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Polarization transfer in p-^{3}He elastic scattering

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Angular distributions of the polarization transfer coefficients $K_x^{x'}$, $K_z^{x'}$, and $K_y^{y'}$ (Wolfenstein parameters R, A, and D) were measured at 16.2 MeV for ^{3}He($\vec{p},\vec{p}$) elastic scattering. The angular range covered was 30° to 90° (lab) in 10° steps. The experiment used a polarized proton beam of 150 to 200 nA, a liquid-nitrogen cooled ^{3}He gas target, and a ^{4}He-filled proton polarimeter. The transfer coefficients are well described by an R-matrix analysis of the p-^{3}He system.

[NUCLEAR REACTIONS ^{3}He(p,p): E = 16.2 MeV, θ = 30–90° (10° steps); measured polarization transfer coefficients.]

I. INTRODUCTION

More accurate data over wider energy ranges are continually being added to our knowledge of few-nucleon systems. There is a definite need to correlate these data in order to determine a consistent data base, to predict observables where data does not exist, and to point the way to new experiments. Such analyses are underway in this laboratory by Dodder and Hale[1] in the form of multilevel, multichannel R-matrix calculations on the four-, five-, six-, and seven-nucleon systems. The addition of polarization transfer data proved to be useful in the analysis of the five-nucleon system,[2] and the present experiment was intended to add in a similar way to the four-nucleon system analysis.

In addition to differential cross sections and analyzing powers in four-nucleon reactions,[3] angular distributions of polarization transfer coefficients have been obtained for ^{2}H($\vec{d},\vec{n}$)^{3}He,[4] ^{2}H($\vec{d},\vec{p}$)^{3}H,[5] and ^{3}H($\vec{p},\vec{n}$)^{3}He,[6] and spin correlation coefficients for $^3\vec{\mathrm{He}}$($\vec{p},p$)^{3}He.[7,8] In order to fix the $T=1$ levels in ^{4}Li, it was decided to parametrize the p-^{3}He channel via the single-channel R-matrix formalism before attacking the multichannel case. The present experiment was intended to contribute to this analysis[9] by providing further constraints on the scattering matrix at one energy. We chose the bombarding energy of 16.2 MeV for this experiment because analyzing power data for both polarized beam[10] and polarized target[11] and precise differential cross section data[12] were available at this energy.

II. FORMALISM

The equations for polarization transfer in a reaction with spin structure $\vec{\tfrac{1}{2}}+A \to \vec{\tfrac{1}{2}}+B$ (the spins of A and B are arbitrary) were first written down by Wolfenstein.[13] These equations are directly applicable to this experiment, but we adopt the more modern notation of Refs. 14 and 15. In this notation the outgoing spin $\frac{1}{2}$ polarizations are given by

$$p_{x'}(\theta)I(\theta) = I_0(\theta)[\,p_x K_x^{x'}(\theta) + p_z K_z^{x'}(\theta)]\,, \tag{1}$$

$$p_{y'}(\theta)I(\theta) = I_0(\theta)[P^{y'}(\theta) + p_y K_y^{y'}(\theta)]\,, \tag{2}$$

$$p_{z'}(\theta)I(\theta) = I_0(\theta)[\,p_x K_x^{z'}(\theta) + p_z K_z^{z'}(\theta)]\,, \tag{3}$$

where the differential cross section $I(\theta)$ is related to the unpolarized differential cross section $I_0(\theta)$ by

$$I(\theta) = I_0(\theta)[1 + p_y A_y(\theta)]\,. \tag{4}$$

$A_y(\theta)$ is the analyzing power for the reaction and $P^{y'}(\theta)$ is the polarization function. The coordinate systems used are shown in Fig. 1. The projectile helicity frame is described by the coordinate axes x, y, and z and the outgoing particle laboratory helicity frame is described by the coordinate axes x', y', and z'. The z and z' axes are along the incoming particle momentum ($\vec{k}_{\mathrm{in}}$) and the outgoing momentum ($\vec{k}_{\mathrm{out}}$) respectively, while y and y' are along $\vec{k}_{\mathrm{in}} \times \vec{k}_{\mathrm{out}}$. The p_i in Eqs. (1)–(4) represent

