assume the values listed in column 7 of Table II. The largest contribution to χ^2(rest) is 8.20 from the $2_3^+ \rightarrow 0_1^+$ branching ratio. Since the χ^2 associated with single-nucleon transfer is not much larger than in previous fits, it is clear that the structure of the wave functions is not grossly different from that given in Tables III–V.

Alternatively, if the pre-Brookhaven data are used for $B(E2; 0_2^+ \rightarrow 2_1^+)$ [Eq. (18b)] and the larger value [Eq. (19b)] for $\delta(2_3^+ \rightarrow 2_1^+)$, one obtains

$$Q(2_1^+)/e = -6 \text{ fm}^2$$

and

$$B(E2; 2_1^+ \rightarrow 0_1^+) = 7.83\ e^2 \text{fm}^4 .$$

The best fit $E2$ matrix elements and values of χ^2 are given in the last column of Table II. With the exception of $\chi^2[\delta(2_3^+ \rightarrow 2_1^+)]$, the fit to the data is roughly as good as obtained in the constrained II calculation. The origin of the large value 0.483 for $\delta(2_3^+ \rightarrow 2_1^+)$ is the fact that q_{22} is now more negative than obtained in the previous fits (it must move in this direction in order to have Q large enough). However, none of the required core $E2$ matrix elements are much more than 10% outside their limiting values.

Thus one concludes that a value of Q/e as large as -7 fm^2 could be tolerated but the overall fit to the data gives a preference for a smaller value, -5 fm^2. This result has also been found by Erikson and Brown[39] and by Engeland and Ellis.[40] Since Q is so sensitive to $B(E2; 2_1^+ \rightarrow 0_1^+)$, it is important to be sure of the value of this quantity and to be sure that no subtle effects have been neglected in the reorientation measurements.

C. Amount of $l = 0$ in the ^{18}O ground state

In all our fits there is a tendency to put more $(s_{1/2})^2{}_0$ in the ground state than seems consistent with the (d, t) data.[20] We have tried various parametrizations of the optical model and for the existing data it is difficult to make the experimental value of $C^2S(0_1^+; l=0/l=2)$ larger than 0.21. Since both the constrained II and unconstrained models fit all the other data to better than two standard deviations it would appear that a remeasurement of this ratio is called for.

D. Lifetime of the 5.25-MeV state

Since there is a possible error[41] in the recent Brookhaven measurement of this lifetime[27] only a limit[31] currently exists. A measurement of this lifetime would provide a test of the predictive properties of our model.

E. Properties of the 7.11-MeV 4_2^+ level

Within our model space of $\delta(4_2^+ \rightarrow 4_1^+)$ always turns out to be small and positive. Experiment confirms that the $E2$ decay is weak compared to the $M1$; however, a negative value for δ is preferred. Since δ is small, any misalignment of the experimental setup could change this sign and so we have not considered this to be a significant disagreement. Our model does lead to a definite prediction for the lifetime of this state. If one allows only $M1$ and $E2$ decays, the various transition probabilities associated with this level using our other model-space states are

$$T(M1; 4_2^+ \rightarrow 4_1^+) = 2.35 \times 10^{14} \text{ sec}^{-1} ,$$

$$T(E2; 4_2^+ \rightarrow 4_1^+) = 4.42 \times 10^{11} \text{ sec}^{-1} ,$$

$$T(E2; 4_2^+ \rightarrow 2_3^+) = 2.33 \times 10^{12} \text{ sec}^{-1} ,$$

$$T(E2; 4_2^+ \rightarrow 2_2^+) = 1.59 \times 10^{13} \text{ sec}^{-1} ,$$

$$T(E2; 4_2^+ \rightarrow 2_1^+) = 8.45 \times 10^{13} \text{ sec}^{-1} ,$$

where the quoted numbers are calculated by use of the constrained II eigenfunctions and the mean lifetime is $\tau_m = 1/T$. Decay to the 5.37-MeV 3_1^+ state is expected to be only a small branch because the $M1$ part can go only via the small admixture coefficient α in Eq. (35). If one uses Eqs. (35) and (36) to describe this state one estimates

$$T(M1; 4_2^+ \rightarrow 3_1^+) = 1.41 \times 10^{12} \text{ sec}^{-1} ,$$

$$T(E2; 4_2^+ \rightarrow 3_1^+) = 4.20 \times 10^{10} \text{ sec}^{-1} .$$

Thus the branch to the 3_1^+ state is less than 1% and this is consistent with the fact it was not seen by Lee, Krone, and Prosser.[30]

