Effects of nonlocal potentials in heavy-ion reactions*

John G. Cramer
Department of Physics, University of Washington, Seattle, Washington 98195
R. M. DeVries
Department of Physics, University of Rochester, Rochester, New York 14627
(Received 30 December 1975)

A nonlocal optical model program has been written in order to investigate the effects of Gaussian nonlocal potentials on heavy-ion scattering and reactions. We have tested the two principal approximations which have been used to handle nonlocal effects in light-ion reactions, the Perey-Buck nonlocal-to-local potential transformation and the local-energy approximation. We find that both of these approximations work very well for the heavy-ion reactions studied up to nonlocal ranges of about $\beta = 0.4$ fm, but fail for larger ranges. Using the local-energy approximation we have tested the sensitivity of distorted-wave Born-approximation calculations to the nonlocal range β. For strongly absorbing potentials, values of $\beta \gtrsim 0.4$ fm cause negligible effects while weakly absorbing potentials with $\beta = 0.4$ fm allow strong modifications to the predicted cross sections.

[NUCLEAR REACTIONS Effects of nonlocal optical potentials in heavy-ion scattering and reactions, exact nonlocal optical model calculations, tested approximations to nonlocal effects, nonlocal DWBA calculations.]

INTRODUCTION

It is now fairly well established that the optical potential which describes the interaction between a proton or neutron and a target nucleus is nonlocal.[1] This nonlocality is responsible for the negative energy dependence of the effective local potential observed in the analysis on nucleon elastic scattering.[1] It is also responsible for the reduction of the wave function in the nuclear interior[2] which had prompted the use of radial cutoffs in distorted-wave Born-approximation (DWBA) calculations which did not include nonlocal corrections.

Nonlocal potentials are functions of two radial variables rather than one. The usual potential term in the Schrödinger equation which has the form $V(\vec{r})\Psi(\vec{r})$ is replaced by an integral of the form $\int V(\vec{r},\vec{r}')\Psi(\vec{r}')d\vec{r}'$. Thus, since the wave function $\Psi(\vec{r})$ usually changes rapidly compared to the potential, this integral has the effect of averaging away some fraction of the potential term, thereby reducing the effective potential. If the averaging interval (or nonlocal "range") becomes comparable to the wavelength of the wave function, this reduction can be sizable and the effective potential may be reduced by orders of magnitude.

Since the wavelengths characteristic of heavy-ion reactions are usually much smaller than in light-ion reactions (due to the larger masses and higher energies involved), nonlocal effects for a given nonlocal range would be expected to be more important for heavy than light ions. Whether such effects are, in fact, important remains an open question which can best be answered by experiment. One must, however, understand which experimentally observable effects would be produced by nonlocal heavy-ion potentials. It is the purpose of this paper to investigate this question and to test the approximations which have been developed for calculating such effects in light-ion reactions.

I. NONLOCAL OPTICAL MODEL PROGRAM

To begin this investigation it was necessary to write an optical model program which could solve the integrodifferential nonlocal Schrödinger equation in the heavy-ion domain. The work of Perey and Buck,[1] based on the assumption of a separable Gaussian form for the nonlocal potential, was taken as a starting point. It was soon found that the methods presented in that work for computing an approximate local potential and for calculating the nonlocal kernel function were divergent in some region of the heavy-ion domain and alternative methods had to be devised. Further, it was found that their method of numerically solving the inhomogeneous Schrödinger equation was highly unstable for short wavelengths, and a new numerical procedure for solving this equation was necessary.

We have therefore written OINK!, an exact non-

local optical model program suitable for the case of heavy-ion scattering. The program will handle 120 partial waves and up to 400 radial integration steps. It initializes for a given partial wave by taking the nonlocal potential, generating a Perey-Buck equivalent local potential,[1,2] and performing an optical model calculation to generate the wave function. The latter is then modified in the interior region using the local-energy approximation.[3,4] The result is taken as the starting value of the nonlocal wave function.

The program then folds the wave function with the nonlocal potential to generate the so-called trivially equivalent local potential,[2] which is then used to solve the Schrödinger equation for a better approximation to the wave function. This procedure is iterated until the S matrix has converged to better than 0.1%. This usually requires only two or three iterations, for the method is very stable.