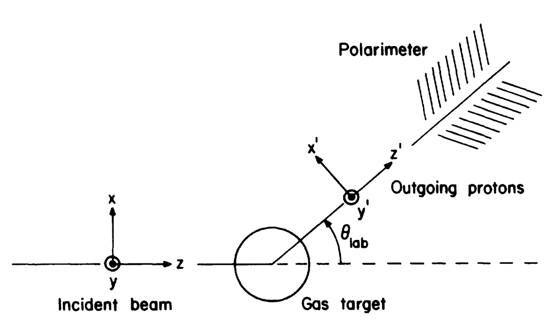

FIG. 1. Illustration of laboratory coordinate systems described in the text.

the beam polarization, while the $p_{i'}$ are the outgoing spin $\frac{1}{2}$ polarizations in their respective reference frames. Although y and y' are identical, we retain the prime in order to differentiate between incoming and outgoing quantities in the primary reaction. $K_i^{j'}(\theta)$ are Cartesian polarization transfer coefficients.[15]

The $K_i^{j'}$ are related to the Wolfenstein parameters as follows:

$$K_x^{x'}=R, \quad K_z^{x'}=A, \quad K_y^{y'}=D,$$
$$K_x^{z'}=R', \quad K_z^{z'}=A'. \tag{5}$$

For elastic scattering we have the relation

$$P^{y'}=A_y, \tag{6}$$

which follows from time reversal invariance.

III. EXPERIMENT

A. Beam and target characteristics

A 2.5 cm diam gas cell covered with 6.3 μm thick Havar[16] foil was suspended in the center of a scattering chamber by a liquid-nitrogen filled cryostat. The cell was cooled to 77 K and pressurized with 99.5% pure ^{3}He to 4.8 atm. The proton bombarding energy of 16.45 MeV gave a mean energy of 16.23 ± 0.12 MeV in the gas cell.

The ^{4}He gas-filled polarimeter used to determine the scattered proton polarization has been described previously.[17] For this experiment it was mounted in the scattering chamber with the front aperture 5.0 cm from the center of the primary target. Square apertures 2.2 mm on a side defined the primary scattering angle to ±2.5° full width at half maximum. After entering the polarimeter through a 12.7 μm thick Havar foil and a 100 μm thick silicon passing detector, the protons were scattered from 20 atm of ^{4}He gas into 1×5 cm side detectors. A series of vanes defined the secondary scattering angle to $60\pm7.5°$ where the analyzing power of ^{4}He is about −0.6 over a broad energy range. The calibration of the polarimeter analyzing power is described in Ref. 17.

The polarized proton beam was produced in the Los Alamos Lamb-shift polarized ion source and accelerated by the tandem Van de Graaff accelerator. A spin precessor between the source and the accelerator allowed orientation of the proton spins in the three directions required for the measurements. The beam polarization was determined several times during each run by inserting a Faraday cup at the high-energy end of the accelerator and measuring the ratio of the beam produced in the $m_I=+\frac{1}{2}$ state by the nuclear spin filter[18] to the unpolarized background. This "quench-ratio" method is accurate to better than 1% for determining the proton polarization.[19] The average beam current on target during the experiment was 190 nA with 88% polarization.

B. Preliminary checks

Several checks were made on the polarized beam and experimental setup prior to starting the ^{3}He($\vec{p},\vec{p}$) runs. For the determination of $K_x^{x'}$ and $K_z^{x'}$ the incident beam polarization must be precisely aligned along the x or z axis, respectively. The polarized source spin precessor had been calibrated several times, but we felt it desirable to check the calibration on the same beam line and under the specific conditions of this experiment. To calibrate the precessor, we used a cubical scattering chamber[20] directly behind the reaction chamber containing the polarimeter. The "cube" was filled to 200 Torr with ^{4}He and detectors were placed left, right, up, and down at $\theta_{\rm lab}=112°$ where the analyzing power of ^{4}He($\vec{p},p$) is large. Four sets of data were obtained in which the spin was precessed in the vertical plane through 90° and 180° and in the horizontal plane through 0° and 180°. Figure 2 shows typical data obtained as the

