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Effective pion-nucleon interaction in nuclear matter*

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We discuss the modification of the interaction between a pion and a nucleon in the presence of an infinite medium of nucleons (nuclear matter). The theory presented here is covariant and is relevant to the calculation of the pion-nucleus optical potential. The specific effects considered are the modifications of the nucleon propagator due to the Pauli principle and the modification of the pion and nucleon propagators due to collisions with nucleons of the medium. We also discuss in detail the pion self-energy in the medium, paying close attention to off-shell effects. These latter effects are particularly important because of the rapid variation with energy of the fundamental pion-nucleon interaction. Numerical results are presented, the main feature being the appearance of a significant damping width for the (3,3) resonance.

[NUCLEAR REACTIONS Effective π-N interaction, propagator modifications in nuclear matter.]

I. INTRODUCTION

In the theory of the scattering of a high-energy particle from a complex target one usually invokes the impulse approximation. In this approximation the scattering amplitude for the projectile and the target particle is taken to be the free two-body amplitude. While this approach has been highly successful in explaining a good deal of experimental data, there are usually significant quantitative disagreements between theory and experiment. As one begins to suggest improvements over the simple approximations one finds that there are a large number of specific effects which should be studied. Among these are the general off-shell aspects of the scattering and various kinematical and dynamical effects arising from the motion of the target particles. In addition, we should consider the modification of the elementary scattering amplitudes from their free-space values. It is these latter effects which we will discuss in this work, considering the situation in nuclear matter for simplicity.

While we will consider the effects of the Pauli principle on the nucleon recoil, as well as some other modifications of the nucleon propagator, our main attention will be directed to the study of the modification of the pion propagator in the medium. We then study the role of the modified propagators in determining an *effective* pion-nucleon amplitude. Intuitively, one expects that such propagator modifications will be important in the calculation of the effective scattering amplitude, particularly since the pion interacts very strongly with the nucleus in the resonance region.

Since a full crossing-symmetric theory becomes quite complicated, we will discuss corrections to the linear theory we have used in the past.[1] As we will see, our equations for the effective pion-nucleon interaction in the medium may be expressed in terms of the free (off-shell) πN amplitudes and various modified propagators. (Equations of this type have often been used to study the effects of propagator modification when evaluating the effective nucleon-nucleon interaction to be used in the calculation of the properties of nuclear matter.) Our approach is fully covariant and, since we do not invoke a static approximation, the ambiguities associated with transformations of kinematic variables often found in calculations using the static model are absent here.

It should be noted that our considerations may be related to those of the isobar model.[2] In that model the pion, upon striking a nucleon, forms an isobar which then propagates in the nuclear medium. In addition to the interactions responsible for the decay of the free isobar, there are further interactions within the nuclear medium which give rise to a shift in the effective isobar mass (i.e., the resonance position) and which modify the isobar lifetime in the medium. While the mass shift and modified lifetime of the isobar in the nucleus may be treated as phenomenological parameters, it is quite likely that these quantities are complicated functions of the isobar energy and the density of the nuclear medium. The model we use in this work enables us to discuss the effective pion-nucleon interaction in the medium without invoking an extremely simplified isobar model. Also, our model enables us to calculate the shift of the resonance position and the modification of the width in the medium on the basis of a dynamical model of the underlying pion-nucleon interaction.

In Sec. II we present our covariant theory of the *effective* pion-nucleon interaction. An essential quantity needed for our analysis is the pion self-energy: the calculation of that quantity is discussed in Sec. III. In Sec. IV we show that the effective interaction may be obtained relatively simply if we use a separable interaction for the pion-nucleon system. This separable-interaction model is covariant and includes the full dynamical effects of the nucleon motion. In Sec. V, we discuss the interpretation of our equations when we wish to consider the effects of inelasticity in the π-nucleon scattering. In Sec. VI, we present the results of our calculations and discuss some of their implications for calculations of pionic interactions with nuclei. Finally, in Sec. VII we present a short summary of our work.

II. PION-NUCLEON INTERACTION IN NUCLEAR MATTER

In our previous work[1] we discussed the pion-nucleon amplitude in free space within the framework of a covariant equation of the Bethe-Salpeter type,

$$\hat{M}_f = \hat{K}_f + \hat{K}_f \hat{G}_f \hat{M}_f . \tag{2.1}$$

Here $\hat{M}_f$ is the free space invariant pion-nucleon amplitude and $\hat{K}_f$ is an invariant irreducible interaction. The quantity $\hat{G}_f$ is the (renormalized) propagator for the pion-nucleon system in free space. We denote the four-vector of the pion by k and the four-vector of the nucleon by p. Our notation, defined in an earlier work,[1] is such that the matrix elements of $\hat{M}_f$ (or $\hat{K}_f$) in an arbitrary frame are related to the matrix elements expressed in terms of the center-of-mass variables by

$$\begin{aligned}\langle p'k' | \hat{M} | pk \rangle &= \delta^{(4)}(p'+k'-p-k)[(2\pi)^3 \langle p'k' | M | pk \rangle] \\ &= \delta^{(4)}(p'+k'-p-k)\langle k_c' | M(\sqrt{s}) | k_c \rangle .\end{aligned} \tag{2.2}$$

We now assume that we may write an equation similar to Eq. (2.1) for the effective pion-nucleon scattering amplitude $\hat{M}_N$ in the nuclear medium:

$$\hat{M}_N = \hat{K}_N + \hat{K}_N \hat{G}_N \hat{M}_N . \tag{2.3}$$

In Eq. (2.3) $\hat{K}_N$ is the irreducible interaction as modified by the medium and $\hat{G}_N$ is the propagator for the pion-nucleon system, also modified by the medium.

It is possible to separate the effects of the propagator modification from those effects arising from the change in the basic irreducible interaction by writing Eq. (2.3) as two equivalent equations:

$$\hat{M}_N = \hat{M}_N' + \hat{M}_N'(\hat{G}_N - \hat{G}_f)\hat{M}_N , \tag{2.4}$$

where

$$\hat{M}_N' = \hat{K}_N + \hat{K}_N \hat{G}_f \hat{M}_N' . \tag{2.5}$$

Ultimately, we will focus our attention on the difference of the propagators in and out of the medium, $\hat{d} \equiv \hat{G}_N - \hat{G}_f$. In terms of the quantity $\hat{d}$, we write Eq. (2.4) as

$$\hat{M}_N = \hat{M}_N' + \hat{M}_N' \hat{d} \hat{M}_N . \tag{2.6}$$

As discussed extensively in our previous work, it is useful to convert the four-dimensional equations such as Eqs. (2.5) and (2.6) to equivalent three-dimensional systems. Again, this is accomplished by writing two equations which are equivalent to Eq. (2.6):

$$\hat{M}_N = \hat{U}_D + \hat{U}_D \tilde{d} \hat{M}_N , \tag{2.7}$$

$$\hat{U}_D = \hat{M}_N' + \hat{M}_N'(\hat{d} - \tilde{d})\hat{U}_D , \tag{2.8}$$

with

$$\tilde{d} = \tilde{G}_N - \tilde{G}_f \tag{2.9}$$

being the difference of two appropriately chosen modified propagators $\tilde{G}_N$ and $\tilde{G}_f$ which are defined in the following discussion.

We expect that the sequence of approximations, $\hat{U}_D \to \hat{M}_N' \to \hat{M}_f$, will be useful. The latter approximation, $\hat{M}_N' \to \hat{M}_f$, implies that $\hat{K}_N = \hat{K}_f$. Some discussion of the deviation of $\hat{K}_N$ from $\hat{K}_f$ due to Pauli blocking of intermediate nucleon lines which arise when calculating $\hat{K}_f$ from a field theory appears in the literature[3]; however, in this work we will take $\hat{K}_N = \hat{K}_f$. Thus we finally direct our attention to the equation

$$\hat{M}_N = \hat{M}_f + \hat{M}_f \tilde{d} \hat{M}_N . \tag{2.10}$$

[Our choice of $\tilde{d}$ will be given in Eq. (2.23).]

