Tensor analyzing power measurements in the $^{16}O(\vec{d},\alpha)^{14}N$ reaction at 0°†

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The tensor analyzing power near 0° is measured in the $^{16}O(\vec{d},\alpha)^{14}N$ reaction at beam energies of 11, 12, 13, and 14 MeV. Spin-parity assignments for states in ^{14}N from 4.91 to 9.13 MeV are deduced. In particular, $J^{\pi}=0^{-}$ assignments are made to the level at 4.91 MeV as well as one member of the doublet at 9.13 MeV. The method is discussed, with emphasis on the technique of energy averaging of the cross sections and on the effects of small deviations of the detection angle from 0°. Comparisons of these predictions are made also with data for the $^{12}C(\vec{d},\alpha)^{10}B$ reaction.

[NUCLEAR REACTIONS $^{16}O(\vec{d},\alpha)$, $E=11$, 12, 13, 14 MeV; measured $T_{20}(0°)$, deduced J,π.]

INTRODUCTION

It has been pointed out[1-3] that the $(\vec{d},\alpha)$ reaction, where the deuterons are incident on an even-even target nucleus and the α particles are detected at 0° or 180°, is a useful technique for determining spin-parity combinations in residual odd-odd nuclei. It will be shown later, using only conservation of angular momentum and parity, that for 100% tensor polarized deuterons ($t_{20}=-\sqrt{2}$) with the quantization axis along the beam direction, the $(\vec{d},\alpha)$ reaction can have yield at 0° or 180° only for levels of spin J in the final nucleus which have unnatural parity, $\pi=(-1)^{J+1}$. This fact can be used to determine the parity of a state if the spin is known or to determine the spin in some cases if there is other experimental evidence limiting the allowed assignments. This method has been used recently in preliminary studies of ^{10}B (Ref. 1) and ^{14}N (Ref. 2) and also in ^{30}P (Ref. 3). This paper reports additional measurements for states in ^{14}N resulting from the $^{16}O(d,\alpha)^{14}N$ reaction. In addition the effects of target thickness and energy averaging of the results and the effects of small deviations of the detection angle from 0° or 180° are discussed.

METHOD

The cross section for the $(\vec{d},\alpha)$ reaction on an even-even target nucleus ($J^{\pi}=0^{+}$) with alpha particles detected at 0° or 180° to the beam direction can be written in terms of the tensor polarization of the beam, t_{20}, and the tensor analyzing power of the reaction, T_{20}, as

$$\frac{d\sigma}{d\Omega}=\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}}(1+T_{20}t_{20}), \tag{1}$$

where $(d\sigma/d\Omega)_{\text{unpol}}$ is the cross section for an unpolarized beam. It is convenient to use the helicity frame adopted at the Madison conference[4] for a description of the reaction. In this frame, the z axis is chosen individually for the description of each particle in the direction of its momentum in the center of mass system. Each particle has a common y axis perpendicular to the reaction plane in the direction $\vec{p}_{\text{in}}\times\vec{p}_{\text{out}}$, where $\vec{p}_{\text{in}}$ and $\vec{p}_{\text{out}}$ are the linear momenta of the deuteron and α particle, respectively. Scattering amplitudes are then denoted by $F^{\lambda_d}_{\lambda_J}$, where λ_d is the component of spin of the deuteron along the beam direction and λ_J is the component of spin of the residual nucleus along its direction of motion. In this representation the tensor analyzing power and the unpolarized cross section are[5]

$$T_{20}(0°\text{ or }180°)=\frac{1}{\sqrt{2}}\,\frac{|F^{-1}_{-1}|^2-2|F^{0}_{0}|^2+|F^{1}_{1}|^2}{|F^{-1}_{-1}|^2+|F^{0}_{0}|^2+|F^{1}_{1}|^2} \tag{2}$$

and

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{unpol}}=\tfrac{1}{3}\left(|F^{-1}_{-1}|^2+|F^{0}_{0}|^2+|F^{1}_{1}|^2\right),$$

since detection of the α particles at 0° or 180° restricts the values of λ_J to $\lambda_J=\lambda_d$. The condition imposed on the scattering amplitudes by parity conservation in the reaction is[5,6]