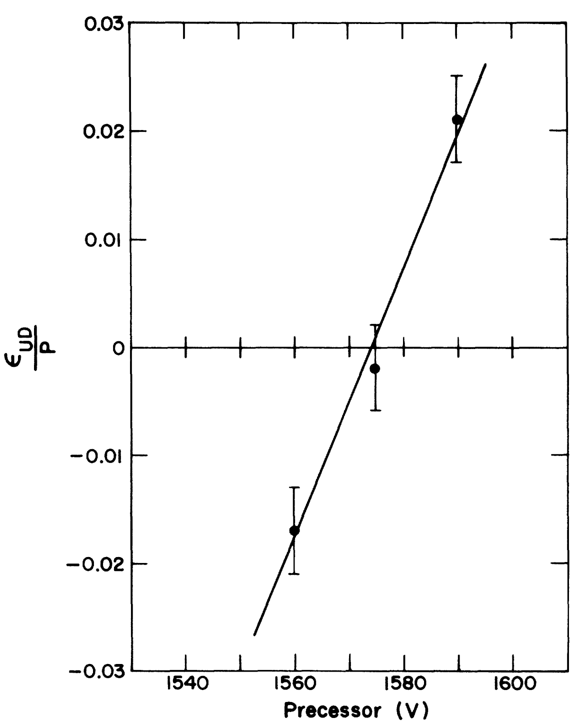

FIG. 2. Typical results are shown for the preliminary check described in Sec. III B to calibrate the spin precessor. These experimental results show that a precessor voltage of 1574 ± 5 V aligns the polarization with the z axis in Fig. 1 to ±0.3°.

spin was precessed through 0° in the horizontal plane. The "up-down" asymmetry (determined by two runs with the spin direction flipped at the ion source between runs) divided by the beam polarization is plotted versus the voltage calibration on the spin precessor, corresponding to 15 V per degree. A precessor setting of 1574 ± 5 V then corresponds to a spin orientation of $0.0 \pm 0.3°$ and a maximum ratio of p_x to p_z of 0.006. This ensures that errors in the determinations of $K_x^{x'}$ and $K_z^{x'}$ due to spin misalignment would be negligible.

A second check prior to starting the ${}^3\mathrm{He}(\vec{p},\vec{p})$ runs was to measure $K_y^{y'}$ for ${}^4\mathrm{He}(\vec{p},\vec{p})$ under identical experimental conditions. Since ^{4}He is a spin-zero nucleus, $K_y^{y'}$ must be unity.[13,15] Experimental runs at $\theta_{\mathrm{lab}} = 30°$ and 40° gave this result within the statistical errors.

C. Data acquisition and reduction

For the incident proton spin in the x or z direction, Eqs. (1) and (4) show that the outgoing proton polarization in the x' direction is directly related to the beam polarization; thus a simple reversal of the incident spin direction reverses the polarization measured by the polarimeter. False asymmetries can easily be canceled by reversing the incident spin between runs and taking a geometric average of the up-down detector counts to obtain $p_{x'}$. For example, if we denote the number of counts in the up and down polarimeter detectors obtained with the beam polarization in the $+z$ direction by U^+ and D^+ and with the incident spin in the $-z$ direction by U^- and D^-, we have from Eqs. (1) and (4) with $p_x = p_y = 0$,

$$K_z^{x'} = \frac{p_{x'}}{p_z} = \frac{1}{\bar{p}A_p}\left(\frac{r-1}{r+1}\right), \tag{7}$$

where

$$r = \left(\frac{D^+U^-}{U^+D^-}\right)^{1/2}. \tag{8}$$

A_p is the analyzing power of the polarimeter and $\bar{p}$ is the average beam polarization for the two runs. The data acquisition for $K_x^{x'}$ and $K_z^{x'}$ consisted of two series of such runs with the beam polarization along the x and z directions, respectively.

The procedure for measuring $K_y^{y'}$ is more difficult because $p_{y'}$ is not directly related to p_y; therefore reversing the beam polarization does not in general reverse the outgoing proton polarization. Equations relating $K_y^{y'}$ (or D) to observed second-scattered counting rates for the present spin structure have appeared in several references.[13,21] These simple expressions, however, are dependent on current integration to first order and do not cancel instrumental asymmetries in the analyzing detectors. The methods we shall derive below are extensions of the ratio technique, which has long been used for polarization and analyzing power measurements[20] and has been extended to polarization transfer[22] for spin structures other than the one we are considering here. Alternative methods may be applicable when a quadrupole triplet[2] or a spin precession solenoid[6] is used between the primary scattering and the polarimeter.

