Magnetic moment by hyperfine interaction and the lifetime of the 1^- state in ^{22}Na

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The magnetic moment of the 1^-, 2210 keV state in ^{22}Na has been measured by a plunger technique utilizing hyperfine interactions. Fields originating from $2s$ and $2p$ electrons were included. By comparison with previous ^{16}O data, population probabilities of the L subshells seem to be different from statistical. The adopted gyromagnetic factor is $|g| = 0.36 \pm 0.07$, the error representing mostly possible systematic uncertainties involved in the specific atomic model used. As a byproduct, the lifetime of the 1^-, 2210 keV ^{22}Na level was remeasured, yielding $\tau = 22.0 \pm 0.8$ ps.

[NUCLEAR REACTIONS ^{19}F$(\alpha, n\gamma)$, $E_\alpha = 5.17$ MeV; measured $\gamma(\theta, t)$, Doppler shift. ^{22}Na, 2210 keV level deduced $T_{1/2}$, $|g|$.]

I. INTRODUCTION

Utilization of hyperfine fields to determine electromagnetic moments of short-lived excited nuclear states is now extensive. It brought about development of new methods and experimental techniques as well as a better understanding of the atomic environment in matter and in vacuum.

The subject of hyperfine interactions in isolated atoms of light nuclei was recently reviewed by Goldberg.[1] In most experiments, time-integral and time-differential techniques have been employed in hydrogen-like systems. Of special interest, however, is the experiment done by Broude *et al.*[2] working in the region of "intermediate ionization." In this measurement the magnetic moment of the 3^- state in ^{16}O was determined by a time-differential method utilizing the hyperfine fields of the L electrons.

In the following report, a similar measurement on the 1^- state at 2.210 MeV in ^{22}Na is described. The general physical picture is closely related to that in ^{16}O, which was already discussed at length in Ref. 2. However, the data in the present measurement are analyzed in terms of a different approach resulting finally in the inclusion of fields associated with the $2p$ electrons as well.

In the course of the experiment, the mean life of the 1^- state was remeasured with a good accuracy.

II. HYPERFINE FIELDS

As was already indicated, the analysis of the experimental data was carried in the framework of the intermediate ionization model. According to this model a superposition of hyperfine static fields acts on the nucleus during the nuclear lifetime. These fields originate in the different electronic configurations that can be formed in the different ionic states following recoil from the target.

The charge state equilibrium fractions $F_{i\infty}$ of the Na ions of charge i were interpolated from the compiled values of Wittkower and Betz[3] for the recoil velocity of the present experiment ($v = 0.0083c$):

$$F_{1\infty} = 0.065, \quad F_{2\infty} = 0.275, \quad F_{3\infty} = 0.395,$$
$$F_{4\infty} = 0.235, \quad F_{5\infty} = 0.030.$$

It is unlikely, on grounds of velocity matching, that an electron can be found higher than in the L shell.[2] Each degree of ionization i may create spectroscopic terms based on three possible configurations: $(1s)^2(2s)^2(2p)^m$, $(1s)^2(2s)(2p)^{m+1}$, and $(1s)^2(2p)^{m+2}$, where $m = (Z-4) - i = 7 - i$.

Table I shows the possible terms that can be produced from the above configurations.[4] Each column, representating an ionic state of Na, lists the proper configurations and their related terms. The numbers in parentheses are values of the reduced matrix elements W_{12} discussed later.