In the present work we have considered only the Gaussian form of nonlocality suggested by Perey and Buck.[1] In that formulation they assumed that the nonlocal potential could be represented as the product of a function of the variable $\vec{s} = (\vec{r} - \vec{r}')/\beta$ and a function of the variable $p = \frac{1}{2}|\vec{r} + \vec{r}'|$. Further, they approximated the latter by $p \simeq \frac{1}{2}(|\vec{r}| + |\vec{r}'|)$, which has the effect of ignoring possible angular nonlocality effects. It has been pointed out[5] that the Perey effect, i.e., the damping of the wave function in the nuclear interior discussed in Sec. III below, is a consequence of the form of the nonlocal potential which Perey and Buck employed and that other plausible forms of nonlocality may give other results. Further, Fuller has recently pointed out that angular nonlocality, which is not properly treated by Perey and Buck, may be very important in heavy-ion reactions.[6] Thus, the implications of the present work should be viewed as strictly applying only to Gaussian nonlocality, and it should be borne in mind that other forms are likely to lead to other results.

II. PEREY-BUCK NONLOCAL-TO-LOCAL POTENTIAL TRANSFORMATION

Perey and Buck[1] have suggested a transformation by means of which a nonlocal potential may be transformed into an effective local potential which produces the same scattering. The transformation including Coulomb effects which Perey and Buck omitted, has the form:

$$u_L = u_N \exp[-\alpha(\epsilon - u_L)], \quad \text{for } \alpha < 1,$$

and (1)

$$u_L = \frac{1}{\alpha}\ln(u_L/u_N) + \epsilon, \quad \text{for } \alpha \geq 1,$$

where $\alpha = \frac{1}{2}(k\beta)^2$, $u_L = V_L/E_{\text{c.m.}}$, $u_N = V_N/E_{\text{c.m.}}$, and $\epsilon = 1 - V_{\text{Coul}}/E_{\text{c.m.}}$. Here $k = p/\hbar$ is the wave number, β is the nonlocal range, V_L and V_N are the (complex) local and nonlocal optical potentials, $E_{\text{c.m.}}$ is the energy of the system in the center of mass system, and V_{Coul} is the Coulomb potential. Both of these expressions are transcendental, and must be iterated to generate a local potential V_L from a given nonlocal potential V_N. However, given V_L, the nonlocal potential V_N can be determined without iteration.

It can be seen from the exponential form of (1) that, since V_L is normally an attractive (i.e., negative) potential, the exponential factor will be less than 1, so that the local potential will usually be less than the nonlocal potential in magnitude. Further, since $E_{\text{c.m.}}$ enters into various factors in this relation, the reduction factor will depend on energy. Since both V_L and V_{Coul} are functions of the radius, the reduction factor will also depend on radius, so that the shape of V_L will be different from that of V_N.

To test the accuracy of the transformation for heavy ions, we have considered two types of com-

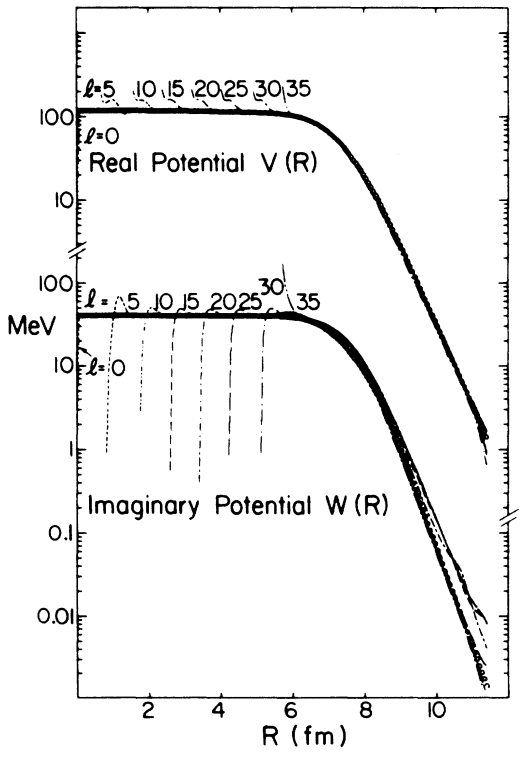

FIG. 1. Comparison of the local equivalent potential (circles) obtained from Eq. (1) with the nonlocal effective potential $V_{\text{eq}}(r,l)$ (dashed curves) for various l values. The case shown is the potential for $^{16}O + ^{48}Ca$ scattering at 48 MeV.

parisons: (a) a comparison of the elastic scattering prediction of the transformed potential with that of an exact nonlocal elastic scattering calculation, and (b) a comparison of the transformed potential with the "trivially equivalent" local potential[2] which is the exact equivalent of a nonlocal potential for a given partial wave. Comparison of elastic scattering cross predictions (a) shows excellent agreement in all cases tested. Since this method of comparison is rather insensitive to the details of nonlocal wave functions, and since a full nonlocal optical model calculation is very time consuming, this method of comparison is rather limited in its utility.