$$F^{-\lambda_d}_{-\lambda_J}=\pi(-)^{J+1}F^{\lambda_d}_{\lambda_J}, \tag{3}$$

where π is the parity and J the spin of the residual nucleus. For a natural parity state $\pi=(-1)^{J}$ so that $F^{0}_{0}=0$ and $F^{-1}_{-1}=-F^{1}_{1}$ and therefore the tensor analyzing power is $1/\sqrt{2}$, independent of the reaction mechanism. A special case is that of a spin zero level, where the amplitudes $F^{\pm1}_{\pm1}$ are not allowed. For $J^{\pi}=0^{+}$ we also have $F^{0}_{0}=0$ and we get the well known result that the cross section is zero. For a $J^{\pi}=0^{-}$ state, on the other hand, the tensor analyzing power is $-\sqrt{2}$. Natural parity states and 0^- states represent the possible limits for T_{20}. In general, a state of unnatural parity can have values of T_{20} anywhere between the two limits including the limits themselves, depending on the ratio F_0^0/F_1^1, and will therefore be model dependent.

In practice, if T_{20} is measured for a state of unknown spin or parity, and it is found to be within but not on the limits, then an unambiguous assignment of unnatural parity can be made. If, however, it is found to be at either limit within error, then an assignment of natural parity or 0^- can only be made if T_{20} remains at the limit for several other energies. With this in mind, it is of some interest to determine what can be expected in terms of the distribution of tensor analyzing powers for an unnatural parity state. In other words, what is the probability that a state with $T_{20}=-\sqrt{2}$ or $1/\sqrt{2}$ has unnatural parity? Boerma *et al.*[3] considered this for a thin target assuming the usual statistical fluctuations of a compound nuclear reaction. They find that the distribution is uniformly distributed between the two limits. These calculations can be extended to include thick targets or measurements at several beam energies if the probability distribution for the amplitudes is modified to include such averaging effects. The probability for each quantity $|F_{\lambda J}^{\lambda d}|^2$ is given by[7]

$$P(x) \propto \left(\frac{x}{\langle x\rangle}\right)^{N-1} \exp(-x/\langle x\rangle)\,, \tag{4}$$

where x is $|F|^2$, and $\langle x\rangle$ the mean value $\langle|F|^2\rangle$. N is related to the target thickness and corresponds roughly to the energy loss of the beam in the target in units of the coherence width. This same distribution of amplitudes would be obtained if one carried out an energy average for N different energies separated by at least the coherence width, which, in this experiment, is roughly 100–200 keV. Indeed, since this method is expected to have its widest range of applicability in light nuclei, targets thick enough to correspond to $N>1$ will, in general, be impractical to use because of resolution effects and, therefore, an average of cross sections for a number of energies will have to be taken.

It is fairly straightforward to show that the distribution of T_{20} in Eq. (2) resulting from the distribution of amplitudes of Eq. (4) is given by

$$P(T_{20})\alpha\frac{[(T_{20}+\sqrt{2})(\sqrt{2}-2T_{20})]^{N-1}}{[\langle|F_1^1|^2\rangle(\sqrt{2}-2T_{20})+\langle|F_0^0|^2\rangle(T_{20}+\sqrt{2})]^{2N}}\,. \tag{5}$$

Using a statistical weight argument, valid for many l values contributing in the reaction, Boerma *et al.*[3] have shown that the mean values of the amplitudes in the compound nuclear model are related by

$$\langle|F_0^0|^2\rangle=2\langle|F_1^1|^2\rangle\,. \tag{6}$$

For the case of a thin target and a single energy, where $N=1$, Eq. (5) then becomes $P(T_{20})=$ constant. For an energy average, on the other hand, the distribution will no longer be uniform and will have a maximum value at $T_{20}=-1/(2\sqrt{2})$. In Fig. 1, the distribution (5) is shown plotted for values of $N=1$, 2, 5, 10, 50. It should be noted that the assumption (6) for the mean values is only approximately correct and for low beam energies the peak in the distribution will shift towards $T_{20}=0$. Using Eq. (5) one can estimate the reliability of assignments of natural parity or 0^- to a state. Obviously, the larger the number of measurements the more confidence one would have in making an assignment. To be more quantitative Table I shows the values of the cumulative probability for deviations of 0.05, 0.1, 0.2, and 0.4 from the limits of $T_{20}(-\sqrt{2}$ or $1/\sqrt{2})$. For example, the table shows that for $N=3$ the probability of obtaining an average value of T_{20} closer than 0.1 to either limit is 0.1%. It is clear that $N\geq 3$ will usually suffice to be confident of an assignment of natural parity or 0^- to a level.