For the polarization of the incident beam along the y axis, the outgoing polarization in the first scattering is, from Eqs. (2) and (4),

$$p_{y'} = \frac{P^{y'} + p_y K_y^{y'}}{1 + p_y A_y}. \tag{9}$$

The second scattering (in the polarimeter) is characterized by the analyzing power A_p and second-scattered intensity given by

$$I' = I_0'[1 + p_{y'}A_p] \tag{10}$$

in a notation analogous to Eq. (4). If we place the polarimeter on the "left side" of the scattering chamber [subscript (l)] and take two runs with the beam polarization alternately in the $+y$ and $-y$ directions (superscripts + and −), the yields in the left and right analyzing detectors (L and R) are proportional to $I(\theta_1)I(\theta_2)$ which from Eqs. (4), (9), and (10) can be written:

$$\begin{aligned}
L_{(l)}^{+} &= n^{+}\Omega_L I_0 I_0'[1 + \bar{p}A_y + (P^{y'} + \bar{p}K_y^{y'})A_p],\\
R_{(l)}^{+} &= n^{+}\Omega_R I_0 I_0'[1 + \bar{p}A_y - (P^{y'} + \bar{p}K_y^{y'})A_p],\\
L_{(l)}^{-} &= n^{-}\Omega_L I_0 I_0'[1 - \bar{p}A_y + (P^{y'} - \bar{p}K_y^{y'})A_p],\\
R_{(l)}^{-} &= n^{-}\Omega_R I_0 I_0'[1 - \bar{p}A_y - (P^{y'} - \bar{p}K_y^{y'})A_p].
\end{aligned} \tag{11}$$

This constitutes a "proper flip" in the notation of Ref. 20, since only the beam polarization has been changed between runs. In these equations $\bar{p}$, the average beam polarization for the runs, can be used without significant error, especially for the slow variations in beam polarization we observe. The factors n^+ and n^- combine current integration and target pressure factors for both primary and secondary scatterings for the two runs. Likewise Ω_L and Ω_R combine detector solid angle and efficiency factors for both scatterings for the two analyzing detectors. For purposes of the following calculations it is not necessary to separate these factors.

From these expressions one can obtain $K_y^{y'}$ in a way that is not dependent on any first-order false asymmetries if A_y and $P^{y'}$ are known from a previous experiment. (We assume the proper averages over geometry have been made.) We form the ratio

$$r = \left(\frac{L_{(l)}^{+}\,R_{(l)}^{-}}{R_{(l)}^{+}\,L_{(l)}^{-}}\right)^{1/2} \tag{12}$$

which is independent of the first four factors in Eqs. (11). From this ratio, after substituting the (known) values for $\bar{p}$, A_p, A_y, and $P^{y'}$, one can solve a quadratic equation for $K_y^{y'}$. We refer to this as the "quadratic method." If data have been obtained on both the left and right side of the scattering chamber, the two ratios can be properly combined and again a quadratic equation for $K_y^{y'}$ results.

Alternately, one can obtain $K_y^{y'}$ without prior knowledge of $P^{y'}$ if other assumptions are made. For the polarimeter positioned on the "right side" of the scattering chamber [subscript (r)] we can write expressions similar to Eq. (11) and, except for different n and Ω factors we have

$$\begin{aligned} L^+_{(l)} &= R^-_{(r)}\,, \\ R^+_{(l)} &= L^-_{(r)}\,, \\ L^-_{(l)} &= R^+_{(r)}\,, \\ R^-_{(l)} &= L^+_{(r)}\,. \end{aligned} \tag{13}$$

There are several alternatives to handling the data thus obtained. The most useful in the present case of elastic scattering, for which Eq. (6) applies, is to first assume that Ω_L and Ω_R do not change when the polarimeter is moved from one side of the scattering chamber to the other. (This is a "nonproper flip" in the notation of Ref. 20.) By defining the geometric means

$$\begin{aligned} L^+ &= (L^+_{(l)} R^-_{(r)})^{1/2}\,, \\ R^+ &= (R^+_{(l)} L^-_{(r)})^{1/2}\,, \\ L^- &= (L^-_{(l)} R^+_{(r)})^{1/2}\,, \\ R^- &= (R^-_{(l)} L^+_{(r)})^{1/2}\,, \end{aligned} \tag{14}$$

the following ratios can be formed which are independent of n, Ω, I_0, and I_0':

$$\epsilon^+ = \frac{L^+ - R^+}{L^+ + R^+} = \left(\frac{P^{y'} + \bar{p}K_y^{y'}}{1 + \bar{p}P^{y'}}\right) A_p\,, \tag{15}$$

$$\epsilon^- = \frac{L^- - R^-}{L^- + R^-} = \left(\frac{P^{y'} - \bar{p}K_y^{y'}}{1 - \bar{p}P^{y'}}\right) A_p\,. \tag{16}$$