The basis of comparison (b) is that an exact equivalent local potential can be defined by $V_{eq}(r,l) \equiv [\int V(r,r')\Psi_l(r')dr']/\Psi_l(r)$. This equivalent potential is strongly l dependent, is not well behaved, and may have cusps and poles when the wave function has minima and zeroes, but it is a local potential which contains *all* of the features of a nonlocal potential and may be directly compared with the transformed potential. Figure 1 shows a comparison between the transformed potential and the exact equivalent l-dependent potential for several different l values.

For this comparison and those which follow we have used the scattering of ^{16}O from ^{48}Ca at 48 MeV as a test case. This system was chosen because it is light enough so that relatively few partial waves are needed in the calculations, because the nonlocal effects are tested more severely with the relatively deep potential ($V = 100$ MeV) used in the elastic scattering analysis, and because significant deviations between experimental transfer data and DWBA calculations have been noted.[7] The potential used is $V = 100$ MeV, $W = 40$ MeV, $r_0 = 1.22$ fm, and $a = 0.49$ fm. For all comparisons we have chosen l values near the grazing l value. As can be seen, the equivalent potentials generally average through the transformed potential. This illustrates the level of approximation which is being used by employing the transformed local potential.

III. LOCAL-ENERGY APPROXIMATION

As mentioned above, one of the effects of a nonlocal potential, sometimes called the Perey effect[2-4] is the reduction of the wave function in the interior of the nucleus. The use of an effective local potential such as the transformed potential described above does not reproduce this effect, since the transformed potential only gives an approximation to the external behavior of the wave function. Thus an additional approximation is required to simulate the reduction of the internal wave function produced by a nonlocal potential. Such an approximation is not required for the analysis of elastic scattering, which depends only on the behavior of the external wave function, but is needed when analyzing transfer reactions where overlaps between bound state wave functions and distorted waves in the nuclear interior make important contributions to the cross sections. The approximation which is conventionally used for light-ion reactions is called the local-energy approximation (LEA).[3,4] Its derivation is given in Ref. 3. It has the form:

$$\Psi_{NL}(r) \simeq \Psi_L(r)\{1 - (\tfrac{1}{2}\beta k)^2[V_L(r)/E_{c.m.}]\}^{-1/2}$$
$$= \Psi_L(r)[1 - \alpha u_L(r)]^{-1/2}. \quad (2)$$

Since $V_L(r)$ is in general an attractive potential and therefore negative, the denominator will be greater than 1 and the wave function will be reduced. Further, since $V_L(r)$ is complex, the approximate nonlocal wave function will not only be reduced in magnitude but also shifted in phase by the transformation. The LEA gives the appearance of being energy dependent, since it explicitly involves $E_{c.m.}$, the center of mass energy of the system. However, since $\alpha \propto E_{c.m.}^{-1}$, the expression is in fact independent of energy, except for the implicit energy dependence of the equivalent local

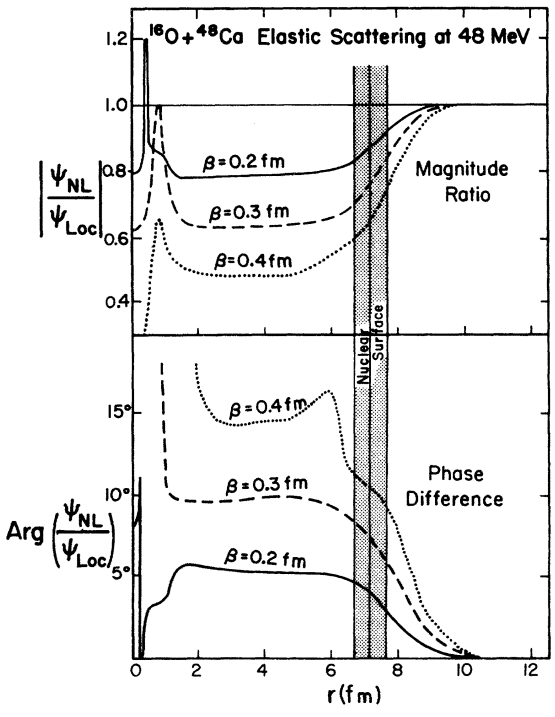

FIG. 2. Comparison of wave functions generated by the local equivalent potential and the nonlocal potential for $^{16}O + ^{48}Ca$ scattering at 48 MeV.