One effect not yet discussed is that of detecting the α particles slightly off axis. In this case, the expression for the tensor analyzing power for natural parity states becomes more complicated and is no longer model independent. In order to estimate the average attenuation of T_{20} that one

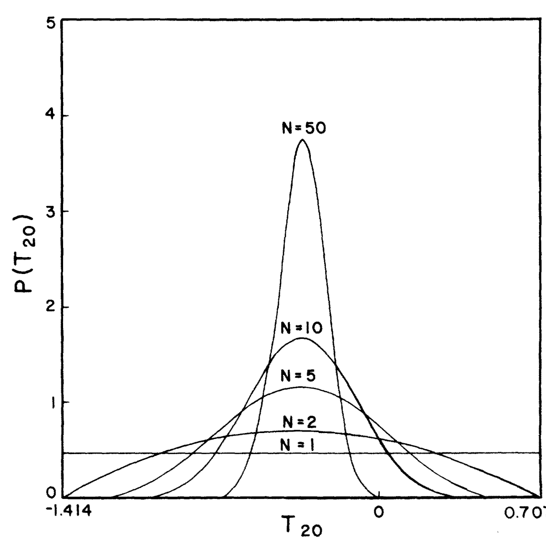

FIG. 1. Probability distribution of the tensor analyzing power for unnatural parity states in the $(\vec{d},\alpha)$ reaction, corresponding to different values of N. N refers to the thickness of the target in units of the coherence width or, alternatively, to the number of different energies for which the data are averaged.

could expect, a statistical model calculation using Coulomb penetrabilities was carried out for incident deuteron beam energies of 5, 10, and 15 MeV on targets of ^{12}C, ^{24}Mg, and ^{40}Ca. The attenuation at 2° (the angle used in this experiment) and at 5° (which corresponds roughly to using an annular counter at 180°) are shown in Table II. Previous measurements of angular distributions have yielded some information in this regard.[1] Figure 2 shows an average of two angular distributions of the tensor analyzing power for the reaction $^{12}C(\vec{d}, \alpha)^{10}B$ at $E_d = 13.6$ and 14.5 MeV and the corresponding calculation. Agreement here is probably somewhat better than expected, but it is apparent that 2° is close enough to 0° to be useful, although in some circumstances 5° may not be.

EXPERIMENTAL PROCEDURE

A polarized deuteron beam was produced by the recently installed McMaster Lamb-shift polarized ion source and accelerated through an FN tandem accelerator. α particles from the reaction were momentum analyzed with an Enge split-pole magnetic spectrograph and detected on the focal plane with a position-sensitive gas proportional counter. The counter enabled us to discriminate against the large background flux of scattered deuterons present in the experiment.

α particle spectra were taken at 2° at beam energies of 11, 12, 13, and 14 MeV. The target used was an 80 $\mu g/cm^2$ GeO_2 layer evaporated onto a 10 $\mu g/cm^2$ ^{12}C backing. The solid angle subtended by the entrance slits of the spectrograph was 0.9 msr and the angular range was 1.2°–2.8°. Because of the large energy dispersion of the spectrograph, only a portion of the spectrum of ^{14}N could be recorded in one run and different bites were taken corresponding to excitation energies in ^{14}N of roughly 4.91 to 7.03 MeV and 7.97 to 9.13 MeV. At the same time, the ^{12}C backing provided α spectra of ^{10}B corresponding to excitation energies from 0 to 4.77 MeV. For each beam energy and bite of the spectrum, runs were taken for deuterons polarized first in the $m = 0$ substate and second for the $m = 1$ substate, where the quantization axis is taken parallel to the beam direction. Each run lasted roughly two hours with an average beam current of 20 nA. All runs were normalized to the same total integrated beam current. In order to compensate for the dead time in the proportional counter and in the on-line computer, the beam was pulsed off for about 200 μsec after each event was detected. The tensor analyzing power of each state was determined by measuring the ratio, R, of the cross sections for beams polarized first in their $m = 0$ substate ($t_{20} = -P\sqrt{2}$) and second in their $m = 1$ substate ($t_{20} = P/\sqrt{2}$) and is given by

TABLE I. Values of the cumulative probability for various deviations Δ from the limits of $T_{20}(-\sqrt{2}$ or $1/\sqrt{2})$.