Solving for $P^{y'}$ and $K_y^{y'}$ gives:

$$P^{y'} = \frac{\epsilon^+ + \epsilon^-}{2A_p - \bar{p}(\epsilon^+ - \epsilon^-)}\,, \tag{17}$$

$$K_y^{y'} = \frac{1}{\bar{p}}\left[\frac{(\epsilon^+ - \epsilon^-) + 2\bar{p}\epsilon^+\epsilon^-/A_p}{2A_p - \bar{p}(\epsilon^+ - \epsilon^-)}\right]. \tag{18}$$

We refer to this calculation as the "ratio method."

If the target chamber and polarimeter are well aligned, the major contribution to changes in Ω_L and Ω_R in moving from one side of the scattering chamber to the other is nonuniform illumination of the polarimeter from the slope of the differential cross section in the primary reaction. The false asymmetries from this effect are calculated in the Appendix and for our data give a maximum effect on $K_y^{y'}$ of 0.025.

In the present experiment we obtained data with the polarimeter on both the left and right sides of the scattering chamber and analyzed our findings using both of the above methods. For the quadratic method the A_y data of Tivol[10] at 16.2 MeV was interpolated for input as A_y and $P^{y'}$. For the ratio method, the false asymmetry calculated in the Appendix was applied to the data. The two calculated values of $K_y^{y'}$ for each angle agreed within the statistical error.

D. Results

The data are listed in Table I with errors that result from a quadratic combination of

(a) counting statistics,

(b) a 0.01 uncertainty assumed for the beam polarization measurements,

(c) a 0.01 uncertainty in the calibration of the polarimeter analyzing power, and

(d) for the $K_y^{y'}$ results only, a 0.01 uncertainty assigned to the interpolated analyzing power data of Ref. 10.

These last three uncertainties together increased the total error by typically 0.006; therefore, the errors listed are primarily from counting statistics.

IV. ANALYSIS AND CONCLUSIONS

Polarization transfer coefficients are sensitive to different scattering matrix elements than are other types of observables and are thus expected to provide new constraints in an analysis. An energy dependent analysis, however, also reduces ambiguities that exist in single-energy phase shift solutions. The analysis of Hale *et al.*[9] had made considerable progress in parametrizing the p-^{3}He system up to 19.5 MeV before the addition of polarization transfer data. It was not surprising therefore that only small shifts in the level parameters were necessary when the present data were included in the search.

The results of Hale's analysis for the polarization transfer coefficients are shown with our data in Fig. 3. Excellent agreement with $K_x^{x'}$ and $K_z^{x'}$ was obtained, with χ^2 per point of 0.3 and 0.9, respectively. The agreement with $K_y^{y'}$ was not as good, having a χ^2 per point of 2.4. It would be interesting to test the calculations at backward angles, but the large kinematic energy loss from scattering off a nucleus as light as ^{3}He makes this extremely difficult for a practical polarimeter. For the range of our measurements, however, the over-

TABLE I. Polarization transfer coefficients for ^{3}He $(\vec{p},\vec{p})\,^3$He at 16.2 MeV.

θ_{lab}	$\theta_{c.m.}$	$K_x^{x'}$	$K_z^{x'}$	$K_y^{y'}$
30.0	39.7	0.658 ±0.031	−0.554 ±0.035	0.880 ±0.027
40.0	52.6	0.544 ±0.031	−0.791 ±0.034	0.906 ±0.027
50.0	65.0	0.308 ±0.030	−0.864 ±0.032	0.899 ±0.031
60.0	77.0	0.022 ±0.033	−0.902 ±0.034	0.916 ±0.027
70.0	88.5	−0.279 ±0.035	−0.872 ±0.035	0.992 ±0.027
80.0	99.4	−0.589 ±0.042	−0.567 ±0.042	0.978 ±0.034
90.0	109.7	−0.736 ±0.058	0.026 ±0.067	0.738 ±0.040

all agreement with the calculations lends added confidence to the analysis of the p-^{3}He system. A paper discussing the complete R-matrix analysis and level parameters in ^{4}Li is in preparation.[1]

ACKNOWLEDGMENTS

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APPENDIX: FALSE ASYMMETRY CALCULATION

As polarization transfer experiments become more precise, it is important to be able to estimate errors that may enter into the calculation of the observables. In this Appendix we will show how one such error can arise and estimate its effect.

