Resonance spectroscopy of ^{30}Si nucleus in the excitation energy range 14.27 to 15.02 MeV†

L. V. Namjoshi, S. K. Gupta, M. K. Mehta, and S. S. Kerekatte
Van de Graaff Laboratory, Nuclear Physics Division, Bhabha Atomic Research Centre, Bombay 400 085, India
(Received 22 July 1975)

Absolute differential cross sections for the elastic scattering of α particles from ^{26}Mg have been measured as a function of bombarding energy between 4.18 and 5.14 MeV at angles corresponding to the c.m. angles 90°, 125°, 141°, and 167°. The data have been analyzed using the multilevel multichannel expression of R matrix theory modified to replace the background hard sphere phase shifts by complex optical model phase shifts. These phase shifts have been obtained by fitting the averaged cross section with the optical model. This modification has resulted in reasonably good fits to excitation functions both in magnitude and shape. Twelve levels in the compound nucleus ^{30}Si have been identified and their spins, parities, and widths have been determined. The possibility of the 0^+ levels being fragmented components of a quartet parent state has been discussed.

[NUCLEAR REACTIONS ^{26}Mg(α,α) $E=4.18$–5.14 MeV; measured $\sigma(\theta,E)$, $\theta_{\text{c.m.}}=90°, 125°, 141°, 167°$; ^{30}Si levels, deduced $\Gamma_\alpha, \gamma_\alpha{}^2, J^\pi$; modified two channel-multilevel R matrix analysis.]

I. INTRODUCTION

In low energy nuclear reactions, the study of elastic scattering has proved to be an important tool to get information about the spectroscopy of the compound nucleus. α particle scattering from even-even nuclei offers additional simplicity because of the zero channel spin and because only natural parity levels having large α widths contribute to cross sections. The study of the reaction ^{26}Mg$(\alpha,\alpha)^{26}$Mg was undertaken to study the details of the reaction mechanism and to obtain information about the levels in ^{30}Si in the excitation energy range of 14–15 MeV.

Russell *et al.*[1] have studied the (α,α), (α,γ), and (α,n) reactions on ^{26}Mg from 2.5 to 4.3 MeV bombarding energy and have analyzed the elastic scattering data to yield spins, parities, and other parameters for a number of levels in ^{30}Si in the excitation energy range 12.8 to 14.4 MeV. We have measured the absolute differential elastic scattering cross section on ^{26}Mg at four angles from 4.18 to 5.14 MeV α particle bombarding energy. This has yielded information about levels in the compound nucleus ^{30}Si in the excitation energy range 14.3 to 15.0 MeV. Bair and Willard[2] have studied the levels of ^{30}Si through the reaction ^{26}Mg$(\alpha,n)^{29}$Si in the excitation energy range 13.3 to 15.3 MeV and have found a large number of close lying resonances.

Section II of this paper describes the details of the experimental arrangement and procedure. The data and data analysis using the multilevel multichannel expression of the R matrix theory,[3] modified to incorporate optical model phase shifts as background phase shifts, is described in Sec. III. The modification was necessary to get the fits to the magnitude of the cross section. In Sec. IV, the implications of the modification have been discussed and a possible explanation of the origin of 0^+ levels is considered.

II. EXPERIMENTAL PROCEDURE

The experiment was performed with a ^{4}He$^+$ beam obtained from the 5.5 MV Van de Graaff accelerator at Trombay. The energy analyzed beam was collimated to 2 mm diameter using a three element tantalum collimator placed at the entrance of the scattering chamber and was focused on the target situated at the centre of the chamber by means of a pair of quadrupole lenses. Surface barrier detectors were used to detect the scattered particles at laboratory angles 80°, 118°, 135°, and 165°. Preamplifiers and active filter amplifiers (model HA502A and model PA563, respectively) made by Electronics Corporation of India, Ltd. were used to amplify and shape the signals from the detectors. The amplified pulses were then fed to a 4096 channel PHA (Nuclear Data Inc.) through a homemade multiplexing unit enabling the spectra from different detectors to be stored in different parts of the memory of the analyzer. The incident beam was collected by means of a Faraday cup and was measured using a current integrator.[4]

Targets of about 5 keV thickness for 4 MeV α particles were prepared by vacuum evaporation of MgO (99% enriched in ^{26}Mg) on thin (~10 μg/cm^2) self-supporting carbon foils, in a silicon-free vacuum system. Special care was taken to avoid deposition of silicon on the target during the

experiment as the elastic peak from the silicon contaminant would interfere with that from ^{26}Mg at the forward angle and would introduce large errors in the yield measurement. The observed pulse height spectrum of scattered α particles at 81° is shown in Fig. 1.