Before considering Eq. (2.10) and $\tilde{d}$ in detail, let us return to Eq. (2.6) and use Eq. (2.2) to write this equation as

$$\begin{aligned}[(2\pi)^3\langle k', p' | M_N | k, p\rangle] &= [(2\pi)^3 \langle k', p' | M_N' | k, p \rangle] \\ &+ \int d^4k'' [(2\pi)^3 \langle k', p' | M_N' | k'', k+p-k''\rangle] d(k'', k+p-k'') [(2\pi)^3 \langle k'', k+p-k'' | M_N | k, p\rangle] ,\end{aligned} \tag{2.11}$$

where

$$d(k'',p+k-k'')=G_N(k'',p+k-k'')-G_f(k'',p+k-k'')$$
$$=\frac{i}{2\pi}\left[\left(\frac{1}{k''^2-m_\pi^2-\Pi(k'')+i\epsilon}\right)\frac{\theta(|\vec{k}+\vec{p}-\vec{k}''|-p_F)}{(\not{k}+\not{p}-\not{k}''-m_N-\Sigma_N(k+p-k'')+i\epsilon}-\left(\frac{1}{k''^2-m_\pi^2+i\epsilon}\right)\left(\frac{1}{\not{k}+\not{p}-\not{k}''-m_N+i\epsilon}\right)\right]. \tag{2.12}$$

We have chosen to work in the special Lorentz frame in which the *nuclear medium* through which the pion is traveling has zero total momentum. In that frame the medium is simply characterized as a Fermi gas with Fermi momentum p_F. (Although the calculation can be done in an arbitrary frame in which the covariance of the result would be manifest, it is clearly simplest to work in the chosen frame.) Note that the modification of the nucleon propagator in Eq. (2.12) involves the restriction of scattering into states above the Fermi sea and the inclusion of a "self-energy" term $\Sigma_N(k+p-k'')$ which represents other effects arising from the presence of the nuclear medium. Further, $\Pi(k'')$ is the "polarization operator" for a pion of four-momentum k''. (We will pay particular attention to the modification of the pion propagator, leaving the detailed discussion of the choice of Σ_N to a later publication.)

We now wish to specify $\tilde{d}=\tilde{G}_N-\tilde{G}_f$ by defining the modified propagators $\tilde{G}_N$ and $\tilde{G}_f$. To this end we adopt a variant of a reduction scheme used previously in the study of the pion-nucleon system. In this case, however, we treat the nucleons of momentum $|\vec{p}|>p_F$ and $|\vec{p}|<p_F$ differently. If $|\vec{p}|>p_F$, we constrain the zeroth component of the nucleon four-vector p^0 to be equal to $(|\vec{p}|^2+m_N^2)^{1/2}$; that is, the nucleon is placed on its mass shell $(\Sigma_N(p)=0)$. If $|\vec{p}|<p_F$ we require $p^0=(|\vec{p}|^2+m_h^2)^{1/2}$, where $m_h<m_N$. The introduction of m_h provides a simple scheme for introducing the binding of the nucleons in the nuclear medium. The constraints discussed above correspond to the choice $\Sigma_N(p)=0$ for $|\vec{p}|>p_F$ and $\Sigma_N(p)=m_h-m_N$ for $|\vec{p}|<p_F$.

For future use we will define

$$\epsilon_{N,\vec{p}}=(\vec{p}^2+m_N^2)^{1/2} \tag{2.13}$$

and

$$\Sigma_N(\vec{p})=\Sigma_N(\vec{p},p^0=\epsilon_{N,\vec{p}})\,. \tag{2.14}$$

In general, we will write the energy of a nucleon in the medium as

$$E_{N,\vec{p}}=\{\vec{p}^2+[m_N+\Sigma_N(\vec{p})]^2\}^{1/2} \tag{2.15}$$
$$\equiv\epsilon_{N,\vec{p}}+V_N(\vec{p})\,, \tag{2.16}$$

where in lowest order $V_N(\vec{p})\simeq(m_N/\epsilon_{N,\vec{p}})\Sigma_N(\vec{p})$.

For the simple model discussed above we have

$$E_{N,\vec{p}}=\epsilon_{N,\vec{p}}\,,\quad |\vec{p}|>p_F \tag{2.17}$$

and

$$E_{N,\vec{p}}=(\vec{p}^2+m_h^2)^{1/2}\,,\quad |\vec{p}|<p_F\,. \tag{2.18}$$

For the case of the pion, we may write the *free* pion propagator as

$$\frac{1}{(k''^0)^2-\omega_{\vec{k}''}^2+i\epsilon}=\frac{1}{2\omega_{\vec{k}''}[k''^0-\omega_{\vec{k}''}-X(\vec{k}'',k''^0)+i\epsilon]}\,, \tag{2.19}$$

where

$$X(\vec{k}'',k''^0)\equiv-[k''^0-\omega_{\vec{k}''}]^2/2\omega_{\vec{k}''}\,. \tag{2.20}$$

Now, considering the propagator in the medium, we write

$$\frac{1}{(k''^0)^2-\omega_{\vec{k}''}^2-\Pi(k'')+i\epsilon}=\frac{1}{2\omega_{\vec{k}''}[k''^0-\omega_{\vec{k}''}-X(\vec{k}'',k''^0)-\Sigma^\pi(\vec{k}'',k''^0)+i\epsilon]}\,, \tag{2.21}$$

where

$$\Sigma^\pi(\vec{k}'',k''^0)\equiv\Pi(k'')/2\omega_{\vec{k}''}\,. \tag{2.22}$$

In our previous work,[1] one of the useful reduction schemes we used involved keeping only the part of the pion propagator which is singular for $k''^0>0$, that is $[(k''^0)^2-\omega_{\vec{k}}^2+i\epsilon]^{-1}\simeq(2\omega_{\vec{k}''})^{-1}\times(k''^0-\omega_{\vec{k}''}+i\epsilon)^{-1}$. This approximation is equivalent to dropping X in Eq. (2.19). We again drop X in Eq. (2.21), keeping in mind that this approximation may be poor if $k''^0\simeq\omega_{\vec{k}''}$ is not a good approximation for the pion propagation. Our current reduction scheme therefore yields an expression for $\tilde{d}$ of the following form:

$$\tilde{d}(k'',p+k-k'')=\frac{1}{2\omega_{\vec{k}''}}\left(\frac{m_N}{\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''}}\right)\Lambda^+(\vec{p}+\vec{k}-\vec{k}'')\delta(k''^0-E_s+E_{N,\vec{p}+\vec{k}-\vec{k}''})\theta(k''^0)\times\left\{\frac{\theta(|\vec{p}+\vec{k}-\vec{k}''|-p_F)}{E_s-[\omega_{\vec{k}''}+E_{N,\vec{p}+\vec{k}-\vec{k}''}+\Sigma^\pi(\vec{k}'',k''^0)-i\epsilon]}-\frac{1}{E_s-(\omega_{\vec{k}''}+\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''}-i\epsilon)}\right\}, \tag{2.23}$$

where

$$E_s = [s + (\vec{p}+\vec{k})^2]^{1/2} = (s + \vec{P}^2)^{1/2} \tag{2.24}$$

and

$$s = (p+k)^2 = (p^0+k^0)^2 - \vec{P}^2 . \tag{2.25}$$

In writing Eq. (2.23) we have dropped terms which are of the order (Σ_N/m_N). In that equation $\Lambda^+(\vec{p}+\vec{k}-\vec{k}'')$ is a projection operator for positive energy nucleon spinors. For the specification of $E_{N,\vec{p}+\vec{k}-\vec{k}''}$ we may use either Eq. (2.15) or Eqs. (2.17) and (2.18). [If we choose to use a complex function for $\Sigma_N(\vec{p}+\vec{k}-\vec{k}'')$ we would replace $\delta(k''^0 - E_s + E_{N,\vec{p}+\vec{k}-\vec{k}''})$ by $\delta(k''^0 - E_s + \mathrm{Re}\,E_{N,\vec{p}+\vec{k}-\vec{k}''})$.] In Appendix A we consider an alternate form for $\tilde{d}$ which may be more useful if the pion does not propagate close to its mass shell. The quantity E_s represents the total energy of the pion-nucleon system and the variable s has the usual interpretation. We remark that $\vec{P}=(\vec{p}+\vec{k})$ is unequal to zero (in general) in the Lorentz frame in which we choose to calculate the effective pion-nucleon interaction.