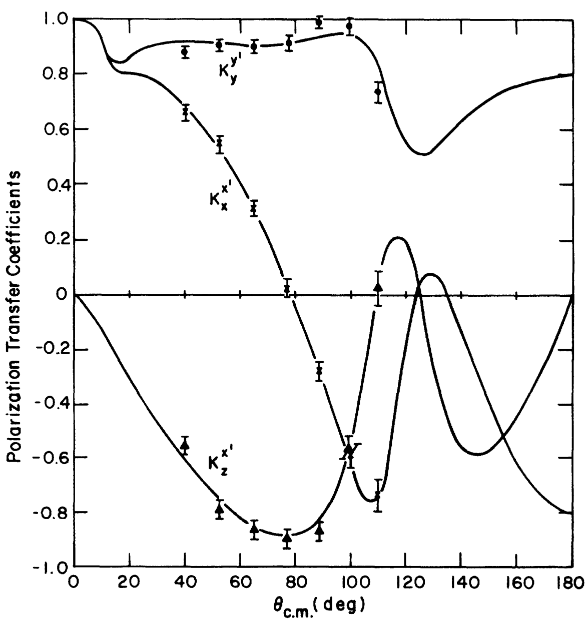

FIG. 3. The experimental results are plotted along with results from the R-matrix analysis of Ref. 9. The $K_y^{y'}$ data are denoted by circles, $K_x^{x'}$ by crosses, and $K_z^{x'}$ by triangles.

Following Ohlsen and Keaton,[20] we define a "proper flip" as a change in the incident beam polarization or the apparatus in such a way that the analyzing detectors remain fixed with respect to the beam position. This is relatively easy to do in a single-scattering experiment where the uniformity of the polarized beam and its position can be controlled. The methods used and the resulting cancellation of first order errors are discussed in Ref. 20.

For spin $\frac{1}{2}$ polarization transfer experiments it is sometimes possible to do a proper flip, as in the determination of $K_x^{x'}$ and $K_z^{x'}$ where a reversal of the incident beam polarization reverses the outgoing beam polarization. In the measurement of $K_y^{y'}$, however, we have shown in Sec. III C that the outgoing polarization is not reversed by reversing the incident beam polarization, and one must either input auxiliary information into the calculation or resort to a "nonproper flip" where the polarimeter analyzing detectors are moved with respect to the beam position. This can be either a rotation of the polarimeter or a movement of the polarimeter to the opposite side of the beam axis (i.e., a "right scattering" instead of a "left scattering"). For outgoing neutrons, a spin precession solenoid can be used, which is equivalent to a proper flip. Ohlsen[25] has discussed the general case.

Although we have shown in Sec. III C that it is possible to calculate $K_y^{y'}$ from a proper flip if one inputs $P^{y'}$ and A_y as determined in another experiment, we would like to investigate the false asymmetry arising from a nonproper flip for the following reasons:

(a) It gives an estimate of the size of the error that can result from improper experimental technique.

(b) In some cases $P^{y'}$ and A_y data may not be available at the energy and angles needed, and, in any case, an average over geometry must usually be taken.

(c) Except for elastic scattering where $P^{y'}$ can be inferred from an A_y experiment, a nonproper flip is usually used to determine $P^{y'}$, and thus false asymmetry corrections are required.

(d) It is useful to be able to calculate $K_y^{y'}$ by two different techniques to see the consistency of the raw data used in the calculation.

The false asymmetry ϵ' is the asymmetry that would be measured if the analyzing power of the polarimeter were to vanish. We define the sign such that an increase in the left detector count rate when the polarimeter is on the left side of the beam axis produces a positive ϵ'. Thus using Eq. (11) we can write, for example,

$$\begin{aligned} L^+_{(l)} &= n^+\Omega_L I_0 I_0'[1+\bar{p}A_y \\ &\quad + (P^{y'}+\bar{p}K_y^{y'})A_p]\ (1+\epsilon'), \\ R^+_{(l)} &= n^+\Omega_R I_0 I_0'[1+\bar{p}A_y \\ &\quad - (P^{y'}+\bar{p}K_y^{y'})A_p]\ (1-\epsilon'). \end{aligned} \tag{A1}$$