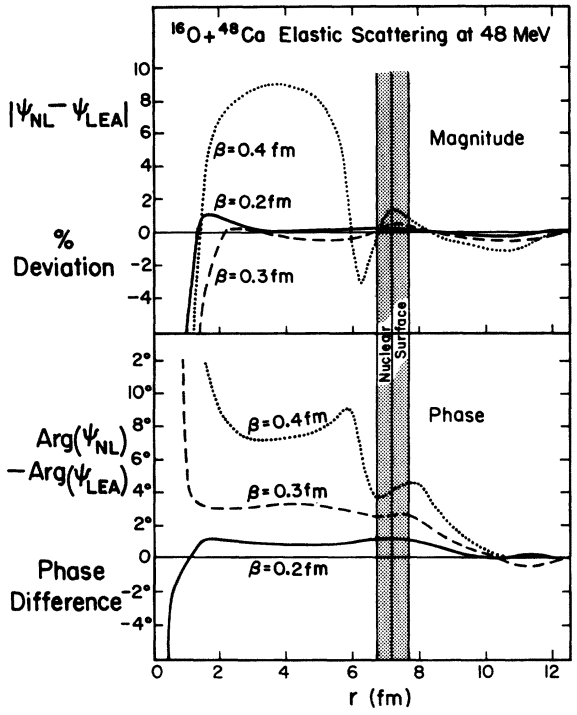

FIG. 3. Accuracy of the local-energy approximation (LEA) in simulating the nonlocal damping arising from the Perey effect for $^{16}O+^{48}Ca$ scattering at 48 MeV.

potential $V_L(r)$.

We have tested the LEA by comparing wave functions calculated with an exact nonlocal potential with wave functions calculated with the effective local potential obtained from the Perey-Buck transformation given in Eq. (1). Figure 2 shows the result of such a comparison. In the upper part of the figure we see the ratio of magnitudes of the nonlocal and local wave functions, while in the lower part the phase difference is shown. The Perey effect is quite obvious in this figure, in that the nonlocal wave function is reduced to between 80% and 50% of the value predicted by the local potential and its phase shifted by between 5° and 15°. Figure 3 shows the accuracy to which these effects can be predicted by the local-energy approximation. The upper part of the figure shows that the accuracy of the magnitude predictions of the LEA are less than 1% until the nonlocal range rises above a value of 0.3 fm, and even at 0.4 fm retains an accuracy of better than 5% in the nuclear surface region. The lower part of the figure shows that the accuracy in phase of the LEA is between 1° and 5° in the nuclear surface region for the nonlocal ranges studied.

IV. DWBA CALCULATIONS

In the previous sections we have investigated the range of applicability of the LEA approximation and found that it works well for heavy-ion reactions as long as $\beta \leq 0.4$ fm. We now can use the LEA in heavy-ion transfer DWBA calculations. We have done this using a modified version of the finite-range DWBA program of *LOLA*.[8] The parameters of Ref. 7 were used for the $^{42}Ca(^{16}O, ^{15}N)^{43}Sc^*$ (1.18 MeV $\frac{3}{2}^-$), a reaction which is not well fitted