Using Eq. (2.23), we may write our basic approximation, as given by Eq. (2.10), suppressing reference to all zeroth components of the four vectors, since these components may be obtained as specific functions of the three-vector (for example, $k''^0 = E_s - \mathrm{Re}\,E_{N,\vec{p}+\vec{k}-\vec{k}''}$, $p^0 = \mathrm{Re}\,E_{N,\vec{p}}$, etc.). Thus, we have

$$\begin{aligned}[(2\pi)^3\langle\vec{k}',\vec{p}'|M_N(E_s)|\vec{k},\vec{p}\rangle] &= [(2\pi)^3\langle\vec{k}',\vec{p}'|M_f(E_s)|\vec{k},\vec{p}\rangle] \\ &+ \int d\vec{k}''[(2\pi)^3\langle\vec{k}',\vec{p}'|M_f(E_s)|\vec{k}'',\vec{p}+\vec{k}-\vec{k}''\rangle]\left(\frac{1}{2\omega_{\vec{k}''}}\frac{m_N}{\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''}}\right) \\ &\times \Lambda^+(\vec{p}+\vec{k}-\vec{k}'')\left\{\frac{\theta(|\vec{p}+\vec{k}-\vec{k}''|-p_F)}{E_s-[\omega_{\vec{k}''}+E_{N,\vec{p}+\vec{k}-\vec{k}''}+\Sigma_{\vec{P},s}^{\pi}(\vec{k}'')-i\epsilon]} - \frac{1}{E_s-(\omega_{\vec{k}''}+\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''}-i\epsilon)}\right\} \\ &\times [(2\pi)^3\langle\vec{k}'',\vec{p}+\vec{k}-\vec{k}''|M_N(E_s)|\vec{k},\vec{p}\rangle] . \end{aligned} \tag{2.26}$$

In Eq. (2.26) we have introduced the quantities

$$\Sigma_{\vec{P},s}^{\pi}(\vec{k}'') = \Sigma^{\pi}(\vec{k}'', k''^0 = E_s - \mathrm{Re}\,E_{N,\vec{p}+\vec{k}-\vec{k}''}) \tag{2.27}$$

and

$$\langle\vec{k}',\vec{p}'|M_N(E_s)|\vec{k},\vec{p}\rangle \equiv \langle k',p'|M_N|k,p\rangle]^{k'^0=E_s-\mathrm{Re}\,E_{N,\vec{p}'};\; p'^0=\mathrm{Re}\,E_{N,\vec{p}'}}_{k^0=E_s-\mathrm{Re}\,E_{N,\vec{p}};\; p^0=\mathrm{Re}\,E_{N,\vec{p}}} , \tag{2.28}$$

etc.

Also, as in our previous work[1] we introduce spinors for off-mass-shell nucleons. We use the notation $u^s(p)$ for such spinors and multiply Eq. (2.26) from the left by $\bar{u}^{s'}(p')$ and from the right by $u^s(p)$. The resulting quantity $\bar{u}^{s'}(p')[(2\pi)^3\langle\vec{k}'\vec{p}'|M_N(E_s)|\vec{k},\vec{p}\rangle]u^s(p)$ is a Lorentz invariant and we may therefore choose to evaluate it in the center-of-mass frame of the pion-nucleon system. We may write, for example,

$$\bar{u}^{s'}(p')[(2\pi)^3\langle\vec{k}',\vec{p}'|M_f(E_s)|\vec{k},\vec{p}\rangle]u^s(p) = \bar{u}^{s'}(p_c')\langle\vec{k}_c'|M_f(\sqrt{s})|\vec{k}_c\rangle u^s(p_c) . \tag{2.29}$$

The notation used in Eq. (2.29) is best understood by reference to Fig. 1. The evaluation of Eq. (2.29) will be discussed at a later stage; however, we note here that we will make use of the approximation of expanding the invariant amplitude about its value for the situation in which the nucleon is on its mass shell, $p_c^2 = p_c'^2 = m_N^2$, as discussed in detail in Ref. 1.

Rather than considering Eq. (2.26) directly, it is useful to introduce a more familiar T-matrix equation. Such an equation may be obtained from Eq. (2.26) after introducing the quantity [see Ref. 1, Eqs. (2.11)–(2.14)]

$$[(2\pi)^3{}_{NR}\langle\vec{k}',\vec{p}',s'|T_N(E_s)|\vec{k},\vec{p},s\rangle_{NR}] = \bar{u}^{s'}(p')[(2\pi)^3\langle\vec{k}',\vec{p}'|M_N(E_s)|\vec{k},\vec{p}\rangle]u^s(p)R^{1/2}(\vec{k}',\vec{p}')R^{1/2}(\vec{k},\vec{p}) , \tag{2.30}$$

where

$$R(\vec{k},\vec{p}) = (m_N/2\omega_{\vec{k}}\epsilon_{N,\vec{p}}) . \tag{2.31}$$

Similarly, we also introduce a quantity T_f related to M_f as T_N is related to M_N. We then obtain from Eqs. (2.20) and (2.24)

$$\begin{aligned}[(2\pi)^3{}_{NR}\langle\vec{k}',\vec{p}',s'|T_N(E_s)|\vec{k},\vec{p},s\rangle_{NR}] &= [(2\pi)^3{}_{NR}\langle\vec{k}',\vec{p}',s'|T_f(E_s)|\vec{k},\vec{p},s\rangle_{NR}] \\ &+ \sum_{s''}\int d\vec{k}''[(2\pi)^3{}_{NR}\langle\vec{k}',\vec{p}',s'|T_f(E_s)|\vec{k}'',\vec{k}+\vec{p}-\vec{k}'',s''\rangle_{NR}] \\ &\times C_s(\vec{k}+\vec{p},\vec{k}'')[(2\pi)^3{}_{NR}\langle\vec{k}'',\vec{p}+\vec{k}-\vec{k}'',s''|T_N(E_s)|\vec{k},\vec{p},s\rangle_{NR}] , \end{aligned} \tag{2.32}$$

with

$$C_s(\vec{k}+\vec{p},\vec{k}'')=\left\{\frac{\theta(|\vec{p}+\vec{k}-\vec{k}''|-p_F)}{E_s-[\omega_{\vec{k}''}+E_{N,\vec{p}+\vec{k}-\vec{k}''}+\Sigma^{\dagger}_{\vec{P},s}(\vec{k}'')+i\epsilon]}-\frac{1}{E_s-[\omega_{\vec{k}''}+\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''}-i\epsilon]}\right\}. \tag{2.33}$$

Equation (2.32) is the basic equation we will study in this work. It is apparent that to proceed with the calculation of $C_s(\vec{P},\vec{k}'')$ of Eq. (2.33), it is necessary to first obtain the pion self-energy $\Sigma_{\vec{P},s}(\vec{k}'')$. We consider the calculation of this quantity in the next section.

III. PION SELF-ENERGY IN NUCLEAR MATTER

In order to evaluate the effective pion-nucleon interaction we need to be able to calculate the pion self-energy $\Sigma_{\vec{P},s}(\vec{k}'')$. [In order to simplify the notation of this section we use Eqs. (2.17) and (2.18) to describe the nucleon dynamics.]

The pion self-energy is then given by the following expression $[(\Sigma_N/m_N)\simeq 1]$,

$$\Sigma^{\dagger}_{\vec{P},s}(\vec{k}'')=\int d\vec{q}\left(\frac{m_N}{2\omega_{\vec{k}''}\epsilon_{N,\vec{q}}}\right)\mathrm{Tr}[(2\pi)^3\langle\vec{k}'',\vec{q}\,|M_N(\bar{s})|\,\vec{k}'',\vec{q}\rangle]\Lambda^+(\vec{q})]\theta(p_F-|\vec{q}|)\,, \tag{3.1}$$

where $\Lambda^+(\vec{q})$ is a projection operator of positive energy spinors having a mass parameter m_h. In Eq. (3.1) we have, with $E_s=(s+\vec{P}^2)^{1/2}$, $\bar{s}=(k''+q)^2$,

$$\langle\vec{k}'',\vec{q}\,|M_N(\bar{s})|\,\vec{k}'',\vec{q}\rangle\equiv\langle k'',q\,|M_N|\,k'',q\rangle\Big|_{\substack{k''^0=E_s-\mathrm{Re}\,E_{N,\vec{P}-\vec{k}''}\\ q^0=\mathrm{Re}\,E_{N,\vec{q}}}}. \tag{3.2}$$

The kinematical variables relevant to calculating the self-energy are displayed in Fig. 2.

The presence of M_N in Eq. (3.1) leads to a complicated self-consistency problem since M_N is a functional of Σ. Therefore, we content ourselves with the simple approximation of replacing M_N by M_f in Eq. (3.1). Thus using Eqs. (4.1) and (4.4) of Ref. 1, we may write, using Eq. (2.18),

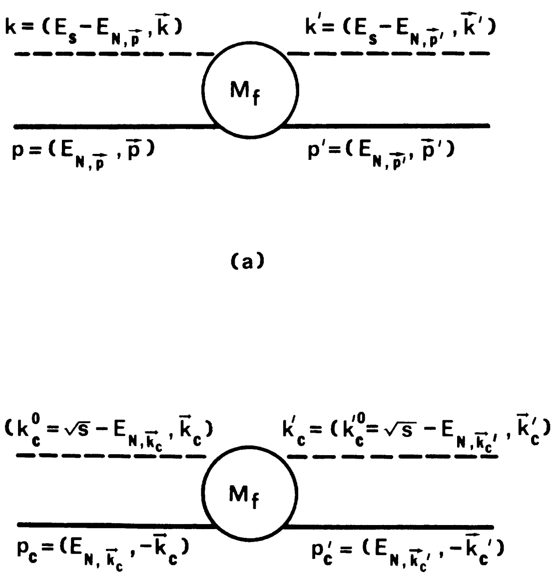

FIG. 1. Kinematic variable for the pion (dashed line) and the nucleon in (a) the Lorentz frame in which the medium is at rest and (b) the center-of-mass frame of the pion-*nucleon* system. If, for example, $|\vec{p}'|<p_F$, we have $E_{N,\vec{p}'}=(\vec{p}'^2+m_h^2)^{1/2}$ and also $E_{N,\vec{k}_c'}=(\vec{k}_c'^2+m_h^2)^{1/2}$, etc., in the simple model of Eqs. (2.17)–(2.18).