Each term component is associated with a magnetic field H interacting with the nuclear magnetic moment, the hyperfine interaction being characterized by the interaction constant A_1.[5] The affected nuclei precess, giving rise to attenuation coefficients $G_k(t)$ in the γ-ray angular distribution[6]

$$W(\theta, t) = \sum_{0,2,4,\ldots} A_k P_k(\cos\theta) G_k(t), \tag{1}$$

$$G_k(t) = \sum_{FF'} \frac{(2F+1)(2F'+1)}{2J+1} \begin{Bmatrix} F & F & k \\ I & I & J \end{Bmatrix}^2 e^{-i\omega_{FF'}t}, \tag{2}$$

where $\omega_{FF'} = (E_F - E_{F'})/\hbar$ and (Ref. 7)

$$E_F = (-1)^{I+J+F} \begin{Bmatrix} F & J & I \\ 1 & I & J \end{Bmatrix} A_1 \Big/ \left[\begin{pmatrix} I & 1 & I \\ -I & 0 & I \end{pmatrix} \Big/ \begin{pmatrix} J & 1 & J \\ -J & 0 & J \end{pmatrix} \right] . \tag{3}$$

The value of A_1 is given by

$$A_1 = \frac{IJ}{[J(J+1)(J+2)]^{1/2}} \langle J \| T_e^1 \| J \rangle , \tag{4}$$

where

$$\begin{aligned} \langle J \| T_e^1 \| J \rangle = {} & \langle \gamma SLJ \| \sum_i l_i \| \gamma SLJ \rangle a_l \\ & - \sqrt{10} \langle \gamma SLJ \| \sum_i (sC^2)_i^1 \| \gamma SLJ \rangle a_{sC} \\ & + \langle \gamma SLJ \| \sum_i s_i \| \gamma SLJ \rangle a_s \delta_{l0} . \end{aligned} \tag{5}$$

The a values are given by

$$a_l = 2\beta\mu_N g \langle r_l^{-3} \rangle ,$$
$$a_{sC} = 2\beta\mu_N g \langle r_{sC}^{-3} \rangle ,$$
$$a_s = 2\beta\mu_N g \langle r_s^{-3} \rangle ,$$

where β is the Bohr magneton, μ_N the nuclear magneton, g the nuclear g factor, and $\langle r_s^{-3} \rangle = (8\pi/3) |\psi(0)|^2$.

The $\langle r^{-3} \rangle$ and $|\psi(0)|^2$ can be calculated using Hartree-Fock methods. The values adopted for the present case were calculated by Bauche.[8] For configurations involving identical electrons, the reduced matrix elements in Eq. (5) can be found in the literature.[7,9,10] For nonidentical electrons, we refer to the chapter on spherical tensors in Ref. 10. Let $\bar{L}, \bar{S}$ be the quantum numbers belonging to the l_c^N core, with l, s those belonging to the $l'(2s)$ electron. Let L, S, J be the total quantum numbers for the (l_c^N, l') configuration. The matrix elements in Eq. (5) are given by

$[X] = 2X + 1$:

$$\begin{aligned} & \langle \gamma SLJ \| \sum_i l_i \| \gamma SLJ \rangle = (-1)^{l+\bar{L}+S+J} [J][L] \begin{Bmatrix} J & 1 & J \\ L & S & L \end{Bmatrix} \begin{Bmatrix} L & 1 & L \\ \bar{L} & l & \bar{L} \end{Bmatrix} [\bar{L}(\bar{L}+1)(2\bar{L}+1)]^{1/2} , \\ & \langle \gamma SLJ \| \sum_i s_i \| \gamma SLJ \rangle = (-1)^{L+J+s+\bar{S}} [S][J] \begin{Bmatrix} J & 1 & J \\ S & L & S \end{Bmatrix} \begin{Bmatrix} S & 1 & S \\ s & \bar{S} & s \end{Bmatrix} [s(s+1)(2s+1)]^{1/2} , \\ & - \sqrt{10} \langle \gamma SLJ \| \sum_i (sC^2)_i^1 \| \gamma SLJ \rangle = (-1)^{s+\bar{S}+\bar{L}+S+L+l+l_c} \sqrt{2} [S][L][J] \begin{Bmatrix} S & 1 & S \\ \bar{S} & s & \bar{S} \end{Bmatrix} \begin{Bmatrix} L & 2 & L \\ \bar{L} & l & \bar{L} \end{Bmatrix} \begin{Bmatrix} S & S & 1 \\ L & L & 2 \\ J & J & 1 \end{Bmatrix} \\ & \qquad \times [s(s+1)(2s+1)]^{1/2} [l_c] \begin{pmatrix} l_c & 2 & l_c \\ 0 & 0 & 0 \end{pmatrix} \langle \gamma \bar{S} \bar{L} \| W_{12} \| \gamma \bar{S} \bar{L} \rangle \quad \text{with } s = \tfrac{1}{2} . \end{aligned} \tag{6}$$