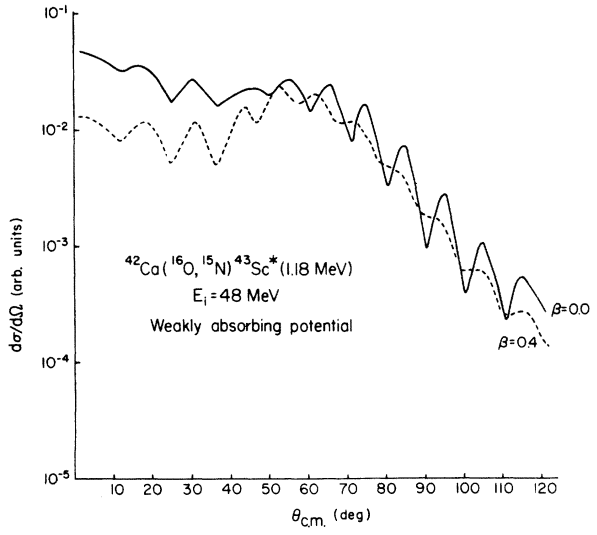

FIG. 4. The effect of Perey damping on the DWBA calculation of $^{42}Ca(^{16}O, ^{15}N)^{43}Sc^*$ at $E=48$ MeV, using a strongly absorbing potential. The potential used is given in Ref. 5 ($W/V=40/100$) and wave functions in the entrance and exit channels were damped using Eq. (2).

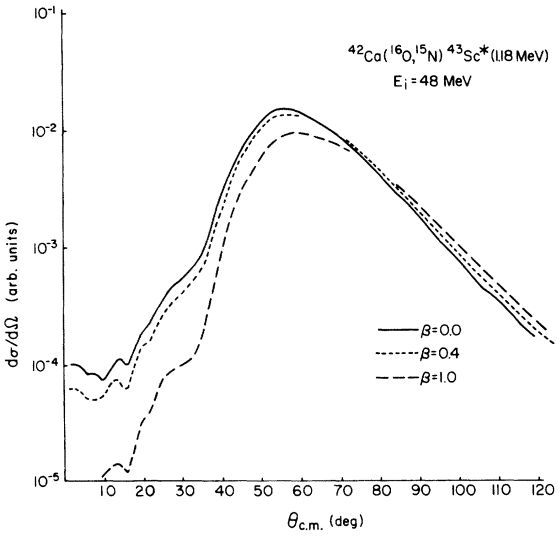

FIG. 5. The effect of Perey damping on a DWBA calculation for $^{42}Ca(^{16}O, ^{15}N)^{43}Sc^*$ at $E=48$ MeV, using a weakly absorbing potential. The potential used is that given in Ref. 5, except that W/V is taken as 5/100.

by the original DWBA calculations, perhaps partly due to its poor momentum and angular momentum matching. Figure 4 shows the predicted angular distributions with $\beta = 0.0$ fm, 0.4 fm, and (unreliable) 1.0 fm values. For β values $\lesssim 0.4$ fm the angular distribution is negligibly changed.

We can understand this result in terms of the amount of absorption in the optical potentials used. Light-ion potentials are often rather weakly absorbing in the interior allowing the Perey effect full sway. For this heavy-ion reaction, however, the interior contributions to the DWBA cross section are strongly damped by the use of a strongly absorbing ($W/V = 40/100$) optical potential. This assertion can be verified by reducing the absorption to $V/W = 5/100$ as shown in Fig. 5. Now, a $\beta = 0.4$ fm is capable of strongly changing the predicted angular distribution. Thus, if heavy-ion reactions are found in which weakly absorbing potentials are appropriate, nonlocality effects might be of concern.

At this point there remains an uncertainty about what nonlocal range is appropriate for heavy ions. We have only the theoretical guidelines of Jackson and Johnson.[9] They derive a simple model which yields a nonlocal range of approximately 0.2 fm for α particles, fitting the observed energy dependence of α elastic scattering potentials rather well. With this model they predict heavy-ion β values to be given by

$$\beta \simeq \beta_{\text{nucleon}}/A_{\text{ion}} \quad \text{or} \quad \beta \simeq 0.05 \text{ fm} \quad \text{for } {}^{16}\text{O}.$$

These small β parameters, of course, suggest that the energy dependence of heavy-ion optical potentials should be rather small. There is some evidence for this conclusion based on a recent analysis of $^{16}O + {}^{28}Si$ elastic scattering between 33 and 215 MeV,[10] where a shallow energy-independent optical potential is found to give good fits to the data over this large energy range.

We conclude, then, that radial nonlocality effects are quite small in heavy-ion transfer DWBA calculations using strongly absorbing optical potentials. We find that the Perey-Buck transformation and LEA work surprisingly well for heavy-ion reactions provided $\beta \leq 0.4$ fm.

The authors are indebted to Dr. F. G. Perey for helpful discussions while this work was in progress.

---

*Work supported in part by U. S. Energy Research and Development Administration.