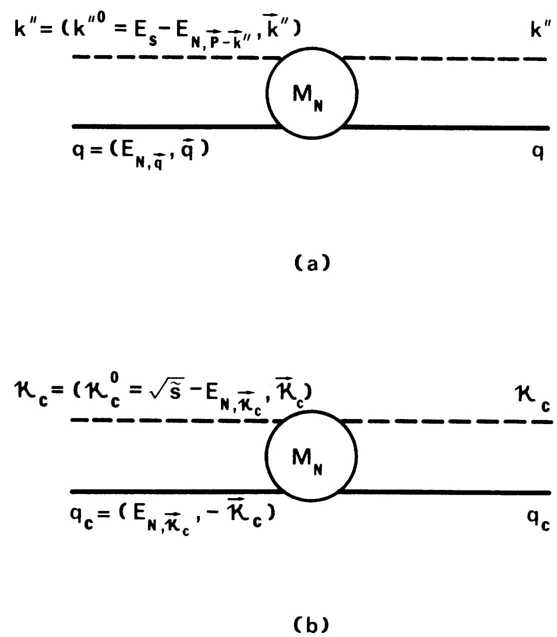

FIG. 2. Kinematic variable for the calculation of the pion self-energy in (a) the Lorentz frame in which the medium is at rest ($|\vec{q}|<p_F$), and in (b) the center-of-mass frame of the pion and the nucleon. Recall that $E_s=(s+\vec{P}^2)^{1/2}$, and $E_{N,\vec{k}_c}=(\vec{k}_c^2+m_h^2)^{1/2}$ if we use Eq. (2.18).

$$\Sigma_{\vec{p},s}(\vec{k}'') = -\frac{2}{(2\pi)^3}\int d\vec{q}\,\theta(p_F - |\vec{q}|)\left(\frac{m_N}{2\omega_{\vec{k}''}\epsilon_{N,\vec{q}}}\right)\mathrm{Tr}\left[(A^{(+)} + B^{(+)}\not{k})\left(\frac{\not{q}+m_h}{2m_h}\right)\right]$$
$$= -\frac{4}{(2\pi)^3}\left(\frac{1}{2\omega_{\vec{k}''}}\right)\int d\vec{q}\,\theta(p_F - |\vec{q}|)\left(\frac{m_N}{\epsilon_{N,\vec{q}}}\right)\left[A^{(+)}(\bar{s},0,k''^2,k''^2,m_h^2,m_h^2) + \frac{\nu}{m_h}B^{(+)}(\bar{s},0,k''^2,k''^2,m_h^2,m_h^2)\right]. \quad (3.3)$$

[Note that there is a factor of 2 on the right-hand side of Eq. (3.3) due to the isospin trace.] The parameter ν is given by

$$\nu = k''\cdot q = \kappa_c^0 E_{N,\vec{\kappa}_c} + \vec{\kappa}_c^{\,2} = \sqrt{\bar{s}}\,E_{N,\vec{\kappa}_c} - m_h^2 . \quad (3.4)$$

Using Eq. (4.31) of Ref. 1, we obtain for forward pion scattering

$$A^{(+)} = 4\pi\left[\frac{(\sqrt{\bar{s}}+m_h)}{E_{N,\vec{\kappa}_c}+m_h}F_1^{(+)} - \frac{(\sqrt{\bar{s}}-m_h)}{E_{N,\vec{\kappa}_c}-m_h}F_2^{(+)}\right], \quad (3.5)$$

$$B^{(+)} = 4\pi\left[\frac{1}{E_{N,\vec{\kappa}_c}+m_h}F_1^{(+)} - \frac{1}{E_{N,\vec{\kappa}_c}-m_h}F_2^{(+)}\right]. \quad (3.6)$$

Using Eqs. (3.4)–(3.6), we obtain

$$\left(A^{(+)} + \frac{\nu}{m_h}B^{(+)}\right) = \frac{4\pi\sqrt{\bar{s}}}{m_h}(F_1^{(+)} + F_2^{(+)}). \quad (3.7)$$

We may also use Eqs. (4.17) and (4.30) of Ref. 1 to relate this last quantity to the forward scattering amplitude:

$$f_0(\bar{s},0,\vec{\kappa}_c^{\,2},\vec{\kappa}_c^{\,2},\kappa_c^0,\kappa_c^0) \equiv \bar{f}_0(\bar{s},\vec{\kappa}_c^{\,2}), \qquad \left[A^{(+)} + \frac{\nu}{m_h}B^{(+)}\right] = \frac{4\pi\sqrt{\bar{s}}}{m_h|\vec{\kappa}_c|}\bar{f}_0(\bar{s},\vec{\kappa}_c^{\,2}). \quad (3.8)$$

With the separable potential model of Londergan, McVoy, and Moniz (LMM),[4] we have upon using Eqs. (4.14) and (4.23) of Ref. 1 ($m_N/m_h \simeq 1$):

$$-\frac{2\pi\sqrt{\bar{s}}\,\bar{f}_0(\bar{s},\vec{\kappa}_c^{\,2})}{\omega_{\vec{\kappa}_c}\epsilon_{N,\vec{\kappa}_c}|\vec{\kappa}_c|} = \sum_{l=0}^{\infty}\sum_{T=1/2,3/2}\sum_{\alpha=\pm1}\tfrac{1}{3}[\tfrac{1}{2}(2T+1)\tfrac{1}{2}(2l+1+\alpha)]\lambda^l_{2T,2l+\alpha}|v^l_{2T,2l+\alpha}(|\vec{\kappa}_c|)|^2/D^l_{2T,2l+\alpha}(\bar{s}). \quad (3.9)$$

Finally, we obtain {with $(m_N/m_h)\simeq 1$}

$$\Sigma_{\vec{p},s}^{\mathrm{I}}(\vec{k}'') = \frac{1}{3(2\pi)^3}\int d\vec{q}\,\theta(p_F - |\vec{q}|)\left(\frac{\omega_{\vec{\kappa}_c}\epsilon_{N,\vec{\kappa}_c}}{\omega_{\vec{k}''}\epsilon_{N,\vec{q}}}\right)\sum_{l=0}^{\infty}\sum_{T=1/2,3/2}\sum_{\alpha=\pm1}(2T+1)(2l+1+\alpha)\frac{\lambda^l_{2T,2l+\alpha}|v^l_{2T,2l+\alpha}(|\vec{\kappa}_c|)|^2}{D^l_{2T,2l+\alpha}(\bar{s})}, \quad (3.10)$$

where

$$|\vec{\kappa}_c|^2 = \frac{(k''\cdot q)^2 - k''^2 m_h^2}{\bar{s}}, \quad (3.11)$$

$$(k'')^2 = (E_s - E_{N,\vec{p}-\vec{k}''})^2 - (\vec{k}'')^2, \quad (3.12)$$

$$(k''\cdot q)^2 = [E_{N,\vec{q}}(E_s - E_{N,\vec{p}-\vec{k}''}) - \vec{k}''\cdot\vec{q}]^2, \quad (3.13)$$

and

$$\bar{s} = (E_{N,\vec{q}} + E_s - E_{N,\vec{p}-\vec{k}''})^2 - (\vec{k}'' + \vec{q})^2. \quad (3.14)$$

We remark that $\Sigma_{\vec{p},s}(\vec{k}'')$ is actually best displayed as a function of the two variables $|\vec{k}''|$ and $k''^0 = E_s - E_{N,\vec{p}-\vec{k}''}$.