TABLE I. The terms that can be produced from configurations belonging to various Na ions. Numbers in parentheses are values of W_{12} discussed later in the text.

Na^{1+}	Na^{2+}	Na^{3+}	Na^{4+}	Na^{5+}
$1s^2 2s^2 2p^6$	$1s^2 2s^2 2p^5$	$1s^2 2s^2 2p^4$	$1s^2 2s^2 2p^3$	$1s^2 2s^2 2p^2$
$^1S_0$ (0)	$^2P_{3/2,1/2}$ (3.873)	$^1S_0$ (0)	$^2P_{3/2,1/2}$ (3.873)	$^1S_0$ (0)
		$^1D_2$ (0)	$^2D_{5/2,3/2}$ (−5.918)	$^1D_2$ (0)
		$^3P_{2,1,0}$ (−3.873)	$^4S_{3/2}$ (0)	$^3P_{2,1,0}$ (−3.783)
		$1s^2 2s 2p^5$	$1s^2 2s 2p^4$	$1s^2 2s 2p^3$
		$^1P_1$ (3.873)	$^2S_{1/2}$ (0)	$^1P_1$ (3.873)
		$^3P_{2,1,0}$ (3.873)	$^2D_{5/2,3/2}$ (0)	$^3P_{2,1,0}$ (3.873)
			$^4P_{5/2,3/2,1/2}$ (−3.873)	$^1D_2$ (−5.918)
			$^2P_{3/2,1/2}$ (−3.873)	$^3D_{3,2,1}$ (−5.918)
				$^5S_2$ (0)
				$^3S_1$ (0)
		$1s^2 2p^6$	$1s^2 2p^5$	$1s^2 2p^4$
		$^1S_0$ (0)	$^2P_{3/2,1/2}$ (3.873)	$^1S_0$ (0)
				$^1D_2$ (0)
				$^3P_{2,1,0}$ (−3.873)

The values of the reduced matrix elements W_{12} can be calculated from fractional parentage coefficients[10] and are listed in Table I.

Table II gives the values of $\langle r^{-3}\rangle$, $\langle r^{-3}\rangle = \langle r_l{}^{-3}\rangle = \langle r_{sC}{}^{-3}\rangle$, and those of $|\psi(0)|^2$ which were introduced into the calculation. By using the relation

$$H_s(\mathrm{MG}) = 0.525\,|\psi(0)|^2 \tag{7}$$

one can estimate the nonprojected, or maximum, field that a $2s$ electron can generate at the nucleus. This field in Na is around 16 MG. It should be realized, however, that due to coupling with the p electrons, this field is projected and hence the effective field acting on the nucleus is attenuated.

III. TIME-DIFFERENTIAL MEASUREMENT

The determination of the g factor of the 1^- state is derived from the temporal behavior of the angular distribution of the decaying γ rays. The attenuation coefficient $G_k(t)$ in Eq. (1) is now, in view of the preceding section, a properly weighted superposition of attenuation coefficients:

$$G_k(t) = \sum_{i\,\alpha} P_{i\alpha}\, G_k^{i\alpha}(t). \tag{8}$$

$G_k^{i\alpha}(t)$ is the attenuation pertinent to the term α, $P_{i\alpha}$ being the fractional population of any term α within the charge state i. In the present calculation, following the measurement in ^{16}O,[2] we assume that the stripping process is independent of the quantum numbers within the L shell. The $P_{i\alpha}$ are thus proportional to the statistical weight $(2J+1)$. We shall comment on this assumption in the discussion.