The analyzing magnet was calibrated using the 3.612 MeV resonance in ^{26}Mg$(\alpha,\alpha)^{26}$Mg reaction observed by Russell *et al.*[1] At higher energies, the observed energies were corrected to account for the nonlinearity in the analyzing magnet arising because of the saturation of the magnetic core.[5] The absolute error in the α particle energy is estimated to be ±15 to ±20 keV in the energy range studied. The relative error from resonance to resonance is, however, less than 5 keV.

Excitation functions were measured at the laboratory angles mentioned above from 4.18 to 5.14 MeV in steps of 5 keV. Two runs were made for the excitation functions utilizing two separate targets. Reproducibility of the data at all angles was very good as indicated in Fig. 2, for two angles. To convert the yields to absolute cross sections, excitation functions at the same four angles were measured from 2.9 to 3.2 MeV in 5 keV steps, without disturbing the geometry. In this range the excitation functions were structureless and followed the $1/E^2$ dependence expected for the Rutherford scattering. The angular distribution at 3 MeV was also found to follow the $\mathrm{cosec}^4\frac{1}{2}\theta$ dependence. Thus assuming all the yield in this range to be due to Rutherford scattering, the normalizing factors (target thickness × solid angle for each detector) for the present measurement were determined and were utilized to convert the yields between 4.18 and 5.14 MeV to absolute cross sections. The error in the normalizing factors consisting of the target thickness, the solid angle, and the number of α particles incident, has been estimated from the departure of the measured cross sections in the range 2.95 to 3.2 MeV from Rutherford cross sections and is about ±5%. The error due to counting statistics is around ±5% on the average, going to ±20% at the minima in the excitation functions. The error on the measured absolute cross sections, therefore, is ±7% except at the minima in the excitation functions where it would go up to ±20%.

III. DATA AND ANALYSIS

Figure 3 shows the measured absolute differential cross sections in the center of mass system as a function of energy at the laboratory angles 81°, 118°, 135°, and 165°. These angles correspond to 90°, 125°, 141° and 167°, respectively,

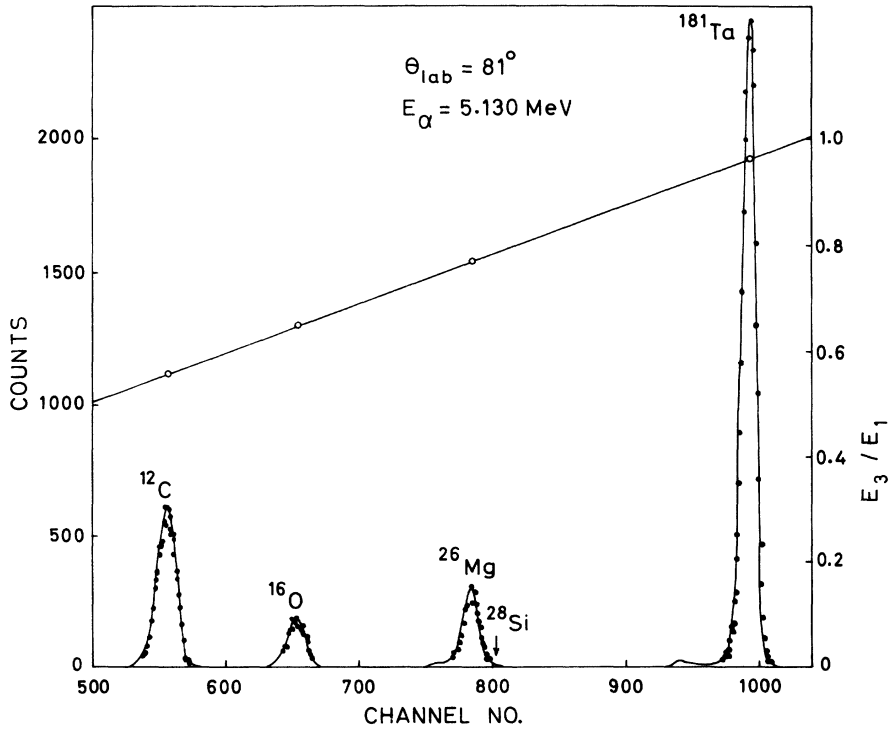

FIG. 1. Observed pulse height spectrum of scattered α particles at E_α = 5.130 MeV. The straight line is drawn according to the reaction kinematics to identify various peaks.