IV. EFFECTIVE PION-NUCLEON INTERACTION IN A SEPARABLE COVARIANT MODEL

We consider, in this section, the solution of Eq. (2.26) in the case the free T matrix T_f has a separable form (see Fig. 1)

$$[(2\pi)^3{}_{NR}\langle\vec{k}',\vec{p}',s'|T_f(E_s)|\vec{k},\vec{p},s\rangle_{NR}]$$
$$= \sum_{JMl} C^{\,l\ \frac{1}{2}\ J}_{M-s'\ s'\ M}C^{\,l\ \frac{1}{2}\ J}_{M-s\ s\ M}\left(\frac{\epsilon_{N,\vec{k}_c}\omega_{\vec{k}_c}\epsilon_{N,\vec{k}_c'}\omega_{\vec{k}_c'}}{\epsilon_{N,\vec{p}}\omega_{\vec{k}}\epsilon_{N,\vec{p}'}\omega_{\vec{k}'}}\right)^{1/2}$$
$$\times\frac{4\pi}{(2\pi)^3}Y_{l,M-s'}(\hat{k}_c')\lambda^l_{2T,2J}$$
$$\times\frac{v^l_{2T,2J}(\vec{k}_c'^2)v^l_{2T,2J}(\vec{k}_c^2)}{D^l_{2T,2J}(\sqrt{s})}Y^*_{l,M-s}(\hat{k}_c), \quad (4.1)$$

with

$$\vec{k}_c^2 = [(k\cdot p)^2 - k^2p^2]/s \quad (4.2)$$

and

$$\vec{k}_c'^2 = [(k'\cdot p')^2 - k'^2p'^2]/s. \quad (4.3)$$

We note that $\vec{k}_c^2$ and $\vec{k}_c'^2$ are the squares of the relative momentum vectors in the πN center-of-mass frame. Our dynamical (covariant) model assumes that the function $v^l_{2T,2J}$ depends on the four-vectors k and p only through the invariant $\vec{k}_c^{\,2}$. The factor in Eq. (4.1) involving the nucleon and pion energies $\epsilon_{N,\vec{k}_c}, \omega_{\vec{k}_c}, \ldots$ etc., arises because the T matrix is evaluated in a general Lorentz frame—see Eq. (2.26). We remark that this factor is equal to 1 in the πN center of mass frame where $|\vec{p}| = |\vec{k}| = |\vec{k}_c|$. We further remark that the decomposition in Eq. (4.1) is made for a particular isospin T.

In order to solve Eq. (2.26) we assume that T_N, the T matrix in the medium, is given by

$$[(2\pi)^3{}_{NR}\langle\vec{k}',\vec{p}',s'\,|\,T_N(E_s)\,|\,\vec{k},\vec{p},s\rangle_{NR}] = \sum_{ll'JJ'MM'} C_{M'-s'\;s'\;M'}^{\;l'\;\;\frac{1}{2}\;J'} C_{M-s\;s\;M}^{\;l\;\;\frac{1}{2}\;J}\left(\frac{\epsilon_{N,\vec{k}_c}\omega_{\vec{k}_c}\epsilon_{N,\vec{k}_c'}\omega_{\vec{k}_c'}}{\epsilon_{N,\vec{p}}\,\omega_{\vec{k}}\,\epsilon_{N,\vec{p}'}\,\omega_{\vec{k}'}}\right)^{1/2}$$
$$\times\frac{4\pi}{(2\pi)^3}Y_{l',M'-s'}(\hat{k}_c')v^{l'}_{2T,2J'}(\vec{k}_c'^{\,2})A^T_{l'J'M',lJM}(\vec{P},\sqrt{s})$$
$$\times\, v^l_{2T,2J}(\vec{k}_c^{\,2})Y^*_{l,M-s}(\hat{k}_c)\,. \qquad (4.4)$$

Inserting Eqs. (4.1) and (4.4) into Eq. (2.26), we obtain

$$A^T_{l'J'M',lJM}(\vec{P},\sqrt{s}) = \frac{\delta_{ll'}\,\delta_{JJ'}\,\delta_{MM'}}{D^l_{2T,2J}(\sqrt{s})}\,\lambda^l_{2T,2J} + \sum_{l''J''M''}\frac{\lambda^{l'}_{2T,2J'}}{D^{l'}_{2T,2J'}(\sqrt{s})}\,\Lambda^T_{l'J'M',l''J''M''}(\vec{P},\sqrt{s})\,A^T_{l''J''M'',lJM}(\vec{P},\sqrt{s})\,, \qquad (4.5)$$

with

$$\Lambda^T_{l'J'M',l''J''M''}(\vec{P},\sqrt{s}) = \frac{1}{2\pi^2}\sum_{s''}\int d\vec{k}''\,Y^*_{l',M'-s''}(\hat{k}_c'')v^{l'}_{2T,2J'}(\vec{k}_c''^{\,2})$$
$$\times\left(\frac{\epsilon_{N,\vec{k}_c''}\omega_{\vec{k}_c''}}{\epsilon_{N,\vec{P}-\vec{k}''}\omega_{\vec{k}''}}\right)C_s(\vec{P},\vec{k}'')v^{l''}_{2T,2J''}(\vec{k}_c''^{\,2})\,Y_{l'',M''-s''}(\hat{k}_c'')C_{M'-s''\;s''\;M'}^{\;l'\;\;\frac{1}{2}\;J'}C_{M''-s''\;s''\;M''}^{\;l''\;\;\frac{1}{2}\;J''}\,. \qquad (4.6)$$

We note that the vector $\hat{k}_c''$ may be obtained by performing a Lorentz transformation to the center of mass of the pion-nucleon system. This vector gives the direction of the three-momentum of the intermediate pion of momentum $\vec{k}''$ [see Eq. (2.26)] in the Lorentz frame in which $\vec{P}=0$. It is convenient to choose the z axis along $\vec{P}$; it is then readily seen that the matrices A and Λ are diagonal in M. We define

$$A^T_{l'J'M',lJM}(\vec{P},\sqrt{s}) = \delta_{MM'}\hat{A}^T_{l'J',lJ}(P,\sqrt{s},M) \qquad (4.7)$$

and

$$\Lambda^T_{l'J'M',lJM}(\vec{P},\sqrt{s}) = \delta_{MM'}\hat{\Lambda}^T_{l'J',lJ}(P,\sqrt{s},M)\,. \qquad (4.8)$$

Finally, we find

$$\hat{A}^T_{l'J',lJ}(P,\sqrt{s},M) = \frac{\delta_{ll'}\delta_{JJ'}}{D^{l'}_{2T,2J'}(\sqrt{s})}\lambda^{l'}_{2T,2J'} + \sum_{l''J''}\frac{\lambda^{l'}_{2T,2J'}}{D^{l'}_{2T,2J'}(\sqrt{s})}\Lambda^T_{l'J',l''J''}(P,\sqrt{s},M)\hat{A}^T_{l''J'',lJ}(P,\sqrt{s},M)\,, \qquad (4.9)$$

where

$$\hat{\Lambda}^T_{l'J',l''J''}(P,\sqrt{s},M) = \frac{1}{2\pi^2}\sum_L \hat{\eta}_L(l'J';l''J'';M)\left[\frac{(2l'+1)(2l''+1)}{4\pi(2L+1)}\right]^{1/2}C_{0\;0\;0}^{\;l''\;l'\;L}$$
$$\times\int d\vec{k}''\,Y_{L0}(\hat{k}_c'')v^{l'}_{2T,2J'}(\vec{k}_c''^{\,2})\left(\frac{\epsilon_{N,\vec{k}_c''}\omega_{\vec{k}_c''}}{\epsilon_{N,\vec{P}-\vec{k}''}\omega_{\vec{k}''}}\right)C_s(\vec{P},\vec{k}'')v^{l''}_{2T,2J''}(\vec{k}_c''^{\,2})\,, \qquad (4.10)$$

and

$$\hat{\eta}_L(l'J';l''J'';M) = \sum_{s''}(-1)^{M-s''}C_{s''-M\;M-s''\;0}^{\;l'\;\;\;l''\;\;\;L}C_{M-s''\;s''\;M}^{\;l'\;\;\frac{1}{2}\;J'}C_{M-s''\;s''\;M}^{\;l''\;\;\frac{1}{2}\;J''}\,. \qquad (4.11)$$

We remark that in the special case that l and J are (approximately) good quantum numbers we have

$$\hat{A}_{l'J',lJ}(P,\sqrt{s},M) \simeq \frac{\delta_{ll'}\delta_{JJ'}\lambda^l_{2T,2J}}{D^l_{2T,2J}(\sqrt{s}) - \lambda^l_{2T,2J}\hat{\Lambda}^T_{lJ,lJ}(P,\sqrt{s},M)} \qquad (4.12)$$

$$\equiv \frac{\delta_{ll'}\delta_{JJ'}\lambda^l_{2T,2J}}{\mathfrak{D}^l_{2T,2J}(P,\sqrt{s},M)}\,. \qquad (4.13)$$

In our numerical calculations we found that the expression given in Eq. (4.13) was a rather good approximation and we will later present results for the quantity $\mathfrak{D}^{I}_{2T,2J}(P,\sqrt{s},M)$. (The shortened notation $\mathfrak{D}_{2T,2J}(\sqrt{s})$ will be used if P and M are specified elsewhere.)

Finally, we note that once the modified T matrix T_N of Eq. (4.4) is obtained it is possible to recalculate the pion self-energy. Further iteration could be used to obtain self-consistency. We did not undertake such self-consistent calculations.

V. ROLE OF PION-NUCLEON INELASTIC CHANNELS

The discussion presented thus far is adequate for the situation in which we neglect inelasticities in the π-nucleon system. However, only small modifications of our treatment are needed if we wish to take into account inelastic effects using the model introduced in Ref. 4. We recall some of the procedures of that work.