When one combines these predicted transition rates one finds that the γ width of the 7.11-MeV state should be

$$\Gamma_\gamma = 0.22 \text{ eV}.$$

The only data on the width of this state gives[30]

$$\frac{\Gamma_\gamma \Gamma_\alpha}{\Gamma} = 0.042 \text{ eV} .$$

Thus if our estimate for Γ_γ is correct the level decays approximately 20% of the time by α emission. Consequently, if one were to populate this level by say the reaction ^{14}C(^{7}Li, t)^{18}O and count the emitted α's in coincidence with the tritons, a value of Γ_α/Γ could be obtained. This could then be combined with the existing data to check our prediction for Γ_γ.

VII. CONCLUSIONS

We have shown that a good fit to all the existing data can be obtained using state-independent effective electromagnetic operators with matrix elements close to those observed in ^{16}O, ^{17}O, and ^{20}Ne. The wave functions for all positive-parity states below 7.2 MeV are determined to within rather narrow limits, and, except for the 7.11-MeV 4^+ level, the major components of them (>95% in all cases) arise from the $(1d_{5/2}, 2s_{1/2})$ model space plus one collective state of each of the spins 0^+, 2^+, and 4^+ (i.e., there are only small $d_{3/2}$ admixtures in these states). Overall, the data favor a small value for the quadrupole moment of the 1.98-MeV 2^+ state $Q(2_1^+)/e \cong -5$ fm^2. If this moment is much more negative than about -7 fm^2 the structure of the low-lying ^{18}O states must be much more complicated than considered here. Furthermore, within the assumed model space we are able to obtain matrix elements of the Hamiltonian that give not only the energy eigenvalues but also, to a high degree of accuracy, the observed dynamic nuclear properties. We show that the matrix elements deduced by Kuo[8] from the Hamada-Johnston potential are, in many cases, in reasonable agreement with the empirical values. In addition we show that the required unperturbed positions of the core-excited states are quite close to where they would be predicted from simple binding-energy arguments. Finally, we have mentioned a variety of experiments that would be useful in refining and checking such a model description of ^{18}O.