in the center of mass system. The first three of these are zeros of the Legendre polynomials of order 1, 2, and 3, respectively, and the last one is the backward-most angle possible in the scattering chamber. As is well known, this particular selection helps in assigning spin and parity values to the compound nuclear levels by mere inspection of the data from the vanishing of an anomaly at a particular angle.[6] All the anomalies will be clearly observed at 165° as interference due to the Rutherford scattering will be negligible at such a backward angle. This simple criterion is very useful in making the first guesses for the J^{π} values of the levels. It is evident from the data, however, that such single level interpretation of the observed anomalies would not be suitable and it would be necessary to carry out a multilevel multichannel R matrix calculation to extract the level parameters. The multilevel multichannel expression of the R matrix theory as given by Lane and Thomas[3] (expression No. 2.6 on page 292) has been coded in a computer program MULTI by Sellin.[7] This program was adapted to the computer BESM-6 at Bhabha Atomic Research Centre. To obtain initial values for input to MULTI, the shapes of the anomalies at the above angles due to various l values were worked out using a single level single channel R matrix program RMATRX .[8] These shapes are shown in Fig. 4. The values of the level parameters were adjusted by comparing the output from MULTI with the measured excitation functions till the shapes of all anomalies were reproduced. In our calculation only two channels, namely, (α, α) and (α, n) with zero channel spin for the neutron channel have been included. Hence the values of neutron widths would not be meaningful. The value of r_0 used in the calculation is 1.45 fm.

With this procedure it was possible to extract spin values and parity values and determine widths for 12 levels corresponding to the major observed anomalies in the excitation function. No effort was

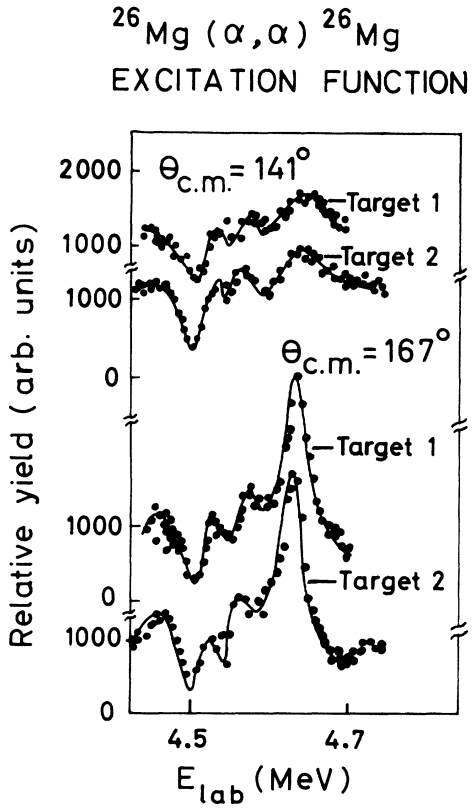

FIG. 2. Reproducibility of excitation functions in the reaction ^{26}Mg(α, α) using two different targets is shown at $\theta_{c.m.} = 141°$ and 167°.