If the experimental phase shift and inelasticity parameter are denoted by δ and η, one defines a quasiphase $\hat{\delta}$ and a parameter $\hat{\eta}$ by means of the equation

$$1-\eta e^{2i\delta}=\hat{\eta}(1-e^{2i\hat{\delta}})\,. \tag{5.1}$$

If one writes the T matrix in the channel in question as

$$\langle k_c|\,T(\sqrt{s})\,|k_c'\rangle=\lambda\frac{v(|\vec{k}_c|)v(|\vec{k}_c'|)}{D(\sqrt{s})}, \tag{5.2}$$

one obtains a solution to the inverse scattering problem expressed in terms of $\hat{\eta}$ and $\hat{\delta}$,

$$D(\sqrt{s})=\exp\left(\frac{1}{\pi}\int\frac{\hat{\delta}(\sqrt{s}')}{\sqrt{s}-\sqrt{s}'+i\epsilon}d(\sqrt{s}')\right) \tag{5.3}$$

and

$$v^2(|\vec{k}|)=\frac{i\pi\lambda}{k}\left[\frac{\epsilon_{N,\vec{k}}+\omega_{\vec{k}}}{\omega_{\vec{k}}\epsilon_{N,\vec{k}}}\right]D(\sqrt{s})\hat{\eta}(e^{2i\hat{\delta}}-1) \tag{5.4}$$

$$=-\frac{2\lambda\pi}{k}\left[\frac{\epsilon_{N,\vec{k}}+\omega_{\vec{k}}}{\omega_{\vec{k}}\epsilon_{N,\vec{k}}}\right]\times\exp\left(\frac{1}{\pi}P\int\frac{\hat{\delta}(\sqrt{s}')\,d(\sqrt{s}')}{\sqrt{s}-\sqrt{s}'}\right)\hat{\eta}\sin\hat{\delta}\,. \tag{5.5}$$

One may show that[4]

$$D(\sqrt{s})=1-\frac{\lambda}{2\pi^2}\int_0^\infty q^2dq\frac{g^2(|\vec{q}|)}{\sqrt{s}-[\epsilon_{N,\vec{q}}+\omega_{\vec{q}}]+i\epsilon}, \tag{5.6}$$

where

$$g^2(|\vec{q}|)=v^2(|\vec{q}|)/\hat{\eta}(|\vec{q}|)\,. \tag{5.7}$$

We see that the T matrix of Eq. (5.2) may be thought of as a solution of a Lippmann-Schwinger equation (with relativistic kinematics) containing an *effective* propagator,

$$\langle\vec{k}|G_{\text{eff}}^{(+)}(\sqrt{s})|\vec{k}'\rangle=\frac{\delta(\vec{k}-\vec{k}')\eta^{-1}(|\vec{k}|)}{\sqrt{s}-(\epsilon_{N,\vec{k}}+\omega_{\vec{k}})+i\epsilon}\,. \tag{5.8}$$

It is useful to separate this propagator into portions that describe the propagation in the elastic and inelastic π-N channels, respectively,

$$\langle\vec{k}|G_{\text{eff}}^{(+)}(\sqrt{s})|\vec{k}'\rangle=\delta(\vec{k}-\vec{k}')\left[\frac{1}{\sqrt{s}-(\epsilon_{N,\vec{k}}+\omega_{\vec{k}})+i\epsilon}+\frac{\eta^{-1}(|\vec{k}|)-1}{\sqrt{s}-(\epsilon_{N,\vec{k}}+\omega_{\vec{k}})+i\epsilon}\right]. \tag{5.9}$$

[While it is not apparent from Eq. (5.9) that the second term does describe the propagation in the inelastic channels, that may be seen to be the correct interpretation of this quantity if reference is made to Eqs. (3.5), (3.10), and (3.11) of Ref. 4.]

In our model of the modified propagator we insert the pion and nucleon self-energy terms only in the propagator for the elastic π-N channel. If we also insert the Pauli principle restriction in the elastic channel only, we see that the second term in Eq. (5.9) is canceled when the propagator *difference* is formed as in Eq. (2.23).

With these considerations in mind, the equations of the previous sections are valid for a model which includes the effects of inelasticity, as long as the form factors $v(|\vec{k}|)$ and the denominator functions $D(\sqrt{s})$ for each channel are obtained by the procedures outlined in Eqs. (5.1)–(5.7) of this section. These expressions have been evaluated and form factors so obtained have been exhibited in Ref. 4. The solutions are characterized by functions which have very great extensions in momentum space (to about 10–30 GeV/c). This feature reflects the fact that the work of Ref. 4 provides a separable T-matrix fit to phase shifts and inelasticity parameters extrapolated to very high energies. While a model based on a separable form for the T matrix is reasonable in the region dominated by resonance structures, the use of such a model to fit phase shifts extended into the multi-GeV range seems unreasonable. Also, we expect that the dynamics of the pion-nucleon interaction in the nuclear medium (at low energy) should not be sensitive to the assumed π-N phase shifts above 1 GeV. For these reasons we felt that a fit to the data below 1 GeV with form factors which had much smaller extensions in momentum space should be preferred. We therefore used Eqs. (5.1)–(5.7), but with different specifications of the high energy behavior of η and δ than those used in Ref. 4. Some results of our model are compared to those of Ref. 4 in Figs. 3 and 4. In Fig. 3 we compare the $\delta_{33}(k)$ and $\eta_{33}(k)$ for $0<k<1$ GeV/c. We remark that in this case the results for these quantities taken from Ref. 4 correspond to phase and inelasticities obtained from a detailed analysis of π-N scattering. Our model provides an adequate fit to these quantities in the region of interest and also exhibits form factors which are constrained such that $v(k)=0$ for $k>1$ GeV/c. In Fig. 4 we compare our form factor for the (3,3) channel with that of Ref. 4. While both models give essentially similar on-shell T matrices for energies less than 1 GeV, they are quite different above that energy. Further, the corresponding T matrices will be quite different when compared for highly "off-shell" values of their parameters.

We have obtained a set of form factors and denominator functions based on a model in which $\eta(k)=1$ and $\delta(k)=0$ or $\delta(k)=\pi$ for $k>1$ GeV/c. Results for the effective π-N interaction in nuclear matter based upon this model are presented in the next sections.

VI. RESULTS OF NUMERICAL CALCULATIONS

We have chosen to calculate the effective pion-nucleon interaction at several values of p_F, corresponding to one-quarter, one-half, and full nuclear density, since we are ultimately interested in the situation in finite nuclei. (In the latter case, the interaction often takes place in the nuclear surface.) Our results for the real and imaginary parts of $\lambda[D_{33}(\sqrt{s})]^{-1}$ and $\lambda[\mathfrak{D}_{33}(\sqrt{s})]^{-1}$ are shown in Figs. 5–7. We note a small shift in the resonance position and a rather significant increase in the width of the resonance, particularly at high density. The curves shown are for $P=207$ MeV/c and $M=\frac{3}{2}$. The dependence of our results on P and M, for various values of p_F, is presented in Table I, where we have presented several parameters defined in the following discussion.

It is useful to parametrize $D_{33}(\sqrt{s})$ by

$$D_{33}(\sqrt{s})=-N_0(\sqrt{s}-E_R+\tfrac{1}{2}i\Gamma_R) \tag{6.1}$$

in the vicinity of the resonance energy. One finds $N_0=1.73\times10^{-3}$ (MeV)$^{-1}$, $E_R=1236$ MeV, and $\Gamma_R=127$ MeV. Further, for given P, M, and p_F we write

$$\mathfrak{D}_{33}(\sqrt{s})=-N\left\{\sqrt{s}-E_R'+i\left[\left(\frac{N_0}{N}\right)\frac{\Gamma_R}{2}+\frac{\Gamma^{\downarrow}}{2}\right]\right\} \tag{6.2}$$

$$=-N\{\sqrt{s}-E_R'+\tfrac{1}{2}i\Gamma\}, \tag{6.3}$$

where $\Gamma\equiv(N_0/N)\Gamma_R+\Gamma^{\downarrow}$. This parametrization is motivated by the fact that the quantity $\Gamma^{\downarrow}$ provides a measure of the deviation from unitarity of the T matrix in the medium T_N. Indeed, if $\Gamma^{\downarrow}=0$, the effect of the medium may be expressed through a "coupling constant renormalization," expressible as $(\lambda/N_0)\to(\lambda/N)$. Clearly, E_R' is the energy of the resonance in the medium. We note that the parametrization given in Equations (6.2) and (6.3) and Table I is accurate only in the vicinity of the resonance. For values away from the resonance energy one may refer to Figs. 5–7.