The 1^- state in ^{22}Na was populated by the $^{19}F(\alpha, n\gamma)$ reaction at $E_\alpha = 5.170$ MeV which is just above threshold. This choice provides good kinematical conditions for a plunger-type time-differential measurement. A conventional plunger arrangement has been used, the stopper being a stretched 3 mg/cm^2 gold foil and the target consisting of a 1 mg/cm^2 gold backing supporting 50 μg/cm^2 CaF_2.

TABLE II. Values in atomic units of $\langle r^{-3}\rangle \simeq \langle r_l{}^{-3}\rangle \simeq \langle r_{sC}{}^{-3}\rangle$ and $|\psi(0)|^2$ that were introduced into the calculation.

Na charge state	$\lvert\psi 0\rvert^2$	$\langle r^{-3}\rangle$	Electronic configuration
8^+	38.34	...	$1s^2 2s^1$
7^+	35.83	32.21	$1s^2 2s^1 2p^1$
6^+	33.44	29.11	$1s^2 2s^1 2p^2$
5^+	31.20	26.25	$1s^2 2s^1 2p^3$
4^+	29.09	23.60	$1s^2 2s^1 2p^4$
3^+	27.18	21.17	$1s^2 2s^1 2p^5$
2^+	25.47	18.94	$1s^2 2s^1 2p^6$

The γ-ray transition takes place between the 1^- state and the 0^+ state at 656 keV. The transition, being of an $E1$ nature, simplifies Eq. (1) by terminating it with $k=2$. All the relevant attenuation is now in $G_2(t)$ in Eq. (8).

Two Ge(Li) detectors were placed at 0° and 125° [$P_2(\cos\theta)=0$]. Let N_θ be the measured stopped γ-ray intensity at the angle θ. We define

$$R(t) = \left[\frac{N_0}{N_{125}}(t)\right]_{\text{stopped}} \Big/ \left[\frac{N_0}{N_{125}}(t=0)\right]_{\text{stopped}}. \tag{9}$$

This ratio is independent of any efficiency and solid angle corrections related to the Ge(Li) detectors. It can be shown that

$$R(t) = \frac{1 + A_2 Q_2(\theta=0°)\, G_2(t)}{1 + A_2 Q_2(\theta=0°)},$$

where Q_2 is the geometrical attenuation coefficient of the detector positioned around 0°. From the above expression we obtain

$$G_2(t) = 1 + [R(t) - 1]\,\frac{1 + A_2 Q_2(\theta=0°)}{A_2 Q_2(\theta=0°)}. \tag{10}$$

The value of $A_2 Q_2$ was determined by performing an angular distribution measurement in which the recoiling Na nuclei were stopped in a 3 mg/cm^2 gold backing, assumed to be hyperfine field free. The mechanical asymmetries have been corrected by use of the isotropic 1.552 keV γ line emitted by the first 0^+ state in ^{30}Si which is fed by the reaction $^{27}Al(\alpha, p)^{30}Si$ at $E_\alpha = 3.90$ MeV.[11]

IV. RESULTS AND DISCUSSION

The lifetime of the 1^- state was determined from the Doppler shifted intensity I_S and the stopped peak I_0 in the detector positioned at 125°. Comparison of the shifted versus stopped lines of the 1.554 MeV γ-ray transition energy confirmed the calculated recoil velocity of $v = 0.0083c$. Figure 1 presents the ratio R,

$$R = \frac{I_0}{I_0 + I_S}, \tag{11}$$

as function of relative plunger displacement. The mean life deduced from this ratio, following the procedure described in Ref. 11, was found to be

$$\tau = 22.0 \pm 0.8 \text{ ps}.$$

This value can be compared to two previous measurements which yielded

$$\tau = 22.4 \pm 1.5 \quad \text{(Ref. 12)},$$

$$\tau = 20.8 \pm 1.0 \quad \text{(Ref. 13)}$$

and is found to be in good agreement with them.