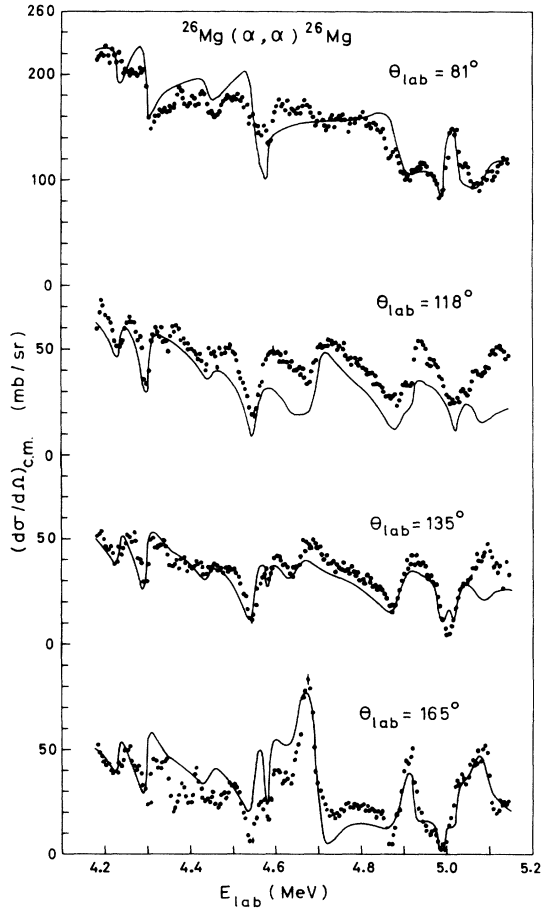

FIG. 3. Differential cross-sections in c.m. system for the reaction ^{26}Mg(α, α) from E_{α}=4.185 to 5.145 MeV at four laboratory angles. Dots are the experimental points. Solid lines are the fits to the experimental data with the multilevel two-channel calculation of the R matrix theory modified to incorporate optical model phase shifts in place of hard sphere phase shifts.