[1]F. G. Perey and B. Buck, Nucl. Phys. 32, 353 (1962).

[2]F. G. Perey in *Direct Interactions and Nuclear Reaction Mechanisms*, edited by E. Clementel and C. Villi (Gordon and Breach, New York, 1963), p. 125.

[3]N. Austern, Phys. Rev. 137, B752 (1965); *Direct Nuclear Reaction Theories* (Wiley, New York, 1970), p. 111.

[4]G. R. Satchler, in *Lectures in Theoretical Physics*, edited by P. D. Kunz, D. A. Lind, and W. E. Brittin (Univ. of Colorado Press, Boulder, Colorado, 1966), Vol. IX, p. 151.

[5]W. Von Dijk and D. Kiang, Nucl. Phys. A181, 106 (1972).

[6]R. C. Fuller, Bull. Am. Phys. Soc. 20, 1167 (1975).

[7]H. J. Körner, G. C. Morrison, L. R. Greenwood, and R. H. Siemssen, Phys. Rev. C 1, 107 (1973).

[8]R. M. DeVries, Phys. Rev. C 8, 951 (1973).

[9]D. F. Jackson and R. C. Johnson, Phys. Lett. B49, 249 (1974).

[10]J. G. Cramer, R. M. DeVries, D. A. Goldberg, M. S. Zisman, and C. McGuire (unpublished).

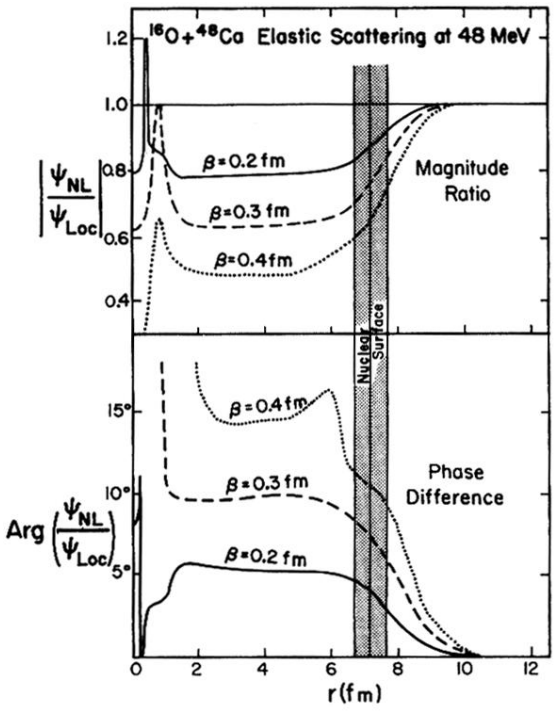

FIG. 2. Comparison of wave functions generated by the local equivalent potential and the nonlocal potential for $^{16}O+^{48}Ca$ scattering at 48 MeV.

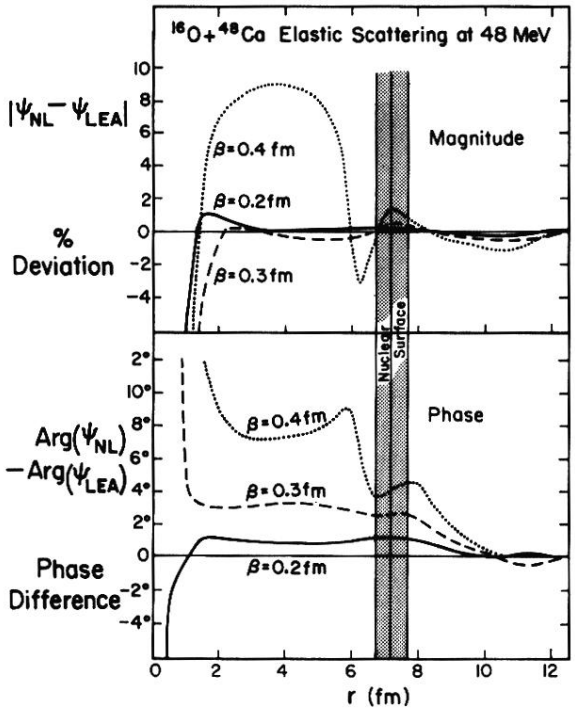

FIG. 3. Accuracy of the local-energy approximation (LEA) in simulating the nonlocal damping arising from the Perey effect for $^{16}O+^{48}Ca$ scattering at 48 MeV.