The real and imaginary parts of $\lambda[D(\sqrt{s})]^{-1}$ and $\lambda[\mathfrak{D}(\sqrt{s})]^{-1}$ are presented for the P_{11} and S_{31} channels in Figs. 8 and 9. We note rather large changes in the P_{11} channels, but we feel that our results in this channel may be less reliable than in other channels. The presence of a resonance at about 1.5 GeV in the P_{11} channel and the shape of the form factor in this channel (a relatively narrow peak at $k\sim550$ MeV/c) make the calcula-

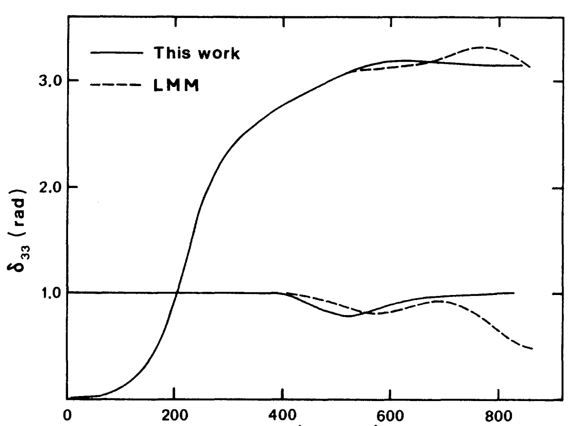

FIG. 3. The phase shifts δ and inelasticity parameters η for our model and that of Ref. 4 (LMM). The latter model provides an accurate fit to the experimentally determined π-N phase shifts.

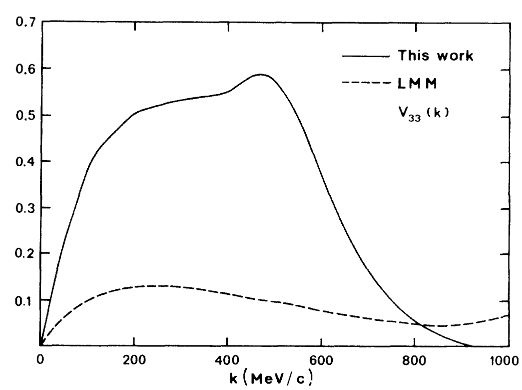

FIG. 4. Form factors $v_{33}(k)$ for the model of this work compared to that of Ref. 4 (LMM), given in units of m_π^{-1}.

TABLE I. Parameters of the (3,3) resonance in the nuclear medium, tabulated quantities are given in MeV. Free space values: $E_R = 1236$ MeV, $N_0 = 1.73\times10^{-3}$ (MeV)$^{-1}$, $\Gamma_R = 127$ MeV.

$P\left(\frac{\text{MeV}}{c}\right)$		$p_F=1.36$ fm^{-1} $M=\frac{3}{2}$	$M=\frac{1}{2}$	$p_F=1.08$ fm^{-1} $M=\frac{3}{2}$	$M=\frac{1}{2}$	$p_F=0.86$ fm^{-1} $M=\frac{3}{2}$	$M=\frac{1}{2}$
130							
	E_R'	1249	1252	1240	1240	1237	1236
	$N\times10^3$	1.66	1.58	1.79	1.79	1.78	1.83
	Γ	182	188	143	137	133	120
	$\Gamma^{\downarrow}$	50	49	20	14	9	0
207							
	E_R'	1252	1261	1242	1245	1239	1239
	$N\times10^3$	1.61	1.46	1.79	1.78	1.82	1.90
	Γ	193	203	143	135	129	113
	$\Gamma^{\downarrow}$	56	52	20	11	9	−3
375							
	E_R'	1259	1272	1246	1255	1242	1249
	$N\times10^3$	1.73	1.79	1.95	1.97	1.97	2.03
	Γ	196	160	138	120	127	111
	$\Gamma^{\downarrow}$	70	38	26	8	15	3

tion of the effects in this channel sensitive to the details of our self-energy calculations at higher energies, where the effects of inelasticity are quite important. Finally, we remark that we have only small effects in the S_{31} channel (Fig. 9) and in the P_{31} and S_{11} channels (not shown).

In the calculations we have described we have included a nucleon self-energy. This was represented by a real potential which was approximately −70 MeV for nucleon momentum $p=0$ and which then decreased linearly with energy, reaching a value of 0 at 350 MeV (nucleon energy). An imaginary potential of about −20 MeV was also included in the nucleon self-energy. These values were used for $p_F = 1.36$ fm^{-1}, and reduced by a factor $(p_F/1.36)^3$ when other values of p_F were considered. It was found that, except for the values of $\sqrt{s}$ near 1100–1150 MeV, the results were not significantly changed if the nucleon self-energy was neglected entirely.

In summary, the main feature of our results appears to be a damping of the resonance in the (3, 3) channel, that is, we have $\Gamma > \Gamma_R$. This effect is rather strongly dependent on the density of the medium.

We remark that our results are largely dominated by the effects of the pion self-energy. For example, we have investigated the effect of dropping the Pauli principle restriction from our mod-

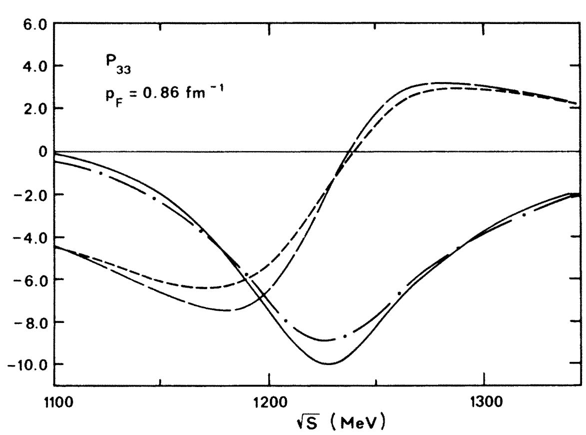

FIG. 5. The real and imaginary parts of $\lambda[L(\sqrt{s})]^{-1}$ and $\lambda[\mathfrak{D}(\sqrt{s})]^{-1}$ for the P_{33} channel calculated for $p_F = 0.86$ fm^{-1}, $P = 207$ MeV/c, and $M=\frac{3}{2}$: ——— ———, Re$\lambda[D(\sqrt{s})]^{-1}$; - - -, Re$\lambda[\mathfrak{D}(\sqrt{s})]^{-1}$; ——, Im$\lambda[D(\sqrt{s})]^{-1}$; —— · —— · ——, Im$\lambda[\mathfrak{D}(\sqrt{s})]^{-1}$.

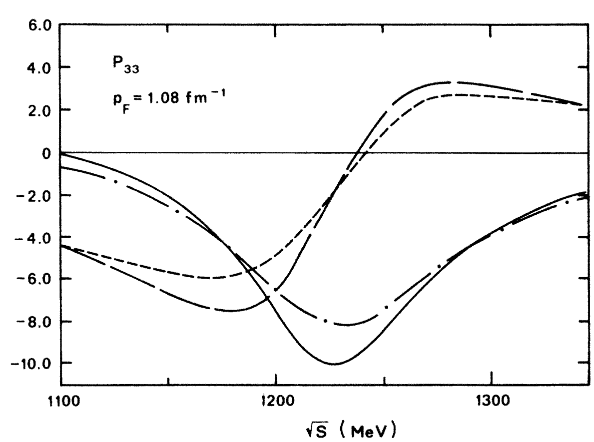

FIG. 6. Same as Fig. 5 except that $p_F = 1.08$ fm^{-1}.

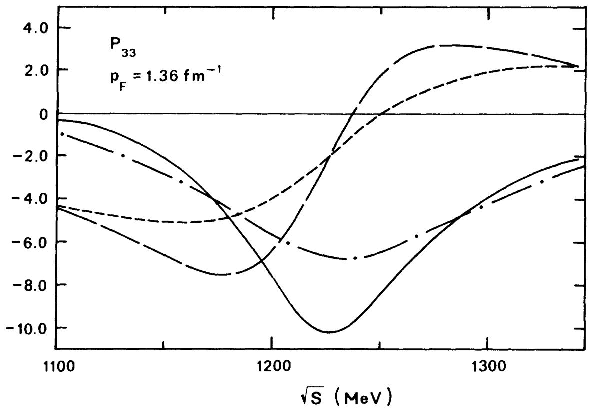

FIG. 7. Same as Fig. 5 except that $p_F = 1.36$ fm^{-1}.

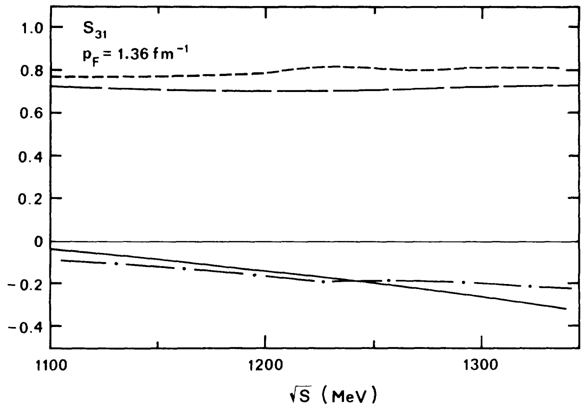

FIG. 9. Same as Fig. 8, but calculated for the S_{31} channel.

ified Green's function, and in Table II we present the values for $\lambda[\mathfrak{D}_{33}(\sqrt{s})]^{-1}$ as calculated with and without the function $\theta(|\vec{p}+\vec{k}-\vec{k}''|-p_F)$ in Eq. (2.33). As may be seen from this table, the results with and without the Pauli principle effects are similar.