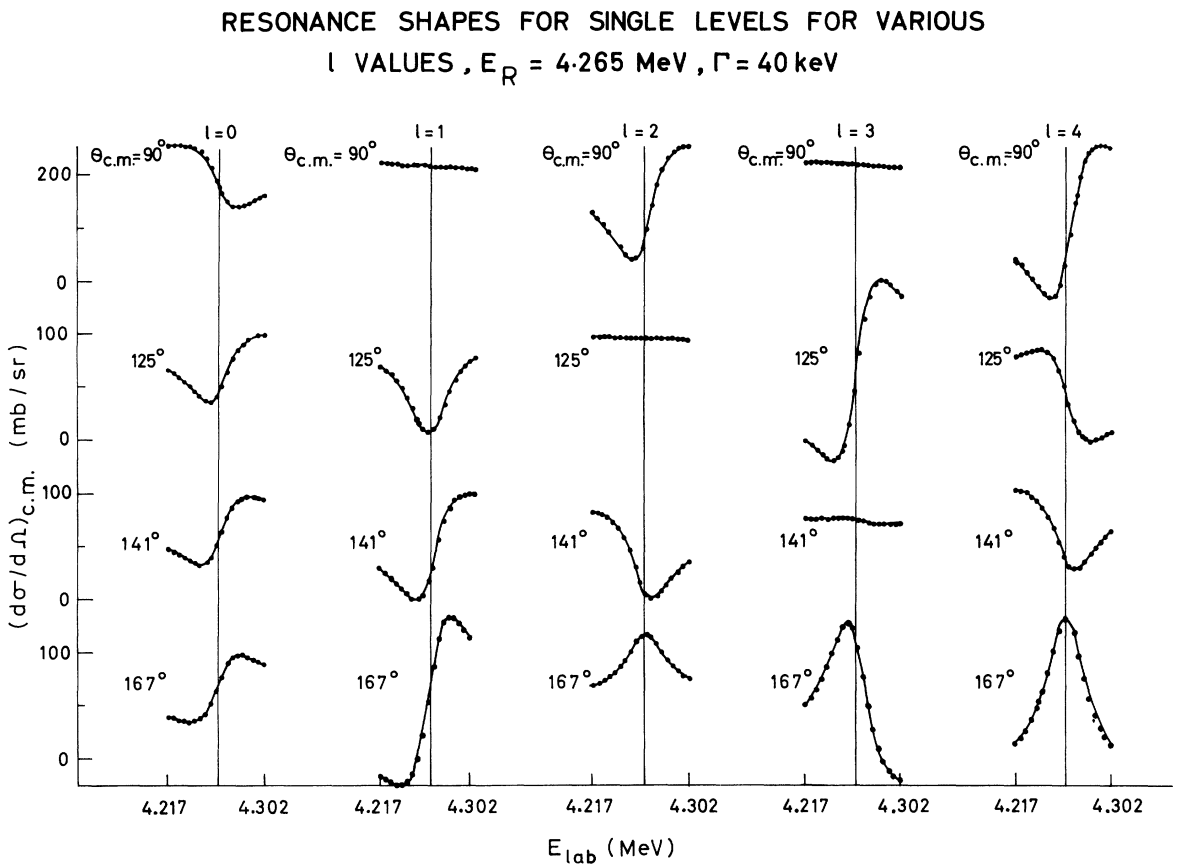

FIG. 4. Typical shapes of anomalies for ^{26}Mg(α, α)^{26}Mg with $l=0, 1, 2, 3$, and 4 at angles $\theta_{c.m.}=90°$, 125°, 141°, and 167° calculated using the single level single-single channel R matrix formula.

expanded in fitting minor deviations in shapes which may be due to the presence of a number of weak levels. Though the shapes of the major anomalies were well reproduced by MULTI, the magnitude of the cross sections was considerably different from the measured values especially at backward angles as illustrated in Fig. 5. It was realized that the discrepancy in the magnitude of cross section arises because of the unrealistic hard sphere phase shifts which appear as background phase shifts in the R matrix formalism. The formalism of the R matrix theory is rigorous, but the R matrix which connects the internal and the external region of configuration space involves an infinite summation over levels given by $R=\sum_\lambda \gamma_\lambda \times \gamma_\lambda/(E_\lambda - E)$. However, for its application to the experimentally observed data, the R matrix has to be approximated to contain only a finite number of terms. In the vicinity of a single isolated level, only one term of the series is supposed to dominate and the effect of all other levels is lumped into a constant term R_0. However, when a large number of levels are explicitly included,

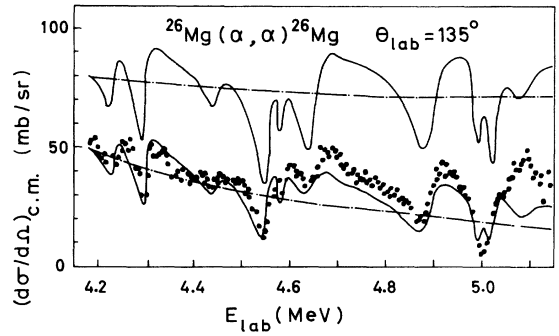

FIG. 5. Differential cross sections for ^{26}Mg(α, α) from 4.185 to 5.145 MeV at $\theta_{lab}=135°$. Dots are the experimental points. The upper continuous curve is a calculation using the conventional R matrix theory. The lower continuous curve is a calculation using R matrix theory modified to incorporate optical model phase shifts in place of hard sphere phase shifts. The upper and lower dot-dashed curves are the cross sections obtained in the absence of resonances in both the conventional R matrix theory and the modified R matrix theory, respectively. The latter is conventional optical model calculation.

R_0 should become redundant. In the present work as many as 19 levels were explicitly included (12 in the present excitation range and 7 from the work of Ref. 1 in the lower excitation range), and it was expected that these would be sufficient to account for the magnitude of the cross section also. The fact that this was not so, led us to conclude that all the nuclear scattering was not accounted for in terms of the large number of levels or in other words the R matrix does not converge fast enough.

To resolve the discrepancy, the scattering matrix expression was modified as follows: In the conventional R matrix theory the scattering matrix is given by

$$U_{CC'} = \Omega_C W_{CC'} \Omega_{C'} ,$$

where $\Omega_C = e^{i(w_C - \phi_C)}$, w_C being the Coulomb phase shift for the channel C and $-\phi_C$ is the hard sphere phase shift. The matrix W is given by

$$W_{CC'} = 1 + 2iP_C^{1/2}[(1 - RL^0)^{-1}]_{CC'} P_{C'}^{1/2} .$$

The external wave functions involved in this expression are Coulomb wave functions.

In the modified form, U is given by

$$U_{CC'} = \Omega'_C W_{CC'} \Omega'_{C'} ,$$

where $\Omega'_C = e^{i(w_C + \delta_C)}$ and δ_C is the optical model phase shift for the channel C as defined in Ref. 9, all other quantities being the same. The external wave functions involved in W_{CC} are still Coulomb wave functions.