We now assume that Ω_L and Ω_R are the detector efficiency and solid angle factors that *remain with the detectors*, whereas ϵ' accounts for changes in these quantities due to nonuniform illumination of the polarimeter. Thus on the right side of the beam we have, for example, from Eq. (11) and (13):

$$\begin{aligned} R^-_{(r)} &= n^-\Omega_R I_0 I_0'[1+\bar{p}A_y \\ &\quad + (P^{y'}+\bar{p}K_y^{y'})A_p]\ (1+\epsilon'), \\ L^-_{(r)} &= n^-\Omega_L I_0 I_0'[1+\bar{p}A_y \\ &\quad - (P^{y'}+\bar{p}K_y^{y'})A_p]\ (1-\epsilon'). \end{aligned} \tag{A2}$$

In taking A_p as constant, we are assuming that variations in the polarimeter analyzing power due to small shifts in energy and angle are accounted for by the uncertainty of 0.01 used for A_p.

Forming the appropriate ratios as in Eqs. (14)–(16), we derive

$$\left(\frac{P^{y'}+\bar{p}K_y^{y'}}{1+\bar{p}P^{y'}}\right)A_p = \frac{\epsilon^+ - \epsilon'}{1-\epsilon^+\epsilon'} \tag{A3}$$

and

$$\left(\frac{P^{y'}-\bar{p}K_y^{y'}}{1-\bar{p}P^{y'}}\right)A_p = \frac{\epsilon^- - \epsilon'}{1-\epsilon^-\epsilon'}. \tag{A4}$$

Note that the same correction is made to both ϵ^+ and ϵ^-; i.e., the false asymmetry is independent of beam polarization to first order.

The expressions for $P^{y'}$ and $K_y^{y'}$ are exactly as in Eqs. (17) and (18), except we replace ϵ^+ and ϵ^- by the corrected values calculated via Eqs. (A3) and (A4). One result is immediately obvious. Whereas the expression for $P^{y'}$ has $\epsilon^+ + \epsilon^-$ in the numerator, resulting in a first order correction $2\epsilon'$, the numerator for $K_y^{y'}$ has $\epsilon^+ - \epsilon^-$, resulting in cancellation of the ϵ' terms to first order, and $\epsilon^-\epsilon^+$ which results in a correction proportional to the measured asymmetry. In general, therefore, the measurement of $K_y^{y'}$ is less susceptable to false asymmetry effects than is $P^{y'}$.

It remains to calculate ϵ' due to nonuniform illumination of the polarimeter. We start with the expression for ϵ' for a gas target as a function of linear and angular displacement of the beam, derived in Ref. 20. [Note: there is an error in Eq. (30) of this reference, which applies to gas target geometry. The correct first-order expression is given below:]

$$\epsilon' = \frac{x_0}{R}\frac{1}{\sin\theta_s} + k_x\cot\theta_s - G_s k_x, \tag{A5}$$

where x_0 is the linear displacement of the beam from the polarimeter axis, k_x is the angular displacement of the beam from the polarimeter axis, θ_s is the secondary scattering angle, G_s is the logarithmic derivative of the secondary scattering cross section, and R is the distance from polarimeter axis to rear aperture along θ_s.

We are using the subscript s to apply to the secondary scattering (in the polarimeter) in order to differentiate from primary scattering expressions which follow. The logarithmic derivative is defined by

$$G = \frac{1}{I_0}\frac{\partial I_0}{\partial\theta}. \tag{A6}$$

The first-order expressions for x_0 and k_x may be obtained by expanding the primary cross section to first order and integrating the properly weighted functions over the solid angle seen by the polarimeter. For the square apertures used in our polarimeter and a primary gas target the results are[23]

$$x_0 = G_p r_0 \psi_0^2/6, \tag{A7}$$

$$k_x = G_p \psi_0^2/6, \tag{A8}$$

where $\psi_0 = \tan^{-1}(2b/h)$, b is the half width of the polarimeter entrance apertures, h is the distance between polarimeter entrance apertures, r_0 is the distance from center of primary target to rear polarimeter aperture along θ_p, and G_p is the logarithmic derivative of the primary scattering cross section. Combining Eqs. (A7) and (A8) with (A5) we obtain

$$\epsilon' = \frac{G_p\psi_0^2}{6}\left[\frac{r_0}{R\sin\theta_s} + \cot\theta_s - G_s\right]. \tag{A9}$$

For the present experiment, the geometrical quantities are listed below along with "worst case" values of the logarithmic derivatives:

$r_0 = 7.5$ cm, $h = 2.5$ cm,

$b = 0.11$ cm, $\theta_s = 60°$,

$R = 2.0$ cm, $G_s = -3$ rad^{-1}, $G_p = -4$ rad^{-1}.