Number of energies N	Cumulative probability[a] $\Delta = 0.05$ (%)	$\Delta = 0.1$ (%)	$\Delta = 0.2$ (%)	$\Delta = 0.4$ (%)
1	2.5	4.5	9.5	19
2	0.1	0.7	2.7	9.5
3		0.1	0.8	5.4
4			0.2	3
5			0.1	1.6
10				0.1

[a] Values not shown are less than 0.1%.

TABLE II. Calculated attenuation of T_{20} for natural parity states when detecting α particles slightly off axis.

Target	Incident energy (MeV)	Attenuation (%) $\theta_{c.m.} = 2°$	$\theta_{c.m.} = 5°$
^{12}C	5	0.4	2.5
^{12}C	10	0.7	5.2
^{12}C	15	1.3	7.6
^{24}Mg	5	0.3	2.0
^{24}Mg	10	0.7	5.0
^{24}Mg	15	1.3	8.0
^{40}Ca	5	0.4	3.1
^{40}Ca	10	1.1	7.6
^{40}Ca	15	2.1	12.0

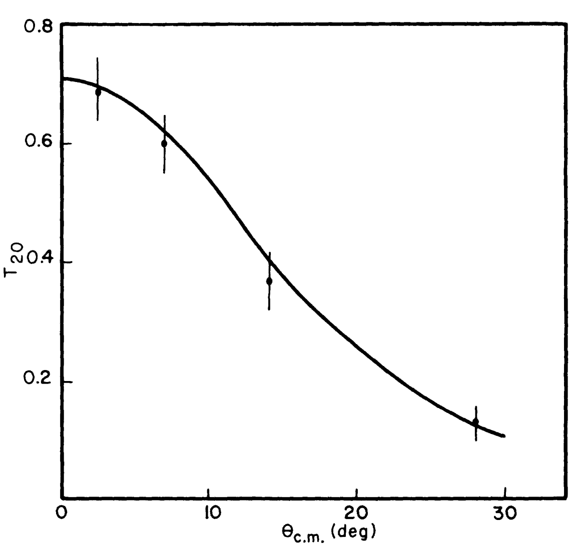

FIG. 2. Calculated and experimental angular distribution near 0° for the tensor analyzing power of the $^{12}C(\vec{d}, \alpha)^{10}B(2^+, 3.59$ MeV) reaction. The solid line corresponds to a Hauser-Feshbach calculation at $E_d = 14$ MeV and the points are data averaged for the energies $E_d = 13.6$ and 14.5 MeV.

$$T_{20}=\frac{1}{\sqrt{2}P}\,\frac{1-R}{1+R/2},$$

where P is the fractional beam polarization. As defined here, P is the same as the P_Q of Ohlsen and Keaton[8] and it represents the fraction of the beam that is polarized in the desired substate; the rest is essentially an unpolarized background. For beams polarized in their $m=1$ substate, there is also a vector polarization. However, since the vector analyzing power is zero at 0°, this will have no effect. The fractional beam polarization was determined by measuring the cross-section ratio for known natural parity states in ^{10}B and ^{14}N, since in this case $T_{20}=1/\sqrt{2}$, and is given by

$$P=\frac{1-R}{1+R/2}.$$

It was found to be constant within 5% during a three day period with an average value of 65%.

The spectra taken at 12 MeV are plotted in Fig. 3. States in ^{14}N are labeled with their excitation energies and those arising from the ^{12}C backing are shown and labeled with the excitation energies. Excitation energies for ^{10}B are taken from Ajzenberg-Selove and Lauritsen[9] while those for ^{14}N are taken from Ajzenberg-Selove.[10] Tensor analyzing powers at 2° for states in ^{14}N from 4.91 to 9.13 MeV are shown plotted in Fig. 4. Tensor analyzing powers were also determined for states in ^{10}B from 0 to