To determine the correct complex phase shifts, the measured cross sections at all angles were averaged over 100 keV intervals and were fitted with the optical model using the computer program OPTIM 13 . A search was performed over the real and imaginary potentials starting with the values given by So, Mayer-Böricke, and Davis[10] for the ^{24}Mg$(\alpha, \alpha)^{24}$Mg reaction. The program MULTI was then modified to replace the hard sphere phase shifts by complex phase shifts determined by OPTIM 13 using the potential parameters obtained from the best fits to the averaged measured excitation functions. Introduction of these phase shifts did not alter much the calculated shapes of the anomalies, but the magnitudes of the anomalies were affected by this modification and required readjustment of the partial widths for the levels. The difference between the use of the hard sphere or optical model for background phase shifts is quite striking as illustrated in Fig. 5 where the upper and lower dot-dashed curves show the cross section as given by the background phase shifts only. With this modification it was possible to get absolute fits to the experimental data as shown in Fig. 3. Again no effort was made to account for the small deviations which may be due to the presence of many weak levels. The level parameters extracted from this analysis and the optical model parameters used are given in Tables I and II, respectively. As can be seen from Table II, the real potential depth was increased by 12 MeV over the value used by So *et al.*[10] and a surface imaginary potential has been used in place of the volume imaginary potential as has been done in Ref. 10.

IV. DISCUSSION

A. Comparison with previous work

Comparison of our results with those of Russell *et al.*[1] brings out two discrepancies. In Ref. 1 authors have reported only relative excitation functions and no absolute cross section is quoted. However, by normalizing their data at 3.2 MeV (their lowest energy) to Rutherford cross sections at all the four angles their excitation functions can be converted to cross sections and compared with our results in the region of overlap. However, this normalization yielded values of cross section at 4.2 MeV for all the four angles much larger than those measured in the present work. The authors[1] themselves have pointed out that their 81° data are expected to have very large errors due to the silicon contamination in the target. But the reason for the discrepancy at other angles cannot be understood. As described earlier, the present data were normalized to the Rutherford cross section between 2.9 and 3.1 MeV after establishing the fact that the cross sections in that range showed Rutherford behavior. Moreover, at 81° an extrapolation of the excitation functions beyond 3.1 MeV according to $1/E^2$ law is completely consistent with the averaged cross section at that angle between 4.2 and 5.2 MeV of the present work, implying that the 81° data are primarily given by the Rutherford cross section. The data between 3.1 and 4.2 MeV were not measured in this work because of the existence of the measurement of Russell *et al.*[1] The discrepancy brought out in the present work and its consistency with the data between 2.9 to 3.1 MeV points out the need for remeasurement of the differential cross sections between 3.1 to 4.2 MeV. Only when these are remeasured an absolute cross-section analysis similar to the one done in the present work would be possible for the lower region of excitation function.

Another discrepancy between our data and the data of Russell *et al.*[1] is in the assignment of the spin to the level observed at 4.235 MeV. This level has been observed by them in both the reactions ^{26}Mg$(\alpha, \alpha)^{26}$Mg and ^{26}Mg$(\alpha, \gamma)^{30}$Si and has been assigned the spin 1^-. This assignment appears reasonable from their (α, γ) data, but their (α, α) data at 81° are not in good agreement with

TABLE I. Resonance parameters for the levels in ^{30}Si observed in the present work through the reaction ^{26}Mg(α, α)^{26}Mg. These values are determined through two-channel-multilevel analysis as discussed in the text. The values for Γ_n would not be accurate because only one channel spin for the neutron channel was included in the calculation. Levels observed by Bair and Willard (Ref. 2) in the reaction ^{26}Mg(α, n)^{29}Si are marked by asterisks.