These numbers in Eq. (A9) give $\epsilon' = -0.04$. As an example, the differences in the calculated values obtained by applying this correction were 0.19 for $P^{y'}$ but only 0.02 for $K_y^{y'}$. As a check on the first-order approximations used in the calculation of ϵ' a numerical integration code[24] was used with the same input data. The false asymmetries calculated with this code were in general slightly less than the analytical results. For most of the $^3He(\vec{p},\vec{p})^3He$ data, the applied corrections reduced the differences between our calculated $P^{y'}$ and the A_y of Ref. 10 and also between the $K_y^{y'}$ values calculated by the two methods described in Sec. III C.

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[1]D. C. Dodder and G. M. Hale (private communication).

[2]R. A. Hardekopf, D. D. Armstrong, W. Gruebler, P. W. Keaton, Jr., and U. Meyer-Berkhout, Phys. Rev. C 8, 1629 (1973).

[3]For a list of references see S. Fiarman and W. E. Meyerhof, Nucl. Phys. A206, 1 (1973).

[4]G. C. Salzman, Gerald G. Ohlsen, J. C. Martin, J. J. Jarmer, and T. R. Donoghue, Nucl. Phys. A222, 512 (1974).

[5]W. Gruebler, R. A. Hardekopf, D. D. Armstrong, and P. W. Keaton, Jr., Nucl. Phys. A230, 353 (1974).

[6]R. C. Haight, J. E. Simmons, and T. R. Donoghue, Phys. Rev. C 5, 1826 (1972).

[7]S. D. Baker, T. A. Cahill, P. Catillon, J. Durand, and D. Garreta, Nucl. Phys. A160, 428 (1971).

[8]D. P. May, Stephen D. Baker, G. G. Ohlsen, and R. A. Hardekopf, Bull. Am. Phys. Soc. 18, 1395 (1973).

[9]G. M. Hale, J. J. Devaney, D. C. Dodder, and K. Witte, Bull. Am. Phys. Soc. 19, 506 (1974).

[10]W. F. Tivol, UCRL Report No. UCRL-18137 (unpublished).

[11]B. E. Watt and W. T. Leland (private communication).

[12]R. L. Hutson, N. Jarmie, J. L. Detch, Jr., and J. H. Jett, Phys. Rev. C 4, 17 (1971).

[13]L. Wolfenstein, Phys. Rev. 96, 1654 (1954).

[14]*Proceedings of the Third International Symposium on Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Wisconsin, 1971), p. xxv.

[15]G. G. Ohlsen, Rep. Prog. Phys. 35, 717 (1972).

[16]Havar is a high tensile strength material available from the Hamilton Watch Company, Lancaster, Pennsylvania.

[17]R. A. Hardekopf, D. D. Armstrong, and P. W. Keaton, Jr., Nucl. Instrum. Methods 114, 17 (1974).

[18]J. L. McKibben, G. P. Lawrence, and G. G. Ohlsen, Phys. Rev. Lett. 20, 1180 (1968).

[19]G. G. Ohlsen, J. L. McKibben, G. P. Lawrence, P. W. Keaton, Jr., and D. D. Armstrong, Phys. Rev. Lett. 27, 599 (1971).

[20]G. G. Ohlsen and P. W. Keaton, Jr., Nucl. Instrum. Methods 109, 41 (1973).

[21]P. Catillon, in *Proceedings of the Third International Symposium on Polarization Phenomena in Nuclear Reactions* (see Ref. 14), p. 657.

[22]G. C. Salzman, C. K. Mitchell, and G. G. Ohlsen, Nucl. Instrum. Methods 109, 61 (1973).

[23]P. W. Keaton, Jr., D. D. Armstrong, L. L. Catlin, and G. G. Ohlsen (unpublished).

[24]D. D. Armstrong (unpublished).

[25]G. G. Ohlsen, Proceedings of the Fourth International Symposium on Polarization Phenomena in Nuclear Reactions, Zurich, 25–29 August 1975 (Birkhäuser-Verlag, Basel, 1976, to be published).