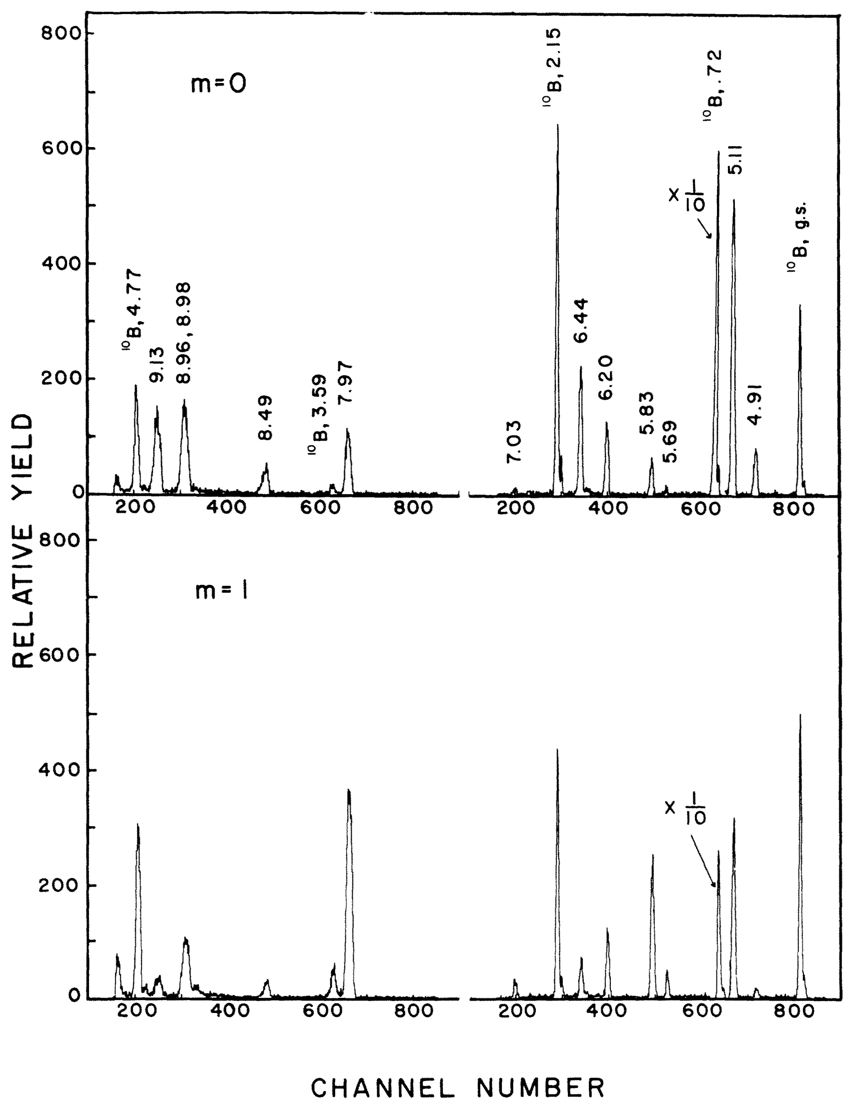

FIG. 3. Spectra for the ^{16}O($\vec{d}$, α)^{14}N reaction at $E_d=12$ MeV and $\theta_{lab}=2°$. The upper spectra are taken for the incident deuteron beam polarized in its $m=0$ substate and the lower spectra for the $m=1$ substate, where the quantization axis is aligned along the incident beam direction.

4.77 MeV and are shown in Fig. 5. States with known natural parity have T_{20} of $1/\sqrt{2}$. In the figures, T_{20} is plotted from left to right for beam energies from 11 to 14 MeV, with the exception of the 6.20 and 7.03 MeV levels for which data are only available for three energies. The errors shown are based only on the statistical uncertainty in the peak areas and on the uncertainty in the beam polarization. It should be pointed out that the probability distributions for the tensor analyzing power derived earlier apply when the cross sections for $m=0$ and $m=1$ are averaged before T_{20} is calculated, and not for an average of T_{20} calculated at each energy. However, the difference involved is slight.

The statistical sample size for the data in this experiment is too small to make a definitive comparison with calculated distributions of the tensor analyzing power; however, it appears that the fluctuations of T_{20} for unnatural parity states are consistent with the calculations.

DISCUSSION

The results for the tensor analyzing powers of states in ^{14}N from 4.91 to 9.13 MeV are consistent with all known spin-parity assignments. Isospin forbidden $T=1$ states were not observed in this reaction. The 6.20 MeV state was not observed at 14 MeV and the 7.03 MeV state was not observed at 11 MeV. An assignment of 0^- to the state at 4.91 MeV state confirms an earlier measurement by Jones *et al.*[2] using the $(\vec{d},\alpha)$ reaction. The states at 7.97, 8.49, and 9.13 MeV are discussed below.

A. 7.97 MeV

This level has been assigned $J^\pi=2^-$ by Balamuth and Noé[11] using model dependent arguments to rule out the possibilities of 1^- and 2^+. Our results show unambiguously that this is an unnatural parity state and thus confirm the assignment of $J^\pi=2^-$.