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[1]G. E. Brown, in *Proceedings of the International Congress on Nuclear Physics, Paris, 1964*, edited by P. Gugenberger (Centre National de la Recherche Scientifique, Paris, 1964), Vol. I, p. 129; T. Engeland, Nucl. Phys. 72, 68 (1965).
[2]See for example P. J. Ellis and T. Engeland, Nucl. Phys. A144, 161 (1970); T. Engeland and P. J. Ellis, *ibid.* A181, 368 (1972), and the references quoted therein.
[3]D. Kurath, in *Nuclear and Particle Physics*, edited by B. Margolis and C. S. Lam (Benjamin, New York, 1968), p. 199.
[4]P. Federman and I. Talmi, Phys. Lett. 19, 490 (1965).
[5]H. T. Fortune and S. C. Headley, Phys. Lett. 51B, 136 (1974).
[6]F. Ajzenberg-Selove, Nucl. Phys. A166, 1 (1971).
[7]A. H. Wapstra and N. B. Gove, Nucl. Data A9, 265 (1971).
[8]T. T. S. Kuo, Nucl. Phys. A103, 71 (1967).
[9]F. Ajzenberg-Selove, Nucl. Phys. A190, 1 (1972).
[10]M. Forterre, J. Gerber, J. P. Vivien, M. B. Goldberg, and K. H. Speidel, Phys. Lett. 55B, 56 (1975); K. H. Speidel, M. B. Goldberg, K. Hagemeyer, G. J. Kumbartzki, G. Goldring, Z. Shekedi, M. Schramm, G. Kraft, and H. A. Doubt, *ibid.* 57B, 143 (1975).
[11]G. H. Fuller and V. W. Cohen, Nucl. Data A5, 433 (1969).
[12]P. Goldhammer and F. W. Prosser, Jr., Phys. Rev. 163, 950 (1967).
[13]S. H. Chew and J. Lowe, Nucl. Phys. A252, 8 (1975).
[14]S. G. Nilsson, K. Dan. Vidensk. Selsk. Mat.-Fys. Medd. 29, No. 16 (1955).
[15]G. E. Brown and A. M. Green, Nucl. Phys. 85, 87 (1966).
[16]J. B. McGrory and B. H. Wildenthal, Phys. Rev. C 7, 974 (1973).
[17]H. T. Fortune and S. C. Headley (unpublished).
[18]J. L. Wiza, R. Middleton, and P. V. Hewka, Phys. Rev. 141, 975 (1966).
[19]T. K. Li, D. Dehnhard, R. E. Brown, and P. J. Ellis, Phys. Rev. C 13, 55 (1976).
[20]J. C. Armstrong and K. S. Quisenberry, Phys. Rev. 122, 150 (1961).
[21]R. Middleton and D. J. Pullen, Nucl. Phys. 51, 63 (1964).
[22]P. D. Kunz (unpublished).
[23]Z. Berant, C. Broude, S. Dima, G. Goldring, M. Hass, Z. Shkedi, D. F. H. Start, and Y. Wolfson, Nucl. Phys. A235, 410 (1974).
[24]A. Christy and O. Haüsser, Nucl. Data A11, 281 (1972).
[25]A. M. Kleinfeld, K. P. Lieb, D. Werdecker, and U. Smilansky, Phys. Rev. Lett. 35, 1329 (1975).
[26]Z. Berant, C. Broude, G. Engler, and D. F. H. Start, Nucl. Phys. A225, 55 (1975).
[27]J. W. Olness, E. K. Warburton, and J. A. Becker, Phys. Rev. C 7, 2239 (1973).
[28]L. F. Chase, Jr., J. A. Becker, D. A. Kohler, and R. E. McDonald, in *Nuclear Physics: An International Conference*, edited by R. L. Becker, C. D. Goodman, P. H. Stelson, and A. Zuker (Academic, New York, 1967).
[29]Z. Berant, M. B. Goldberg, M. Popp, J. S. Sokolowski, P. N. Tandon, and Y. Wolfson, Nucl. Phys. A173, 401 (1971).
[30]F. D. Lee, R. W. Krone, and F. W. Prosser, Nucl. Phys. A96, 209 (1967).
[31]P. G. Lawson, Ph.D. thesis, Oxford University, 1968 (unpublished).
[32]A. E. Litherland, M. J. L. Yates, B. M. Hinds, and D. Eccleschall, Nucl. Phys. 44, 220 (1963).
[33]J. S. Lopes, O. Haüsser, R. D. Gill, and H. J. Rose, Nucl. Phys. 89, 127 (1966).
[34]J. R. Erskine, R. E. Holland, R. D. Lawson, M. H. Macfarlane, and J. P. Schiffer, Phys. Rev. Lett. 14, 915 (1965).
[35]H. G. Benson and J. M. Irvine, Proc. Phys. Soc. (London) 89, 249 (1966).
[36]S. Kahana, H. C. Lee, and C. K. Scott, Phys. Rev. 180, 956 (1969).
[37]H. T. Fortune, S. C. Headley, J. N. Bishop, and L. R. Medsker (unpublished).

[38]A. B. McDonald, T. K. Alexander, O. Haüsser, G. J. Costa, J. S. Forster, and A. Olin, Can. J. Phys. 52, 1381 (1974).
[39]T. Erikson and G. E. Brown (unpublished).
[40]T. Engeland and P. J. Ellis, Phys. Rev. Lett. 36, 994 (1976).
[41]J. W. Olness and E. K. Warburton (private communication).