Resonance number	E_α (MeV)	E_x (MeV)	J^π	Γ_α (keV)	Γ_n (keV)	γ_α^2 (MeV)
1*	4.235	14.320	0^+	5	12	4.61×10^{-3}
2*	4.300	14.376	0^+	12	8	9.93×10^{-3}
3	4.450	14.506	0^+	6	24	3.96×10^{-3}
4	4.555	14.597	0^+	30	5	1.73×10^{-2}
5*	4.590	14.627	2^+	1.8	8.2	2.38×10^{-3}
6*	4.645	14.675	1^-	12	28	7.91×10^{-3}
7*	4.695	14.718	3^-	15	30	2.22×10^{-2}
8*	4.900	14.895	0^+	40	20	1.55×10^{-2}
9	4.925	14.917	3^-	3	17	4.60×10^{-3}
10*	4.990	14.973	2^+	4	11	2.74×10^{-3}
11*	5.030	15.008	0^+	20	10	6.85×10^{-3}
12*	5.100	15.069	4^+	5	45	1.58×10^{-2}

the above assignment. Our data favor the assignment of 0^+ to the above level in contradiction to the (α, γ) data of Russell *et al.* It is likely that there are two close lying levels, one 0^+ and another 1^- with the 1^- having a large γ width and a small α width thus not appearing in the (α, α) data, while the 0^+ level has a large α width and a small γ width which makes it prominent in the elastic scattering data only.

B. 0^+ levels

An interesting finding of the present investigation is the presence of many 0^+ levels in ^{30}Si within 14–15 MeV excitation region. Reduced α widths of these levels show a distribution peaked around 14.7 MeV excitation as shown in Fig. 6, suggesting that these levels may be fragmented components of a strong 0^+ level. An estimate of the total strength can be obtained by taking the ratio of the reduced widths to the "Wigner limit" assuming the α particle to be a single entity. The sum of such strengths for all the 0^+ levels is 14% of the Wigner limit indicating a fairly large α particle strength. The large α particle strength and the spin and parity 0^+ makes such a level a good candidate to be classified as a level due to "quartet" excitation[11,12] as a strong four particle correlation is indicated. Satpathy, Schmid, and Faessler[12] have carried out calculations for intrashell quartet excitation in s-d shell for $4n$ nuclides, but no such calculations are available for a $4n+2$ nuclide like ^{30}Si. However, if it is assumed that

TABLE II. Optical model parameters used in the analysis of ^{26}Mg(α, α)^{26}Mg data:

$V(r) = V_0 f_s(r) + i[w_s f_I(r) + w_d\, d f_I(r)/dr]$,

where

$f_x = \{1+\exp[(r-r_x)/A_x]\}^{-1}$.

V_0 = 122.2 MeV	R_c = 1.33 fm
R_s = 1.8 fm	A_s = 0.48 fm
w_s = 0.0 MeV	w_D = 4.0 MeV
R_I = 1.8 fm	A_I = 0.48 fm

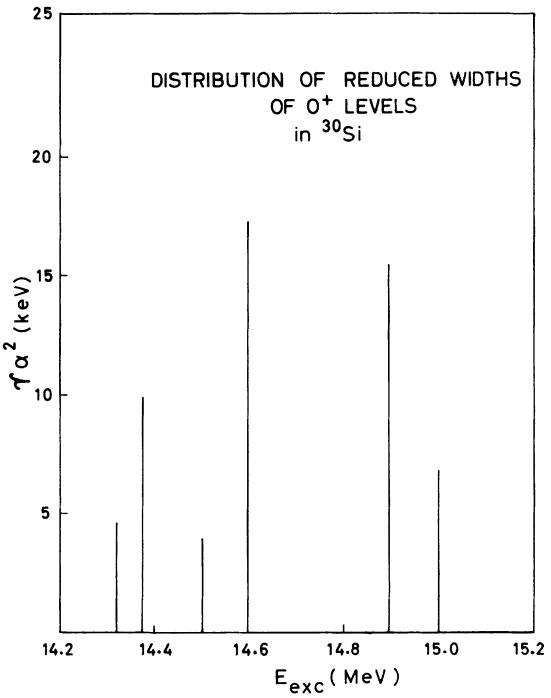

FIG. 6. Distribution of reduced α widths of the 0^+ levels in ^{30}Si as a function of excitation energy. The distribution is peaked around 14.7 MeV excitation.

the extra two neutrons do not play a significant role in the quartet excitation and only affect the fragmentation of the total strength, the excitation energy of 14.7 MeV in ^{30}Si of such a state would be consistent qualitatively with the calculations of Ref. 12 which predict the presence of intrashell quartet states at 11.4, 13.4, and 18.1 MeV. Rigorous calculations are, however, necessary before making any more comparison; e.g. an explicit calculation of reduced widths based on the quartet model would be very useful.