We now turn to our main aim, which is the determination of the magnetic moment of the 1^-, 2210 keV state. Figures 2 and 3 present the deduced values of $G_2(t)$ [expression (10)] from the measured ratios $R(t)$ [expression (9)]. The computed curves stem from two different approaches as will be explained later.

The justification for considering a superposition of static interactions proved itself valid when the measurement in ^{16}O was compared with the measurement of Randolph *et al.*[14] This group in Oxford utilized the field of a hydrogen-like oxygen ion and obtained $g = 0.55 \pm 0.03$. In the analysis given in Ref. 2, Broude *et al.* got 0.54 ± 0.05 neglecting the hyperfine contribution coming from the $2p$ electrons that take part in forming the excited atomic systems of the different ions. An analysis for ^{22}Na that follows the same approach is exhibited in Fig. 2. In such an analysis a_l and a_{sc} in Eq. (5) are set equal to zero. Under this assumption, the g factor is 0.36 ± 0.04.

In Fig. 3 the same ^{22}Na experimental data are shown compared with the realistic calculation given in Sec. II. Here the spin and orbital contributions originating in the participating $2p$ electrons are also included. The g factor deduced in this way decreased to $g = 0.28 \pm 0.03$.

Such a realistic calculation was repeated for ^{16}O for the experimental conditions given in Ref. 2. (By setting $a_l = a_{sc} = 0$, very good agreement with the curves of Ref. 2 was obtained.) The most probable value of this g factor decreased as well,

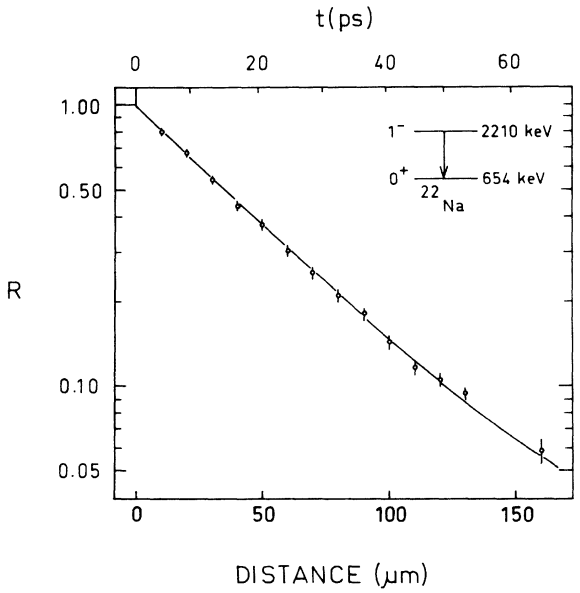

FIG. 1. The ratio of stopped peak intensity to that of stopped plus shifted peak intensity as a function of plunger displacement.

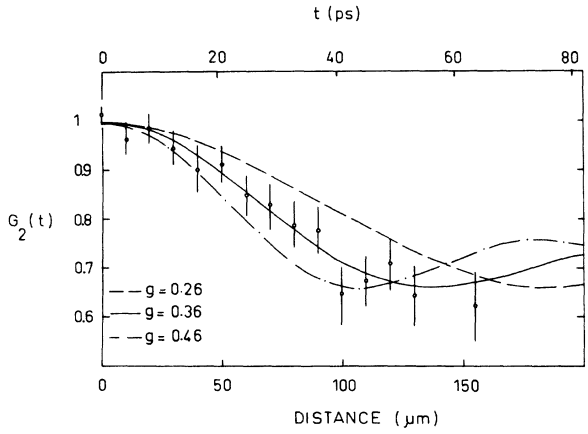

FIG. 2. The temporal behavior of $G_2(t)$. The theoretical curves were calculated neglecting contributions from $2p$ electrons to the hyperfine interaction. Each curve corresponds to a given g factor.

reaching 0.45 ± 0.05. In view of the underlying assumptions in the intermediate ionization model, and since the $2p$ contribution should be in principle included, even the new value $g = 0.45$ can be regarded as being in fair agreement with the value of Ref. 14.