B. 8.49 MeV

This level has been assigned $J^\pi=4^-$ by Noé, Balamuth, and Zurmühle[12] using model dependent arguments to rule out the possibility of 4^+. Our results show unambiguously that this is an unnatural parity state and thus confirm the assignment of $J^\pi=4^-$.

C. 9.13 MeV

The 9.13 MeV level was observed by Detenbeck *et al.*[13] as a weak resonance in the ^{13}C$(p,\gamma)^{14}$N reaction and was assigned $J^\pi=2^-$ on the basis of the γ-ray angular distribution. However, there is evidence that this state may actually be an unresolved doublet and Noé *et al.*[12] have concluded that one member might have $J\geq 3$ in order to explain the proton decay angular correlation in the ^{12}C$(^3$He$,p)^{14}$N$(p')^{13}$C reaction. Fortune *et al.*[14] have found an $L=0$ component in the angular distribution of the ^{10}B$(^6$Li$,d)^{14}$N reaction and conclude that one member has $J^\pi=3^+$. They suggest that this is the 3^+ state predicted by Lie[15] but previously unidentified. Our results for the 9.13 MeV level show a T_{20} consistently very close to the limit for a 0^- state, which strongly suggests that one member of the "doublet" has $J^\pi=0^-$. Unidentified states having $J^\pi=0^-$, $T=0$ have also been predicted by

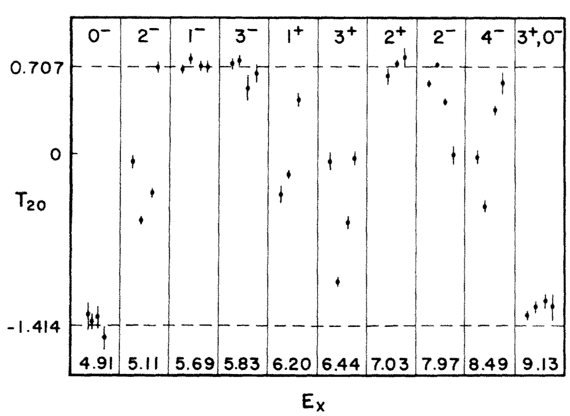

FIG. 4. Tensor analyzing powers at $\theta_{lab}=2°$ for states in ^{14}N from ^{16}O$(\vec{d},\alpha)$ reaction. The data points, from left to right, for each state, correspond to beam energies of 11, 12, 13, and 14 MeV, respectively. Only three measurements are available for the 6.20 and 7.03 MeV states, as noted in the text.

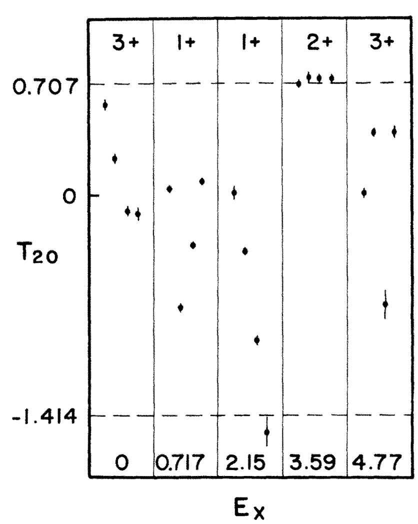

FIG. 5. Tensor analyzing powers at $\theta_{lab}=2°$ for states in ^{10}B from the ^{12}C$(\vec{d},\alpha)^{10}$B reaction.

Lie but at a somewhat higher excitation energy (13 MeV). One is tempted to conclude that the two members of the doublet have $J^\pi = 3^+$ and $J^\pi = 0^-$.

CONCLUSIONS

It has been shown that the $(\vec{d},\alpha)$ reaction on an even-even target nucleus is a useful method for determining spin-parity combinations in residual odd-odd nuclei. $J^\pi = 0^-$ assignments have been made to the level at 4.91 MeV and to one member of the doublet at 9.13 MeV in ^{14}N. The fluctuations of the tensor analyzing power with energy for unnatural parity states has been calculated using the statistical compound nucleus model and it is shown that, for most cases, an average of the cross sections for three energies will be sufficient for unambiguously assigning natural parity or 0^- to a level. It has also been shown that the expected attenuation of the tensor analyzing power for natural parity states is negligible when the α particles are detected at 2°.

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