C. Modification of the *R* matrix theory

The investigation of the reaction ^{26}Mg$(\alpha, \alpha)^{26}$Mg has clearly brought out the well known drawbacks of the conventional *R* matrix theory from the point of view of its application. The fact that conventional *R* matrix theory using the hard sphere phase shifts as background was not successful in predicting the absolute value of the cross sections underscores the already known unphysical role of the hard sphere phase shifts. The modification of the conventional *R* matrix analysis brought about by substituting optical model phase shifts for the hard sphere phase shifts has been successful in predicting the absolute magnitude of the cross sections correctly. This modification may appear to be *ad hoc*. However, it can be justified on the basis of the *K* matrix theory based on the shell model approach to nuclear reactions.[13] It has been shown by Mahaux and Weidenmüller[13-15] that the scattering matrix expressions arrived at in both the theories are analogous in form; the difference being that the expression involving *K* matrix involves only a finite number of terms and optical model phase shifts appear naturally in the *K* matrix expression in place of hard sphere phase shifts in the *R* matrix expression. The analogous quantities in the two expressions have similar energy dependence in the absence of a single particle resonance in the continuum. Thus the present analysis using the modified *R* matrix expression is effectively the *K* matrix analysis. The parameters obtained by such an analysis can be interpreted using either of the two theories. Recently, Cugnon[16] has also come to a conclusion similar to ours on the basis of a theoretical analysis of the *R* matrix theory using the projection operator formalism. He finds that hard sphere phase shifts are just mathematical in nature and can be replaced by any other type of phase shifts. Gupta[17] has further shown that the results obtained by Cugnon can also be derived within the standard *R* matrix theory. He has also given an expression for the collision matrix *U* on the basis of the standard *R* matrix theory similar to the modified expression given above.

ACKNOWLEDGMENTS

The authors wish to thank Mr. C. V. Fernandes, Mr. V. Tambwekar, and Mr. S. Saini for their help in the experimental preparations and data taking. The authors are thankful to Mr. Arvind Jain for his help in adapting the computer program MULTI to the computer BESM-6 at Bhabha Atomic Research Centre. The help of Mrs. Sheela Mukherjee in running the computer program OPTIM 13 is duly acknowledged. The cooperation of the Van de Graaff crew in efficient operation of the machine is greatly appreciated. One of the authors (Mrs. L.V. Namjoshi) thanks National Council of Educational Research & Training, India for the award of a scholarship during the course of this work.

---

†This work is based on a thesis submitted to the University of Bombay for the award of the Ph.D. degree by L. V. Namjoshi.

[1]J. P. Russell, W. E. Taylor, F. E. Dunnam, and H. A. Van Rinsvelt, Nucl. Phys. A187, 449 (1972).
[2]J. K. Bair and H. B. Willard, Phys. Rev. 128, 299 (1962).
[3]A. M. Lane and R. G. Thomas, Rev. Mod. Phys. 30, 257 (1958).
[4]S. K. Gupta, Nucl. Instrum. Methods, 60, 323 (1968).
[5]M. Balakrishnan (private communication).
[6]J. R. Cameron, Phys. Rev. 90, 839 (1953).
[7]D. L. Sellin (private communication).
[8]M. Balakrishnan, Bhabha Atomic Research Centre Report No. B.A.R.C.-663 1973 (unpublished).
[9]P. E. Hodgson, *Nuclear Reactions and Nuclear Structure* (Clarendon, Oxford, 1971).
[10]S. S. So, C. Mayer-Böricke, and R. H. Davis, Nucl. Phys. 84, 641 (1966).
[11]A. Arima, V. Gillet, and J. Ginocchio, Phys. Rev. Lett. 25, 1043 (1970).
[12]L. Satpathy, K. W. Schmid, and A. Faessler, Phys. Rev. Lett. 28, 832 (1972).
[13]C. Mahaux, Ann. Phys. (N.Y.) 32, 259 (1965).
[14]C. Mahaux and H. A. Weidenmüller, Nucl. Phys. A97, 378 (1967).
[15]C. Mahaux and H. A. Weidenmüller, *Shell Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, London, 1969).
[16]J. Cugnon, Phys. Rev. C 11, 291 (1975).
[17]S. K. Gupta, Phys. Rev. C (to be published).