The availability of the two independent sets of data for ^{16}O can serve certain calibration processes. Until now the population probabilities $P_{l\alpha}$ in Eq. (8) have been assumed to be of a statistical nature, namely proportional to the J multiplicity of the terms α. We raise a question as to whether this assumption is justified. If ionization potentials, or in particular, binding energies of the L electrons, determine the charge state fractions, it seems plausible that relative populations of the L subshells should not be the same, even when multiplicity is taken into account.

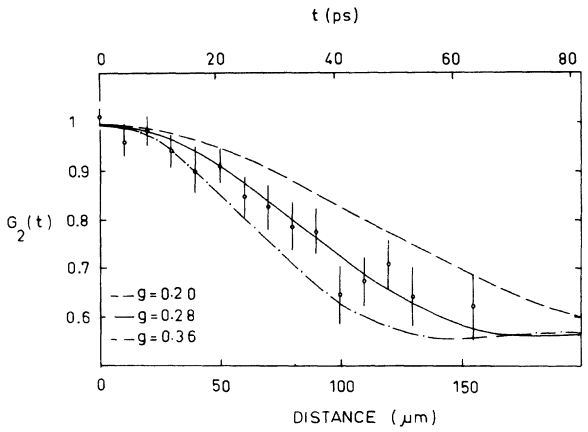

FIG. 3. Same as Fig. 2, the calculations taking into account contributions from $2p$ electrons.

The most probable charge fraction in the ^{16}O measurement is 2^+. Since the binding energies of the $2p$ electrons in such a system could not be found in the literature, it was estimated that the $2S_{1/2}$ electrons were bound 4.5 times more strongly than those in the $2P_{1/2}$ and $2P_{3/2}$ shell. The same argument in Na^{+3} yields a factor of 3 in binding energies. These estimates are based on binding energy extrapolations from nearby atoms. By observing the behavior of the charge state fractions $F_{i\alpha}$ as a function of recoil energy of ^{16}O, we assign an attenuation factor of 0.3 to a term formed from a configuration having a hole in the $2s$ subshell. The equivalent estimation in ^{22}Na is 0.5. It should be emphasized that these estimates are very crude.

With the application of the above procedure, the g factor in ^{16}O moves closer to the Oxford value, becoming now $g = 0.50 \pm 0.05$. An important observation is the substantial improvement in the χ^2 value. In ^{22}Na the g factor climbs back to $g = 0.36$, the χ^2 reaching the previous minimum value at the new value of the g factor.

To conclude, the ^{16}O Ref. 2 data can be interpreted in terms of a realistic calculation including $2s$ and $2p$ contributions. Better agreement with the previous measurement, independent of superposition assumptions, and an improved χ^2 are obtained if attenuation in population probabilities of terms formed by configurations having holes in the L_I subshell are properly altered. While we do believe in the physical picture qualitatively, we are at present far from being able to calculate these $P_{i\alpha}$ attenuations in a satisfactory way. For the g factor in ^{22}Na, we adopt the value

$$g = 0.36 \pm 0.07 .$$

The error presents an estimate of possible systematic uncertainties mentioned above, concurrently with a statistical standard error of 11%.

While the positive levels in ^{23}Na are explained by the Nilsson model as well as by shell model calculations, the negative parity states, as of yet, are not explained. There is experimental evidence, both of energy spacings and transition probabilities, which indicates that the 1^-, 2.211 MeV state is the head of a rotational band.[15] Garret, Middleton, and Fortune[16] conclude from analyzing pickup and stripping reactions leading to ^{22}Na that the levels in the band must have a $(p)^{-1}(s,d)^7$ configuration. However, calculations based on that observation were not carried out. Comparison with theory must then be delayed until detailed theoretical calculations, presumably complex, will be undertaken.

This work is a detailed and renewed account which replaces the abstract published in the Uppsala Conference.[17]

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