A rigorous treatment of the self-energy insertions in the inelastic channels is complicated and beyond the scope of this work. We believe our results for those values of $\sqrt{s}$ where inelasticity is not particularly important ($\sqrt{s} < 1300$ MeV) are reliable and will not be altered significantly by a proper treatment of the self-energies pertaining to the inelastic channels.

If we consider the situation in finite nuclei, we face various complications. For example, since the pion-nucleus interaction is often a surface interaction [particularly in the (3,3) resonance region], it is difficult to specify the effective nuclear density at which the interaction should be calculated.

It is probably a good approximation to identify P with the laboratory momentum of the pion, a result which obtains if one averages over the Fermi motion of the nucleons in the target. This choice is less critical than the choice of p_F since our results are not strongly dependent on the value of P. Finally, we note that the calculation of the value of $\sqrt{s}$ for the π-nucleon collision should take into account the binding energy of the struck nucleon and its momentum. It can be shown that when these effects are taken into account the calculated value of $\sqrt{s}$ is approximately 40–50 MeV less than the value one would calculate using the usual fixed scatterer approximation for the nucleon.

VII. CONCLUSION

Since the pion-nucleon interaction is strongly energy dependent, it is quite possible that it is

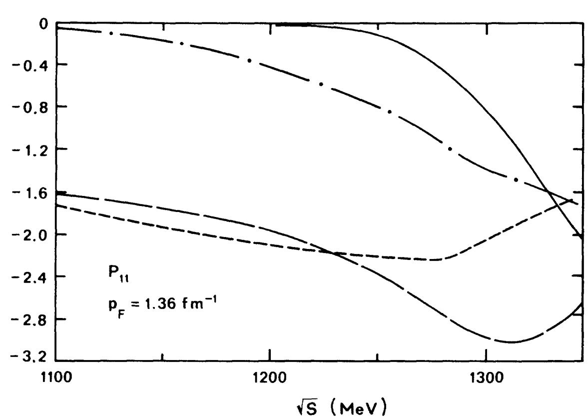

FIG. 8. The real and imaginary parts of $\lambda[D(\sqrt{s})]^{-1}$ and $\lambda[\mathfrak{D}(\sqrt{s})]^{-1}$ for the P_{11} channel calculated with $p_F = 1.36$ fm^{-1}, $P = 207$ MeV/c, and $M = \frac{1}{2}$. See caption of Fig. 5.

TABLE II. Effect of the Pauli principle restriction on the calculation of $\lambda_{33}[\mathfrak{D}_{33}(\sqrt{s})]^{-1}$ for $P = 207$ MeV/c and $p_F = 1.36$ fm^{-1}.

$\sqrt{s}$	$\lambda_{33}[\mathfrak{D}_{33}(\sqrt{s})]^{-1}$ With Pauli principle		$\lambda_{33}[\mathfrak{D}_{33}(\sqrt{s})]^{-1}$ Without Pauli principle	
(MeV)	Real part	Imag. part	Real part	Imag. part
1050	−3.49	−0.24	−3.59	−0.68
1070	−3.83	−0.46	−3.79	−0.94
1090	−4.23	−0.81	−4.01	−1.29
1110	−4.64	−1.32	−4.24	−1.75
1130	−4.91	−2.00	−4.39	−2.30
1150	−5.02	−2.84	−4.42	−2.98
1170	−4.93	−3.92	−4.28	−3.88
1190	−4.44	−4.05	−3.78	−4.81
1210	−3.32	−6.18	−2.77	−5.65
1230	−1.77	−6.69	−1.49	−5.96
1250	−0.15	−6.45	−0.23	−5.72
1270	1.02	−5.65	0.68	−5.13
1290	1.78	−4.61	1.36	−4.36
1310	2.06	−3.77	1.67	−3.71
1330	2.16	−2.98	1.87	−3.04
1350	2.12	−2.37	1.93	−2.49

necessary to obtain a satisfactory theory of the *effective* pion-nucleon interaction in the nucleus before good results may be obtained in the calculation of pion-nucleus scattering. In this work we have outlined such a theory, paying particular attention to the calculation of propagator modifications. As part of this program we have studied the pion self-energy in nuclear matter in the context of a covariant reduction scheme used previously.

As mentioned previously, the main feature of our calculations is a significant increase of the width of the (3, 3) resonance, the resulting effective T matrix exhibiting an important deviation from the unitarity condition $\mathrm{Im}\,T=-\pi|T|^2$. This decrease of the lifetime of the resonance in the medium (collision broadening) may lead to some difficulty in defining a useful isobar model for the study of the role of the (3, 3) resonance in nuclei.

As remarked previously, there are various complications in the application of our results to the calculation of the pion-nucleon interaction in finite nuclei. At this point, we hope that a study of the scattering of pions from nuclei at low energy may be sensitive to the details of the *effective* interaction. We hope to investigate this question in a future publication.

APPENDIX A

In this Appendix we introduce a form for $\tilde{d}$ which we expect will yield a better approximation in the case where the pion does not propagate so as to be close to its mass shell. Specifically, we put

$$\tilde{d}(k'',p+k-k'')=\left(\frac{m_N}{\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''}}\right)\Lambda^+(\vec{p}+\vec{k}-\vec{k}'')\delta(k''^0-E_s+\mathrm{Re}\,E_{N,\vec{p}+\vec{k}-\vec{k}''})$$
$$\times\,\theta(k''^0)\left[\frac{\theta(|\vec{p}+\vec{k}-\vec{k}''|-p_F)}{(E_s-E_{N,\vec{p}+\vec{k}-\vec{k}''})^2-\omega_{\vec{k}''}^2-\Pi(\vec{k}'',k''^0)+i\epsilon}-\frac{1}{(E_s-\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''})^2-\omega_{\vec{k}''}^2+i\epsilon}\right]. \tag{A1}$$

To the extent that the pion remains fairly close to its mass shell, i.e., $k''^0\simeq\omega_{\vec{k}''}$, we can show that Eq. (A1) reduces to Eq. (2.23), since

$$\begin{aligned}(E_s-E_{N,\vec{p}+\vec{k}-\vec{k}''})^2-\omega_{\vec{k}''}^2-\Pi(\vec{k}'',k''^0)+i\epsilon &\simeq[k''^0-\mathrm{Im}\,V_N(\vec{p}+\vec{k}-\vec{k}'')]^2-\omega_{\vec{k}''}^2-2\omega_{\vec{k}''}\Sigma^\pi(\vec{k}'')+i\epsilon\\ &\simeq 2\omega_{\vec{k}''}\left[\frac{(k''^0)^2-\omega_{\vec{k}''}^2-2k''^0\mathrm{Im}\,V_N(\vec{p}+\vec{k}-\vec{k}'')}{2\omega_{\vec{k}''}}-\Sigma^\pi(\vec{k}'')+i\epsilon\right]\\ &\simeq 2\omega_{\vec{k}''}[k''^0-\omega_{\vec{k}''}-\mathrm{Im}\,V_N(\vec{p}+\vec{k}-\vec{k}'')-\Sigma^\pi(\vec{k}'')+i\epsilon]\\ &\simeq 2\omega_{\vec{k}''}[E_s-E_{N,\vec{p}+\vec{k}-\vec{k}''}-\omega_{\vec{k}''}-\Sigma^\pi(\vec{k}'')+i\epsilon]\end{aligned} \tag{A2}$$

and

$$\begin{aligned}(E_s-\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''})^2-\omega_{\vec{k}''}^2+i\epsilon &=[k''^0+\mathrm{Re}\,V_N(\vec{p}+\vec{k}-\vec{k}'')]^2-\omega_{\vec{k}}^2+i\epsilon\\ &\simeq 2\omega_{\vec{k}''}\left[\frac{(k''^0-\omega_{\vec{k}''})(k''^0+\omega_{\vec{k}''})}{2\omega_{\vec{k}''}}+\frac{2k''^0}{2\omega_{\vec{k}''}}\mathrm{Re}\,V_N(\vec{p}+\vec{k}-\vec{k}'')+i\epsilon\right]\\ &\simeq 2\omega_{\vec{k}''}[k''^0-\omega_{\vec{k}''}+\mathrm{Re}\,V_N(\vec{p}+\vec{k}-\vec{k}'')+i\epsilon]\\ &\simeq 2\omega_{\vec{k}''}[E_s-\epsilon_{N,\vec{p}+\vec{k}-\vec{k}''}-\omega_{\vec{k}''}+i\epsilon].\end{aligned} \tag{A3}$$

In general we expect that Eq. (A1) will yield a somewhat better approximation than Eq. (2.23).

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